PROSPERITY ACADEMY

A2 PHYSCIS 9702

Crash Course

RUHAB IQBAL

IDEAL GASES

COMPLETE NOTES

0331 - 2863334

ruhab.prosperityacademics@gmail.com

Ideal gases :-

1 mole = 6.023×10^{23} particles

$N = n\, N_A$ → Avagardo's number : 6.023×10^{23} particles

Number of particles = moles × (6.023×10^{23}) particles

[Important :- 1 dm^3 = 1 litre] *

$n = \frac{mass}{M_r/A_r}$ → mass in g

mol

Some constants :-

K = Boltzmann's constant = 1.38×10^{-23}

N_A = Avagadro's number = 6.023×10^{23}

R = molar gas constant = $(K)(N_A)$ = 8.31

*Note :-

C + 273.15 = K

use this in physics

Q. What is an ideal gas?

Ans. An ideal gas follows the ideal gas equations and conforms to the assumptions of the Kinetic theory.

$\boxed{PV = nRT}$

n: no. of moles
R: molar gas constant
T: temperature in K
P: pressure in Pa
V: volume in m^3

or $PV = \frac{N}{\cancel{N_A}} \times K \times \cancel{N_A} \times T$

$\boxed{PV = NKT}$

N: Number of particles
K: Boltzmann constant
T: temperature in K
P: pressure in Pa
V: volume in m^3

Assumptions of Kinetic Model theory:- Particles are always in random motion

1) No intermolecular forces between particles

2) The time of a specific particulate collision is negligible compared to the time between successive collisions.

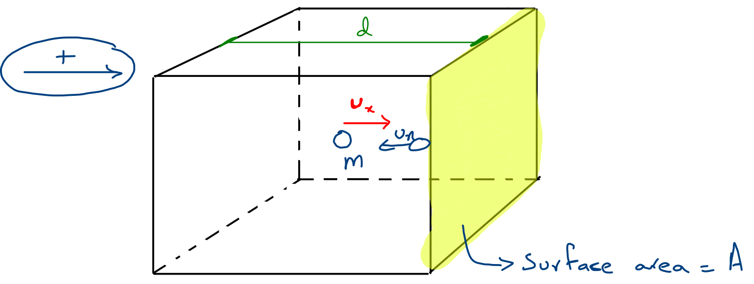
3) The collisions are perfectly elastic

4) The volume of the particles themselves are negligible compared to the volumes of gas

5) There are always a large number of particles present for statistical approximation to be applied on the population.

Derivation of ideal gas equation

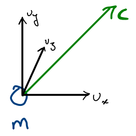
1) $PV = nRT$
2) $PV = NKT$] we already Know



The time between collisions:-

$\Delta t = \frac{s}{v} = \frac{2d}{u_x}$

In actuality, a particle will have a speed c, which is going to be a 3D vector



$c = \sqrt{u_x^2 + u_y^2 + u_z^2}$ Important

$\therefore c_{rms} = \sqrt{u_{Nx}^2 + u_{Ny}^2 + u_{Nz}^2}$

(root mean square)
(Average velocity of all particles)

Based on assumption 5,
$u_{Nx}^2 = u_{Ny}^2 = u_{Nz}^2$

So,
$c_{rms} = \sqrt{u_{Nx}^2 + u_{Nx}^2 + u_{Nx}^2}$

$\langle c^2 \rangle = 3 u_{Nx}^2$

mean square speed of all particles

$u_{Nx}^2 = \frac{\langle c^2 \rangle}{3}$

Change of momentum of particle:- (using assumption 3)

$\Delta p = p_f - p_i$

$\Delta p = m(-u_x) - m(u_x)$

$\Delta p = -2mu_x$

$|\Delta p| = 2mu_x$

A real gas with N particles will have a change of momentum:

$\Delta p_{N \text{ particles in } x \text{ direction}} = N(2mu_x)$

Problems:
- Assuming all in x direction
- Assuming same speed for all particles

Force exerted:-

$\Sigma F_{N \text{ particles}} = \frac{\Delta p}{\Delta t}$ → The time of collision is negligible, we cannot use this.

Instead what we are concerned is the average time between collisions

$\Sigma F_{N \text{ particles}} = \frac{\Delta p}{2d/u_x} = \frac{N(\cancel{2}mu_x)}{\cancel{2}d/u_x} = \frac{Nm(u_x)^2}{d}$

Pressure exerted on the wall

$P = \frac{F}{A} = \frac{\frac{mN(u_x)^2}{d}}{A} = \frac{mN(u_x)^2}{dA} = \frac{mN(u_x)^2}{V}$ → volume of gas

therefore

$PV = mN(u_x)^2$

Assumption:-
All particles travel at the same speed and in the same x direction

$PV = \frac{1}{3} Nm \langle c^2 \rangle$

$\langle c^2 \rangle$ = mean square speed

Comparing $PV = NKT$ and $PV = \frac{1}{3} Nm\langle c^2\rangle$

$$\cancel{N}KT = \frac{1}{3}\cancel{N}m\langle c^2\rangle$$

$$\frac{1}{2}\times 3KT = \underbrace{\frac{1}{2}m\langle c^2\rangle}_{E_K \text{ of one particle}}$$

$$\boxed{E_K \text{ of one particle} = \frac{3}{2}KT}$$

$$\boxed{E_K \text{ of } N \text{ particles} = \frac{3}{2}NKT}$$

Internal Energy :- (U)

$$U = \Sigma K.E + \Sigma P.E$$

(Ideal gases have no intermolecular forces $\therefore$ $P.E = 0$)

$$U = \Sigma K.E + 0$$

$$\boxed{U = \frac{3}{2}NKT}$$

Comparing $PV = nRT$ and $PV = \frac{1}{3}Nm\langle c^2\rangle$

$$nRT = \frac{1}{3}Nm\langle c^2\rangle$$

$$\underbrace{\frac{3nR}{Nm}}_{\text{const}} \times T = \langle c^2\rangle$$

$$T \propto \langle c^2\rangle \Rightarrow \frac{T_1}{\langle c^2\rangle_1} = K = \frac{T_2}{\langle c^2\rangle_2}$$

$\langle c^2\rangle$: mean square speed

$c_{r.m.s}$: root mean square speed

$c_{r.m.s} = \sqrt{\langle c^2\rangle}$

$$\boxed{\frac{T_1}{\langle c^2\rangle_1} = \frac{T_2}{\langle c^2\rangle_2}}$$

Another equation :-

$$PV = \frac{1}{3}Nm\langle c^2\rangle$$

$$P = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle$$

total mass

Volume

$\rho = \frac{m}{V}$

$\rho = \frac{Nm}{V}$

$$\boxed{P = \frac{1}{3}\rho\langle c^2\rangle}$$

3 (a) (i) The kinetic theory of gases leads to the equation

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT \rightarrow 0 = 0$$

Explain the significance of the quantity $\tfrac{1}{2}m\langle c^2\rangle$.

mean Kinetic energy of one particle

(ii) Use the equation to suggest what is meant by the absolute zero of temperature. 0K

At $T = 0K$, the Kinetic energy is also zero

[3]

(b) Two insulated gas cylinders **A** and **B** are connected by a tube of negligible volume, as shown in Fig. 3.1.

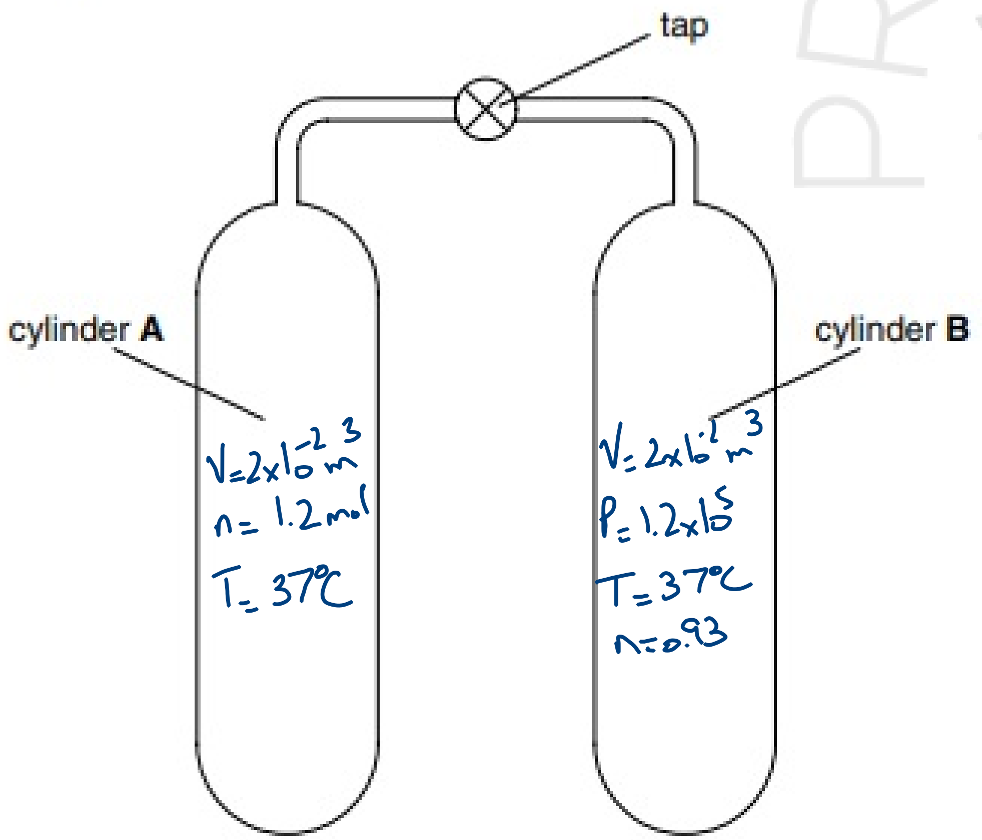


Fig. 3.1

Each cylinder has an internal volume of $2.0 \times 10^{-2}\,m^3$. Initially, the tap is closed and cylinder **A** contains 1.2 mol of an ideal gas at a temperature of 37 °C. Cylinder **B** contains the same ideal gas at pressure 1.2×10^5 Pa and temperature 37 °C.

(i) Calculate the amount, in mol, of the gas in cylinder **B**.

$$PV = nRT$$

$$(1.2\times10^5)(2\times10^{-2}) = n(8.31)(37+273.15)$$

$$n = 0.9311$$

amount = 0.93 mol

(ii) The tap is opened and some gas flows from cylinder **A** to cylinder **B**. Using the fact that the total amount of gas is constant, determine the final pressure of the gas in the cylinders.

$$pV = nRT$$

$$P(4\times10^{-2}) = (1.2+0.93)(8.31)(37+273.15)$$

$$P = 137243.7$$

$$P = 1.37\times10^5$$

pressure = 1.37×10^5 Pa

[6]

2 The pressure p of an ideal gas is given by the expression

$$p = \frac{1}{3}\frac{Nm}{V}\langle c^2\rangle.$$

ρ

(a) Explain the meaning of the symbol $\langle c^2\rangle$.

mean square speed of all particles in the gas [2]

(b) The ideal gas has a density of $2.4\,\text{kg m}^{-3}$ at a pressure of 2.0×10^5 Pa and a temperature of 300 K.

(i) Determine the root-mean-square (r.m.s.) speed of the gas atoms at 300 K.

$P = \frac{1}{3}\rho\langle c^2\rangle$

$2\times10^5 = \frac{1}{3}\times(2.4)\langle c^2\rangle$

$\sqrt{\langle c^2\rangle} = \sqrt{250\,000}$

$c_{r.m.s} = 500$

r.m.s. speed = 500 m s^{-1} [3]

(ii) Calculate the temperature of the gas for the atoms to have an r.m.s. speed that is twice that calculated in **(i)**.

$\sqrt{\frac{T_1}{\langle c_1^2\rangle}} = \sqrt{\frac{T_2}{\langle c_2^2\rangle}}$

$\frac{\sqrt{T_1}}{c_{r.m.s_1}} = \frac{\sqrt{T_2}}{c_{r.m.s_2}}$

$\frac{\sqrt{300}}{\cancel{500}} = \frac{\sqrt{T_2}}{\cancel{1000}\,2}$

$(2\sqrt{300})^2 = (\sqrt{T_2})^2$

$1200 = T_2$

temperature = 1200 K [3]

2 (a) The equation

$$pV = \text{constant} \times T$$

NK and nR

relates the pressure p and volume V of a gas to its kelvin (thermodynamic) temperature T.

State two conditions for the equation to be valid.

1. The gas must be ideal

2. Amount of gas must remain constant

[2]

(b) A gas cylinder contains $4.00 \times 10^4\,\text{cm}^3$ of hydrogen at a pressure of 2.50×10^7 Pa and a temperature of 290 K.

$\text{cm}^3 \to \text{m}^3$ $(10^{-2})^3$

The cylinder is to be used to fill balloons. Each balloon, when filled, contains $7.24 \times 10^3\,\text{cm}^3$ of hydrogen at a pressure of 1.85×10^5 Pa and a temperature of 290 K.

Calculate, assuming that the hydrogen obeys the equation in **(a)**,

(i) the total amount of hydrogen in the cylinder,

$PV = nRT$

$(2.5\times10^7)(4\times10^4\times10^{-6}) = n(8.31)(290)$

$n = 414.95$

amount = 415 mol [3]

(ii) the number of balloons that can be filled from the cylinder.

$PV = nRT$

$(1.85\times10^5)(7.24\times10^3\times10^{-6}) = n(8.31)(290)$

n of 1 balloon = 0.55579

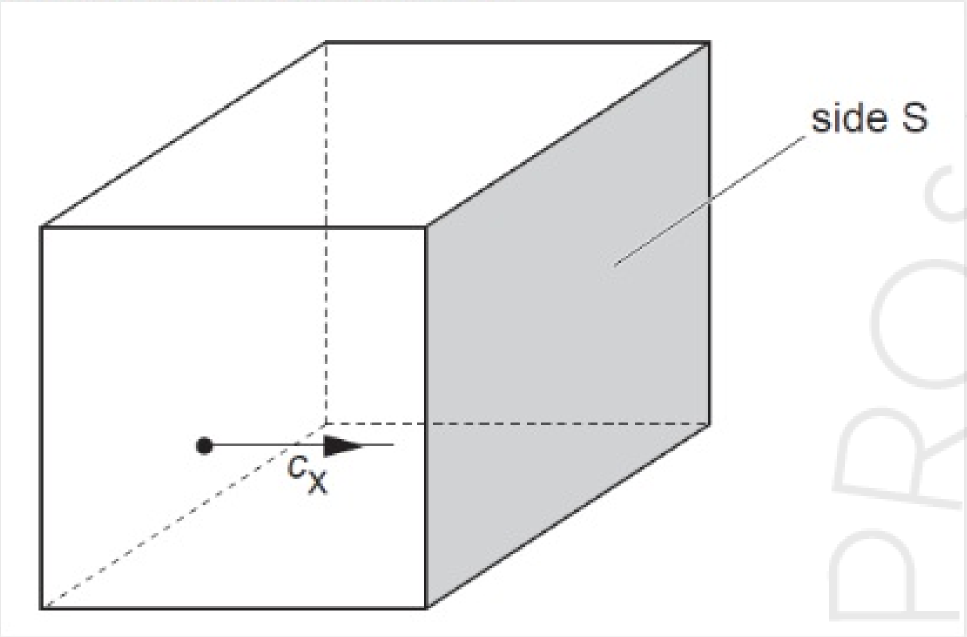
no. of balloons:- $415 \div 0.55579 = 746.68$ balloons

746 balloons

Ans

(a) Kinetic theory of gases is based on some simplifying assumptions.
Molecules of the gas are assumed to behave as hard elastic identical spheres.
State assumption about ideal gas molecules based on
(i) nature of their movement Particles are always in random motion
(ii) their volume The volume of the particles is negligible compared to the volume of the container they occupy.

(b) Cube of volume V contains N molecules of an ideal gas. Each molecule has a component c_X of velocity normal to one side S of the cube, as shown.



Pressure p of the gas due to component c_X of velocity is given by expression

$$pV = Nmc_X^2$$

where m is mass of a molecule.
Explain how expression leads to the relation

$$pV = (1/3)\,Nm\langle c^2\rangle$$

where $\langle c^2\rangle$ is mean square speed of the molecules.

c is the mean square speed so

$$c = \sqrt{c_x^2 + c_y^2 + c_z^2}$$

There is always a large number of particles for statistical approximations to apply $\therefore$ $c_x = c_y = c_z$

$$(c)^2 = \left(\sqrt{c_x^2 + c_x^2 + c_x^2}\right)^2$$

$$\langle c^2\rangle = 3c_x^2$$

$$c_x^2 = \frac{1}{3}\langle c^2\rangle$$

(c) Molecules of an ideal gas have a root-mean-square (r.m.s.) speed of 520 m s^{-1} at a temperature of 27 °C.
Calculate r.m.s. speed of the molecules at a temperature of 100 °C.

$$\sqrt{\frac{T_1}{\langle c_1\rangle^2}} = \sqrt{\frac{T_2}{\langle c_2\rangle^2}}$$

$$\frac{\sqrt{T_1}}{c_{r.m.s_1}} = \frac{\sqrt{T_2}}{c_{r.m.s_2}}$$

$$\frac{\sqrt{27+273.15}}{520} = \frac{\sqrt{100+273.15}}{c_{r.m.s_2}}$$

$$c_{r.m.s_2} = 580\ ms^{-1}$$

2 **(a)** State what is meant by an *ideal gas*.

A gas that obeys all of the ideal gas equations and conforms to the assumptions of the Kinetic model theory. It obeys the equation of the form $PV \propto T$ where P is pressure in Pa, V is volume in m^3, and T is temperature in Kelvins [3]

(b) Two cylinders A and B are connected by a tube of negligible volume, as shown in Fig. 2.1.

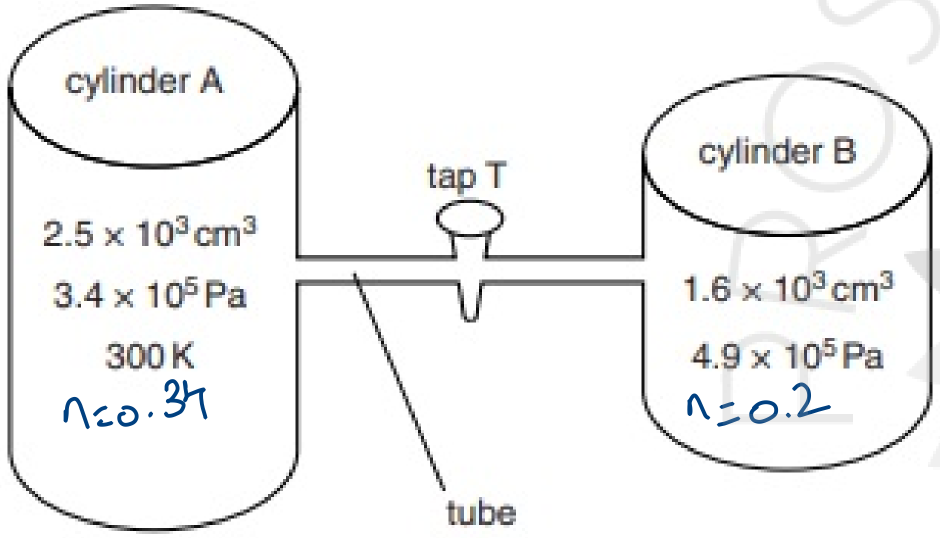


Fig. 2.1

Initially, tap T is closed. The cylinders contain an ideal gas at different pressures.

(i) Cylinder A has a constant volume of $2.5 \times 10^3\,cm^3$ and contains gas at pressure $3.4 \times 10^5\,Pa$ and temperature 300 K.

Show that cylinder A contains 0.34 mol of gas.

$PV = nRT$

$(3.4\times10^5)(2.5\times10^3\times10^{-6}) = n(8.31)(300)$

$n = 0.34$ mol

[1]

(ii) Cylinder B has a constant volume of $1.6 \times 10^3\,cm^3$ and contains 0.20 mol of gas.
When tap T is opened, the pressure of the gas in both cylinders is $3.9 \times 10^5\,Pa$.
No thermal energy enters or leaves the gas.

Determine the final temperature of the gas.

$PV = nRT$

$(3.9\times10^5)((2.5+1.6)\times10^3\times10^{-6}) = (0.2+0.34)(8.31)(T)$

$T = 356\,K$

temperature = 360 K [2]

(c) By reference to work done and change in internal energy, suggest why the temperature of the gas in cylinder A has changed.

As there is a change in volume, the work done has increased and therefore the internal energy has increased. As ideal gases have no intermolecular forces, the Kinetic energy of the particles has increased and therefore the temperature has also increased. [3]

$\uparrow \Delta U = \overset{\rightarrow 0}{Q} + W\uparrow$

$V\uparrow \rightarrow W\uparrow$

$\rightarrow \Sigma K.E\uparrow \rightarrow \langle c^2\rangle\uparrow \rightarrow T\uparrow$

2 (a) The kinetic theory of gases is based on a number of assumptions about the molecules of a gas.

State the assumption that is related to the volume of the molecules of the gas.

Volume of the molecules of a gas occupy a negligible volume compared to the volume of the container of gas. [2]

(b) An ideal gas occupies a volume of $2.40 \times 10^{-2}\,m^3$ at a pressure of $4.60 \times 10^5\,Pa$ and a temperature of 23 °C.

(i) Calculate the number of molecules in the gas.

$pV = NKT$

$(4.6\times10^5)(2.4\times10^{-2}) = N(1.38\times10^{-23})(23+273.15)$

$N = 2.7\times10^{24}$

number = 2.7×10^{24} [3]

(ii) Each molecule has a diameter of approximately $3 \times 10^{-10}\,m$.

Estimate the total volume of the gas molecules.

1.5×10^{-10}

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\pi\times(1.5\times10^{-10}) = 1.41\times10^{-29}$

$2.7\times10^{14} \times 1.41\times10^{-29} = 4\times10^{-6}$

volume = 4×10^{-6} m^3 [3]

(c) By reference to your answer in **(b)(ii)**, suggest why the assumption in **(a)** is justified.

Yes, the assumption is justified as $4\times10^{-6} << 2.7\times10^{24}$ [1]

[Total: 9]

2 (a) A square box of volume V contains N molecules of an ideal gas. Each molecule has mass m.

Using the kinetic theory of ideal gases, it can be shown that, if all the molecules are moving with speed v at right angles to one face of the box, the pressure p exerted on the face of the box is given by the expression

$$pV = Nmv^2. \qquad \text{(equation 1)}$$

This expression leads to the formula

$$p = \frac{1}{3}\rho\langle c^2\rangle \qquad \text{(equation 2)}$$

for the pressure p of an ideal gas, where ρ is the density of the gas and $\langle c^2\rangle$ is the mean-square speed of the molecules.

Explain how each of the following terms in equation 2 is derived from equation 1:

ρ: $P = \frac{Nm}{V}(v^2)$. $\rho = \frac{Nm}{V} = \left(\frac{\text{Total mass}}{\text{Total volume}}\right)$

$\frac{1}{3}$: We are considering all 3 dimensions

$\langle c^2\rangle = 3v^2$

$\langle c^2\rangle$: $c = \sqrt{v_x^2+v_y^2+v_z^2}$ $\quad v_x = v_y = v_z = v$

$(c)^2 = \left(\sqrt{3v^2}\right)^2 \Rightarrow \langle c^2\rangle = 3v^2$

[4]

(b) An ideal gas has volume, pressure and temperature as shown in Fig. 2.1.

volume $6.0 \times 10^{-3}\,m^3$
pressure $3.0 \times 10^5\,Pa$
temperature 17 °C

Fig. 2.1

The mass of the gas is 20.7 g. (Total mass)

Calculate the mass of one molecule of the gas. (mass of one molecule)

① $PV = nRT$

$(3\times10^5)(6\times10^{-3}) = n(8.31)(17+273.15)$

$n = 0.746$ mol

$n = \frac{\text{Mass}}{M_r/A_r} \Rightarrow$ mass number $\Rightarrow 0.746 = \frac{20.7}{\text{Mass number}} \Rightarrow$ Mass number $= 27.747$ amu

$27.747 \times 1.66\times10^{-27} = 4.6\times10^{-26}$ Kg $\times 10^3 \Rightarrow 4.6\times10^{-26}$ g

② $PV = NKT$

$(3\times10^5)(6\times10^{-3}) = N(1.38\times10^{-23})(17+273.15)$

$N = 4.495\times10^{23}$ (no. of particles)

mass of 1 particle $= \frac{20.7}{4.495\times10^{23}} = 4.6\times10^{-23}$ g

Boyle's law:-

T const $\quad P \propto \frac{1}{V}$

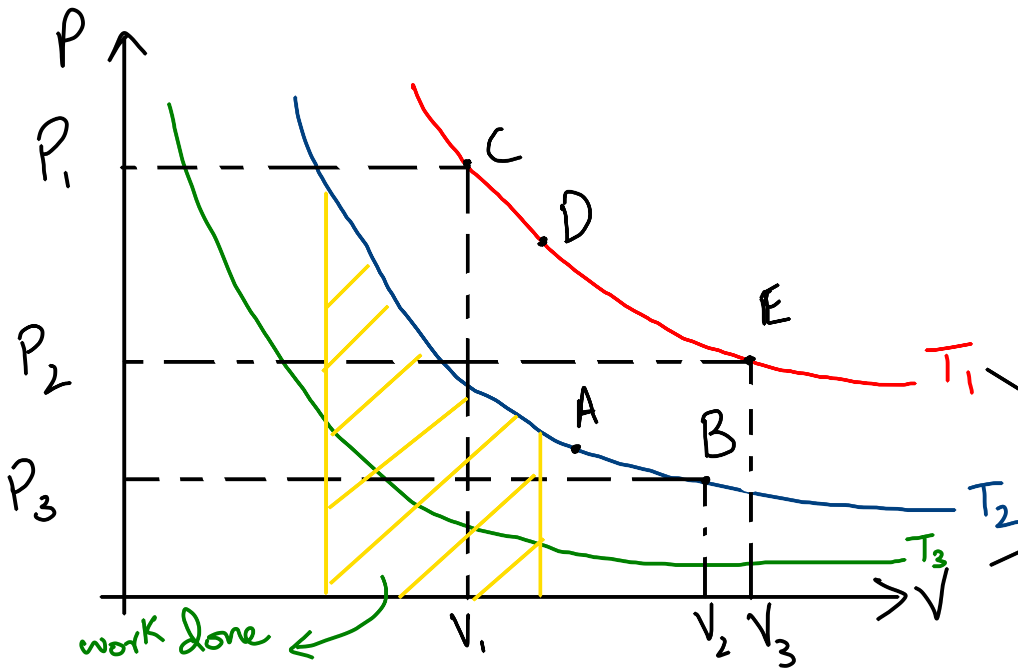
Explaining Boyle's Law

Why does the pressure of a gas increase when volume decreases?

- If the volume is reduced molecules have less distance to travel between collisions with a wall of the container
- They collide with the walls more often
- There is a greater change in momentum per second
- There is a greater force and a greater pressure

For an ideal gas at constant temperature, $P \propto \frac{1}{V}$

$P = \frac{K}{V} \Rightarrow P_1 V_1 = K = P_2 V_2 \Rightarrow \boxed{P_1 V_1 = P_2 V_2}$



$T_1 > T_2 > T_3$

each of these lines is an isothermic graphs

Temp of $C = D = E = T_1$ $\qquad$ Temp of $A = B = T_2$

$P_1 V_1 = P_2 V_3 \qquad P_2 V_3 \neq P_3 V_2$

$\Delta U = Q + W$

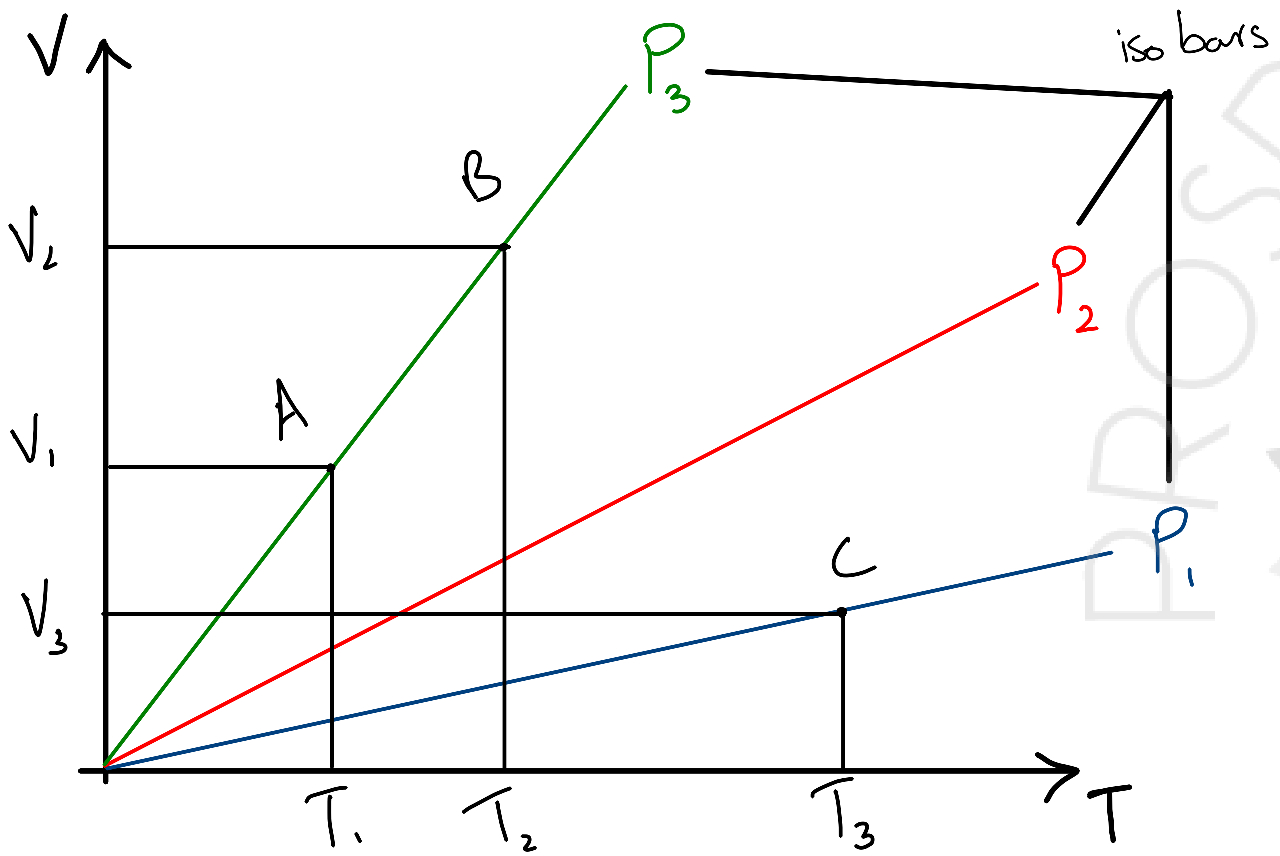
$\Delta U = \frac{3}{2} N K \Delta T$ ($\Delta T = 0$ for an isotherm)

$\Delta U = 0$

$\boxed{Q = -W}$

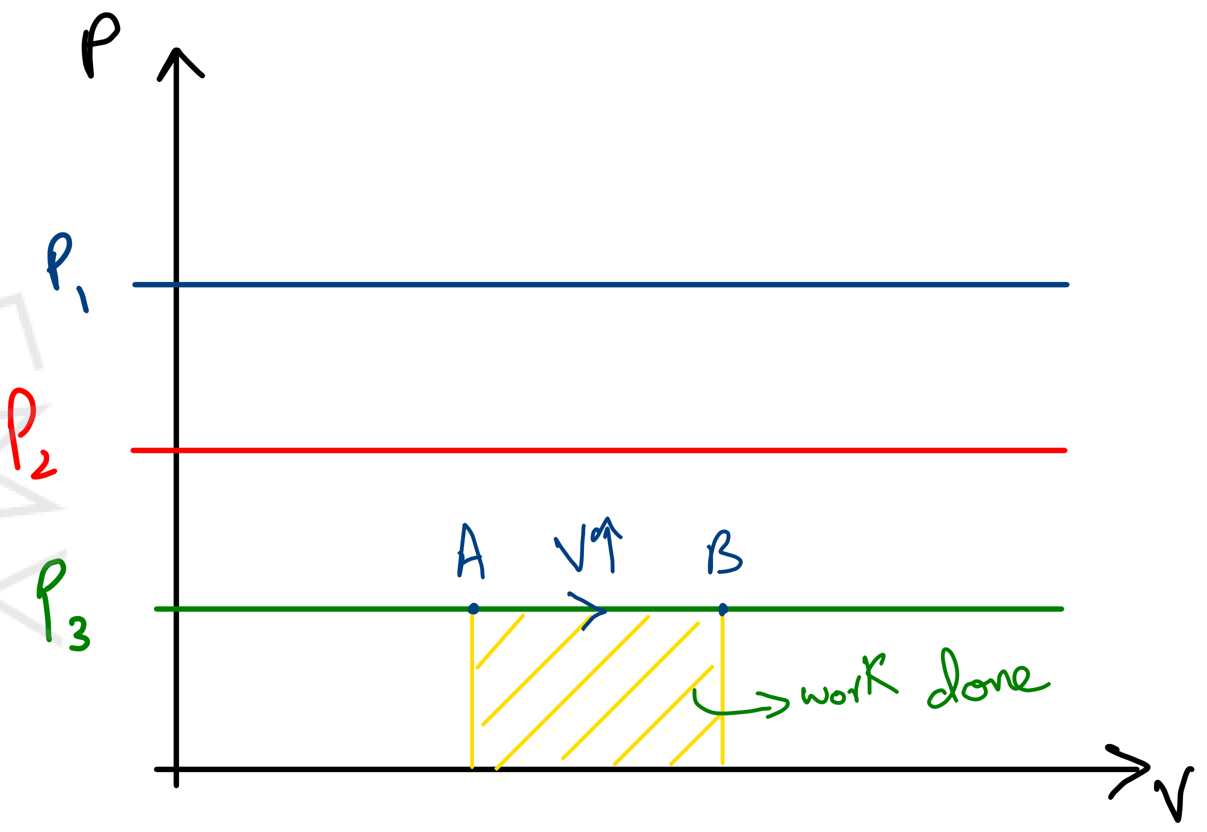
Charle's law :-

For an ideal gas under constant pressure, $V \propto T \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$



$P_1 > P_2 > P_3$

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ $\qquad$ $\frac{V_1}{T_1} \neq \frac{V_3}{T_3}$



$PV = nRT$

$V = \frac{nR}{P} T + 0$

$y = m x + c$

$\uparrow m = \frac{nR}{P\downarrow}$

P const $\quad V \propto T$

Explaining Charles' Law

Why does the volume increase with temperature when pressure is constant?

- Increased temperature increases the molecular speed
    - The momentum of each molecule increases.
    - Force for each collision increases
    - To keep pressure constant there must be fewer collisions with the walls each second
    - The molecules need to move further between collisions
    - This is achieved by increasing volume

Constant pressure $\Rightarrow W = P \times \Delta V$

If $V\uparrow$, W will be negative

$\Delta U = Q - W$

$\Delta U = Q - P\Delta V$

Pressure law:-

For an ideal gas with constant volume, $P \propto T$

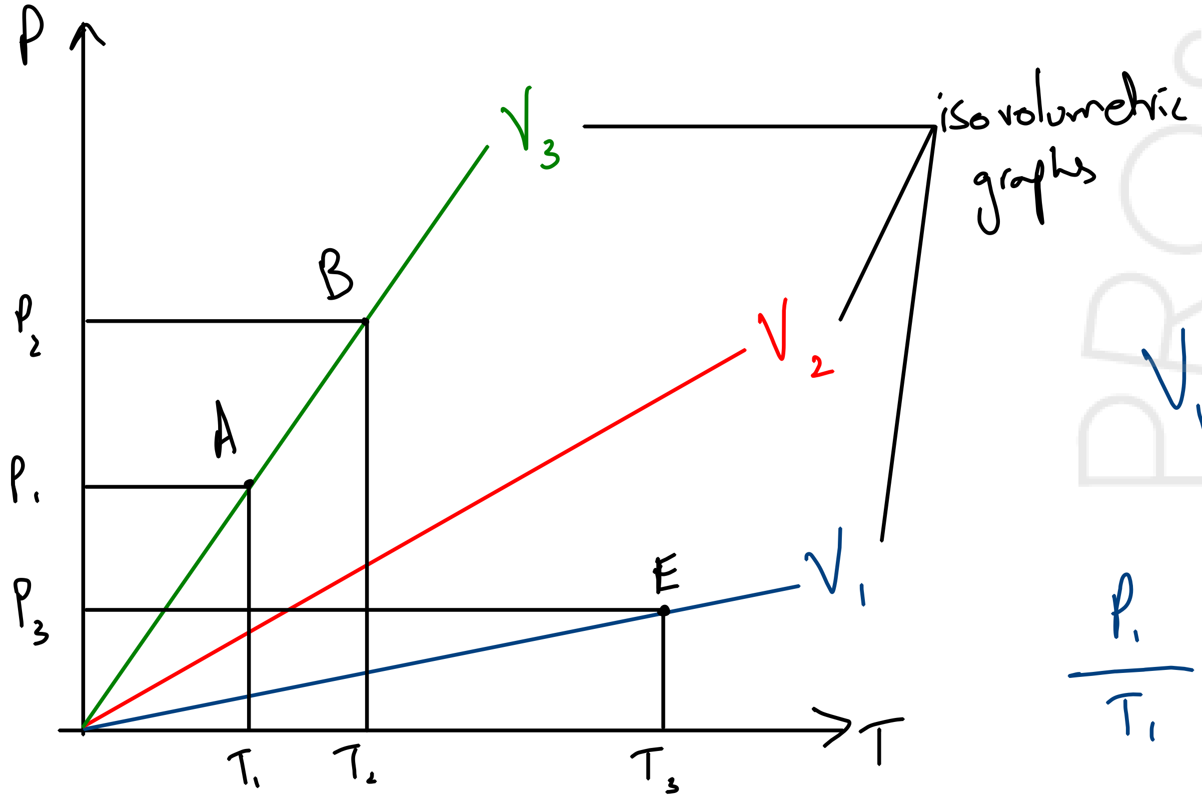
$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

V_{const}, $P \propto T$

Explaining the pressure Law

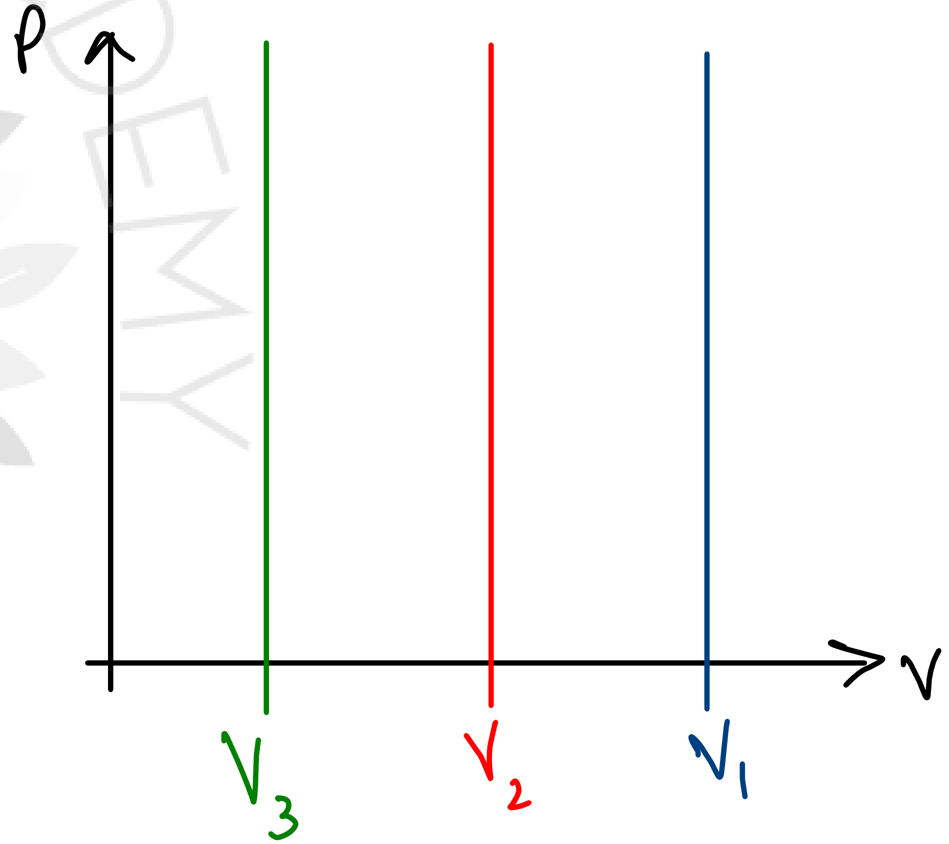
Why does the pressure increase with temperature when volume is constant?

- Increased temperature increases molecular speeds.
    - The momentum of each molecule increases
    - Each molecule collides more frequently with the walls of the container
    - Each of these effects increases the momentum change per second
    - There is greater force and therefore a greater pressure.



$V_1 > V_2 > V_3$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad \frac{P_1}{T_1} \neq \frac{P_3}{T_3}$$



$PV = nRT$

$$P = \frac{nR}{V} \times T + 0$$

$$y = m \quad x + c$$

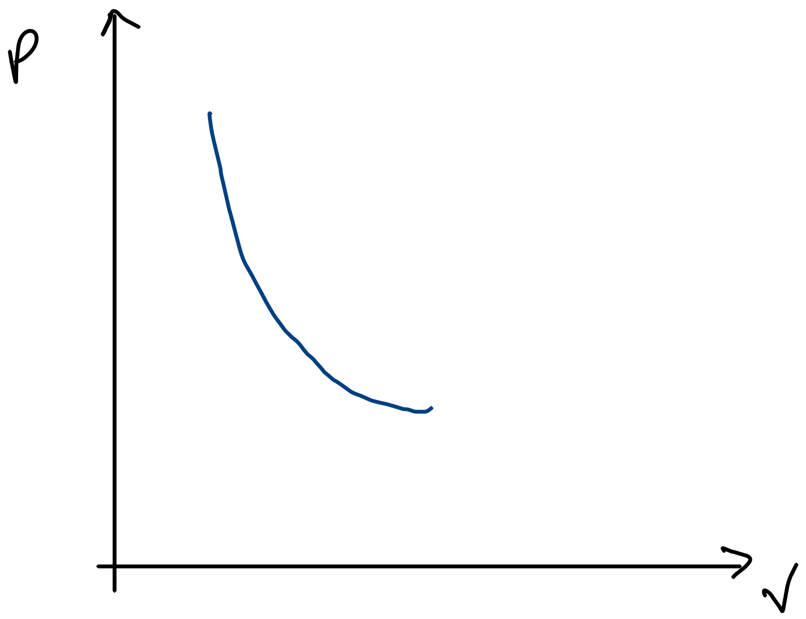
$$\uparrow m = \frac{nR}{V\downarrow}$$

$\Delta U = Q + W$

if V is constant, Area under graph = 0 $\therefore W = 0$

$\Delta U = Q$

Adiabatic Process:- No heat exchange takes place in this process ($Q = 0$)



$$\Delta U = \overset{0}{Q} + W$$

$$\Delta U = W$$