A LEVEL

12 Motion in a circle → Mechanics !

CIRCULAR MOTION

Learning outcomes

By the end of this topic, you will be able to:

12.1 Kinematics of uniform circular motion

1. define the radian and express angular displacement in radians
2. understand and use the concept of angular speed
3. recall and use $\omega = 2\pi/T$ and $v = r\omega$

12.2 Centripetal acceleration

1. understand that a force of constant magnitude that is always perpendicular to the direction of motion causes centripetal acceleration
2. understand that centripetal acceleration causes circular motion with a constant angular speed
3. recall and use $a = r\omega^2$ and $a = v^2/r$
4. recall and use $F = mr\omega^2$ and $F = mv^2/r$

A2-PHYSICS ⇒ The Introduction

Q) How to define the Kinematics of a CIRCLE

① Linear Displacement :-

- denote :- (s)
- define :- displacement in a straight line
- units :- (m)

② Linear Velocity :- $v = \frac{s}{t}$ *

- denote :- (v)
- define :- rate of change of displacement
- units :- (m/s)

③ Angular Displacement :-

- denote :- (θ)
- define :- the angle made by the object at the centre as it performs CIRCULAR MOTION
- units :- (radians) → defined next pg

④ Angular Velocity :- $\omega = \theta/t$

- define :- Rate of change of angular displacement
- denote :- ω → Omega symbol
- units :- (rad/s)

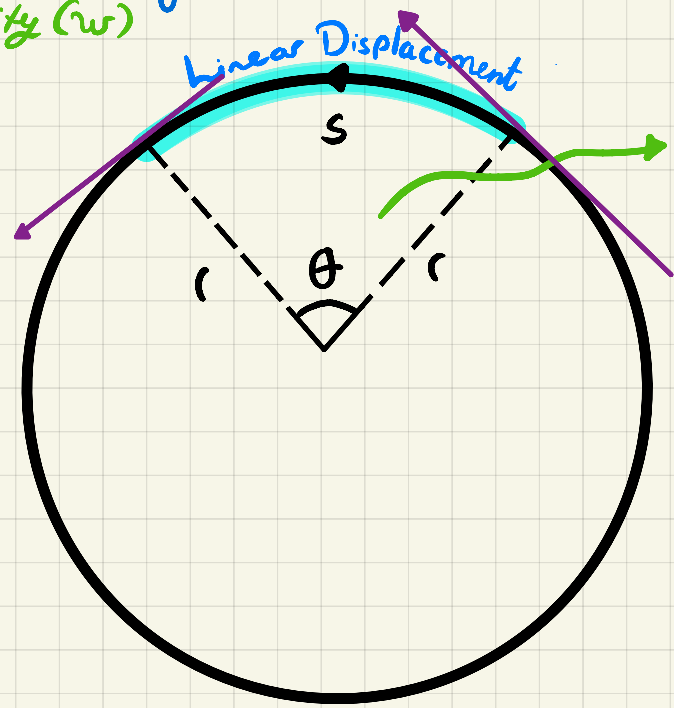
Now, lets visualize the Kinematics of a CIRCLE $\Rightarrow$

* Relationship b/w Linear velocity (v) and angular velocity (ω)

$s = r\theta$

$\frac{s}{t} = \frac{r\theta}{t}$

$\boxed{v = r\omega}$



Angular Displacement

Tangential velocity ~ LATER

Define Radian

•) the angle substended at the centre of the circle

•) where its arc length is equal to its radius

$360 \equiv 2\pi$ rad

* An object completes an entire circle as it moves

$\theta = 2\pi \quad t = T \quad \ldots \quad \omega = \frac{2\pi}{T}$

2π ~ angle for one cycle

T ~ time/period of one cycle

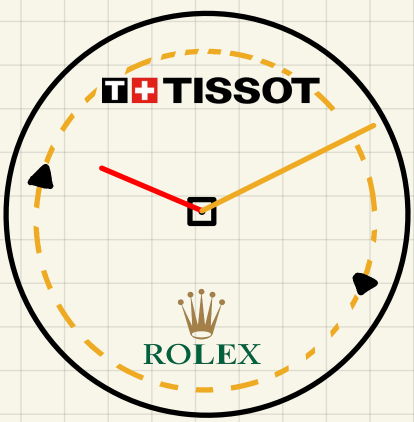
* Uniform angular velocity is achieved if an object subtends equal angles in equal intervals of time

→ Calculating angular velocity (ω) :-

Q) Calculate angular velocity for the minute hand of a clock?

$\omega = \frac{\theta}{t}$; $\theta \to 2\pi$, $t \to 60\text{min}\ (1\text{hr})$; $\omega = \frac{2\pi}{60 \times 60}$ rad/s

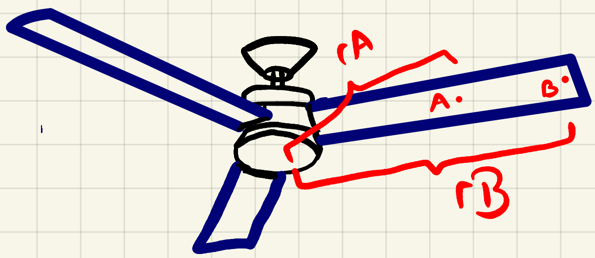
1.7×10^{-3} rad/s



The Formula "$v = r\omega$" Applied!

★ $v \propto r$, if ω constant ★ $v \propto \omega$, if r constant

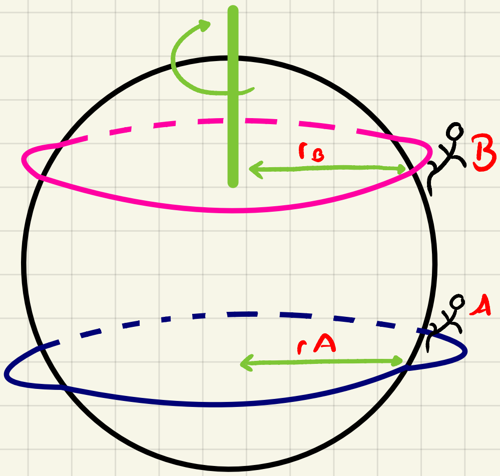
Examples: **Blades of a fan.**



When the blade rotates, both points A & B will have the same angular velocity, HOWEVER

$\therefore$ $r_B > r_A$ → linear velocity of both points will be Different!

! $V_B > V_A$!



Earth

ω = Same

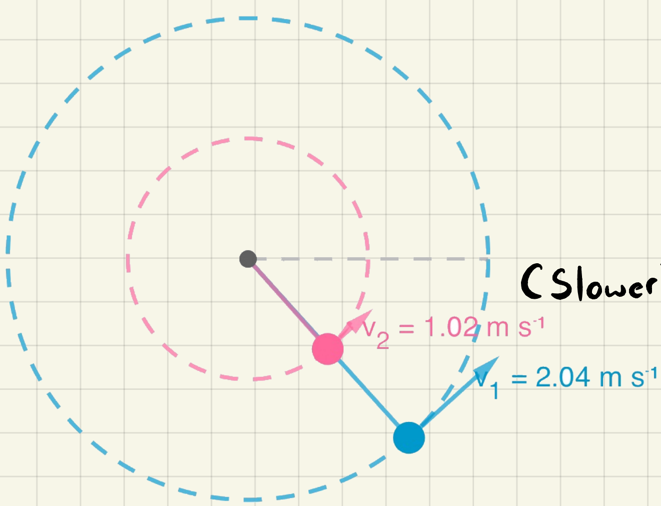
$r_A > r_B$

$V_A > V_B$

Q) What about linear velocity?

It is possible for different points to have the same angular velocity, however depending on their position (value of r), their linear velocity may or may not be the same. NICE!

$\uparrow V = \uparrow r \, \omega$ → constant



(Slower)

(Faster)

The Case of CENTRIPETAL Force / Acceleration

→ **Key points** :

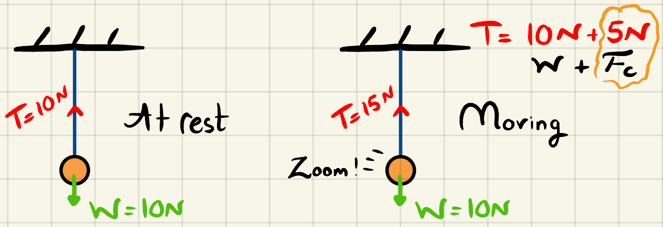
★ The Force that tends to rotate an object

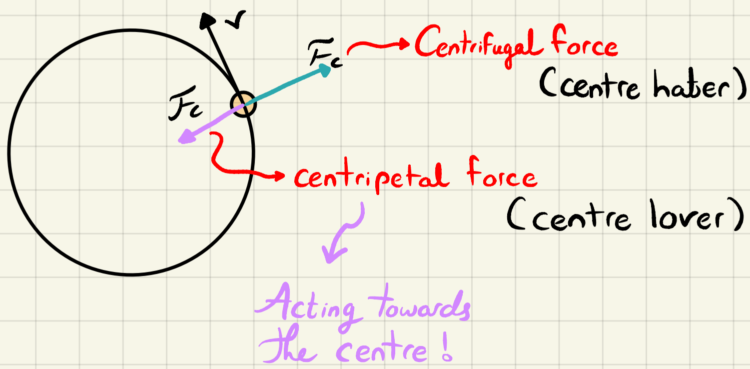
★ The Force that reassures a body will move in a circle

When there's a Yin, there is always a Yang !

Following Newton's 3rd Law … the Equal & Opposite Force

CENTRIFUGAL → ★ moves the object out of the circular orbit.





The extra force is the Centripetal force

List of Forces acting as Centripetal Force →

(i) Planetary motion / Satellite → Gravitational force
(ii) Car making a turn about a corner → Friction force
(iii) Stone tied to a thread → Tension force
(iv) Water in a bucket in a vertical circle → Weight + Contact Force

Centripetal Acceleration → Rate of change of Linear velocity about a point

GENERAL FORMULA = $F_c = \dfrac{mv^2}{r}$ or $mr\omega^2$

F is centripetal force in N

M is mass of the body in kg

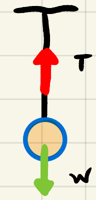
V is linear velocity in m/s

r is radius of the path in m

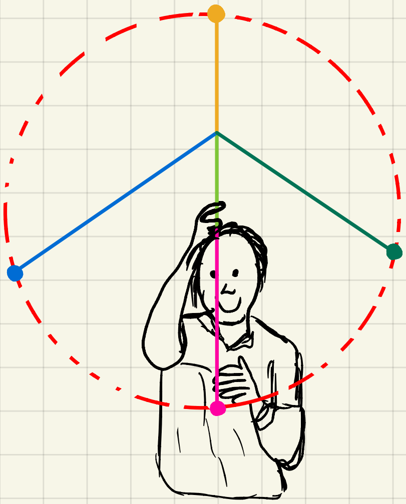
Conceptualize → Vertical Circular Motion

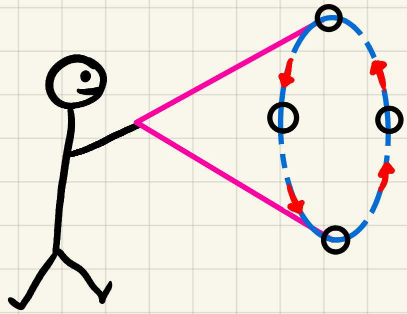
When the altitude of a body is changing with its position

i) to depict this notion → A body tied to an inextensible string the forces acting on it are :-



Tension (i)
Weight (ii)



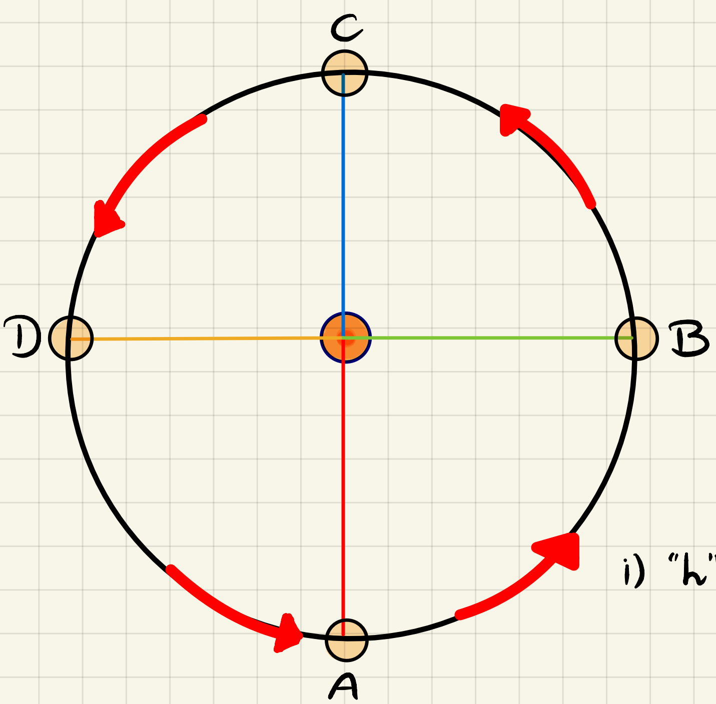


☆ Circular Path → Altitude changing !

☆ GPE changes to KE

Keypoints :-

1. GPE changes
2. Speed / K.E also changes
3. $F_c = mv^2/r$, F_c also changes
4. Tension at every point also changes



Lets Analyze these Facts.

i) "h" decreases from C to D & D to A

$\downarrow GPE = m \times g \times h \downarrow$

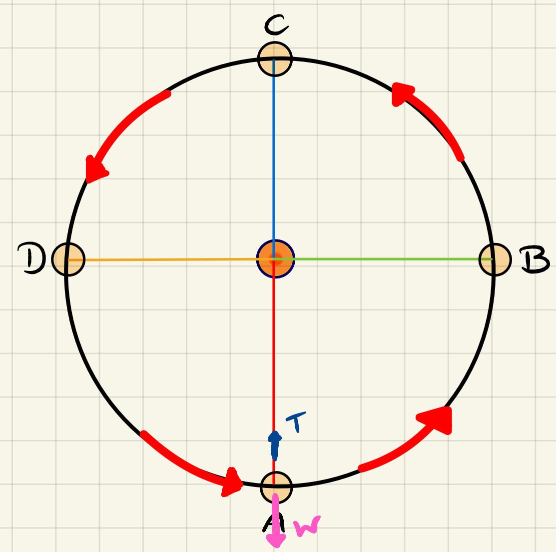
$\uparrow KE = 1/2 \times m \times v^2 \uparrow$

ii) h increases from A to B & B to C

$\uparrow GPE = m \times g \times h \uparrow$

$\downarrow KE = 1/2 \times m \times v^2 \downarrow$

THE POSITIONS

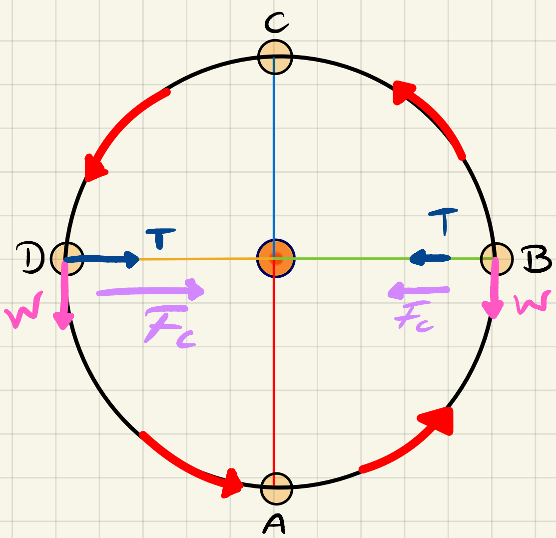


Position "A" : Lowest Position

Resultant force $\mathcal{F}_c$ acts towards the centre

$$\text{Resultant} = T - W$$
$$\mathcal{F}_c = T - W$$

$$T = \mathcal{F}_c + W \quad \text{or} \quad T = \frac{mv^2}{r} + W$$

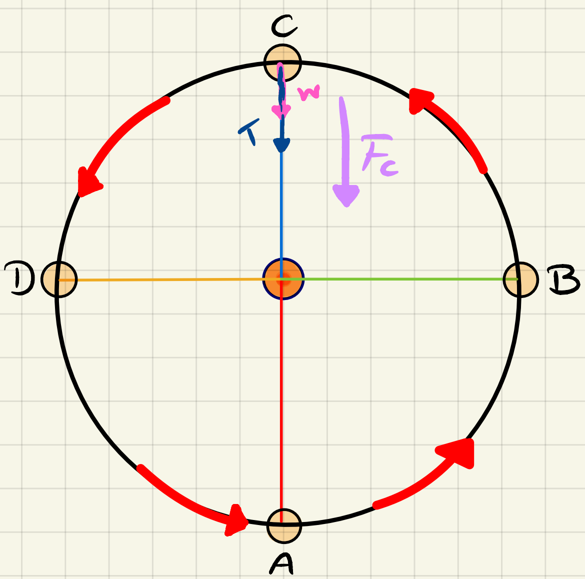


Position "B" & Position "D"

Intermediate !

W is neither → acting along T
→ acting against T

$$\mathcal{F}_c = T \quad \text{or} \quad T = \frac{mv^2}{r}$$



Position "C" : Highest position

W & T acting in the same direction towards the centre

$$\text{Resultant} = W + T$$
$$\mathcal{F}_c = W + T$$

$$T = \mathcal{F}_c - W \quad \text{or}$$
$$T = \frac{mv^2}{r} - W$$

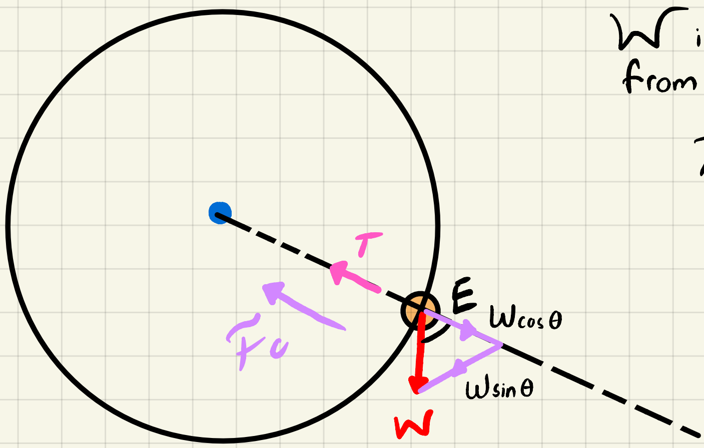
Conclusions For Clarity !

Position A	Position B	Position C	Position D
$\frac{mv^2}{r} + W$	$\frac{mv^2}{r}$	$\frac{mv^2}{r} - W$	$\frac{mv^2}{r}$
T max		T min	

T max → string most likely to BREAK!

Graph next Page!

What if → Body at some other position



W is neither acting towards nor away from the centre, it is acting at an angle...

Thus, It can be Resolved !

$$F_c = T - W\cos\theta$$

or

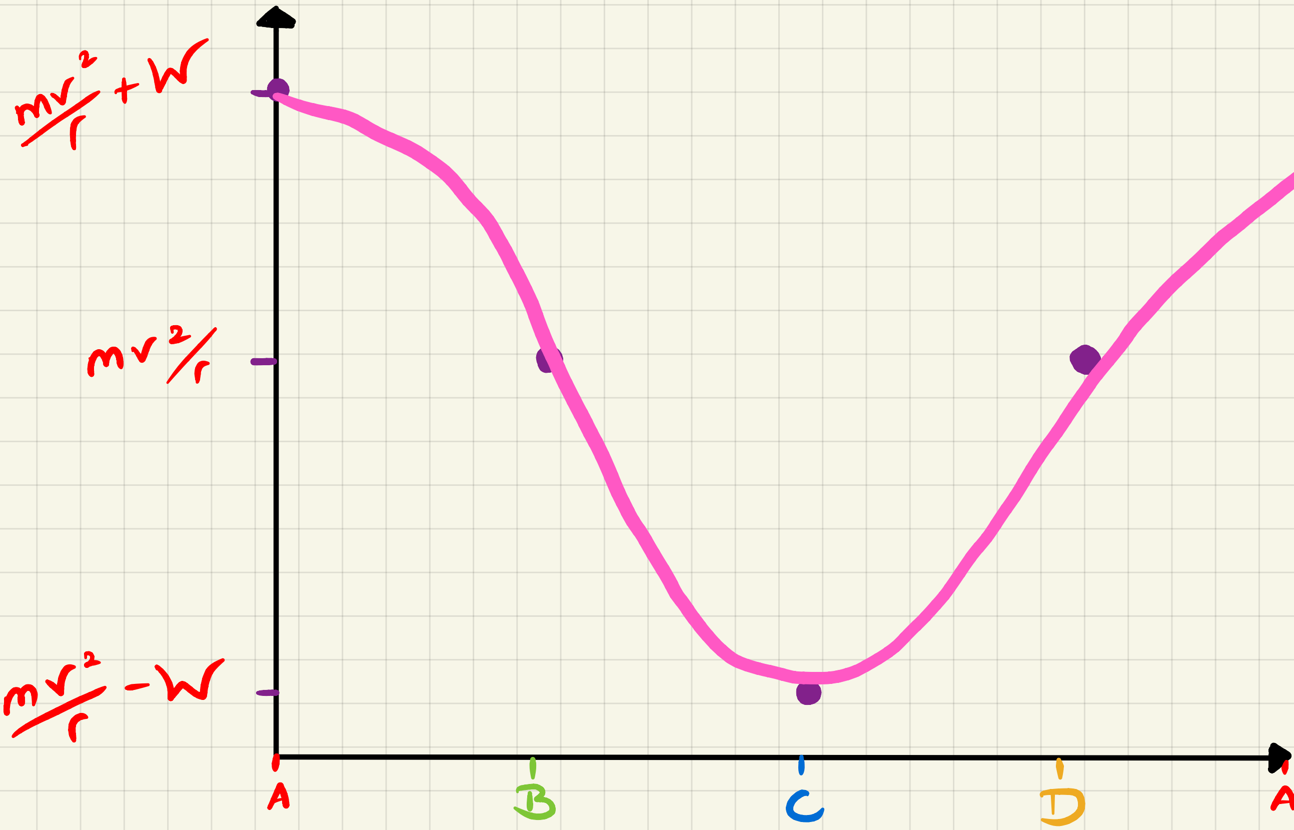
$$T = F_c + W\cos\theta$$

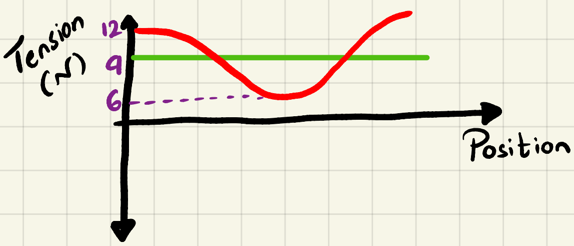
$$\boxed{T = \frac{mv^2}{r} + W\cos\theta}$$

↳ General Expression → Function of $\cos\theta$, HENCE

Spoiler Alert → Graph is Cosine Graph!

GRAPHICAL Questionnaire {4!





(i) using info. from the graph & formulas Calculate ...:

(a) mass "m" of the object

$T_{max} = 12$ {lowest pt}

$12 = \cancel{\frac{mv^2}{r}} + mg$ — (1)

$T_{min} = 6$ {Highest pt}

$-6 = -\cancel{\frac{mv^2}{r}} \pm mg$ — (2)

$6 = 2mg$ $\therefore m = 0.3\,kg$

$g \approx 10$

b) If $r = 0.6m$. Cal Linear velocity (v)

use Eq 1 or 2

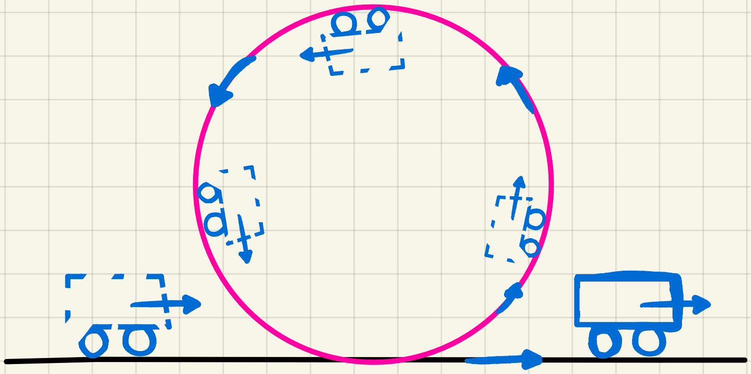
$12 = \frac{mv^2}{r} + mg$

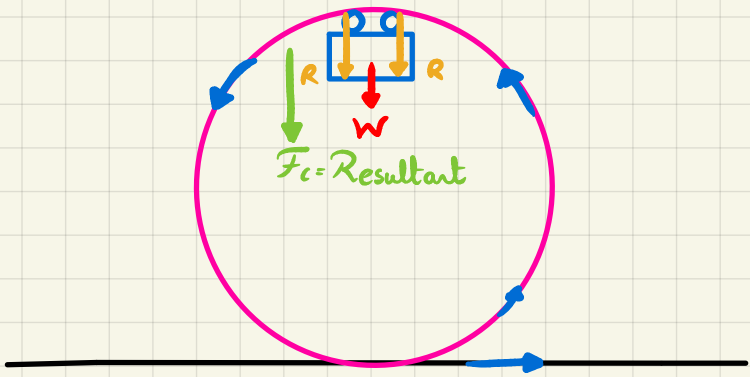
$12 = \frac{0.3 \times v^2}{0.6} + 0.3 \times 10$

$v = 4.24\,m/s$

Vertical Motion of a Trolley on a Circular Track!

Keypoints :-
- ☆ No strings attached "Literally" ⟶ No Tension !
- ☆ It all happens on a track ! → No Friction, Only Contact !



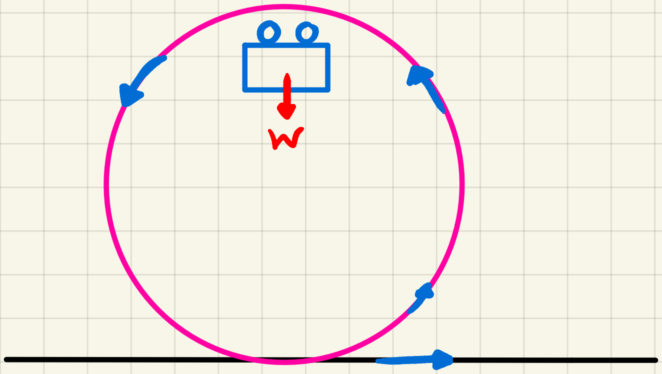


Forces acting on trolley are ⟶ (i) Weight "W" (ii) Normal / Contact "R"

Both of these forces are acting towards the centre

$$\boxed{F_c = W + R}$$ → If in contact with surface !

WHAT IF ⟶ Trolley loses it's contact with surface —!



$$F_c = W + R \rightarrow \text{Zero}$$

$$F_c = W$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m} \times g$$

$$v^2 = rg$$

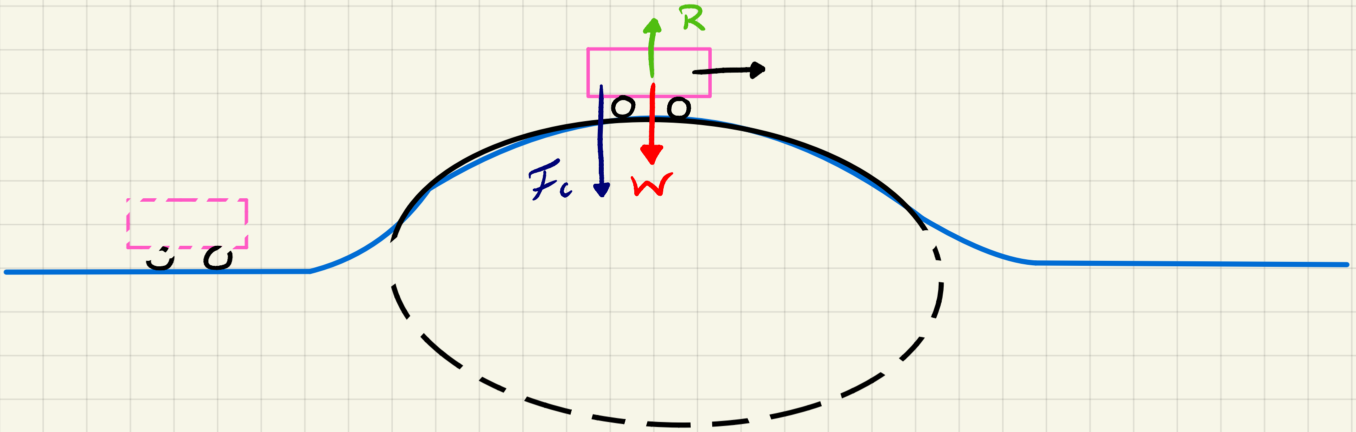
$$\boxed{v = \sqrt{rg}}$$

In order to maintain contact with the surface, the speed must be more than $\sqrt{rg}$

$$\boxed{V_{safe} > \sqrt{rg}}$$

The Hump-Back Bridge

Consists of a spherical path → Circular !



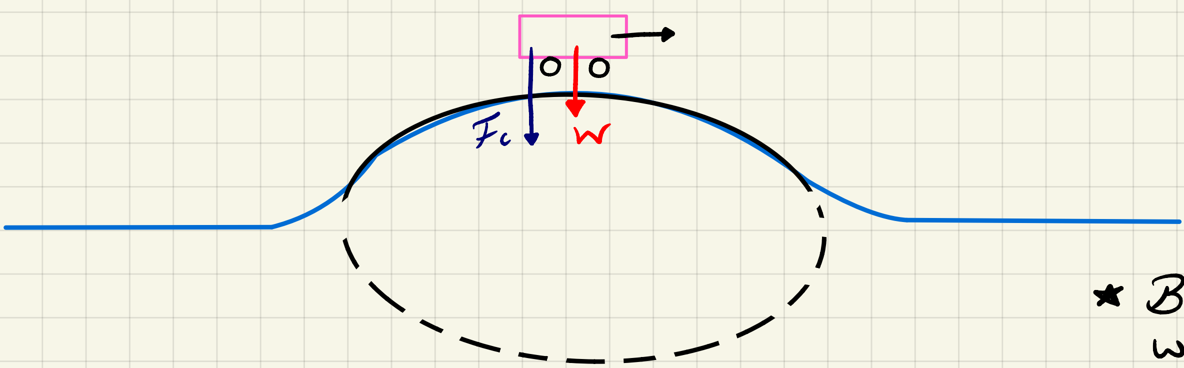
At the top, Forces acting on the body are:-

(i) Weight (ii) Contact (iii) F_c (Resultant)

★ F_c acting towards the centre, hence $W > R$

Resultant $= W - R$
$F_c = W - R$

What if → Body loses it's contact with surface —!



★ Body has to follow path with smaller radius in order to remain in contact with the surface.

$$F_c = W - R \longrightarrow F_c = W - 0$$

$$\frac{\cancel{m}v^2}{r} = \cancel{m} \times g \rightarrow v^2 = rg \rightarrow v = \sqrt{rg} \rightarrow \text{not safe}$$

$$v < \sqrt{rg} \rightarrow \text{safe !}$$