Assumptions for an Ideal Gas according to Kinetic Molecular Theory of Matter

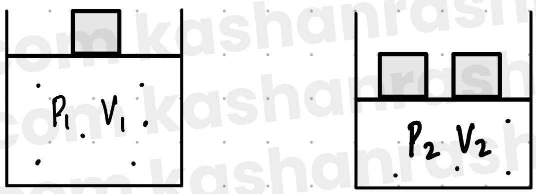
1. Gas molecules are in a continuous random motion.
2. They move in straight line.
3. Molecules have negligible inter-molecular force of attraction.
4. The gas molecules behave as hard, elastic spheres
5. The collisions between/of molecules is perfectly elastic.
6. Time of collision is negligible compared to time between collisions.
7. Volume of gas molecules is negligible compared to volume of gas.

Boyle's Law (P-V)

☑ For a fixed mass of gas at constant Temperature

☑ Volume of gas is inversely proportional to its pressure.

$V \propto \frac{1}{P} \rightarrow V = \frac{k}{P}$ i.e. $PV = $ constant



$\boxed{P_1V_1 = P_2V_2}$

When volume decreases

molecules come closer to one another

↓

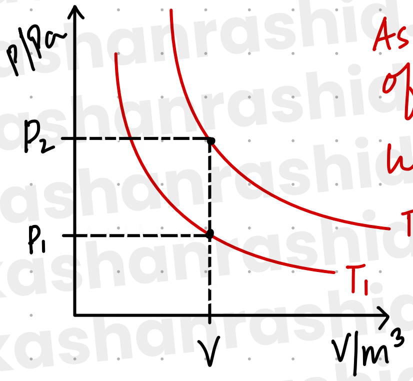
frequency of collisions will wall of container increases

↓

Force exerted by molecules increase

↓

Hence pressure increase



As for same volume of gas, $P_2 > P_1$

hence $T_2 > T_1$

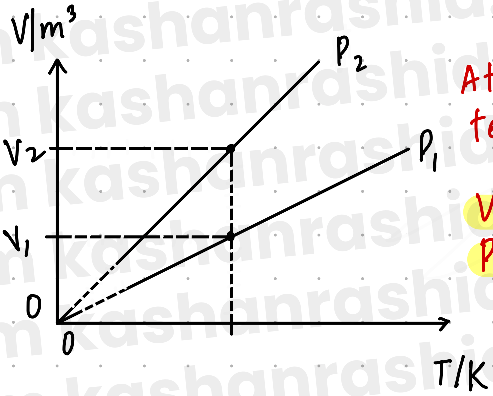
Charle's Law (V-T)

☑ For a fixed mass of gas at constant pressure

☑ Volume of gas is directly proportional to absolute temperature.

$V \propto T \rightarrow V = KT$ i.e. $\frac{V}{T} = $ constant

$\boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$ where T is in Kelvin



At the same temperature

$V_1 < V_2$ so

$P_1 > P_2$

Avagadro's Law (V–n)

- For a gas at constant temp and pressure
- Volume of gas is directly proportional to no. of moles.

$V \propto n \rightarrow V = kn$ i.e. $\frac{V}{n} = \text{constant}$

Ideal Gas Equation

$V \propto \frac{1}{P}$ $\quad V \propto T \quad$ $V \propto n$

$$V \propto \frac{nT}{P}$$

$$V = R \times \frac{nT}{P}$$

$$\boxed{PV = nRT}$$

P: pressure (Pa) $\quad$ n: no. of moles (mol)

V: volume (m^3) $\quad$ T: Absolute temp (K)

$\boxed{\text{R: Molar Gas Constant } \quad R = 8.314 \text{ J/mol.K}}$

- $n = \frac{\text{mass}}{\text{Ar or Mr}}$ → (in grams)

  n → no. of moles

- $N = n \times N_A$

  N → no. of particles (atoms / molecules)

  N_A → Avagadro's number $N_A = 6.02 \times 10^{23}$

as $PV = nRT$ and $n = \frac{N}{N_A}$

$$PV = \left(\frac{N}{N_A}\right)RT$$

$\boxed{PV = NkT}$ where $k = \frac{R}{N_A}$

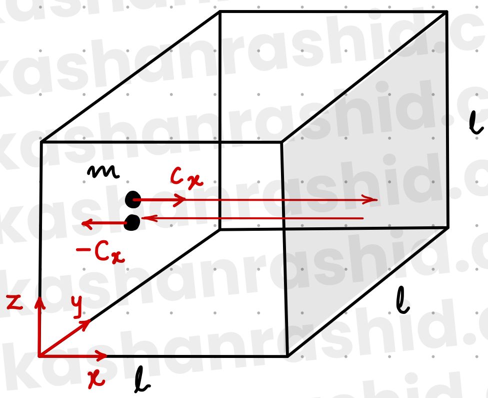
$\boxed{\text{Boltzmann Constant } (k) = 1.38 \times 10^{-23}}$

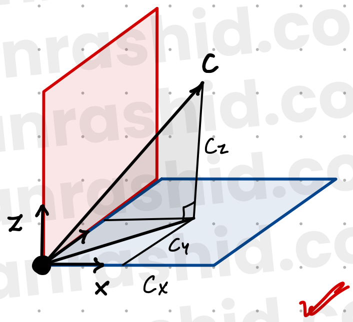
Define Ideal Gas: Any gas that obeys $PV = nRT$ or $PV = NkT$ at all temperatures, pressures and volumes is called Ideal gas.

How gases exert pressure?

- Gas molecules are in a continuous random motion
- They collide with one another and with walls of container
- The molecules undergo rate of change of momentum which produces force
- This force per unit area is the pressure of gas.

Deriving $P = \frac{1}{3}\rho\langle c^2\rangle$





Momentum → change in momentum → $\boxed{\text{force} \to \text{Pressure}}$

Momentum

$p = m \times v$ so along x-axis

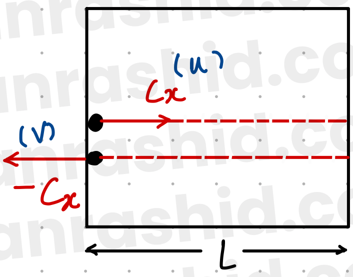
$p_x = mc_x$ $\quad \{ p_y = mc_y \quad p_z = mc_z \}$

Change of Momentum

$\Delta p = m(v - u)$

$\Delta p = m(-c_x - c_x)$

$\Delta p_x = -2mc_x$



Force

$F = \frac{\Delta p}{\Delta t}$ so $F = -\frac{2mc_x}{\Delta t}$

replacing Δt in eq.

$\Delta t \Rightarrow$ time between successive collisions

$v = \frac{d}{t} \rightarrow c_x = \frac{2L}{\Delta t} \rightarrow \Delta t = \frac{2L}{c_x}$

hence

$$F_x = -\frac{\cancel{2}mc_x}{\frac{\cancel{2}L}{c_x}}$$

$$F_x = -\frac{mc_x^2}{L}$$

For pressure, we take magnitude of force so $\boxed{F_x = \frac{mc_x^2}{L}}$

Pressure

$P = \frac{F}{A}$ so $P = \frac{mc_x^2}{L \times L^2}$

$P = \frac{mc_x^2}{L^3}$ or $P = \frac{mc_x^2}{V}$

for 1 molecules

for N no. of molecules

$P = \frac{Nmc_x^2}{V}$ $\quad \boxed{PV = Nmc_x^2}$

pressure of gas due to all of its molecules having speed in x-axis.

Adjusting for speed of molecules

$$\vec{c} = \vec{c_x} + \vec{c_y} + \vec{c_z}$$

for magnitude of velocity.

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

Due to random motion, as molecules are equally likely to move in all directions, their average velocity along x-axis, y-axis and z-axis are same.

"This is why we get the same gas pressure on all the walls of container."

$$c_x^2 \simeq c_y^2 \simeq c_z^2$$

$$\langle c^2 \rangle = c_x^2 + c_x^2 + c_x^2$$

$$\langle c^2 \rangle = 3\,c_x^2$$

$$\boxed{c_x^2 = \frac{1}{3}\langle c^2 \rangle}$$

As $PV = Nmc_x^2$ and $c_x^2 = \frac{1}{3}\langle c^2 \rangle$

$$PV = \frac{1}{3}Nm\langle c^2 \rangle$$

$$P = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$$

as $\rho = \frac{Nm}{V}$ → total mass of gas, V → total volume

$$\boxed{P = \frac{1}{3}\rho\langle c^2 \rangle}$$

P: pressure (Pa) ρ: density (kgm⁻³)

$\langle c^2 \rangle$: mean-square speed of gas molecules

$$\sqrt{\langle c^2 \rangle} = c_{rms}$$

root-mean-square speed of gas molecules!

values: −4, 3, 5, −6, 1

3rd root, 2nd mean, 1st square

$$\sqrt{\frac{(-4)^2 + (3)^2 + (5)^2 + (-6)^2 + (1)^2}{5}}$$

Simple average of negative values will provide a misleading conclusion for the average/mean speed of all gas molecules.

Therefore rms was taken!

preffered when effect of −ve sign has to be eliminated from mean

Relating mean kinetic energy of gas molecules with temp of gas

$$P = \frac{1}{3}\rho\langle c^2 \rangle$$

$$P = \frac{1}{3}\frac{Nm}{V}\langle c^2 \rangle$$

$$3PV = Nm\langle c^2 \rangle$$

as $PV = NkT$ so

$$3\cancel{N}kT = \cancel{N}m\langle c^2 \rangle$$

$$3kT = m\langle c^2 \rangle \longrightarrow T \propto \langle c^2 \rangle$$

$$\boxed{\sqrt{T} \propto c_{rms}}$$

$$\frac{3}{2}kT = \boxed{\frac{1}{2}m\langle c^2 \rangle}$$

$$\boxed{\langle E_K \rangle = \frac{3}{2}kT}$$

Temperature is the measurement of mean kinetic energy of gas molecules.

Example

Gas molecules at a temperature of 100°C have a C_{rms} of $450\ ms^{-1}$. What will be the C_{rms} at a temperature of 250°C.

$100 + 273 = 373K$
$250 + 273 = 523K$

$\sqrt{T} \propto C_{rms}$

$\sqrt{373}$ — 450
$\sqrt{523}$ — x

$\sqrt{373} \times x = \sqrt{523} \times 450$

$x = 532.8 \simeq 533\ ms^{-1}$

Thermodynamics

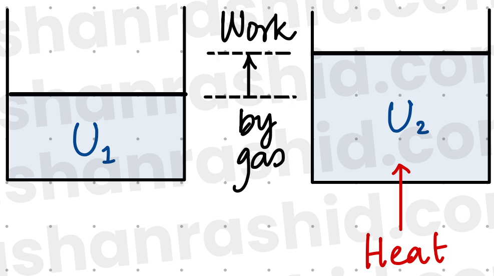
heat   work

Study of processes and systems that involve heat and work and conversion between them.

1st Law of Thermodynamics

Statement #1

Heat supplied to a system is equal to the sum of increase in its internal energy and workdone by the system.

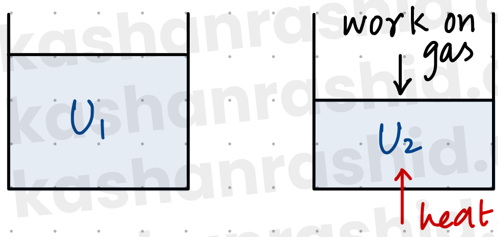


$$\boxed{Q = \Delta U + W}$$

Q → heat supplied to gas
ΔU → increase in int. energy
W → workdone by gas

Statemen #2

The sum of heat supplied to a system and workdone on the system is equal to the increase in its internal energy.



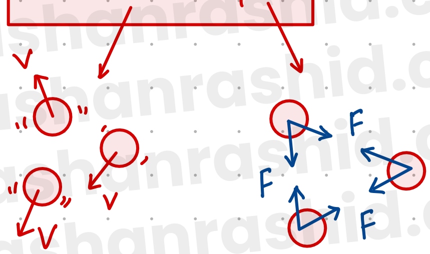
$$\boxed{Q + W = \Delta U}$$

Q → heat supplied to gas
W → workdone on gas
ΔU → increase in internal energy

• Internal Energy (U)

The sum of microscopic kinetic and potential energy of (gas) molecules due to random motion.

$\boxed{U = k.e + p.e}$



speed of motion

Intermolecular force of attraction

- ☑ For an ideal gas there is negligible intermolecular force of attraction.
- ☑ Hence there is no/negligible P.E between ideal gas molecules
- ☑ Internal energy of ideal gas only depends on K.E of molecules
- ☑ $\langle K.E \rangle = \frac{3}{2} kT \qquad \Delta \langle K.E \rangle \propto \Delta T$

As the change of K.E is directly proportional to change in temp, so change in internal energy is also directly proportional to the change in temperature.

$\boxed{\Delta U \propto \Delta T}$ for ideal gas

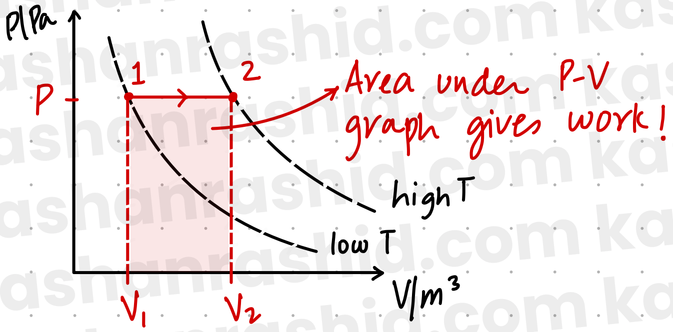
• Thermodynamic Processes

1. Constant Pressure (Isobaric)

$\boxed{W = P\Delta V}$ for workdone at constant pressure

$Q = \Delta U + W$ OR $Q + W = \Delta U$

$Q = \Delta U + P\Delta V$ $\qquad$ $Q + P\Delta V = \Delta U$



2. Constant Volume (Isochoric)

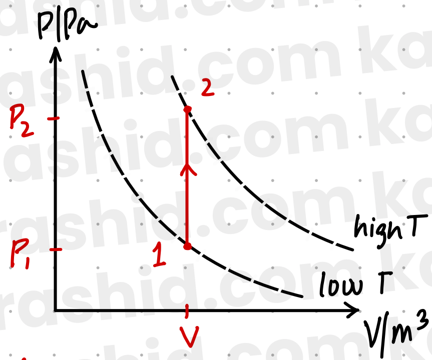
$\Delta V = 0$ so $W = P\cancel{\Delta V}^{0}$

$W = 0$ (no work)

$Q = \Delta U + \cancel{W}^{0}$

$\boxed{Q = \Delta U}$

- All heat stored as internal.
- Temp rises
- Pressures rises to increase in molecular speed.



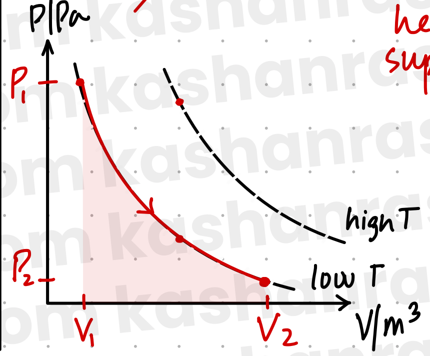
3. Constant Temperature (Isothermal)

$T = \text{constant}$ so $\Delta T = 0$

$\Delta U \propto \Delta T$ so $\Delta U = 0$

$Q = \cancel{\Delta U}^{0} + W$ so $\boxed{Q = W}$

heat supplied $\qquad$ workdone by gas



4 No heat transfer (Adiabatic)

$Q = 0$

$\cancel{Q}^{\,0} = \Delta U + W$

$0 = +\Delta U + W$

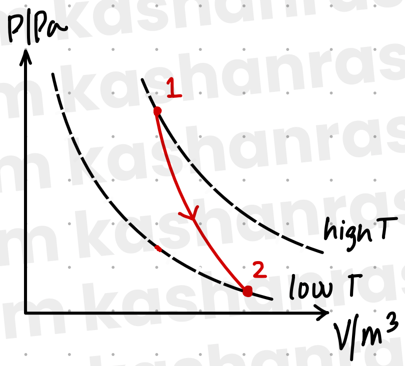
$$\boxed{-\Delta U = W}$$

decrease in internal energy

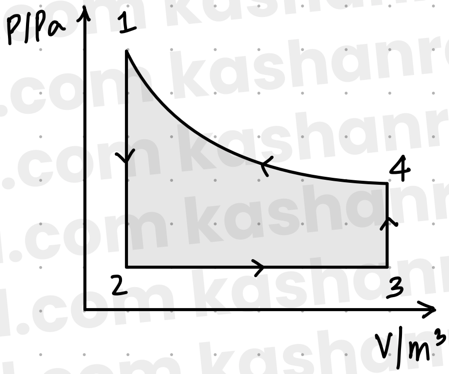
workdone by gas

temp decreases!

- Gas cools down when it does work.
- Gas heats up when work is done on it



In a thermodynamic cycle, $\Delta U_{cycle} = 0$



- A cycle starts and ends at the same temperature. So $\Delta T_{cycle} = 0$
- As $\Delta U \propto \Delta T$ so $\Delta U_{cycle} = 0$

$$\Delta U_{1-2} + \Delta U_{2-3} + \Delta U_{3-4} + \Delta U_{4-1} = 0$$

2 **(a)** Explain qualitatively how molecular movement causes the pressure exerted by a gas.

Gas molecules are in a continuous random motion. Upon collision with one another and walls of container they undergo rate of change of momentum. This produces force and the force per unit area is pressure. [3]

For Examiner's Use

(b) The density of neon gas at a temperature of 273 K and a pressure of 1.02×10^5 Pa is $0.900\,kg\,m^{-3}$. Neon may be assumed to be an ideal gas.

Calculate the root-mean-square (r.m.s.) speed of neon atoms at

(i) 273 K,

$P = \frac{1}{3}\rho\langle c^2\rangle$

$1.02 \times 10^5 = \frac{1}{3} \times 0.9 \times \langle c^2\rangle$

$C_{rms} = 583\ ms^{-1}$

speed = 583 $m\,s^{-1}$ [3]

(ii) 546 K.

$\sqrt{T} \propto C_{rms}$

$\sqrt{273}$ — 583

$\sqrt{546}$ — x

$x = \frac{583 \times \sqrt{546}}{\sqrt{273}}$

$x = 824.5 \simeq 825$

speed = 825 $m\,s^{-1}$ [2]

$P = \frac{1}{3}\rho\langle c^2\rangle$ → Temperature

For Examiner's Use

(c) The calculations in **(b)** are based on the density for neon being 0.900 kg m^{-3}.
Suggest the effect, if any, on the root-mean-square speed of changing the density at constant temperature.

c_{rms} stays same as it only depends on temperature.
Changing density will change the pressure of gas.

.... [2]

For Examiner's Use

2 **(a)** The kinetic theory of gases is based on some simplifying assumptions.
The molecules of the gas are assumed to behave as hard elastic identical spheres.
State the assumption about ideal gas molecules based on

(i) the nature of their movement,

....

.... [1]

(ii) their volume.

....

....

.... [2]

For Examiner's Use

(b) A cube of volume V contains N molecules of an ideal gas. Each molecule has a component c_X of velocity normal to one side S of the cube, as shown in Fig. 2.1.

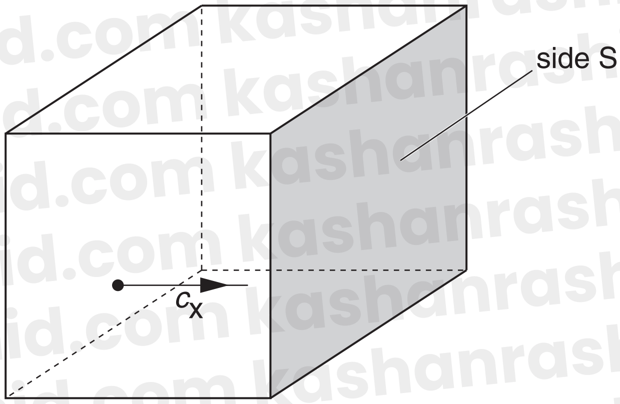


Fig. 2.1

The pressure p of the gas due to the component c_X of velocity is given by the expression

$$pV = Nmc_X^2$$

where m is the mass of a molecule.

Explain how the expression leads to the relation

$$pV = \frac{1}{3}Nm<c^2>$$

where $<c^2>$ is the mean square speed of the molecules.

[3]

(c) The molecules of an ideal gas have a root-mean-square (r.m.s.) speed of 520 m s^{-1} at a temperature of 27 °C.

Calculate the r.m.s. speed of the molecules at a temperature of 100 °C.

r.m.s. speed = ... m s^{-1} [3]

For Examiner's Use

2 (a) (i) State the basic assumption of the kinetic theory of gases that leads to the conclusion that the potential energy between the atoms of an ideal gas is zero.

There is negligible intermolecular force of attraction.

.... [1]

(ii) State what is meant by the *internal energy* of a substance.

It is the sum of microscopic kinetic and potential energy of molecules, due to random motion. [2]

(iii) Explain why an increase in internal energy of an ideal gas is directly related to a rise in temperature of the gas.

Internal energy only depends on kinetic energy of molecules and change in kinetic energy is directly proportional to change in temperature. [2]

(b) A fixed mass of an ideal gas undergoes a cycle PQRP of changes as shown in Fig. 2.1.

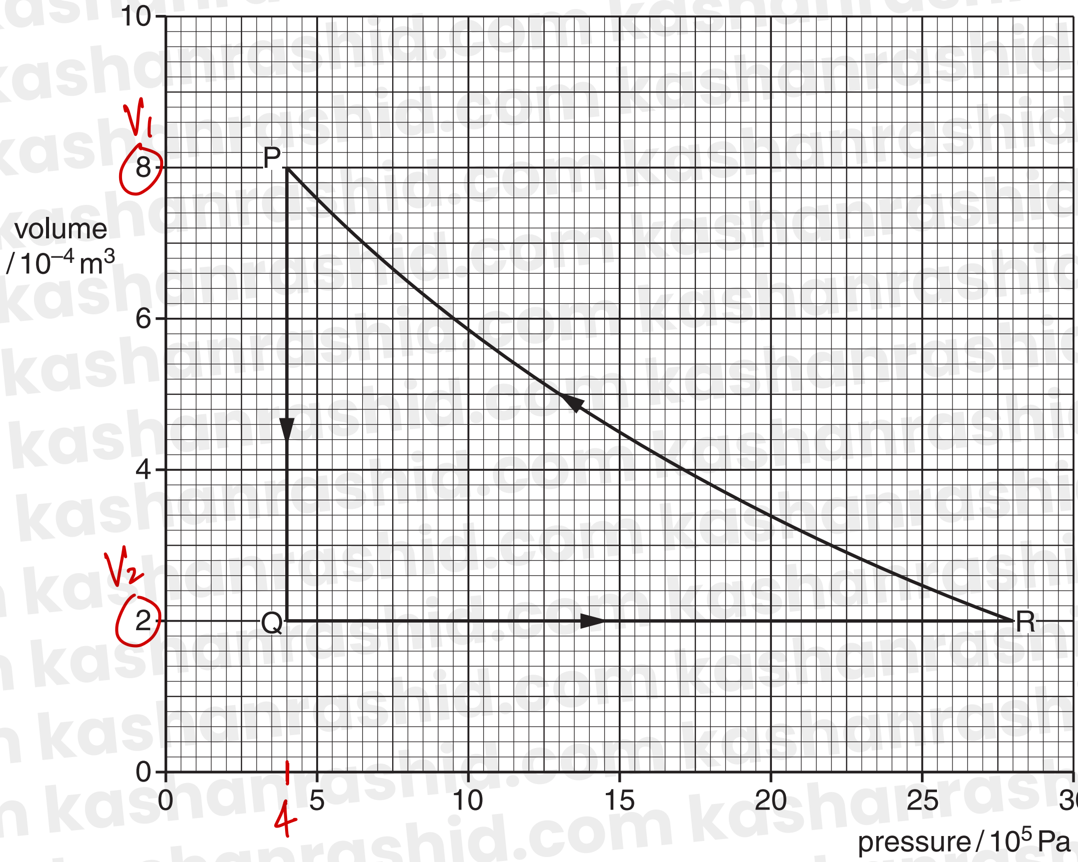


Fig. 2.1

For Examiner's Use

(i) State the change in internal energy of the gas during one complete cycle PQRP.

change = ...0.......... J [1]

(ii) Calculate the work done on the gas during the change from P to Q.

$W = P\Delta V$

$W = P(V_2 - V_1)$

$W = (4\times10^5)(2-8)\times10^{-4}$

$W = 240\ J$

work done = ...240.......... J [2]

(iii) Some energy changes during the cycle PQRP are shown in Fig. 2.2.

change	work done on gas / J	heating supplied to gas / J	increase in internal energy / J
P → Q	240	–600	–360
Q → R	0	+720	+720
R → P	–840	+480	–360

Fig. 2.2

Complete Fig. 2.2 to show all of the energy changes. [3]

$Q + W = \Delta U$

$-600 + 240 = \Delta U$

$Q + W = \Delta U$

$720 + 0 = \Delta U$

$-360 + 720 + x = 0$

$360 + x = 0$

$x = -360$

$Q + W = \Delta U$

$480 + W = -360$

$W = -360 - 480$

$W = -840\ J$

2 **(a)** Use one of the assumptions of the kinetic theory of gases to explain why the potential energy of the molecules of an ideal gas is zero.

....

.... [1]

(b) The average translational kinetic energy E_K of a molecule of an ideal gas is given by the expression

$$E_K = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$$

where m is the mass of a molecule and k is the Boltzmann constant.

State the meaning of the symbol

(i) $\langle c^2\rangle$,

.... [1]

(ii) T.

.... [1]

(c) A cylinder of constant volume $4.7 \times 10^4\,\text{cm}^3$ contains an ideal gas at pressure $2.6 \times 10^5\,\text{Pa}$ and temperature 173 °C.

The gas is heated. The thermal energy transferred to the gas is 2900 J. The final temperature and pressure of the gas are T and p, as illustrated in Fig. 2.1.

Fig. 2.1

(i) Calculate

1. the number N of molecules in the cylinder,

N = ... [3]

2 **(a)** State what is meant by an *ideal gas*.

Any gas that obey's $PV = nRT$ / $PV = NkT$ at all temperatures, pressures and volumes.

[2]

(b) An ideal gas comprised of single atoms is contained in a cylinder and has a volume of $1.84 \times 10^{-2}\,m^3$ at a pressure of $2.12 \times 10^7\,Pa$.
The mass of gas in the cylinder is 3.20 kg.

(i) Determine, to three significant figures, the root-mean-square (r.m.s.) speed of the atoms of the gas.

$$P = \frac{1}{3}\rho \langle c^2 \rangle$$

$$2.12 \times 10^7 = \frac{1}{3} \times \left(\frac{3.2}{1.84 \times 10^{-2}}\right) \langle c^2 \rangle$$

$$c_{rms} = 604.73$$

r.m.s. speed = 605 $m\,s^{-1}$ [3]

$22 + 273 = 295\,K$

(ii) The temperature of the gas in the cylinder is 22 °C.

Determine, to three significant figures,

1. the amount, in mol, of the gas,

$PV = nRT$

$$\frac{(2.12 \times 10^7)(1.84 \times 10^{-2})}{8.314 \times 295} = n$$

$n = 159.04$

amount = ... 159 .... mol [2]

2. the mass of one atom of the gas.

$(N = n \times N_A)$

$$\text{mass of 1 atom} = \frac{3.20}{159.04 \times 6.02 \times 10^{23}} \rightarrow \text{no. of atoms}$$

$= 3.34 \times 10^{-26}$

mass = ... 3.34×10^{-26} .... kg [2]

(c) Use your answer in **(b)(ii) part 2** to determine the nucleon number *A* of an atom of the gas.

$1\,u = 1.66 \times 10^{-27}\,kg$

$$\text{no. of nucleon (nucleon number)} = \frac{3.34 \times 10^{-26}}{1.66 \times 10^{-27}}$$

$= 20$

A = ... 20 .... [1]

[Total: 10]

2 **(a)** The pressure p and volume V of an ideal gas are related to the density ρ of the gas by the expression

$$p = \tfrac{1}{3}\rho\langle c^2\rangle.$$

(i) State what is meant by the symbol $\langle c^2\rangle$.

Mean square speed of gas molecules

.... [1]

(ii) Use the expression to show that the mean kinetic energy E_K of a gas molecule is given by

$$E_K = \frac{3}{2}kT$$

where k is the Boltzmann constant and T is the thermodynamic temperature.

$P = \frac{1}{3}\rho\langle c^2\rangle$

$P = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$

$3PV = Nm\langle c^2\rangle$

$3\cancel{N}kT = \cancel{N}m\langle c^2\rangle$

$3kT = m\langle c^2\rangle$

$\frac{3}{2}kT = \underbrace{\frac{1}{2}m\langle c^2\rangle}$

$E_K = \frac{3}{2}kT$

[3]

(b) **(i)** An ideal gas containing 1.0 mol of molecules is heated at constant volume.
Use the expression in **(a)(ii)** to show that the thermal energy required to raise the temperature of the gas by 1.0 K has a value of $\frac{3}{2}R$, where R is the molar gas constant.

$\Delta V = 0$ so $W = 0$

$Q = \Delta U + \cancel{W}^{0}$

$\boxed{\cdot\ Q = \Delta U}$

$\cdot$ As $U = K.E + \cancel{P.E}^{0}$

$U = K.E$

$\Delta U = \Delta K.E$

$\Delta U = \frac{3}{2}k\Delta T$

$\boxed{\Delta U = N \times \frac{3}{2}k\Delta T}$

$Q = \frac{3}{2}Nk\Delta T$

for $\Delta T = 1K$ and 1mol of molecules $N = N_A$

$Q = \frac{3}{2}\cancel{N_A} \times \frac{R}{\cancel{N_A}} \times 1$

$Q = \frac{3}{2}R$

[3]

7

$R = 8.314\ J\,mol^{-1}\,K^{-1}$ $M_r = 28$

(ii) Nitrogen may be assumed to be an ideal gas. The molar mass of nitrogen gas is $28\,g\,mol^{-1}$. Use the answer in **(b)(i)** to calculate a value for the specific heat capacity, in $J\,kg^{-1}\,K^{-1}$, at constant volume for nitrogen.

$c = \frac{Q}{m\Delta T}$

$c = \frac{3}{2}R \div m(1)$

$c = \frac{3R}{2m}$

$n = \frac{m}{M_r}$

$1 = \frac{m}{28}$

$m = 28g$

$m = 0.028kg$

1mol or 0.028kg

$c = \frac{3R}{2 \times 0.028}$

$c = \frac{3 \times 8.314}{2 \times 0.028}$

$c = 445.39$

specific heat capacity = ...450........ $J\,kg^{-1}\,K^{-1}$ [2]

[Total: 9]

$$Q + W = \Delta U$$

heat supplied to a system + workdone on the system = increase in internal energy

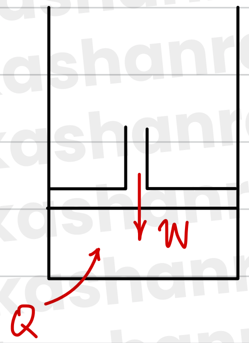
gas compress

microscopic

$I.E = KE + PE$

KE → speed of molecule → Temp

PE → Intermolecular force of attraction



Work = $P\Delta V$

$$Q = \Delta U + W$$

heat supplied to a system = increase in internal energy + workdone by the system

gas expands

- **Isobaric** – constant pressure $\{ Q + W = \Delta U \}$ | $\{ Q = \Delta U + W \}$
- **Isochoric** – constant volume $\Delta V = 0$; $W = 0$ $\{ Q = \Delta U \}$
- **Isothermal** – constant temperature $\Delta T = 0$; $\Delta U = 0$ $\{ Q = W.D \text{ by gas} \}$
- **Adiabatic** – no heat transfer $Q = 0$ $\{$ W.D on gas = inc. in IE $\}$
  W.D by gas = dec. in IE

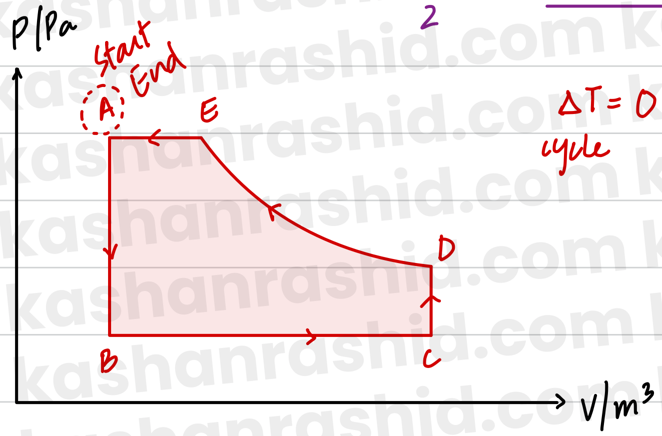
$IE = KE + \cancel{PE}$

$KE \to \frac{3}{2}kT$, $PE \to$ ~~IMF~~ $\to 0$

for an ideal gas

$IE = KE$

$IE = \frac{3}{2}kT \rightarrow \underline{\Delta IE \propto \Delta T}$



$\Delta T = 0$ cycle

$\Delta U = 0$ cycle

$$\Delta U_{AB} + \Delta U_{BC} + \Delta U_{CD} + \Delta U_{DE} + \Delta U_{EA} = 0$$