9702 C12 Motion in a circle

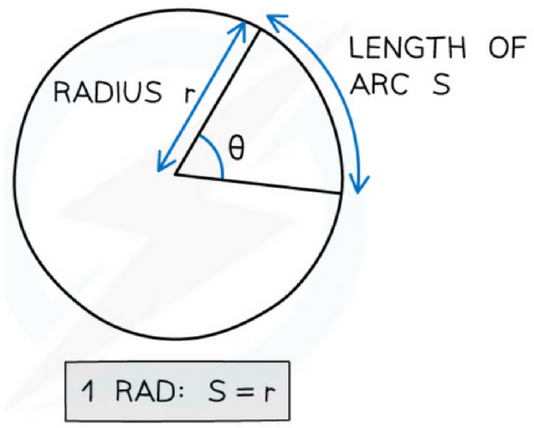
Radians & Angular Displacement

-->we use radians rather than degree in circular motion equation (as it is more convenient)

radian(rad) = angle subtended at the centre of circle when arc length = radius

angular displacement(θ) = change in angles, in radians, of a body as it rotates around a circle

$$\Delta\theta = \frac{\Delta s}{r}$$



Where:

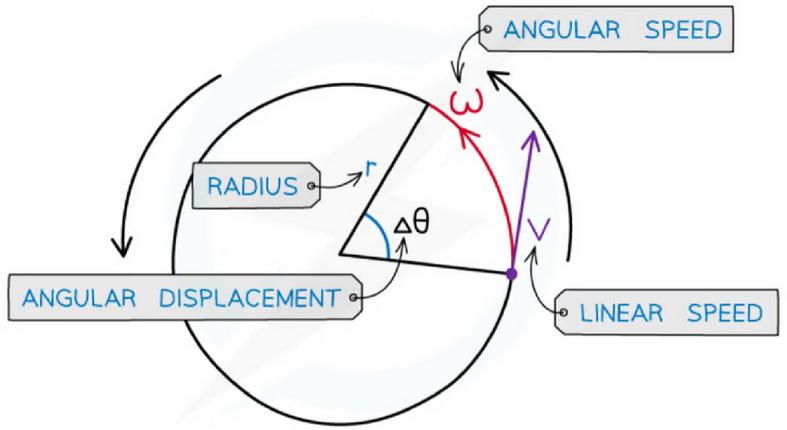
- $\Delta\theta$ = angular displacement, or angle of rotation (radians)
- s = length of the arc, or the distance travelled around the circle (m)
- r = radius of the circle (m)

Angular Speed

Any object travelling in a uniform circular motion at the same speed travels with a **constantly changing velocity**

-->this is because it is constantly changing direction(velocity is vector), and is therefore accelerating

The **angular speed(ω)** of a body in circular motion = **the rate of change in angular displacement with respect to time**



$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T} = 2\pi f$$

Where:

- $\Delta\theta$ = change in angular displacement (radians)
- Δt = time interval (s)
- T = the time period (s)
- f = frequency (Hz)

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{v}{r}$$

$$v = \omega r$$

Where:

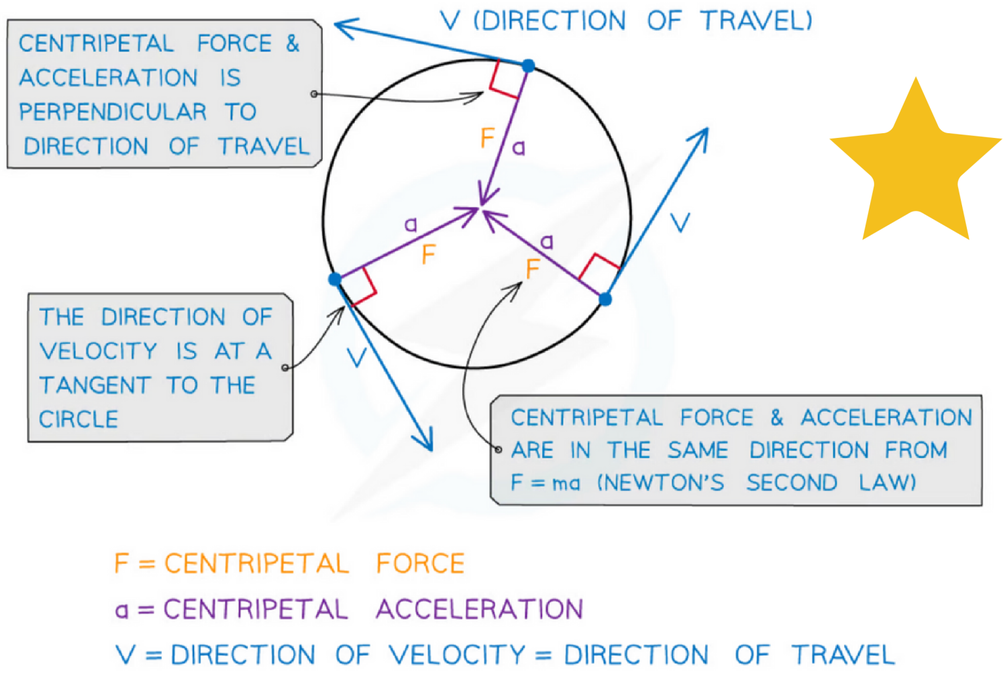
- v is the linear speed ($m\,s^{-1}$)
- r is the radius of orbit (m)

This equation tells us"
-the greater the rotation angle in a given amount of time, the greater the angular velocty
-an object rotating further from the centre of the circle(larger r) moves with a smallar angular velocity(smaller ω)

Centripetal Acceleration

An object in uniform circular motion is **continuously changing direction**, and **therefore is constantly changing velocity**. This means the **object must therefore be accelerating**.

This is called the **centripetal acceleration and is perpendicular to the direction of the linear speed**.
(**centripetal** means it **acts towards the centre** of the circular path)



Centripetal acceleration is caused by a constant magnitude of centripetal force that acts perpendicular towards the centre. So the centripetal acceleration and force act in the same direction

Centripetal acceleration and force

centripetal acceleration = acceleration of an object towards the centre of a circle when an object is in motion(rotating) around a circle at a constant speed

$$a = \frac{v^2}{r}$$

From $a = \frac{v^2}{r}$ and $v = r\omega$

$$a = \frac{(r\omega)^2}{r}$$

$$\mathbf{a = r\omega^2}$$

$$a = \frac{v^2}{\left(\frac{v}{\omega}\right)}$$

$$a = v\omega$$

Where:

- a = centripetal acceleration ($m\,s^{-2}$)
- v = linear speed ($m\,s^{-1}$)
- r = radius of the circular orbit (m)

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centripetal force (F) = the resultant force towards the centre of the circle required to keep a body in uniform circular motion. It is always directed towards the centre of the body's rotation

$$F = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$