Motion in a circle

12 Motion in a circle

12.1 Kinematics of uniform circular motion

Candidates should be able to:

1. define the radian and express angular displacement in radians
2. understand and use the concept of angular speed
3. recall and use $\omega = 2\pi / T$ and $v = r\omega$

12.2 Centripetal acceleration

Candidates should be able to:

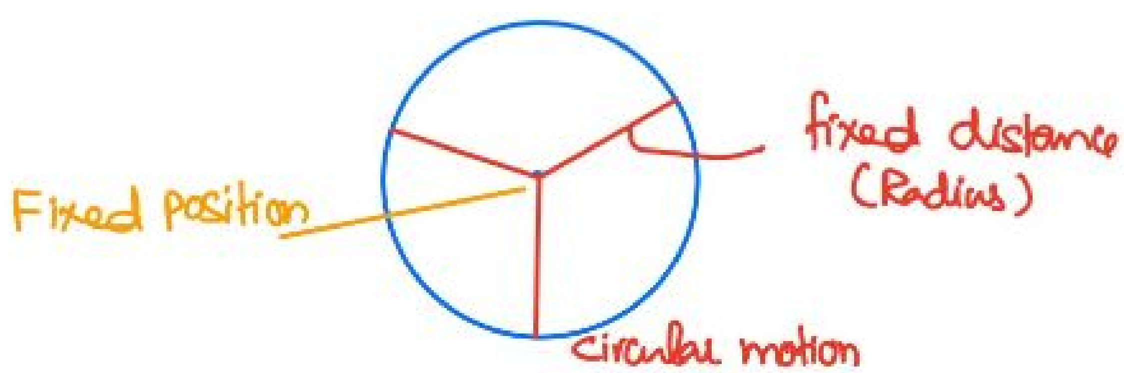
1. understand that a force of constant magnitude that is always perpendicular to the direction of motion causes centripetal acceleration
2. understand that centripetal acceleration causes circular motion with a constant angular speed
3. recall and use $a = r\omega^2$ and $a = v^2 / r$
4. recall and use $F = mr\omega^2$ and $F = mv^2 / r$

CIRCULAR MOTION:-

Identification: Source → Curved path

Position → fixed / Center

distance → fixed ie radius



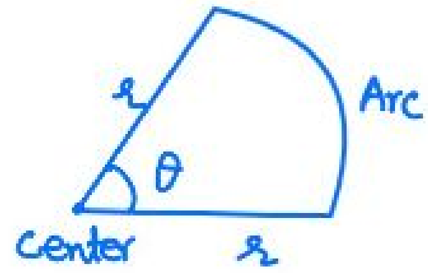
Angular displacement:

Concept: × Angle

→ Source : Arc length / Curved path

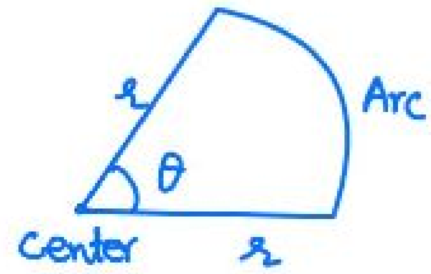
→ Position : center

→ Reference axis : Radii



Def. Angle swept out by an arc at center with radii is angular displacement.

Symbol: θ



Formula:

Arc length $\propto$ angular displacement

$$S \propto \theta$$

$$\boxed{S = r\theta}$$ r – Radius of circle

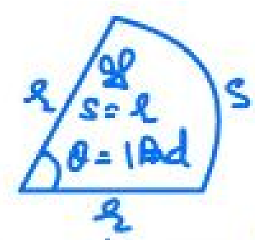
P.S. Vector

Direction: clockwise or anti-clockwise

Units: Degree ($^\circ$) or Radian (rad)

Note:

(1) **Def. of Radian:** $\theta = \frac{S}{r}$



$\theta = 1$ rad iff $S = r$

Unit of angle subtended by an arc at center with radii such that arc length is equal to the radius of circular path.

(2) **Relationship between degree and radian:-**

$$S = r\theta$$

$$2\pi r = r(360^\circ)$$

$$2\pi = 360^\circ$$

$$\boxed{\pi \text{ rad} = 180^\circ}$$

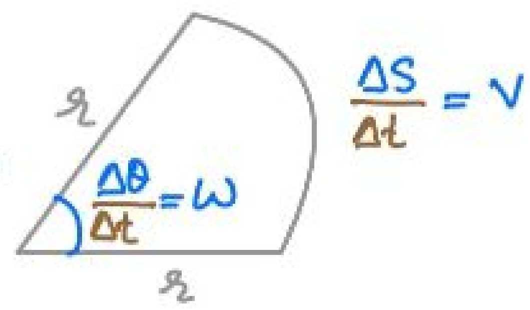
$S = 2\pi r$

$\theta = 360^\circ$

Angular speed:-

Concept:

Def. Change of angle swept out by an arc at center with radii per unit time.



Symbol. ω

Formula: $\omega = \frac{\Delta\theta}{\Delta t}$

For one complete revolution

$\Delta\theta = 2\pi$ and $\Delta t = T$

$$\boxed{\omega = \frac{2\pi}{T}}$$

But $\frac{1}{T} = f$

$$\boxed{\omega = 2\pi f}$$

Units: rad s^{-1}

P.S Scalar

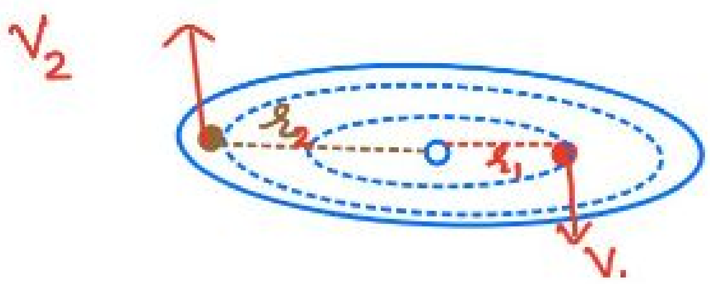
Note: Relationship b/w speed and angular speed:-

$$\Delta S = r\,\Delta\theta$$

Divide both sides by time

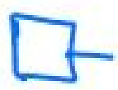
$$\frac{\Delta S}{\Delta t} = r\left(\frac{\Delta\theta}{\Delta t}\right)$$

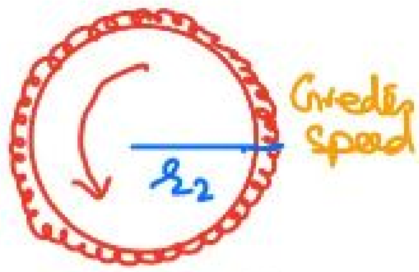
$$\boxed{V = r\omega}$$

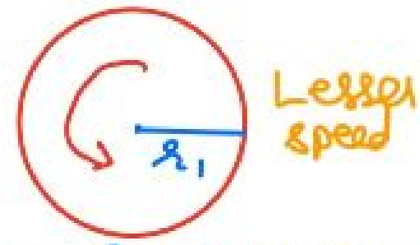


$V_1 < V_2$ because Radius $(r_1 < r_2)$ as ω = Constant.

Speed Camera



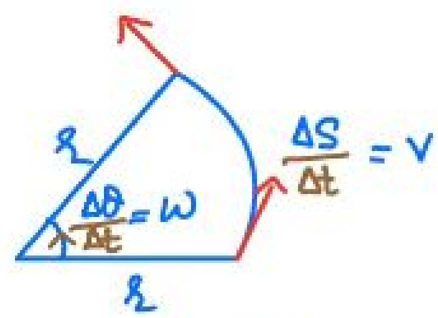




Speed Camera will measure greater speed due to greater radius as torque produced by engine has constant value for both tyres

Angular velocity:-

Def. Change of angle in clockwise or in anticlockwise direction swept out by an arc at center with radii per unit time.

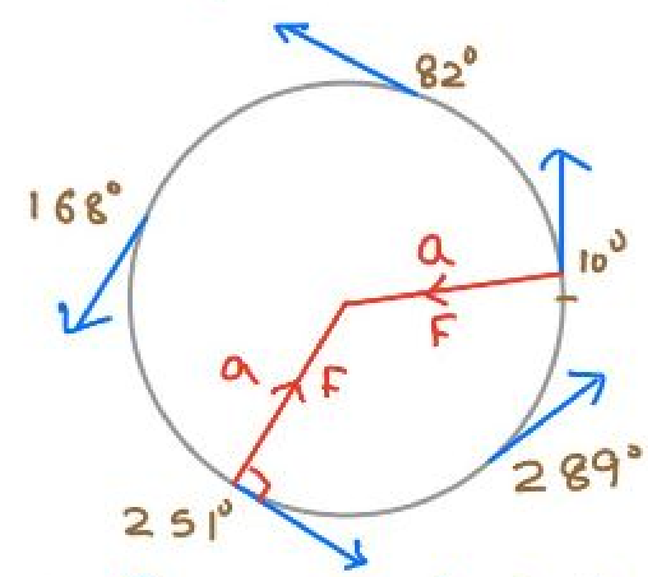


P.S Vector

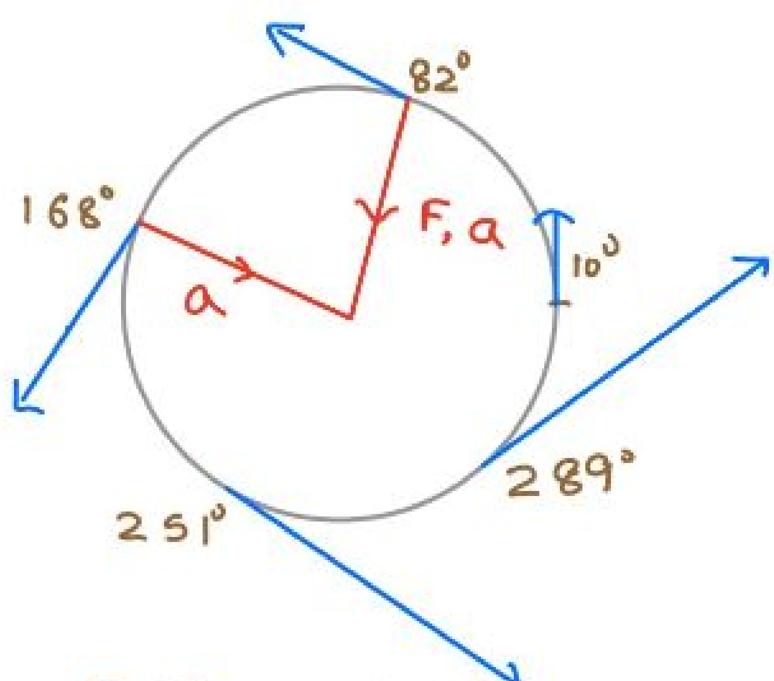
Direction: Clockwise or anticlockwise

Acceleration / Centripetal acceleration:-

Concept:



Uniform speed but variable velocity due to change of direction



Both speed and velocity vary

$$a = \frac{\text{change of velocity}}{\text{time}}$$

$$a = \frac{(\text{Constant speed})(\text{change of direction})}{\text{time}}$$

$$a = |V|\left[\frac{\Delta\theta}{\Delta t}\right] \Rightarrow \boxed{a = V\omega}$$

But $V = r\omega$

$a = (r\omega)(\omega)$	$a = V\left(\frac{V}{r}\right)$
$\boxed{a = r\omega^2}$	$\boxed{a = \frac{V^2}{r}}$

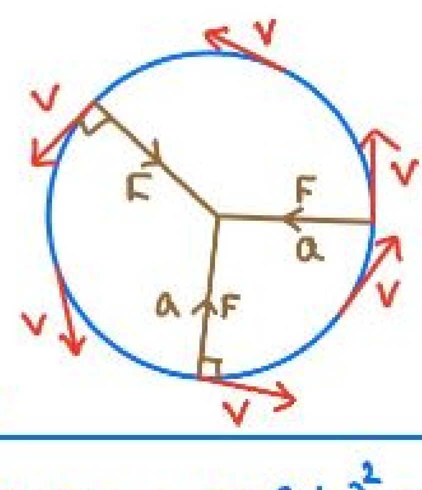
Units: $m s^{-2}$

P.S: Vector

Direction: Towards resultant force/centripetal force i.e towards center of circular path and perpendicular to tangential velocity.

Centripetal Force

Def. Rate of change of momentum which compel an object to move in a circular path.



Formula: $F = ma$

$$\boxed{F = mV\omega = mr\omega^2 = m\frac{V^2}{r}}$$

Direction: towards center of circular path and perpendicular to tangential velocity.

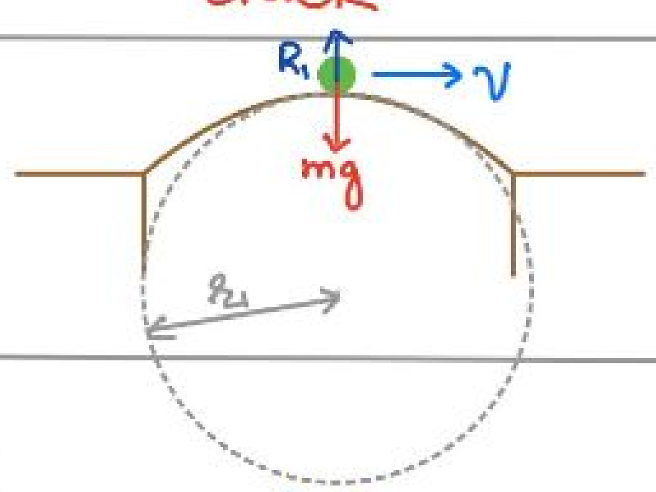
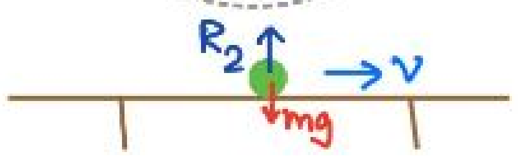
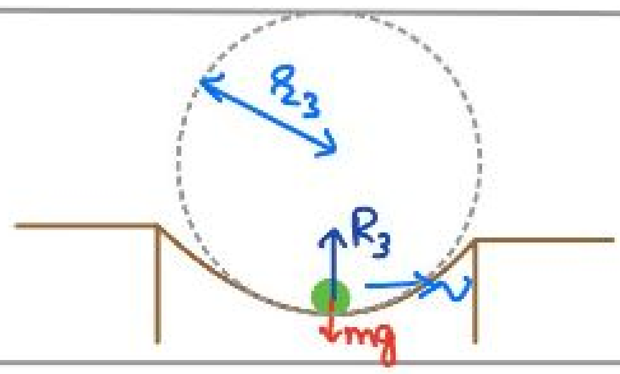
Working rules for problems involving circular motion :-

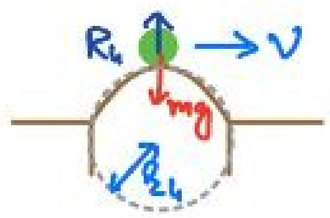
Step 1: Draw a free force diagram representing all the forces acting on the body.

Step 2: Find the resultant force acting towards centre of circular path.

Step 3: Equate the force from step 2 to centripetal force and get an expression with desired physical quantity as subject.

EX. 1: Motion of a vehicle along a curved track :-

S.No.	track	Resultant Force	Equation
1		$mg - R_1$	$mg - R_1 = \frac{mv^2}{r_1}$ $R_1 = mg - \frac{mv^2}{r_1}$
2		Zero	$R_2 = mg$ as centripetal force $= 0$
3.		$R_3 - mg$	$R_3 - mg = \frac{mv^2}{r_3}$ $R_3 = mg + \frac{mv^2}{r_3}$

4.		$mg - R_4$	$mg - R_4 = \frac{mv^2}{r_4}$ $R_4 = mg - \frac{mv^2}{r_4}$

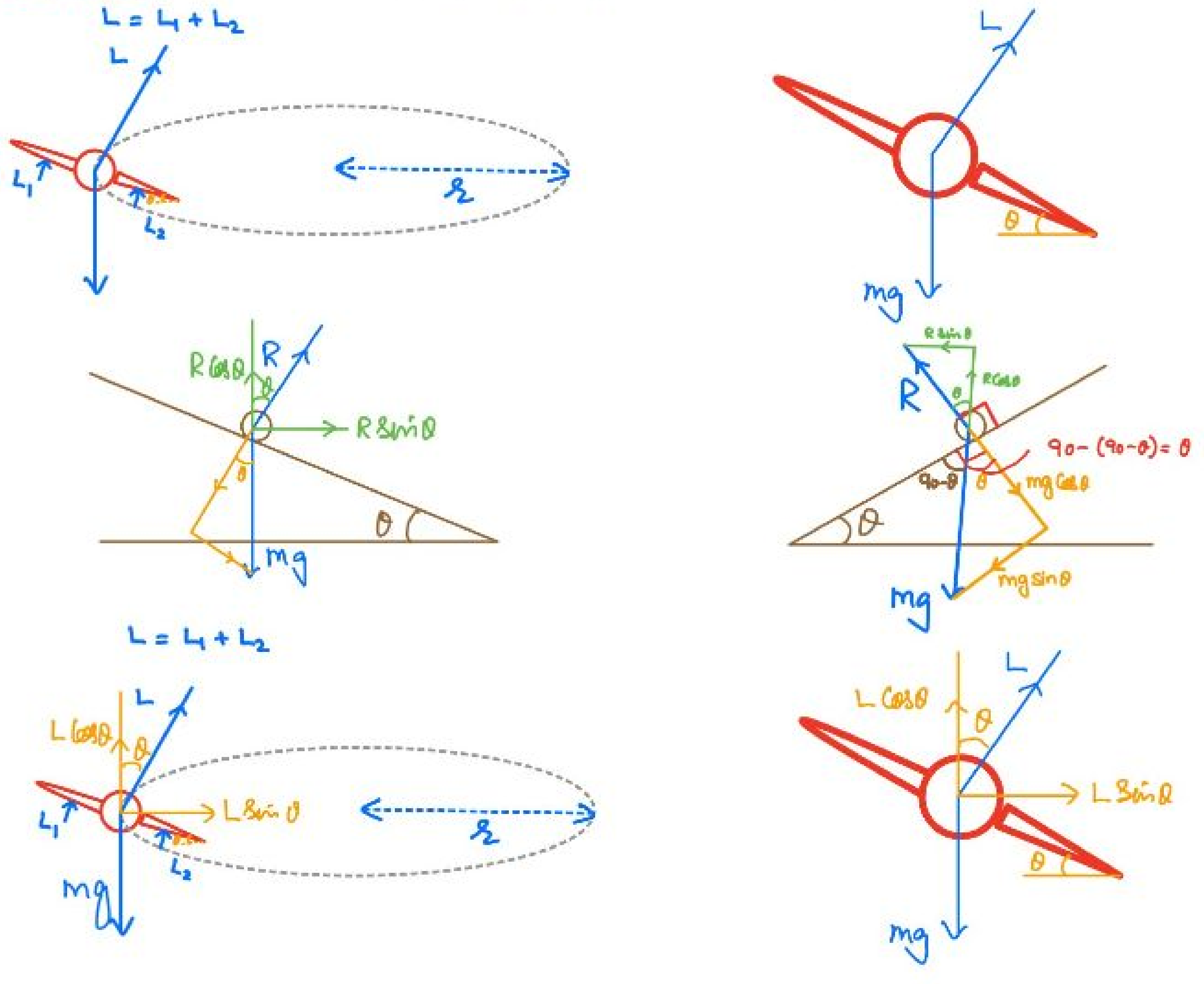
Analysis: Relative order of normal reaction force of track on vehicle:

$$R_3 > R_2 > R_1 > R_4$$

Here $R_1 > R_4$ because radii $(r_1 > r_4)$

$$R\downarrow = mg - \left[\frac{mv^2}{r\downarrow}\right]$$

EX.2: Motion of an aeroplane in a horizontal circle:- ie $\Delta h = 0$ or $\Delta GPE = 0$



Vertical forces: $L\cos\theta = mg$ ---------------- (1)

Horizontal force: $L\sin\theta = \frac{mv^2}{r}$ ------------ (2)

Divide (2) by (1)

$$\frac{\cancel{L}\sin\theta}{\cancel{L}\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

or $\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

Analysis: $\theta\uparrow$ or $[\tan(\theta)]\uparrow$ if

(i) $v\uparrow$ for constant (rg)

(ii) $r\downarrow$ for constant $\left(\frac{v^2}{g}\right)$

$\theta/°$	0	30	45	60	90	$\uparrow$
$\sin(\theta/°)$	0	0.5	0.707	0.866	1	$\uparrow$
$\cos(\theta/°)$	1	0.866	0.707	0.5	0	$\downarrow$

$$(\tan\overset{\uparrow}{\theta})\uparrow = \frac{(\sin\overset{\uparrow}{\theta})\uparrow}{(\cos\overset{\uparrow}{\theta})\downarrow}$$

EX.3: Motion of a sphere in a horizontal circle in a bowl:-

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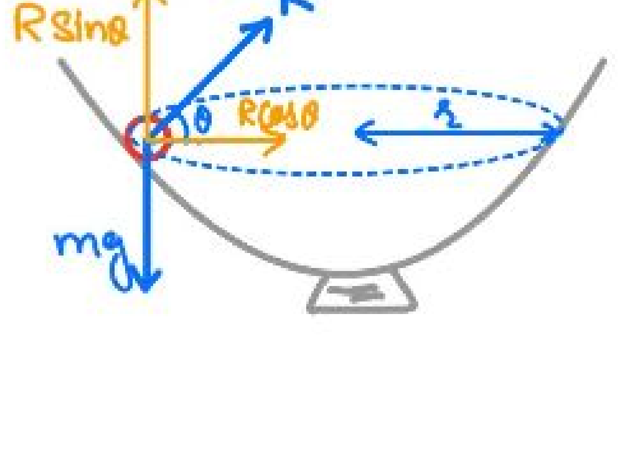
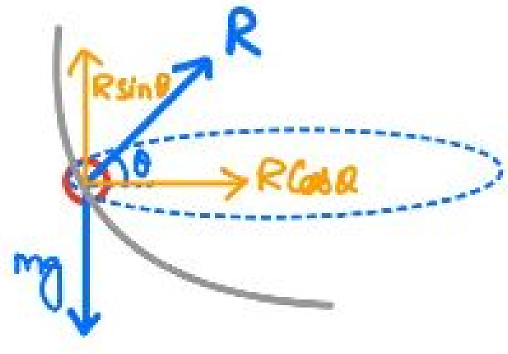




<u>Vertical forces</u>: $R\sin\theta = mg$ ----------------- (1)

<u>Horizontal force</u>: $R\cos\theta = \frac{mv^2}{r}$ ----------------- (2)

Divide (1) by (2)

$$\frac{\cancel{R}\sin\theta}{\cancel{R}\cos\theta} = \frac{\cancel{m}g}{\frac{\cancel{m}v^2}{r}}$$

$$\boxed{\tan\theta = \frac{rg}{v^2}} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{rg}{v^2}\right)$$

Note: If angle of force to be resolved is provided with

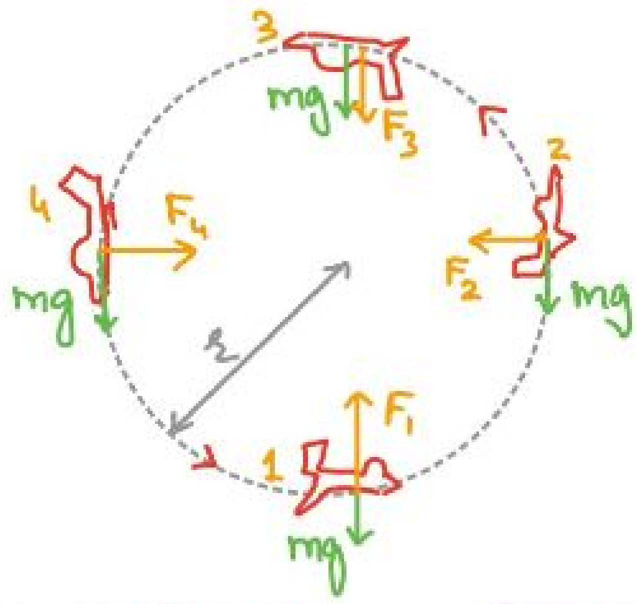
(i) <u>Horizontal</u>:

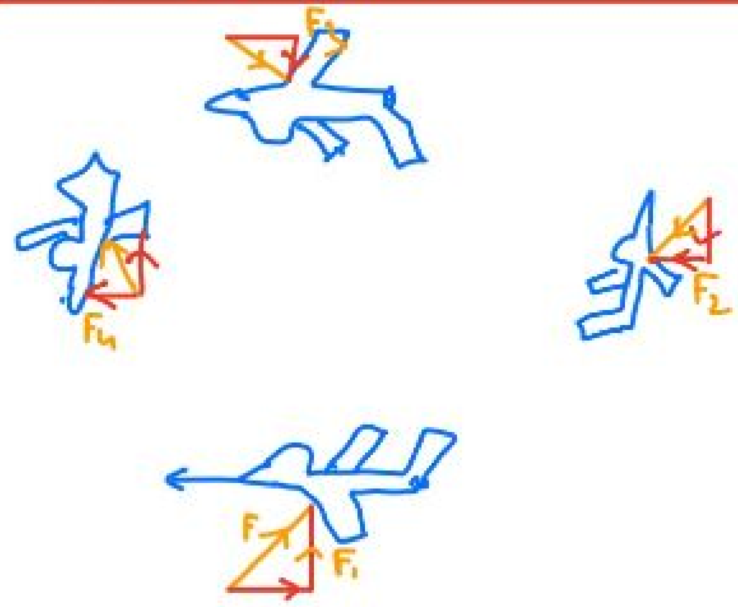
$$\tan\theta = \frac{rg}{v^2}$$

(ii) <u>Vertical</u>:

$$\tan\theta = \frac{v^2}{rg}$$

Ex. 4. Motion of an aeroplane in a vertical circle:-





<u>At position 1</u>: $F_1 - mg = \frac{mv^2}{r} \Rightarrow F_1 = mg + \frac{mv^2}{r}$

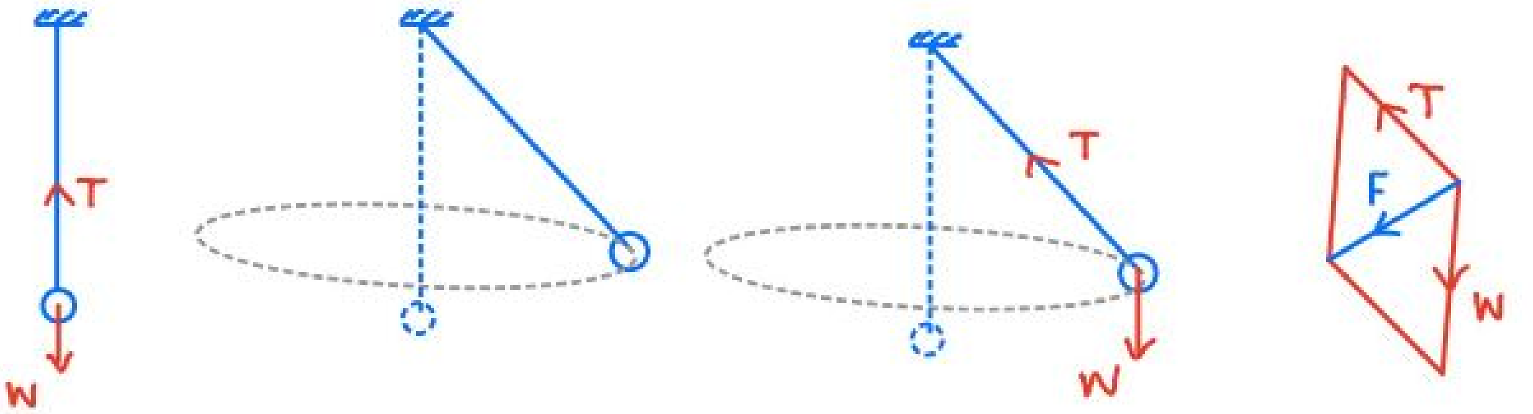
<u>At position 2 and at 4</u>:- $F_2 = \frac{mv^2}{r} = F_4$

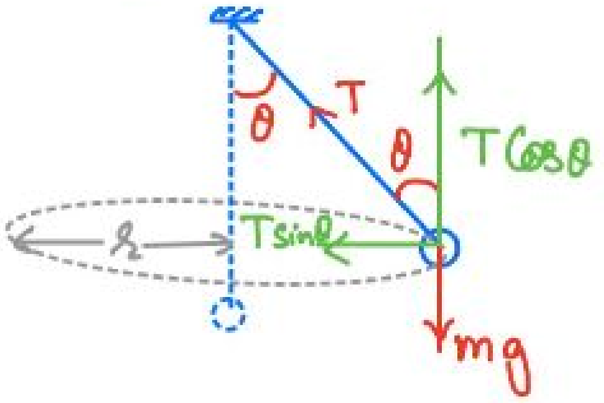
<u>At position 3</u>: $F_3 + mg = \frac{mv^2}{r} \Rightarrow F_3 = \frac{mv^2}{r} - mg$

Analysis: Relative order of thrust of air particles at different positions be:

$$F_1 > (F_2 = F_4) > F_3$$

EX. 5: Conical pendulum:-





Vertical forces: $T\cos\theta = mg$ ......(1)

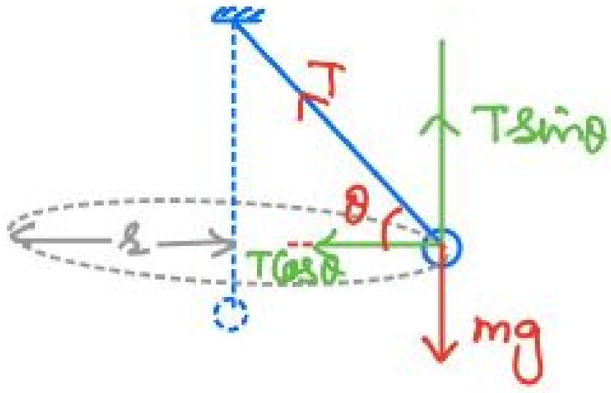
Horizontal forces: $T\sin\theta$ provides centripetal force

$$T\sin\theta = \frac{mv^2}{r} \quad ......(2)$$

Divide (2) by (1)

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$



Vertical forces: $T\sin\theta = mg$ ......(1)

Horizontal forces: $T\cos\theta$ provides centripetal force

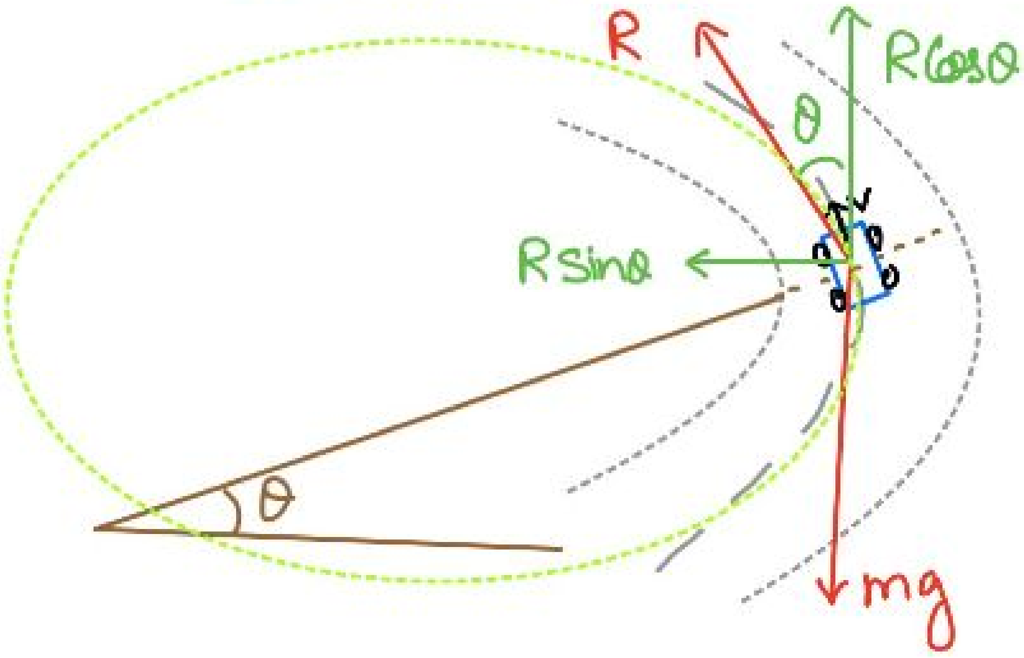
$$T\cos\theta = \frac{mv^2}{r} \quad ......(2)$$

Divide (1) by (2)

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\cancel{m}g}{\frac{\cancel{m}v^2}{r}}$$

$$\boxed{\tan\theta = \frac{rg}{v^2}}$$

EX.6: Banked track:- Curved inclined path which provides centripetal force due to angle of inclination is banked track.



Vertical forces: $R\cos\theta = mg$ ----(1)

Horizontal force: $R\sin\theta$ provides centripetal force

$$R\sin\theta = \frac{mv^2}{r} \text{ ------- } (2)$$

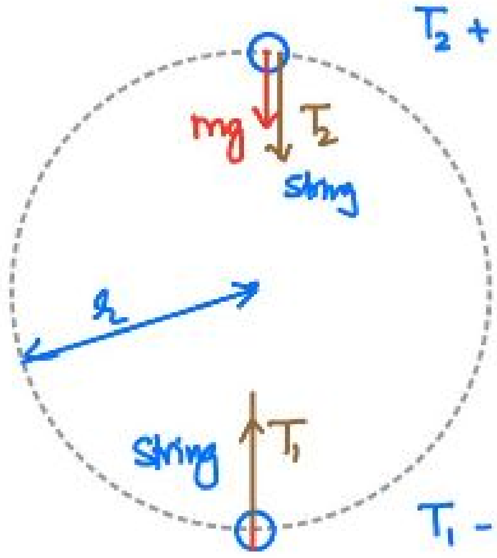
Divide (2) by (1)

$$\frac{\cancel{R}\sin\theta}{\cancel{R}\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$

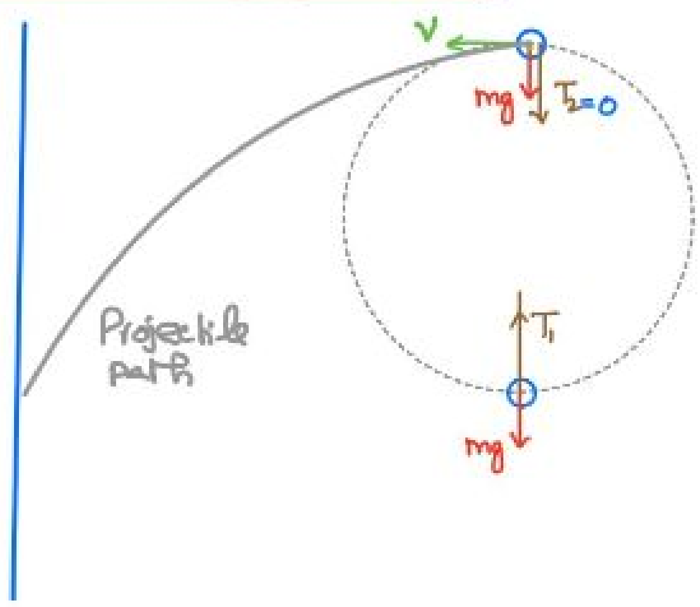
$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

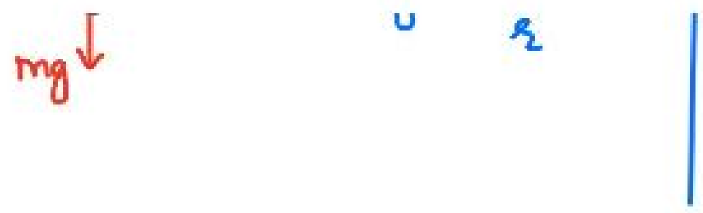
EX.7: Motion of a stone in a vertical circle:

$T_2 + mg = \frac{mv^2}{r}$



$T_1 - mg = mv^2$



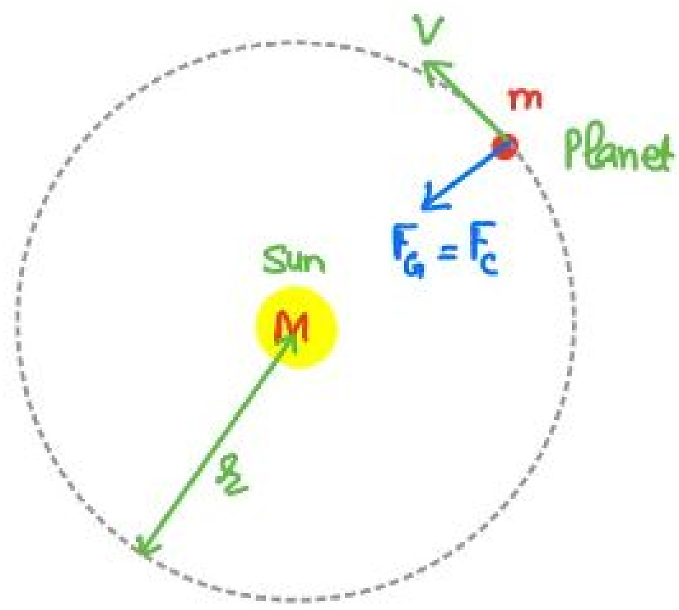


Note: (1) ($T_1 > T_2$), so string may break at bottom / lowest position if its tensile strength is not sufficient enough to bear the tension.

(2) If $a = g$, object will trace a parabollic path if it has velocity at the highest point.

(3) If ($a > g$) then object follows a circular path.

EX.8: Solar system



Gravitational pull of Sun provides centripetal force to planet

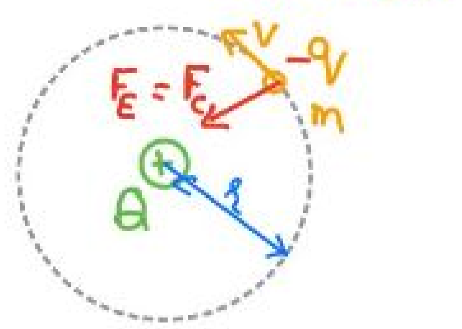
$$F_G = F_c$$

$$\boxed{\frac{GMm}{r^2} = \frac{mv^2}{r}}$$

G – Gravitational Constant $= 6.67 \times 10^{-11}\ \mathrm{N m^2 kg^{-2}}$

gravitational constant, $G = 6.67 \times 10^{-11}\ \mathrm{N\,m^2\,kg^{-2}}$ (Given on data page)

EX.9: Atom



Electric pull of +ve nucleus provides centripetal force to electrons.

$$F_E = F_c$$

$$\boxed{K\frac{Qq}{r^2} = \frac{mv^2}{r}}$$

$K = 8.99 \times 10^{9} \mathrm{m\,F^{-1}}$

$K = 8.99 \times 10^{9}\ \mathrm{N m^2 C^{-2}}$

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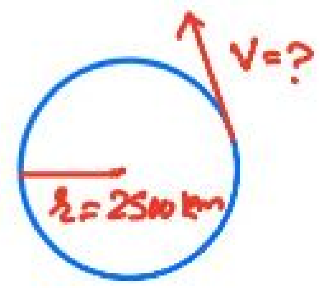
CIRCULAR MOTION

Q.1 What is the tangential velocity of a point on the equator of Mercury which has a rotation period of 59 days and an equatorial radius of 2500 km?

$$v = r\omega$$
$$= r\left(\frac{2\pi}{T}\right) = \frac{2(3.14)(2500\times10^3)}{59\times24\times60\times60}$$
$$= 3.08\ ms^{-1}$$



Q. 2 The Earth rotates once in about 24 h. Calculate its rotation frequency (f) and its angular frequency (ω).

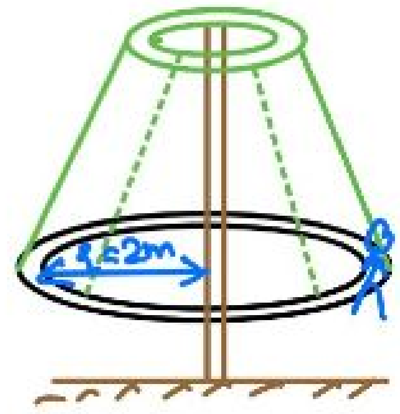
$$f = \frac{n}{t} \Rightarrow f = \frac{1}{T}$$
$$f = \frac{1}{24\times60\times60} = 1.16\times10^{-5}\ s^{-1}$$

$$\omega = \frac{2\pi}{T} \quad \text{or} \quad \omega = 2\pi f$$
$$\omega = 2(3.14)(1.16\times10^{-5})$$
$$\omega = 7.27\times10^{-5}\ rad\ s^{-1}$$

Q. 3 (a) What is the centripetal acceleration of a 40 kg child sitting 2m from the centre of a roundabout which turns once in 5.0 s?

$$a = r\omega^2$$
$$= r\left(\frac{2\pi}{T}\right)^2$$
$$= \frac{4\pi^2 r}{T^2}$$

$$a = \frac{4(3.14)^2(2)}{(5.0)^2}$$
$$a = 3.16\ ms^{-2}$$



(b) What is the resultant horizontal force acting on the child?

$$F = ma$$
$$= (40)(3.16) = 126\ N$$

(c) What is the origin of this force?

Frictional force b/w child and seat underneath him provides centripetal force.

Q. 4 A boy stands at the equator of the Earth having a radius of 6.4×10^6 m. Find its

(a) Angular velocity

$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{24\times60\times60}$$
$$= 7.27\times10^{-5}\ rad\ s^{-1}$$

(b) Linear velocity

$$v = r\omega$$
$$= (6.4\times10^6)(7.27\times10^{-5})$$
$$= 465\ ms^{-1}$$

Q. 5 The spindle motor of CD rotates it at 33.3 revolutions / minute.

(a) What is the angular velocity of the record in rad s^{-1}?

$$\omega = 2\pi f = 2\pi\left(\frac{n}{t}\right) \Rightarrow \omega = \frac{2(3.14)(33.3)}{60}$$
$$\omega = 3.49\ rad\ s^{-1}$$

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(b) What is the speed of a point on CD at a distance of 3.0 cm from its centre?

$$v = r\omega$$
$$= (3.0\times10^{-2})(3.49) = 0.105\ m s^{-1}$$

Q. 6 A pulley wheel rotates at 300 rev min^{-1}. Calculate

(a) its angular velocity in rad s^{-1},

$$\omega = 2\pi f = 2\pi\left(\frac{n}{t}\right) \Rightarrow \omega = \frac{2(3.14)(300)}{60} = 31.4\ rad\ s^{-1}$$

(b) the linear speed of a point on the rim if the pulley has a radius of 150 mm,

$$v = r\omega$$
$$= (150\times10^{-3})(31.4) = 4.71\ m s^{-1}$$

(c) the time for one revolution.

300 rev. complete in = 60 s

1 rev. ______ = $\frac{60}{300}$ = 0.20 s

Q.7 A car moves round a circular track of radius 1.0 km at a constant speed of 129 kmh^{-1}. Calculate its angular velocity in rad s^{-1}.

$$v = r\omega$$
$$\frac{129\times10^{3}}{60\times60} = (1.0\times10^{3})\omega \Rightarrow \omega = 0.036\ rad\ s^{-1}$$

Q.8 A child is sitting on a fairground ride, as shown in fig. 8. The ride turns through one complete revolution every four seconds. If the combined mass of the child and the seat is 40 kg, and the radius of the circular path is 6.0 m, calculate the tension, T, in the support.

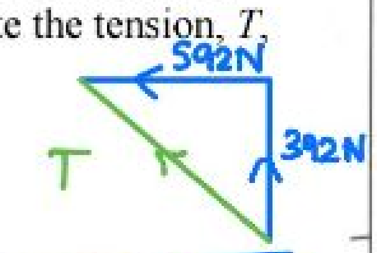
Vertical forces: $T_v = mg$

$$T_v = (40)(9.81) = 392N$$

Horizontal forces: $T_v = mr\omega^2$

$$T_v = mr\left(\frac{2\pi}{T}\right)^2$$

$$T_v = \frac{(40)(6.0)(4)(3.14)^2}{(4)^2} = 592N$$



$$T = \sqrt{(392)^2 + (592)^2}$$
$$T = 710N$$

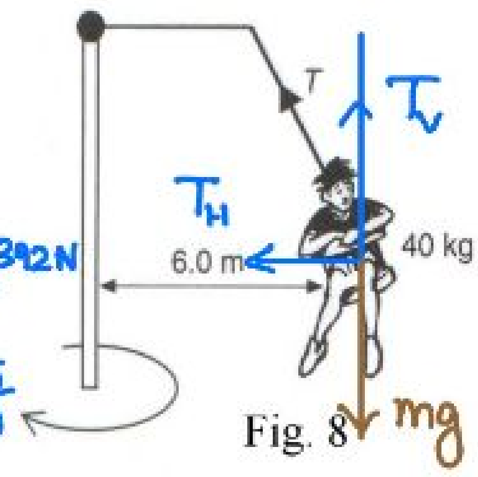


Fig. 8

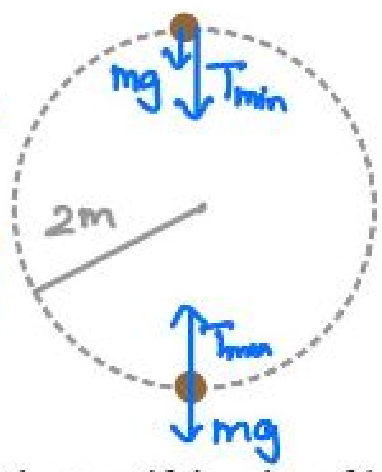
Q. 9 An object of mass 4.0 kg is whirled round in a vertical circle of radius 2.0 m with a speed of 5.0 m s^{-1}. Calculate the maximum and minimum tension in the string connecting the object to the centre of the circle. Assume acceleration due to gravity g = 10 ms^{-2}.

Max. tension at bottom:

$$T_{max} - mg = \frac{mv^2}{r}$$
$$T_{max} - (4.0)(10) = \frac{(4.0)(5.0)^2}{2.0}$$
$$T_{max} = 90$$

Min. tension at top

$$T_{min} + mg = \frac{mv^2}{r}$$
$$T_{min} + (4.0)(10) = \frac{(4.0)(5.0)^2}{2}$$
$$T_{min} = 10N$$



Q. 10 An aircraft is banking as it turns, as shown in fig 10. What is the radius of curvature of the turn if the aircraft's velocity is 200 ms^{-1} and it is banked at 35°?

$$\tan\theta = \frac{v^2}{rg}$$
$$\tan 35 = \frac{(200)^2}{(r)(9.81)}$$
$$r = 5.82\times10^{3}\ m$$

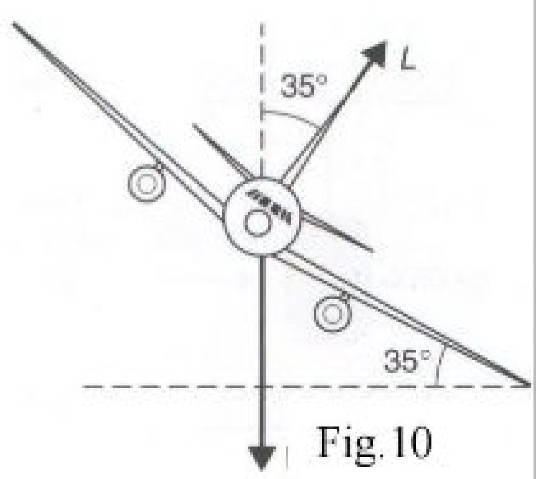


Fig.10

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Q. 11 A pilot flies an aeroplane at a constant angular velocity ω in a circle in a vertical plane. The radius of the circle is r.
What is the difference in the forces experienced by the pilot at the bottom and at the top of the circular loop?

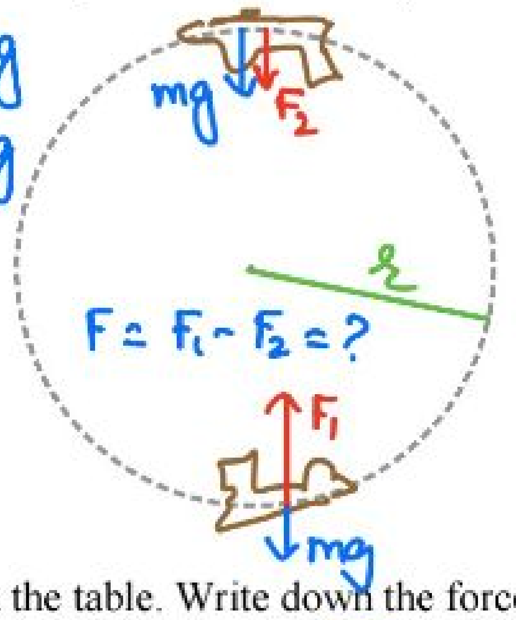
At top: $F_2 + mg = mr\omega^2 \Rightarrow F_2 = mr\omega^2 - mg$

At bottom: $F_1 - mg = mr\omega^2 \Rightarrow F_1 = mr\omega^2 + mg$

$F = F_1 - F_2$

$= \cancel{mr\omega^2} + mg - \cancel{mr\omega^2} + mg$

$F = 2mg$



Q.12 A car of mass m moves with the same speed v over each of the 4 bridges shown in the table. Write down the force equation of the car at each bridge if the normal reaction is R.
In which of the bridges is the force which the car exerts on the bridge the smallest?

S.#	Bridge	Equation
1.	 bridge	
2.	 bridge	
3.	 bridge	
4.	 bridge	
Lesser force bridge =		

Q. 13 In a fairground ride called a 'rotor', a person of mass 60 kg stands against a wall, as shown in fig 13.1, and the wall is rotated. When it is spinning at a suitable speed the floor is dropped so that the person is left 'struck to the wall'.

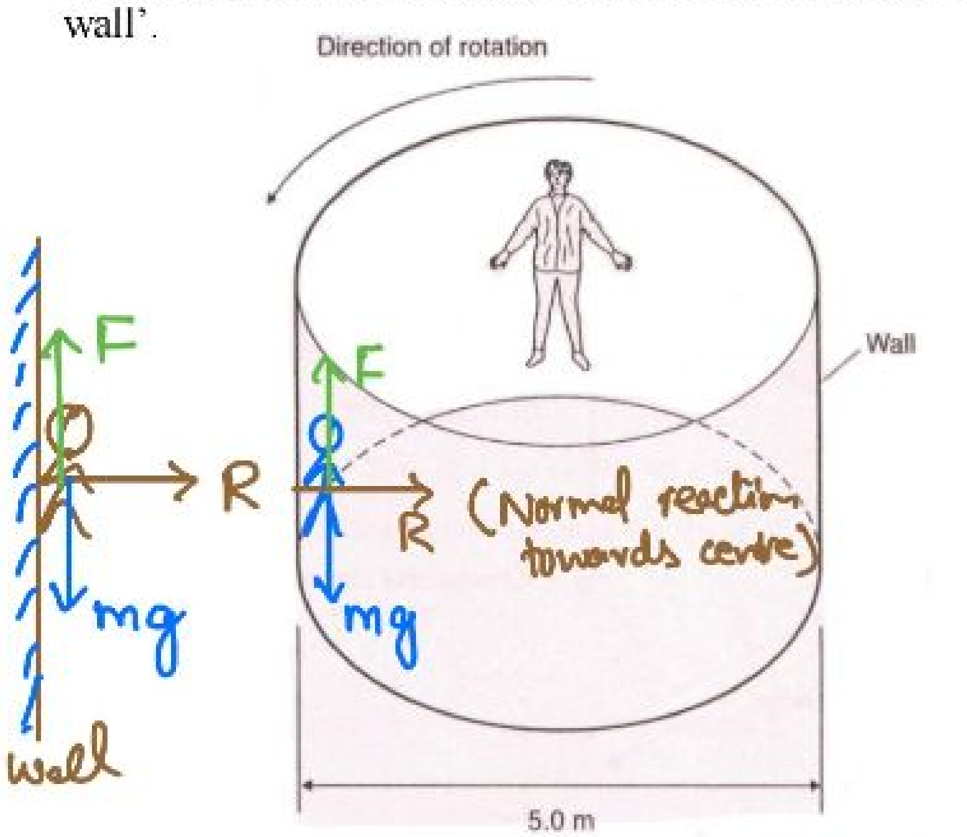


Fig. 13.1

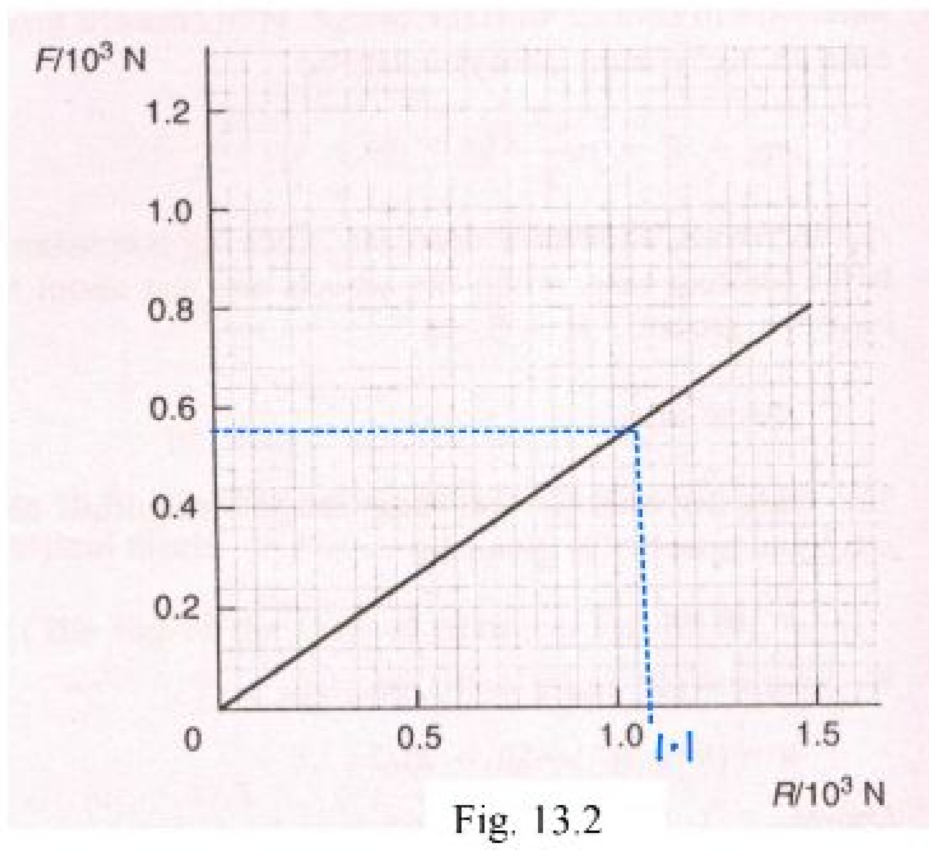


Fig. 13.2

Fig. 13.2 shows the variation in frictional force, F, with normal reaction, R, between the person and the wall.

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Determine;

(a) the normal reaction when the frict~~ional force is equal to the weight of a person o~~f mass 60 kg;

$F = W$

$F = mg = (60)(9.81) = 589N = 0.589kN$

From graph

$R = 1.1 kN$

(b) the minimum angular speed, in rad s^{-1}, at which such a person must be rotated to remain in position when the floor is dropped.

Normal reactional force provides centripetal force

$R = mr\omega^2$

$1.1 \times 10^3 = (60)(2.5)(\omega^2)$

$\omega = 2.71 \text{ rad } s^{-1}$

Q. 14 In a ride at an entertainment park, two people, each of mass 80 kg, sit in cages which travel at constant speed in a vertical circle of radius 8.0 m as shown in fig 14. Each revolution takes 4.2s.

(a) When a cage is at the top of the circle (position **A**) the person in it is upside down. For the person in cage **A** calculate the magnitudes of

(i) the angular velocity

$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{4.2} = 1.5 \text{ rad } s^{-1}$

(ii) the linear velocity

$v = r\omega = (8.0)(1.5) = 12 ms^{-1}$

(iii) the centripetal acceleration

$a = v\omega = (12)(1.5) = 18 ms^{-2}$

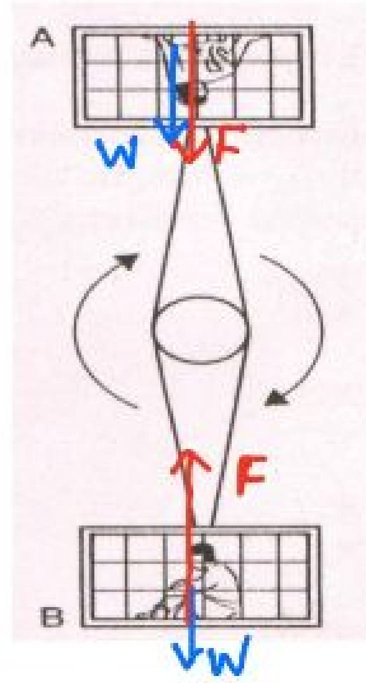

Fig. 14

(b) **(i)** Draw a vector diagram to show the directions of the following forces acting on the person in cage **A** in fig 14

1. the weight *W* of the person,
2. the force *F* exerted by the cage on the person.

(ii) Draw the corresponding diagram for the person at the bottom of the circle (position **B**).

(iii) What must be the value of the resultant of these two forces at both **A** and **B**?

Resultant force at A and B is the centripetal force and remain same

Resultant force: $F = ma$

$= (80)(18) = 1440 N$

(iv) Explain why the person remains on the floor of the cage at the top of the circle.

A Since $(a > g)$ or (Resultant force > weight) so apart from weight, another downward force is also acting due to contact.

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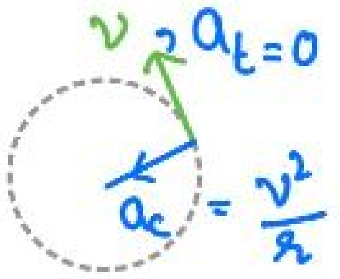
(v) State the position of the cage at which the force it exerts on the person has its maximum value. Calculate the magnitude of this force

Maximum force at position B

$F_B - mg = F$

$F_B - (80)(9.81) = 1440$

$F_B = 2.22 \times 10^3$ N



1. A body moving in a circular path of radius $\boldsymbol{r}$ has tangential acceleration $\boldsymbol{a_t}$ and centripetal acceleration $\boldsymbol{a_c}$. If the body is moving at constant speed $\boldsymbol{v}$, what are the magnitudes of $\boldsymbol{a_t}$ and $\boldsymbol{a_c}$?

	Tangential acceleration $\boldsymbol{a_t}$	Centripetal acceleration $\boldsymbol{a_c}$
A	$r v^2$	0
B	v^2 / r	0
C	0	$r v^2$
D	0	v^2 / r

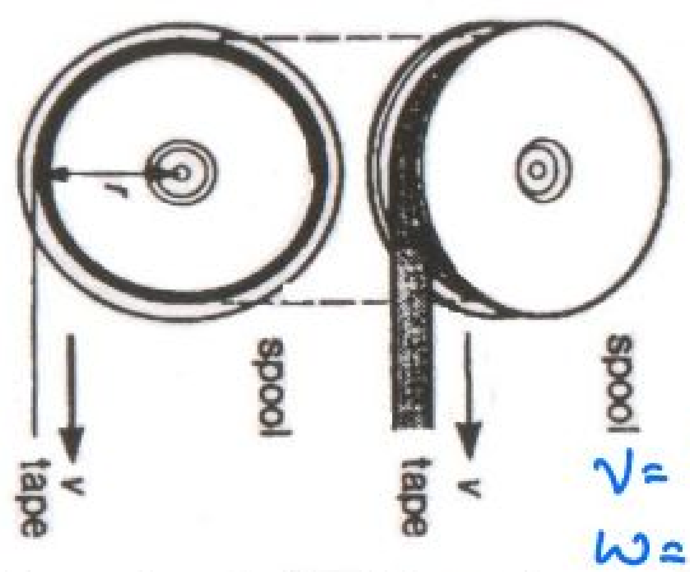
2. An object travels at constant speed around a circle of radius 1.0 m in 1.0 s. What is the magnitude of its acceleration?

$a = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 r}{T^2}$

A zero **B** 1.0 m s^{-2} **C** 2π m s^{-2} **D** $4\pi^2$ m s^{-2}

$a = \frac{4\pi^2(1.0)}{(1.0)^2} = 4\pi^2$

3. In a tape cassette, the tape leaves one spool at a constant speed v and at a variable distance r from the Centre.



$v = r\omega$

$\omega = \frac{v}{r}$

The angular velocity of the spool

A is proportional to $1/r^2$ **B** is proportional to $1/r$

C is proportional to r **D** does not depend on r

$\omega = \frac{\text{constant}}{r} \Rightarrow \omega \propto \frac{1}{r}$

CIE PAST PAPER QUESTIONS

Q. 1. An aircraft flies with its wings tilted as shown in fig. 1.1 in order to fly in a horizontal circle of radius r. The aircraft has mass 4.00×10^4 kg and has a constant speed of 250 ms^{-1}.

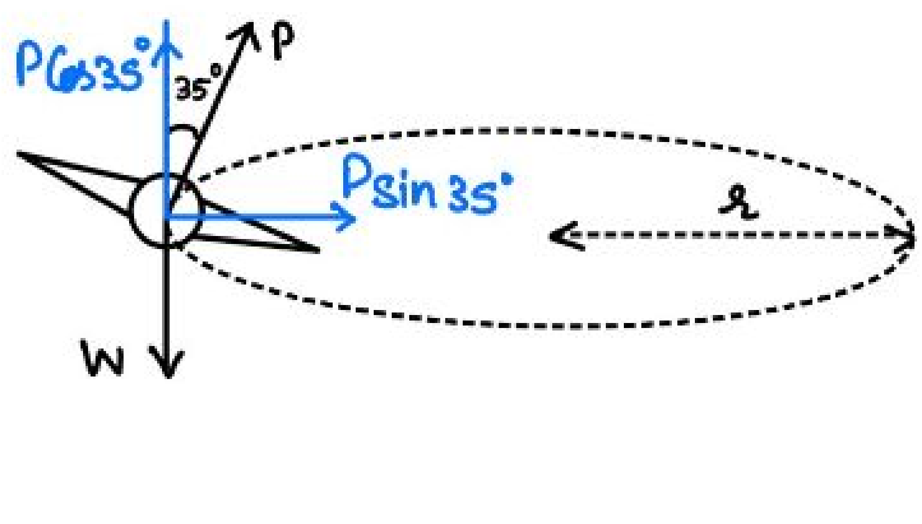


fig. 1.1

With the aircraft flying in this way, two forces acting on the aircraft in the vertical plane are the forces P acting at an angle of 35° to the vertical and the weight W.

(a) State the vertical component of P for the horizontal flight.

$P\cos 35°$, W, mg

vertical component of P = ……………………………………………[1]

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(b) Calculate P.

Consider vertical force

$P\cos 35 = mg$

$$P = \frac{(4.00\times10^4)(9.81)}{\cos 35}$$

P = 4.79×10^5 N [2]

(c) Calculate the horizontal component of P.

$P_H = P\sin 35°$

$= (4.79\times10^5)\sin 35°$

horizontal component of P = 2.74×10^5 N [1]

(d) Use Newton's second law to determine the acceleration of the aircraft towards the centre of the circle.

P_H provides centripetal force

$P_H = ma$

$2.74\times10^5 = (4.00\times10^4)(a)$

acceleration = 6.85 ms^{-2} [2]

(e) Calculate the radius r of the path of the aircraft's flight.

$$a = \frac{v^2}{r} \Rightarrow 6.85 = \frac{(250)^2}{r}$$

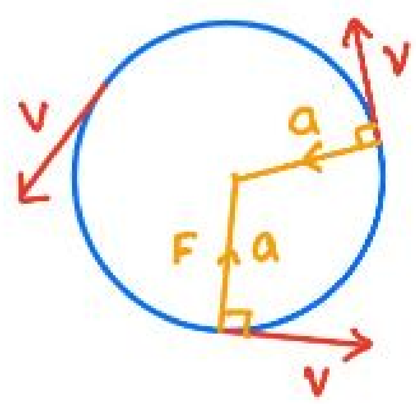
r = 9.10×10^3 m [2]

{*Q.3/June 2000/9243-2*}

Q. 2. (a) An object traveling at a constant speed in a circular path is said to have a centripetal acceleration. Explain, using a diagram,

(i) why there is an acceleration even though the speed is constant,

(ii) the direction of the acceleration. [4]



i) velocity of the object changes per unit time due to change in the direction of motion.

(ii) Towards resultant force i.e. towards center of circular path and perpendicular to the tangential velocity.

(b) A motorway designer plans to have motorists leaving one motorway and joining another by constructing a circular link road, as shown in fig. 2.1.

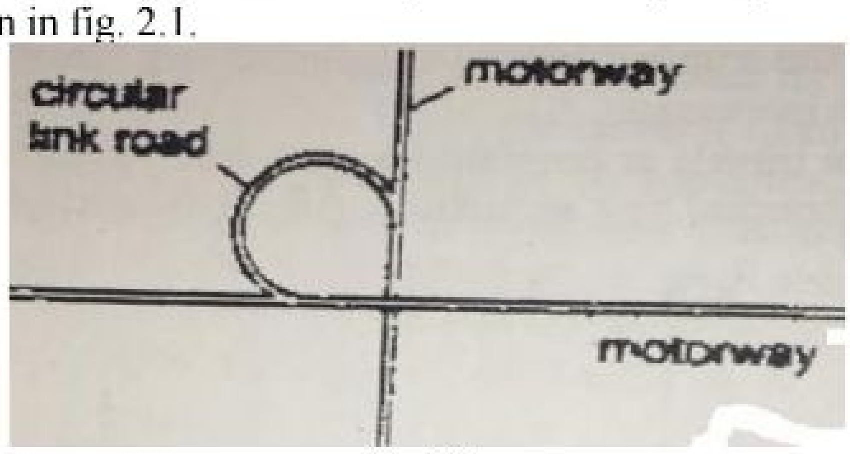


fig. 2.1

In order to use as small an area of land as possible, the designer proposes a speed limit of 25 ms^{-1} for cars on the circular link road.

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(i) Calculate the minimum radius for the circular link road, given that the maximum sideways force between a car and the road is $0.801W$, where W is the weight of a car.

Sideways frictional force provides centripetal force

$$0.801W = \frac{mv^2}{r} \Rightarrow 0.801\cancel{m}g = \frac{\cancel{m}v^2}{r} \Rightarrow r = \frac{v^2}{0.801g}$$

$$r = \frac{(25)^2}{(0.801)(9.81)}$$

radius = ... 79.6 .... m [3]

(ii) Suggest why lorries may have to go at a slower speed than the 25 m s^{-1} limit for cars.

$$0.801W = \frac{mv^2}{r} \Rightarrow \text{Constant} = \frac{m^{\uparrow}\, v^{2\downarrow}}{\text{Constant}}$$

....

$m\uparrow$, $v\downarrow$ so that provided sideways frictional force is able enough to provide centripetal force to lorries. .....[2]

{*Q.3/June 2001/9243-2*}

Q.3. A particle is following a circular path and is observed to have an angular displacement of 10.3°.

(a) Express this angle in radians (rad). Show your working and give your answer to three significant figures.

$$\pi \text{ rad} = 180^\circ$$
$$1^\circ = \frac{\pi}{180} \text{ rad}$$
$$10.3^\circ = \left(\frac{3.14}{180^\circ}\right)(10.3^\circ)$$

angle = ... 0.180 ....rad [2]

(b)(i) Determine tan10.3° to three significant figures.

tan10.3°= .... 0.182 ....

(ii) Hence calculate the percentage error that is made when the angle 10.3°, as measured in radians, is assumed to be equal to tan10.3°.

$$\frac{\Delta\theta}{\theta} \times 100 = \left(\frac{0.182 - 0.180}{0.180}\right)100$$

percentage error = ... 1.11% ....[3]

{*Q.1/Nov 2004/9702-4*}

Q. 4. The orbit of the Earth, mass 6.0×10^{24} kg, may be assumed to be a circle of radius 1.5×10^{11} m with the Sun at its centre, as illustrated in Fig.4.1.

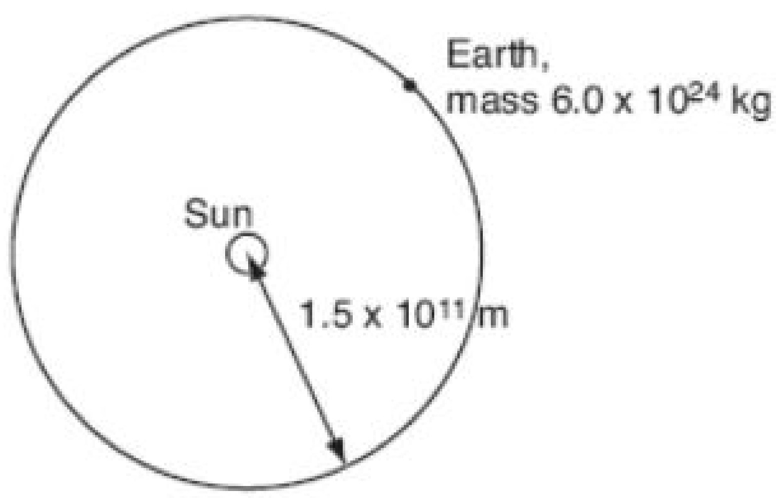

Fig. 4.1

The time taken for one orbit is 3.2×10^7s.

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(a) Calculate

(i) the magnitude of the angular velocity of the Earth about the Sun,

$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{3.2\times10^{7}}$$

angular velocity = 1.96×10^{-7} rads^{-1} [2]

(ii) the magnitude of the centripetal force acting on the Earth.

$$F_c = m r \omega^2$$
$$= (6.0\times10^{24})(1.5\times10^{11})(1.96\times10^{-7})$$
$$= 3.46\times10^{22}$$

force = ...N [2]

(b)(i) State the origin of the centripetal force calculated in (a)(ii)

Gravitational pull of Sun provides centripetal force to Earth. [1]

(ii) Determine the mass of the Sun.

$$F_G = F_c$$

$$\frac{GMm}{r^2} = F_c$$

$$\frac{(6.67\times10^{-11})(M)(6.0\times10^{24})}{(1.5\times10^{11})^2} = 3.46\times10^{22}$$

mass = 1.95×10^{30} kg [3]

{Q.1/June 2005/9702-4}

Q. 5. **(a)** Explain

(i) what is meant by a radian,

....

.... [2]

(ii) why one complete revolution is equivalent to an angular displacement of 2π rad.

....

....[1]

(b) An elastic cord has an unextended length of 13.0 cm. One end of the cord is attached to a fixed point C. A small mass of weight 5.0 N is hung from the free end of the cord. The cord extends to a length of 14.8 cm, as shown in Fig. 5.1.

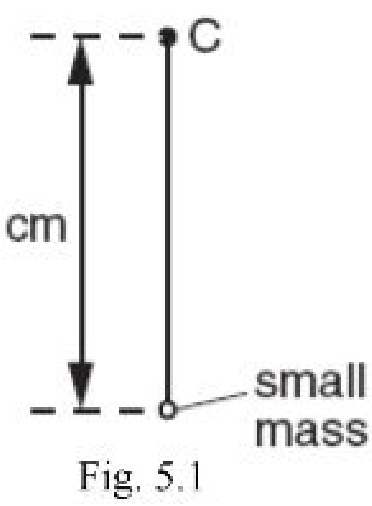

Fig. 5.1

The cord and mass are now made to rotate at constant angular speed ω in a vertical plane about point C. When the cord is vertical and above C, its length is the unextended length of 13.0 cm, as shown in Fig. 5.2.

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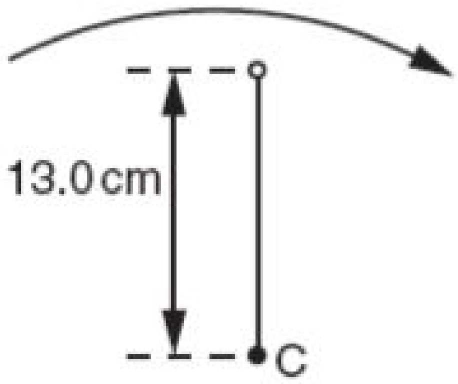

Fig. 5.2

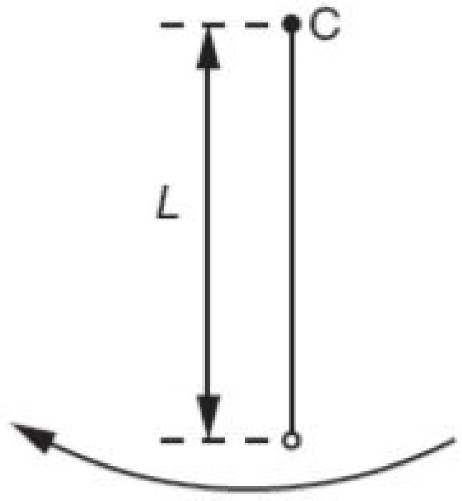

Fig. 5.3

(i) Show that the angular speed ω of the cord and mass is 8.7 rad s^{-1}.

[2]

(ii) The cord and mass rotate so that the cord is vertically below C, as shown in Fig. 5.3.
Calculate the length L of the cord, assuming it obeys Hooke's law.

L = ...cm [4]

{Q.1/Nov.2007/9702-4}

Q. 6.(a) (i) Define the radian.

....

....[2]

(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed, as shown in Fig. 6.1.

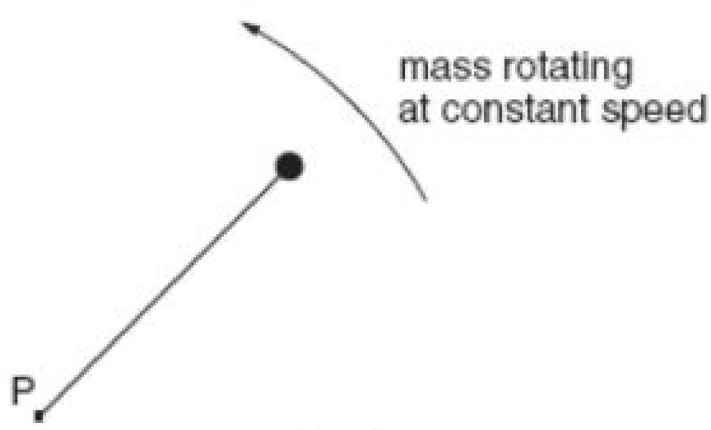

Fig. 6.1

Explain what is meant by the angular speed about point P of the mass.

....

....

....[2]

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(b) A horizontal flat plate is free to rotate about a vertical axis through its centre, as shown in Fig. 6.2.

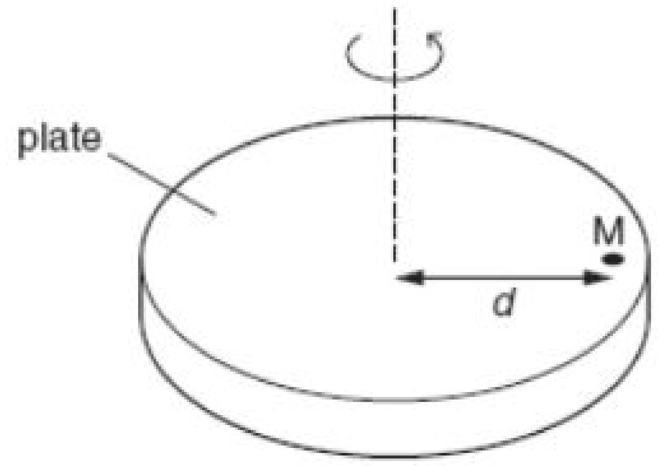

Fig.6.2

A small mass M is placed on the plate, a distance d from the axis of rotation. The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.
The maximum frictional force F between the plate and the mass is given by the expression

$$F = 0.72W,$$

where W is the weight of the mass M. The distance d is 35 cm.
Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate. Explain your working.

number = ... [5]

(c) The plate in (b) is covered, when stationary, with mud. Suggest and explain whether mud near the edge of the plate or near the centre will first leave the plate as the angular speed of the plate is slowly increased.

....

....

....[2]

{Q.1/June 2007/9702-4}

Q. 7.

A large bowl is made from part of a hollow sphere.

A small spherical ball is placed inside the bowl and is given a horizontal speed. The ball follows a horizontal circular path of constant radius, as shown in Fig. 2.1.

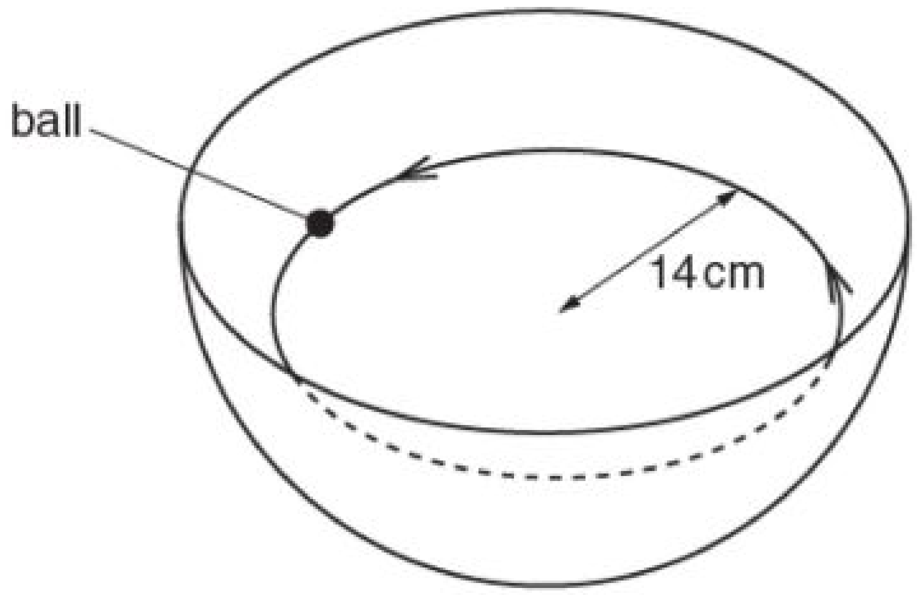

Fig. 2.1

The forces acting on the ball are its weight W and the normal reaction force R of the bowl on the ball, as shown in Fig. 2.2.

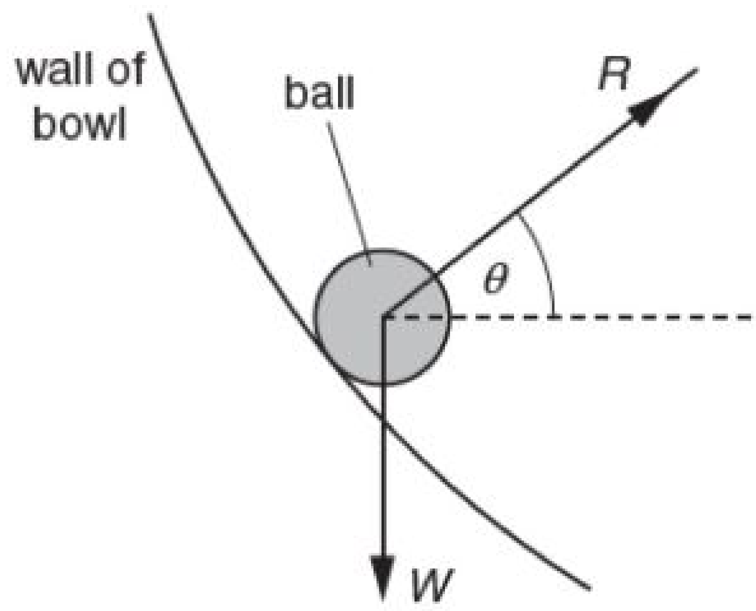

Fig. 2.2

The normal reaction force R is at an angle θ to the horizontal.

(a) **(i)** By resolving the reaction force R into two perpendicular components, show that the resultant force F acting on the ball is given by the expression

$$W = F \tan \theta.$$

[2]

(ii) State the significance of the force F for the motion of the ball in the bowl.

....

.... [1]

(b) The ball moves in a circular path of radius 14 cm. For this radius, the angle θ is 28°.

Calculate the speed of the ball.

speed = ... $m s^{-1}$ [3]

{Q. 2/Nov. 2014/Variant 42}

Q. 8

A telescope gives a clear view of a distant object when the angular displace.. ent between the edges of the object is at least 9.7×10^{-6} rad.

(i) The Moon is approximately 3.8×10^{5} km from Earth.
Estimate the minimum diameter of a circular crater on the Moon's surface that can be seen using the telescope.

diameter = ... km [2]

(ii) Suggest why craters of the same diameter as that calculated in **(i)** but on the surface of Mars are not visible using this telescope.

....

....

.... [2]

{Q. 7/ June 2014/ variant 42}

Section A

Answer **all** the questions in the spaces provided.

For Examiner's Use

9702/41/M/J/10

1 **(a)** Define the *radian*.

$S = r\theta \Rightarrow \theta = \frac{S}{r}$ ie $\theta = 1$ Rad iff $S = r$

Unit of angle swept out by an arc at center with radii such that arc length is equal to the radius of circular path. [2]

(b) A stone of weight 3.0 N is fixed, using glue, to one end P of a rigid rod CP, as shown in Fig. 1.1.

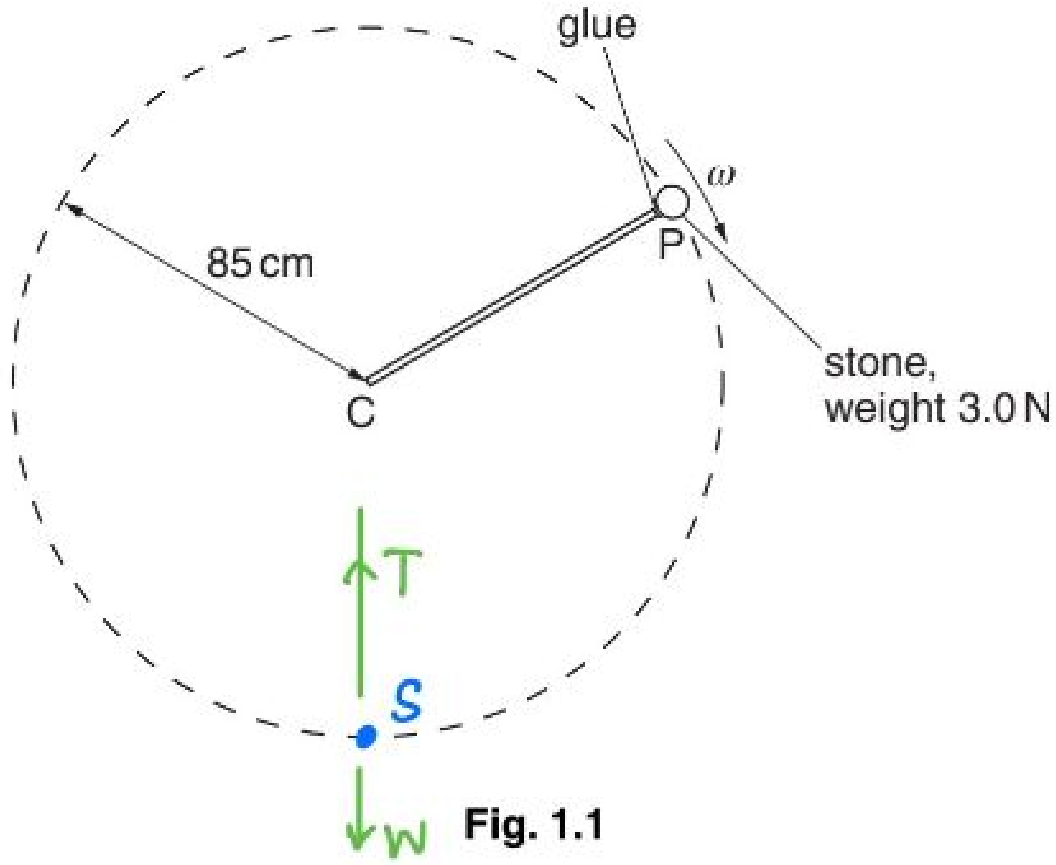

Fig. 1.1

The rod is rotated about end C so that the stone moves in a vertical circle of radius 85 cm.

The angular speed ω of the rod and stone is gradually increased from zero until the glue snaps. The glue fixing the stone snaps when the tension in it is 18 N.

For the position of the stone at which the glue snaps,

(i) on the dotted circle of Fig. 1.1, mark with the letter S the position of the stone, [1]

(ii) calculate the angular speed ω of the stone.

Maximum tension at bottom of circular path

$$T - W = mr\omega^2$$

$$18 - 3.0 = \left(\frac{3.0}{9.81}\right)(85 \times 10^{-2})(\omega^2)$$

$$\omega = \sqrt{\frac{(15)(9.81)}{(3.0)(85 \times 10^{-2})}}$$

angular speed = ... rad s^{-1} [4]

9702/42/O/N/21

Answer **all** the questions in the spaces provided.

1 (a) State what is meant by *centripetal* acceleration.

Comment: Here centripetal is written in italic, so one has to focus on it rather than on acceleration only.

Acceleration towards the center of the circular path and perpendicular to the tangential velocity. [1]

(b) An unpowered toy car moves freely along a smooth track that is initially horizontal. The track contains a vertical circular loop around which the car travels, as shown in Fig. 1.1.

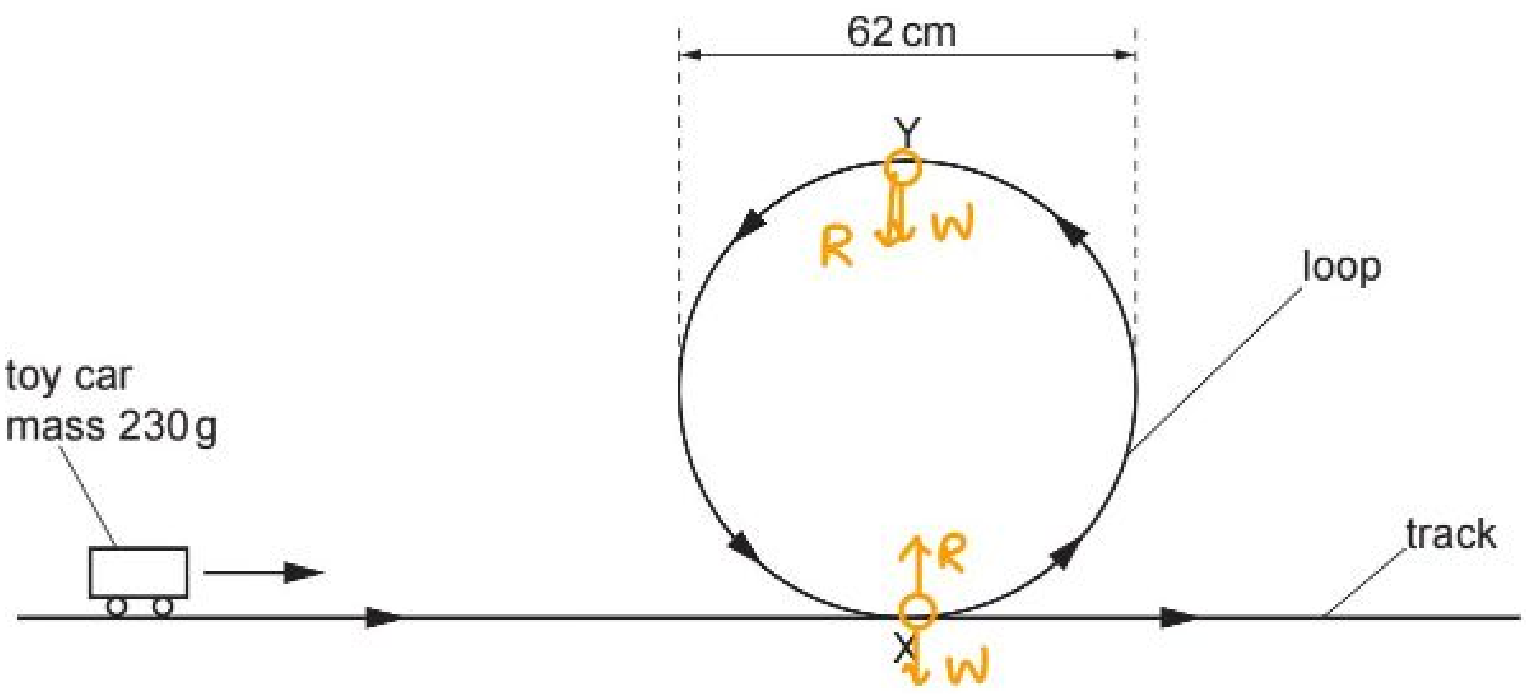

Fig. 1.1

The mass of the car is 230 g and the diameter of the loop is 62 cm. Assume that the resistive forces acting on the car are negligible.

Comment: Speed of car at Y is lesser than X, so centripetal acceleration at Y is lesser than X.

(i) State what happens to the magnitude of the centripetal acceleration of the car as it moves around the loop from X to Y.

$a = \frac{v^2}{r}$, r = constant, $v\downarrow$ so $a\downarrow$

decreases [1]

(ii) Explain, if the car remains in contact with the track, why the centripetal acceleration of the car at point Y must be greater than $9.8\,m\,s^{-2}$.

If $a = g$, car fall freely due to its weight. So $a > g$ to remain in contact with track.

Centripetal force at Y is the resultant of downward weight and normal reactional force of track, while $9.81\,ms^{-2}$ is caused by weight of car only. Hence centripetal accelelation is greater than $9.8\,ms^{-2}$ due to contact force. [2]

(c) The initial speed at which the car in **(b)** moves along the track is $3.8\,\text{m}\,\text{s}^{-1}$.

Determine whether the car is in contact with the track at point Y. Show your working.

By Principle of conservation of energy

K.E. at X = K.E. at Y + G.P.E at Y.

$\frac{1}{2}\cancel{m}V_X^2 = \frac{1}{2}\cancel{m}V_Y^2 + \cancel{m}g\,h_Y$

$\frac{1}{2}(3.8)^2 = \frac{V_Y^2}{2} + (9.81)(62\times10^{-2}) \Rightarrow V_Y = 1.51\,\text{m}\,\text{s}^{-2}$

Acceleration at Y: $a = \frac{V_Y^2}{r} \Rightarrow a = \frac{(1.51)^2}{31\times10^{-2}} = 7.34\,\text{m}\,\text{s}^{-2}$

Since $a < 9.81\,\text{m}\,\text{s}^{-2}$, so car is not in contact with the track.

[3]

(d) Suggest, with a reason but without calculation, whether your conclusion in **(c)** would be different for a car of mass 460 g moving with the same initial speed.

Mass cancels in the equation and centripetal acceleration is independent of mass, so it makes no difference. [1]

[Total: 8]