PROSPERITY ACADEMY

A2 PHYSCIS 9702

Crash Course

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CIRCULAR MOTION

COMPLETE NOTES

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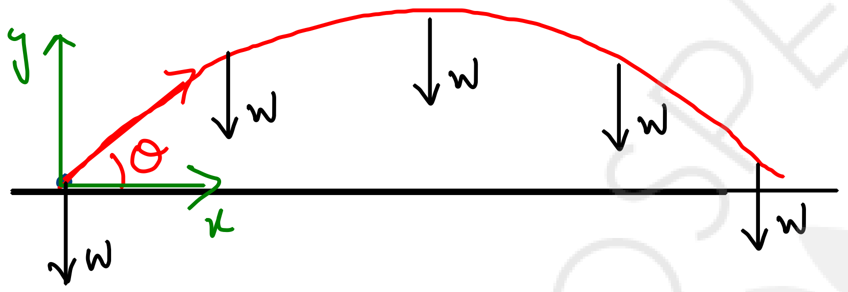
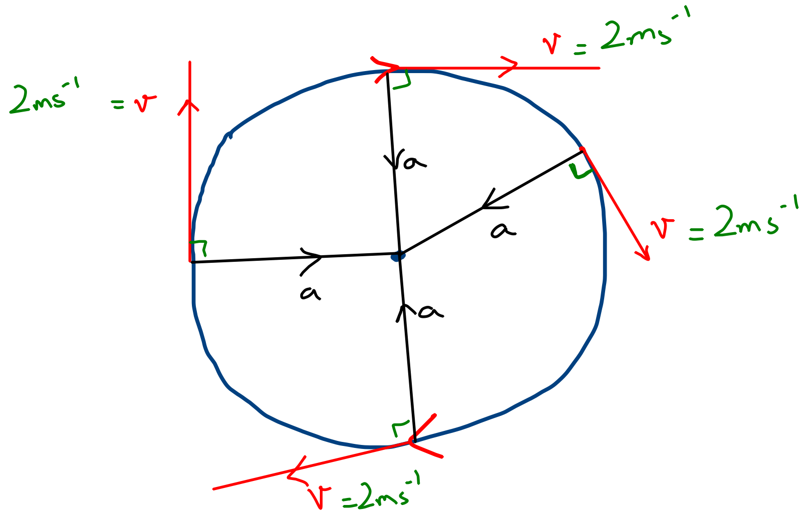
Linear motion:- motion along a line:- e.g. object falling, car driving

Projectile motion:- motion in x & y directions . Force/acceleration is always in one direction only.

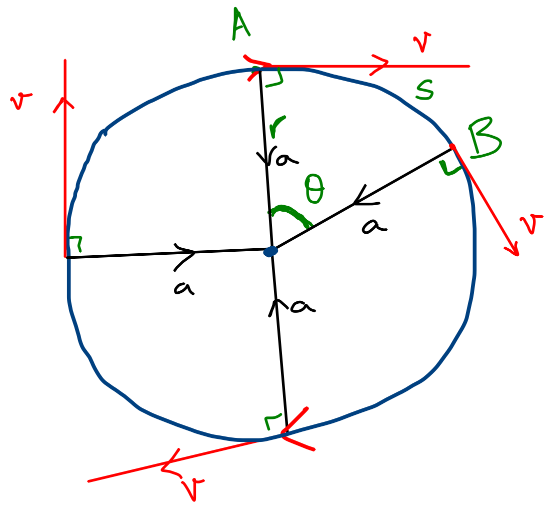


Circular motion:- . motion in one plane . Force/acceleration is always normal/perpendicular to direction of motion.



Uniform Circular motion:- magnitude of velocity/speed is same

Non Uniform Circular motion:- " " " " is not same



Angular displacement (θ) :- Angle swept in circular motion in travelling between 2 points. (θ is in radians)

$\boxed{s = r\theta}$

$\pi : 180°$
$2\pi : 360°$

$\frac{\theta}{360°} \times 2\pi r = s$

$\frac{\theta}{\cancel{2\pi}} \times \cancel{2\pi} r \Rightarrow r\theta = s$

1 radian

(a) (i) Define the *radian.*

the radian is the angle subtended at the centre of the circle where arc length = radius

[2]

$s = r\theta \Rightarrow s = r(1)$
$s = r$

tangential velocity (v) :- Instantaneous velocity

Angular velocity (ω) :- Rate of change of angular displacement

① $\boxed{\omega = \frac{d\theta}{dt}}$ ② $\omega = \frac{2\pi}{T}$

time period: the time to complete 1 revolution around a circle

Q. An object covers $\pi/3$ rad in 2 seconds. What is it's ω?

$$\omega = \frac{d\theta}{dt} = \frac{\pi/3}{2} = \pi/6 \text{ rad s}^{-1}$$

Q. What is it's time period?

$\omega = \pi/6 = \frac{2\pi}{T} \Rightarrow T = 2\cancel{\pi} \times \frac{6}{\cancel{\pi}} = 12$

Centripetal acceleration:- Net acceleration directed towards the centre of the circle which keeps the object in circular motion.

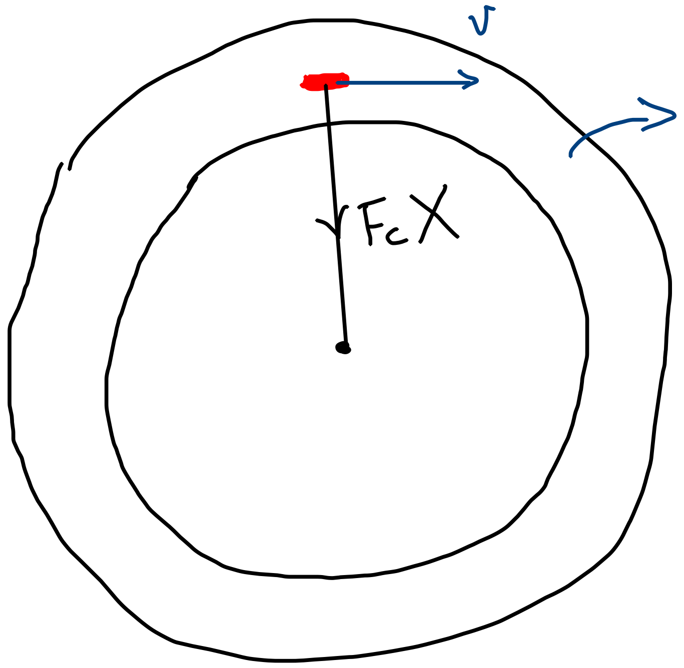
$$a_c = \frac{v^2}{r} \quad \text{or} \quad a_c = r\omega^2$$

Centripetal force :- Net force directed towards the centre of the circle which keeps the object in circular motion.

$$F_c = m a_c$$

$$F_c = \frac{mv^2}{r} \quad \text{or} \quad F_c = mr\omega^2$$

①

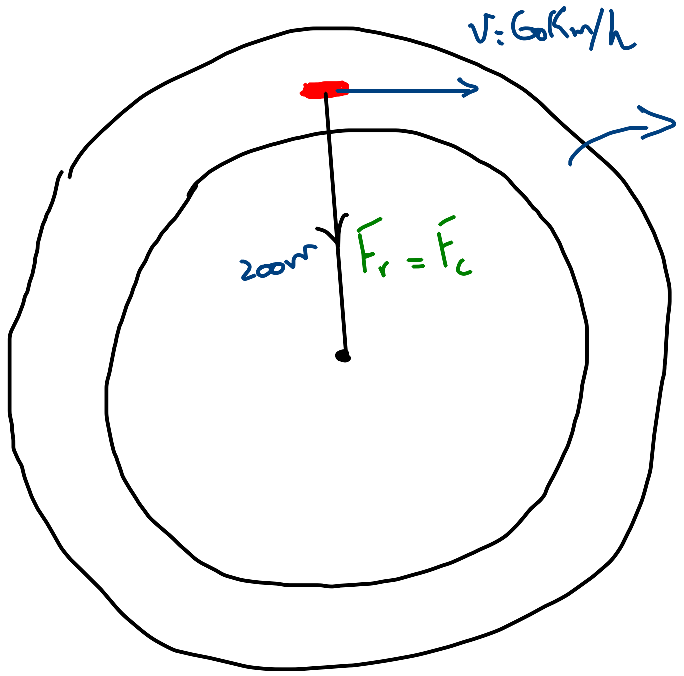


Neglect air resistance

→ smooth road

no friction

②



Neglect air resistance

→ rough road

Q. A car of mass 800Kg is travelling at 60 Km/h on a horizontal track of radius 200m.
What is the total friction force required to keep the object in circular motion?

$$\frac{60 \times 1000}{1 \times 60 \times 60} = \frac{50}{3} \, ms^{-1}$$

$$F_r = F_c = \frac{mv^2}{r}$$

$$\frac{\cancel{800}^{4} \times \left(\frac{50}{3}\right)^2}{\cancel{200}_{1}} = 1111.11 \, N$$

$$= 1000 \, N$$

$\downarrow F_r = \uparrow\downarrow F_c = \frac{mv^2 \downarrow\downarrow}{r \downarrow}$

For smaller speeds, the radius of circular motion gets / can be smaller.

$\uparrow F_r = \downarrow\uparrow F_c = \frac{mv^2 \uparrow\uparrow}{r \uparrow}$

For larger speeds, the radius of circular motion gets / can be larger.

Q. A car of mass 1200 kg is moving along a circular track of radius 150m. The maximum friction offered by the road is 2000N. Deduce the maximum speed the car can travel at without skidding.

$$F_r = F_c = \frac{mv^2}{r}$$

$$2000 = \frac{(1200)v^2}{150} \Rightarrow v = 15.8\,ms^{-1}$$

w21 qp41 q1

Answer **all** the questions in the spaces provided.

1 **(a)** With reference to velocity and acceleration, describe uniform circular motion.

circular motion in which the speed is constant and the acceleration is always normal to direction of motion

.... [2]

(b) Two cars are moving around a horizontal circular track. One car follows path X and the other follows path Y, as shown in Fig. 1.1.

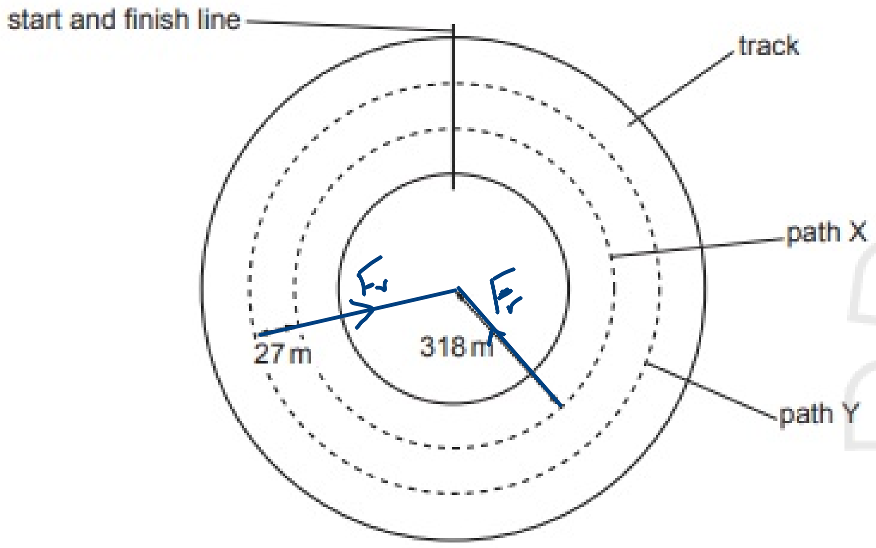


Fig. 1.1 (not to scale)

The radius of path X is 318 m. Path Y is parallel to, and 27 m outside, path X. Both cars have mass 790 kg. The maximum lateral (sideways) friction force *F* that the cars can experience without sliding is the same for both cars.

(i) The maximum speed at which the car on path X can move around the track without sliding is 94 m s^{-1}.

Calculate *F*.

$$F_r = \frac{mv^2}{r} = \frac{(790)(94)^2}{318}$$

$$= 21951$$

F = 22000 N [2]

(ii) Both cars move around the track. Each car has the maximum speed at which it can move without sliding.

Complete Table 1.1, by placing one tick in each row, to indicate how the quantities indicated for the car on path Y compare with the car on path X.

Table 1.1

	Y less than X	Y same as X	Y greater than X
centripetal acceleration		✓	
maximum speed			✓
time taken for one lap of the track (T)			✓

[3]

[Total: 7]

$$F_r = F_c = m a_c$$

const

$$\boxed{F_r} = F_c = \frac{mv^2 \downarrow}{r \downarrow}$$

const

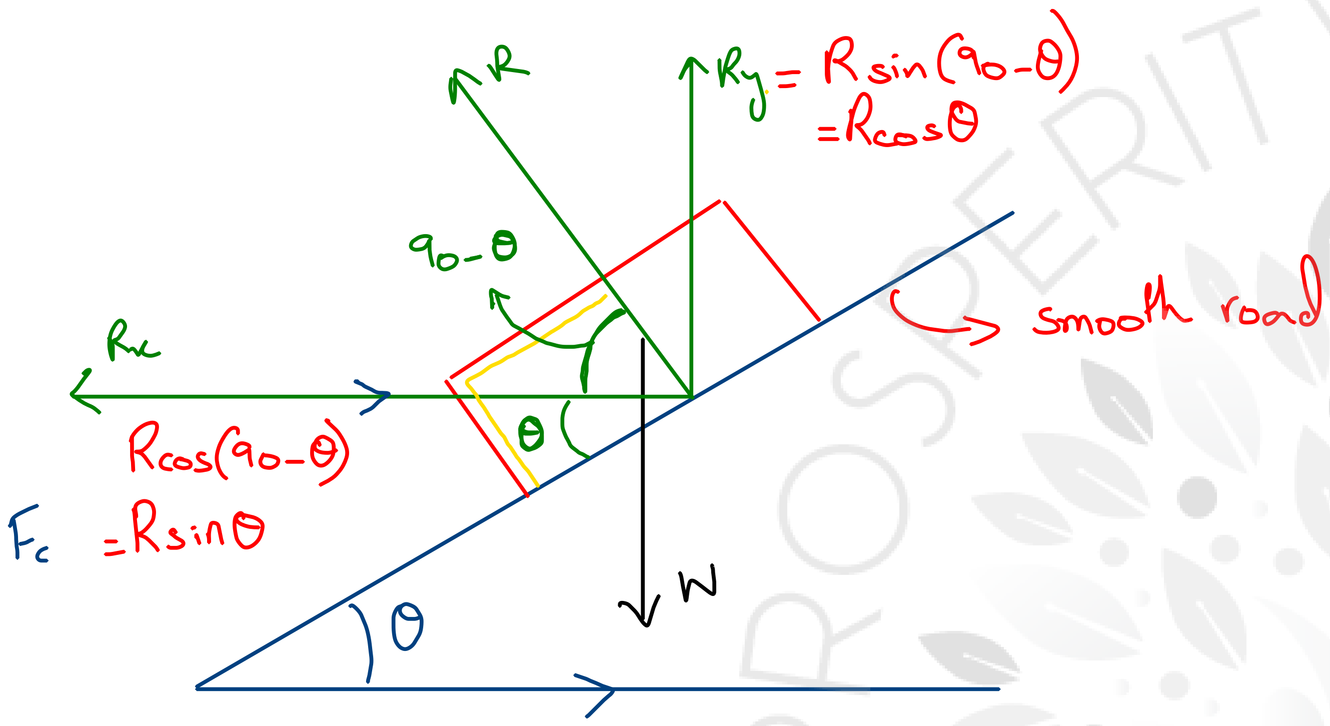
$$\boxed{F_r} = F_c = \frac{mv^2 \uparrow}{r \uparrow}$$

$$F_c = \overset{\text{const}}{m}\, r_y \uparrow \omega_y^2 \downarrow \quad = \quad F_c = \overset{\text{const}}{m}\, r_x \downarrow \omega_x^2 \uparrow$$

$$\downarrow \omega_y = \frac{2\pi}{T_y \uparrow} \qquad \uparrow \omega_x = \frac{2\pi}{T_x \downarrow}$$

Smooth Embanked/Inclined plane :-

$$\boxed{\begin{aligned} \sin(90-\theta) &= \cos\theta \\ \cos(90-\theta) &= \sin\theta \end{aligned}}$$



Q. A car of mass 800 kg is travelling on a smooth circular road embanked at an angle of 10°. If the car is driving at 40 km/h, what is the radius of its circular motion?

Fr ✗

$$R\sin\theta = F_c = \frac{mv^2}{r}$$

$$R\sin(10) = \frac{(800)\left(\frac{40\times1000}{3600}\right)^2}{r}$$

$$r = \frac{800\times\left(\frac{40\times1000}{3600}\right)^2}{7969\sin(10)} = 71.37\text{ m} \approx \boxed{70\text{ m}}$$

Equilibrium

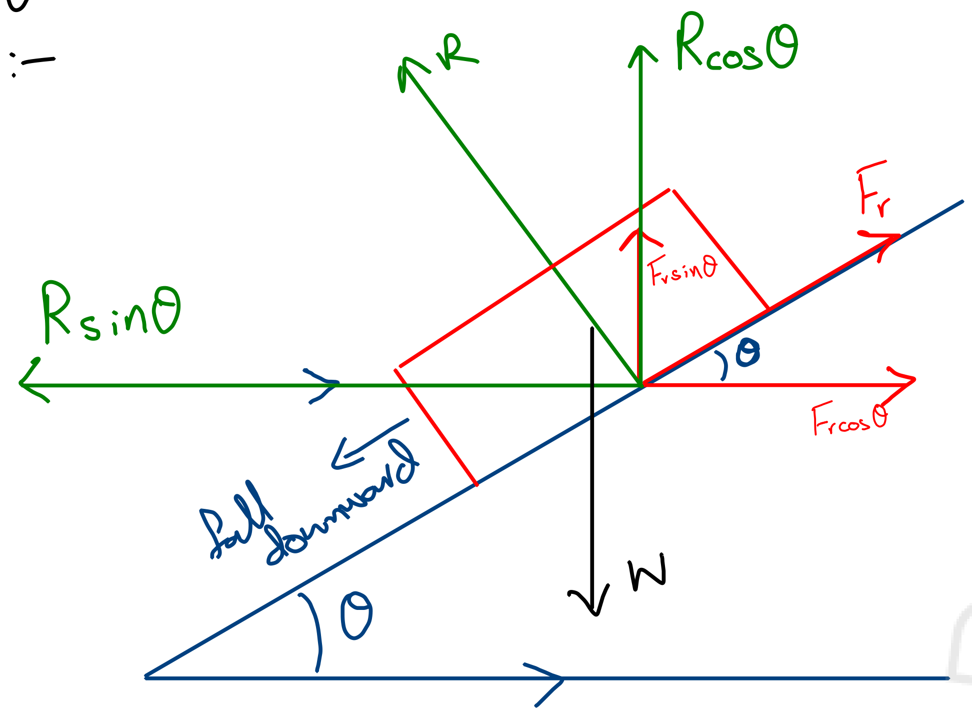
$$R\cos\theta = W$$

$$R\cos(10) = (800)(9.81)$$

$$R = 7969\text{ N}$$

Rough Inclined / Embanked plane:-

Basic:-

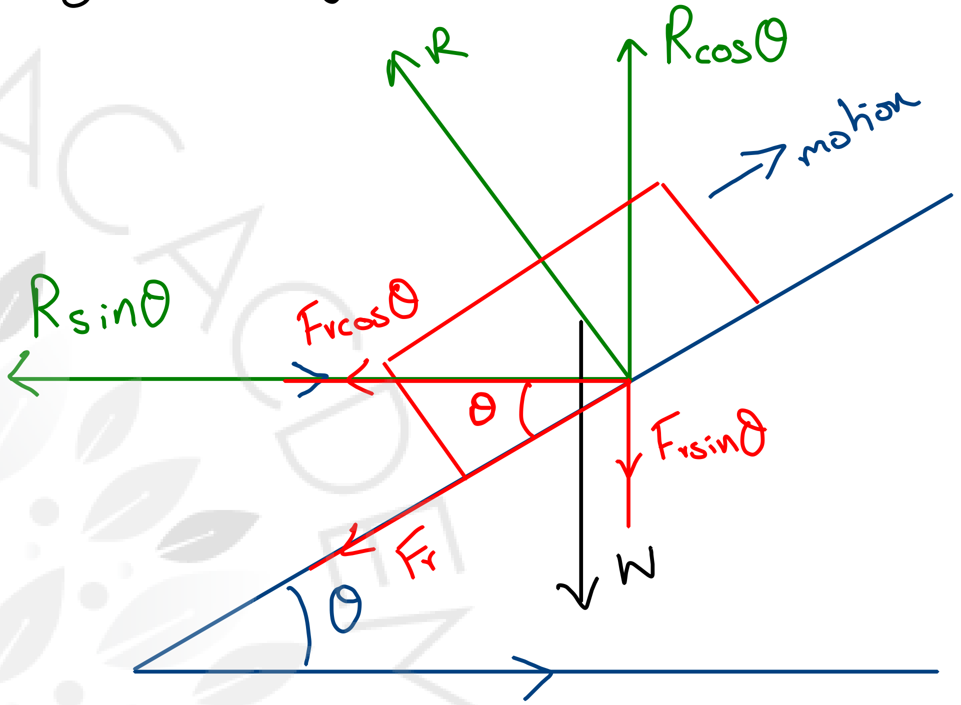


$$F_c = R\sin\theta - F_r\cos\theta$$

$$R\cos\theta + F_r\sin\theta = W$$

Increasing velocity:-

$$\downarrow\uparrow F_c = \frac{mv^2\uparrow}{r\uparrow}$$

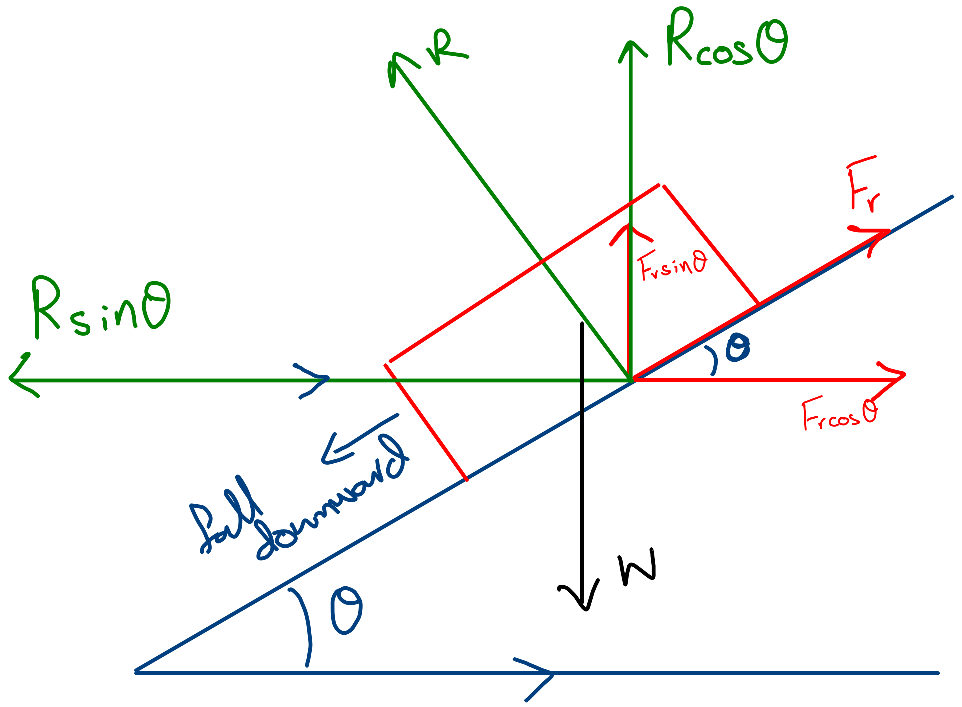


$$F_c = R\sin\theta + F_r\cos\theta$$

$$R\cos\theta = W + F_r\sin\theta$$

Decreasing Velocity:-

$$\uparrow\downarrow F_c = \frac{mv^2\downarrow}{r\downarrow}$$



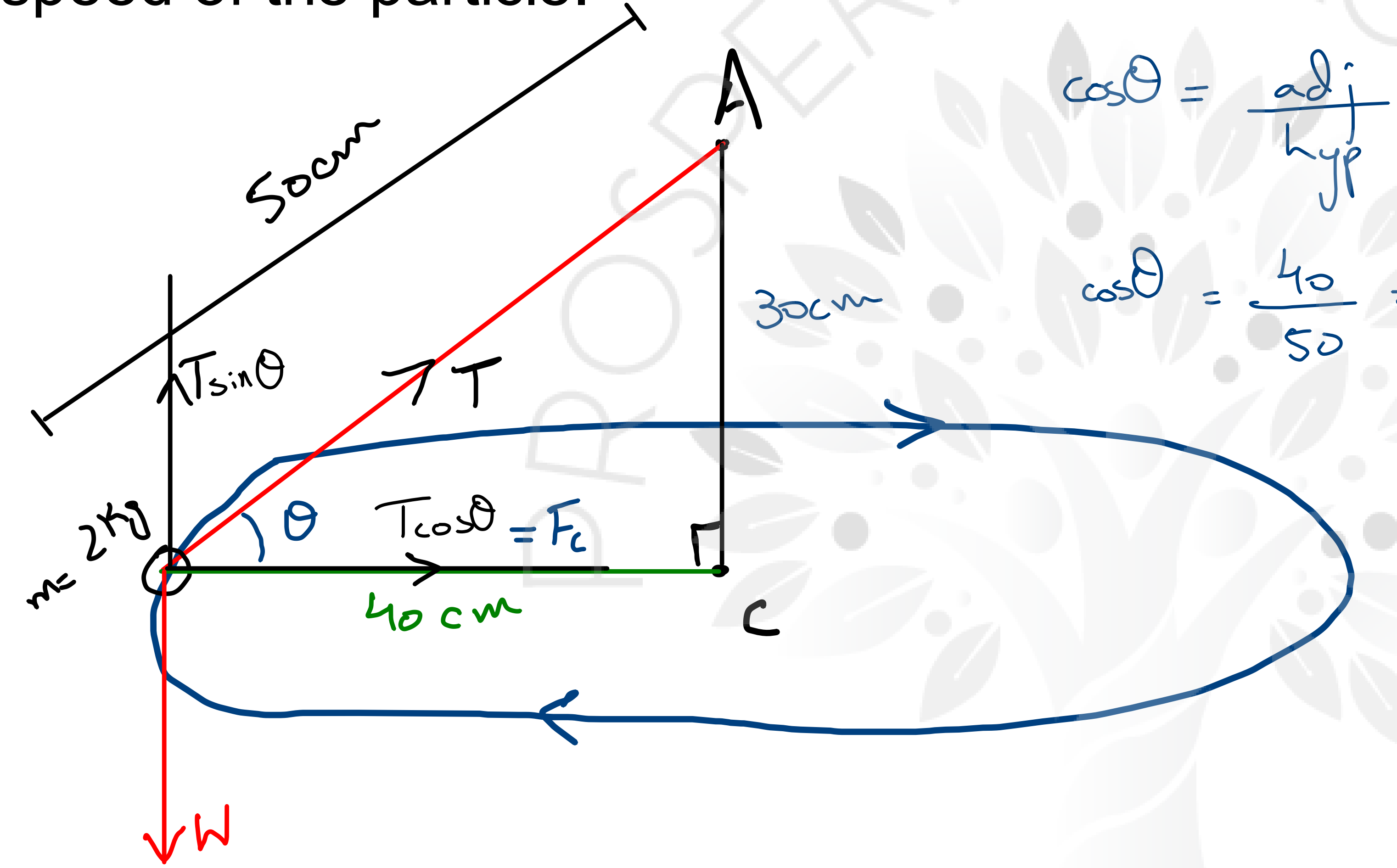
$$\downarrow F_c = R\sin\theta - \uparrow F_r\cos\theta$$

no weight

A particle of mass 2kg is attached to a light inextensible string of length 50cm. The other end of the string is attached to a fixed point A. The particle moves with a constant angular speed in a horizontal circle of radius 40cm. The center of the circle is vertically below A. Calculate the tension in the string and the angular speed of the particle.

$\sqrt{50^2 - 40^2} = \text{opp}$

$30\text{cm} = \text{opp}$



$\cos\theta = \frac{\text{adj}}{\text{hyp}}$

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$

$\cos\theta = \frac{40}{50} \Rightarrow \frac{4}{5}$

$\sin\theta = \frac{30}{50} = \frac{3}{5}$

Equilibrium

$T_y = W$

$T \times \frac{3}{5} = 2 \times 9.81$

$T = 32.7\text{N}$

$\approx 33\text{N}$

$T_x = F_c$

$(32.7) \times \frac{4}{5} = (2)(40 \times 10^{-2})\,\omega^2$

$\omega = \sqrt{\frac{32.7 \times \frac{4}{5}}{2 \times (40 \times 10^{-2})}} = 5.7 \text{ rad s}^{-1}$

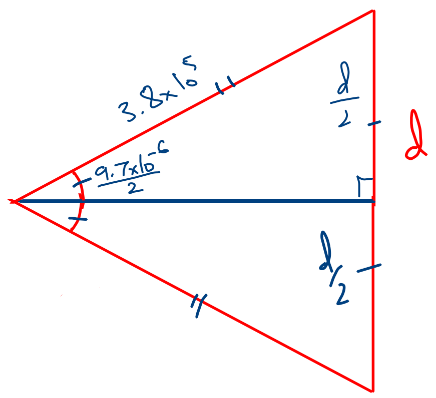
7 **(a)** Define the *radian*.

done in lecture

[2]

(b) A telescope gives a clear view of a distant object when the angular displacement between the edges of the object is at least 9.7×10^{-6} rad.

(i) The Moon is approximately 3.8×10^5 km from Earth.
Estimate the minimum diameter of a circular crater on the Moon's surface that can be seen using the telescope.



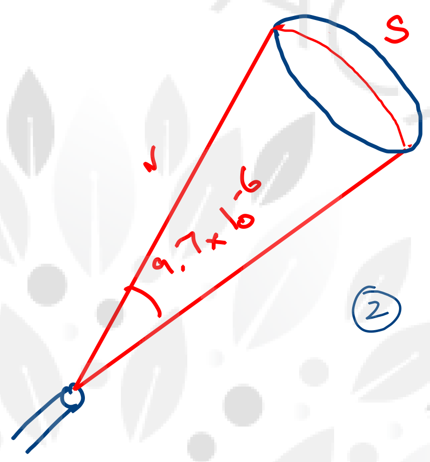
$\sin\left(\frac{9.7\times10^{-6}}{2}\right) = \frac{d/2}{3.8\times10^{5}}$

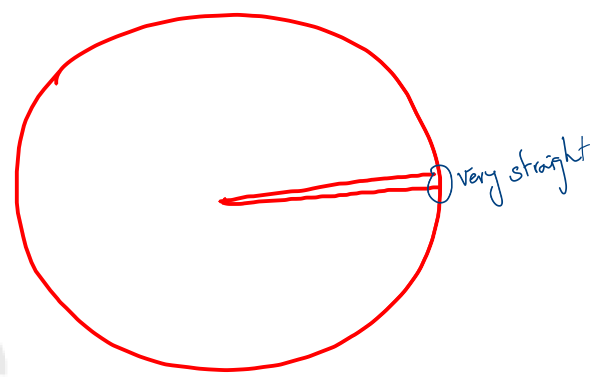
$d = (3.8\times10^{5}) \times \sin\left(\frac{9.7\times10^{-6}}{2}\right) \times 2$

$d = 1.843 \times 2 = 3.7$

diameter = 3.7 km [2]

Alternate:-





② $s = r\theta$

$s = (3.8\times10^{5})(9.7\times10^{-6})$

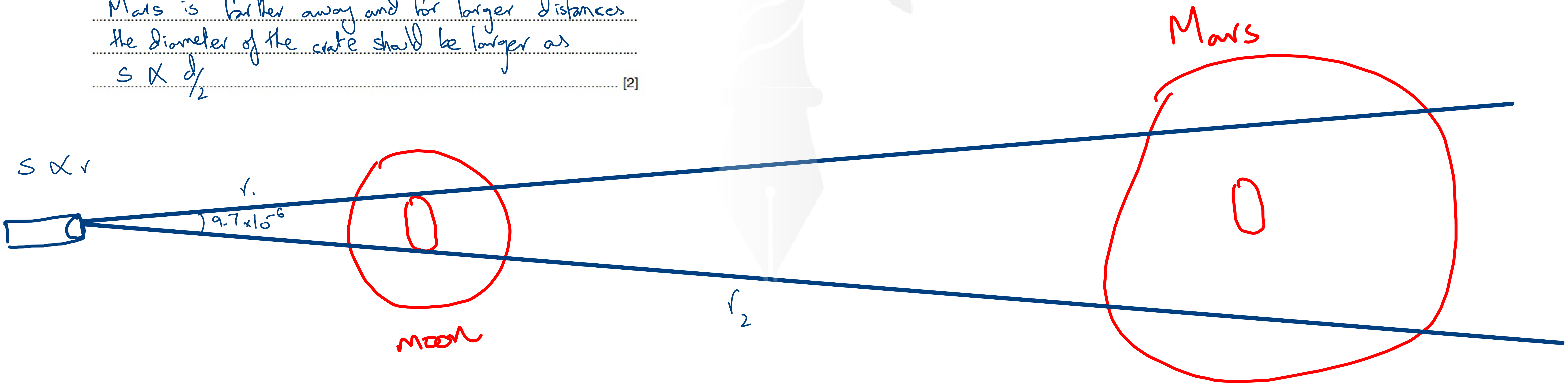
$s = 3.7$ Km

(ii) Suggest why craters of the same diameter as that calculated in **(i)** but on the surface of Mars are not visible using this telescope.

Mars is farther away and for larger distances the diameter of the crate should be larger as $s \propto d/2$

[2]

$s \propto r$



2 A large bowl is made from part of a hollow sphere.

A small spherical ball is placed inside the bowl and is given a horizontal speed. The ball follows a horizontal circular path of constant radius, as shown in Fig. 2.1.

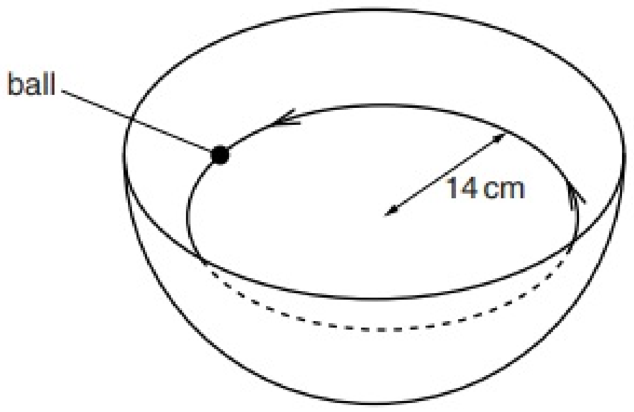


Fig. 2.1

The forces acting on the ball are its weight W and the normal reaction force R of the bowl on the ball, as shown in Fig. 2.2.

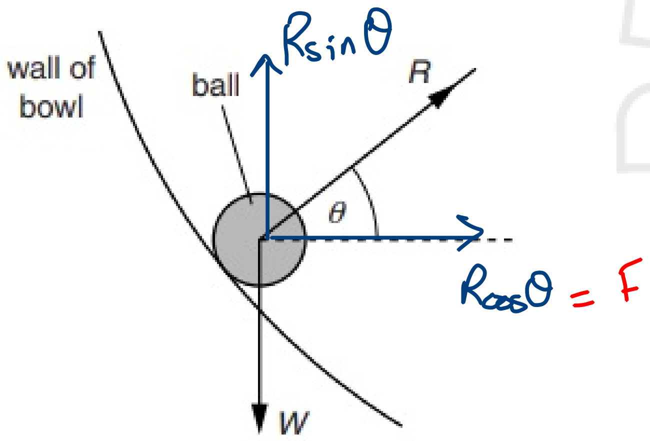


Fig. 2.2

The normal reaction force R is at an angle θ to the horizontal.

(a) (i) By resolving the reaction force R into two perpendicular components, show that the resultant force F acting on the ball is given by the expression

$$W = F\tan\theta.$$

$R\sin\theta = W$ $\quad$ $R\cos\theta = F$

$R = \dfrac{W}{\sin\theta}$ $\quad$ $\dfrac{W}{\sin\theta} \times \cos\theta = F \Rightarrow W = \dfrac{F\sin\theta}{\cos\theta} \Rightarrow \boxed{W = F\tan\theta}$

(ii) State the significance of the force F for the motion of the ball in the bowl.

F is the net force directed towards the center that keeps the object in circular motion. [1]

(b) The ball moves in a circular path of radius 14 cm. For this radius, the angle θ is 28°.

Calculate the speed of the ball.

$F = \dfrac{mv^2}{r}$

$F\tan\theta = W$

$\dfrac{\cancel{m}v^2}{r}\tan\theta = \cancel{m}g$

$\sqrt{v^2} = \sqrt{\dfrac{gr}{\tan\theta}} = \sqrt{\dfrac{(9.81)(14\times10^{-2})}{\tan(28)}} \Rightarrow v = 1.607$

speed = 1.6 ms^{-1} [3]

$S = r(1)$

1 (a) (i) Define the *radian*.

It is the angle subtended at the centre of the circle where arc length = radius.

.... [2]

(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed, as shown in Fig. 1.1.

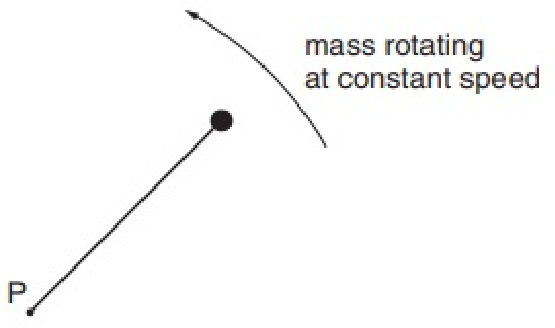


Fig. 1.1

Explain what is meant by the *angular* speed about point P of the mass.

The rate of change of angular displacement of the string with respect to time.

.... [2]

(b) A horizontal flat plate is free to rotate about a vertical axis through its centre, as shown in Fig. 1.2.

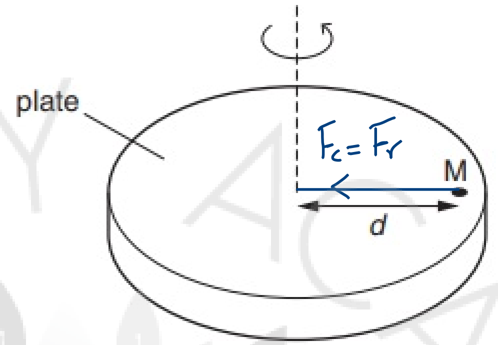


Fig. 1.2

A small mass M is placed on the plate, a distance *d* from the axis of rotation.
The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.

The maximum frictional force *F* between the plate and the mass is given by the expression

$$F = 0.72W,$$

where *W* is the weight of the mass M.
The distance *d* is 35 cm.

Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate. Explain your working. 1

1 At the maximum number of revolutions, max $F_r = F_c$

$0.72\,W = F_c$

$0.72\,\cancel{m}g = \cancel{m}r\omega^2$

$0.72 \times 9.81 = (35 \times 10^{-2}) \times \omega^2$

$\omega = \left(\frac{0.72 \times 9.81}{35 \times 10^{-2}}\right)^{\frac{1}{2}} = 4.49$

$\omega = \frac{2\pi}{T}$

$4.49 = \frac{2\pi}{T} \Rightarrow T = 1.399\ s$

$n = 60 \div 1.399 = 42.88$

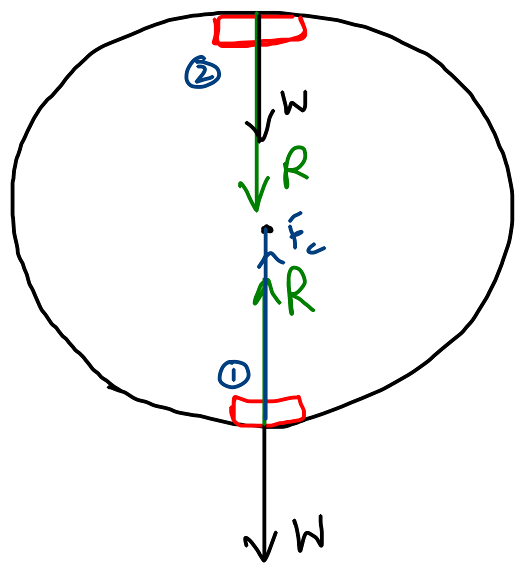
number = 43 [5]

(c) The plate in **(b)** is covered, when stationary, with mud.
Suggest and explain whether mud near the edge of the plate or near the centre will first leave the plate as the angular speed of the plate is slowly increased.

The mud near the edge leaves the plate first as it requires a greater centripetal force due to its large radius.

[2]

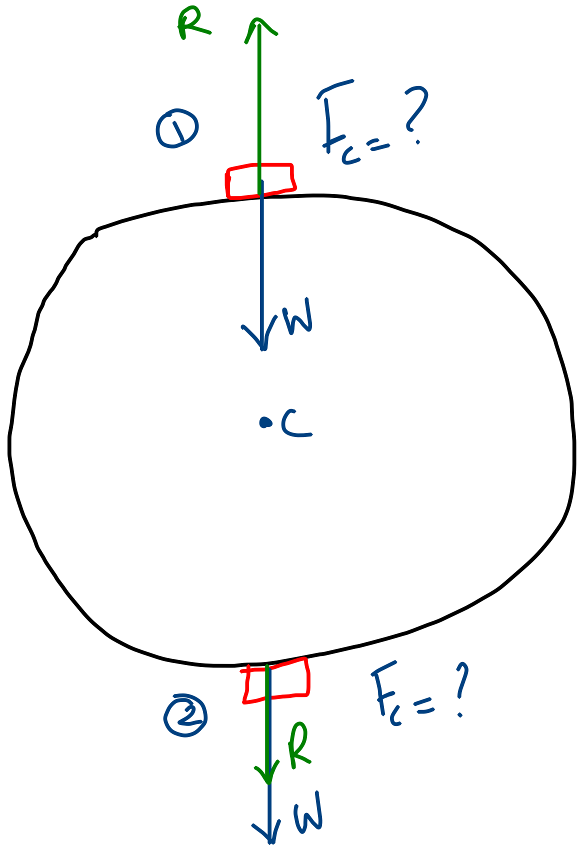
Vertical Circular motion :- Never uniform



smooth
Neglect air resistance

① $F_c = R - W$

② $F_c = R + W$



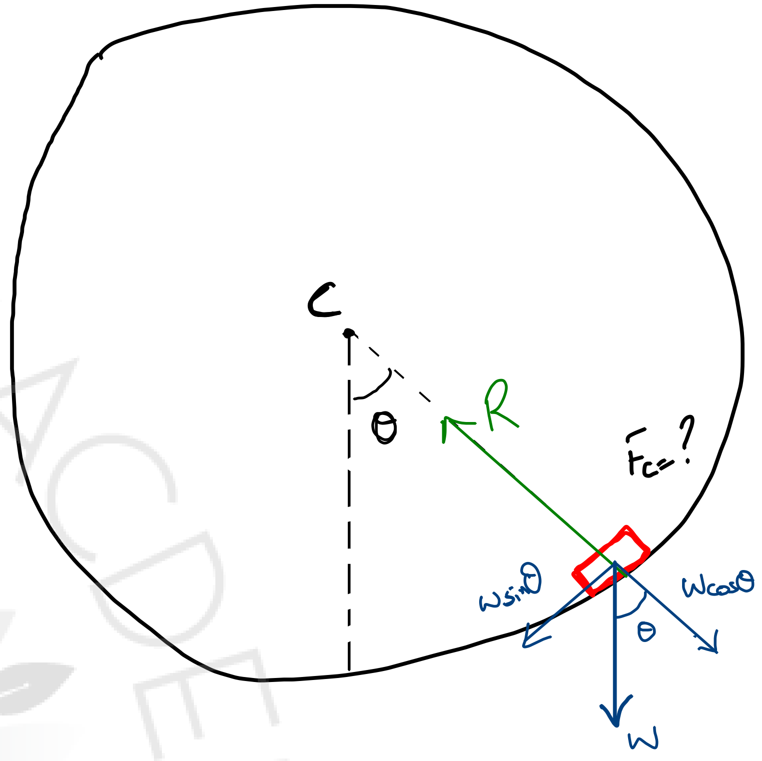
smooth
Neglect air resistance

① $F_c = W - R$

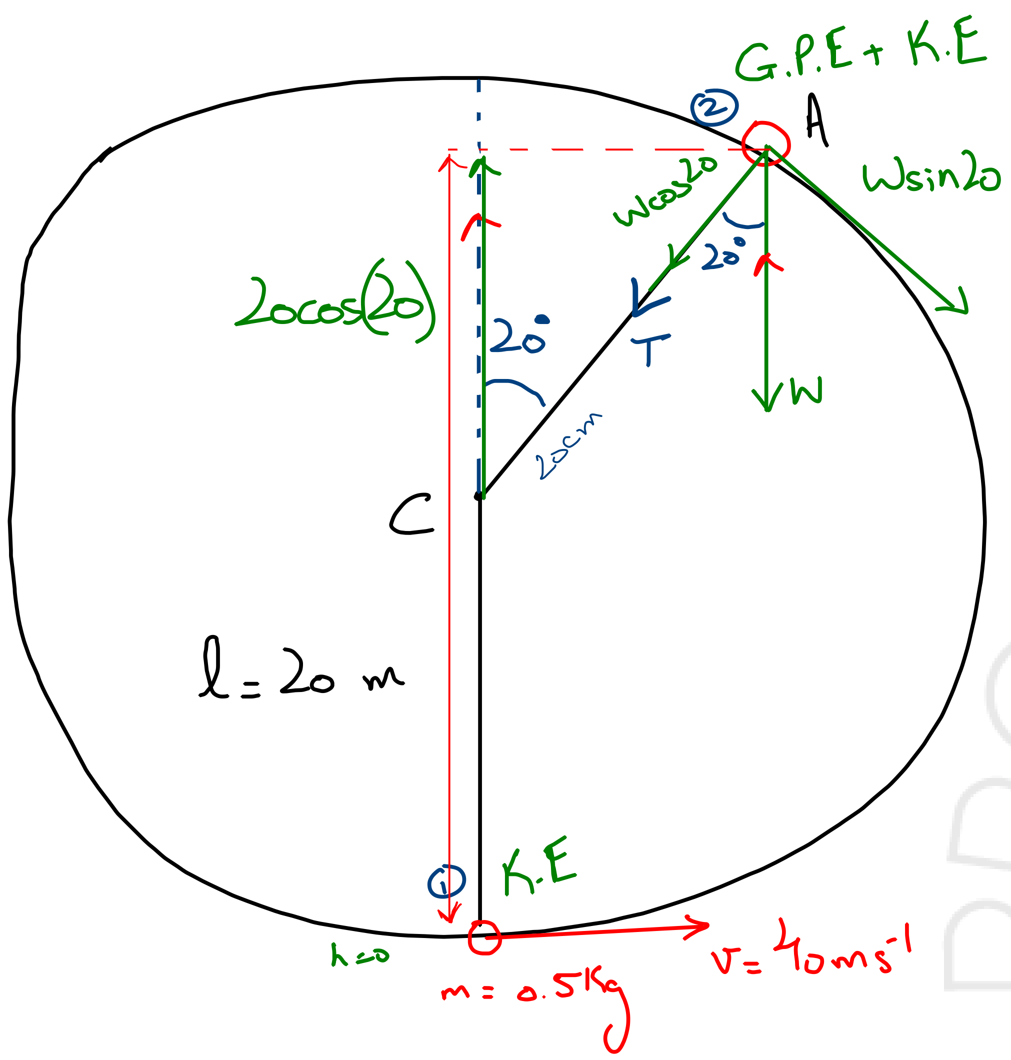
② $F_c = -R - W$

$\boxed{F_c = 0}$

③



$F_c = R - W\cos\theta$



A mass of 0.5 Kg is attached to a light inextensible string of length 20 m. At point A, when the string makes an angle of 20° with the vertical, calculate the tension in the string.

$$F_c = T + mg\cos 20$$

$$\frac{mv^2}{r} = T + mg\cos 20$$

$$\frac{(0.5)(838.864)}{20} = T + (0.5)(9.81)(\cos 20)$$

$$T = 16.4 \text{ N}$$

Energy① = Energy②

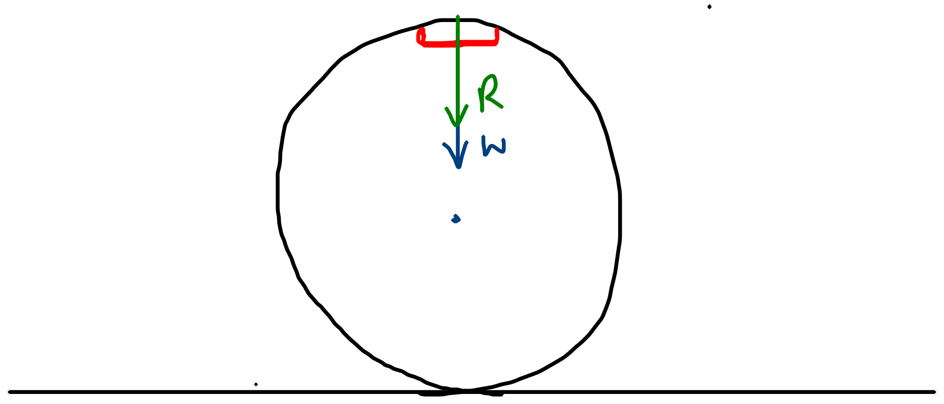
$$K.E_1 = G.P.E_2 + K.E_2$$

$$\frac{1}{2}(\cancel{0.5})(40)^2 = (\cancel{0.5})(9.81)(20 + 20\cos 20) + \frac{1}{2}(\cancel{0.5})v_2^2$$

$$800 = 380.568 + \frac{1}{2}v_2^2$$

$$v_2^2 = 838.864$$

Looping the loop :-

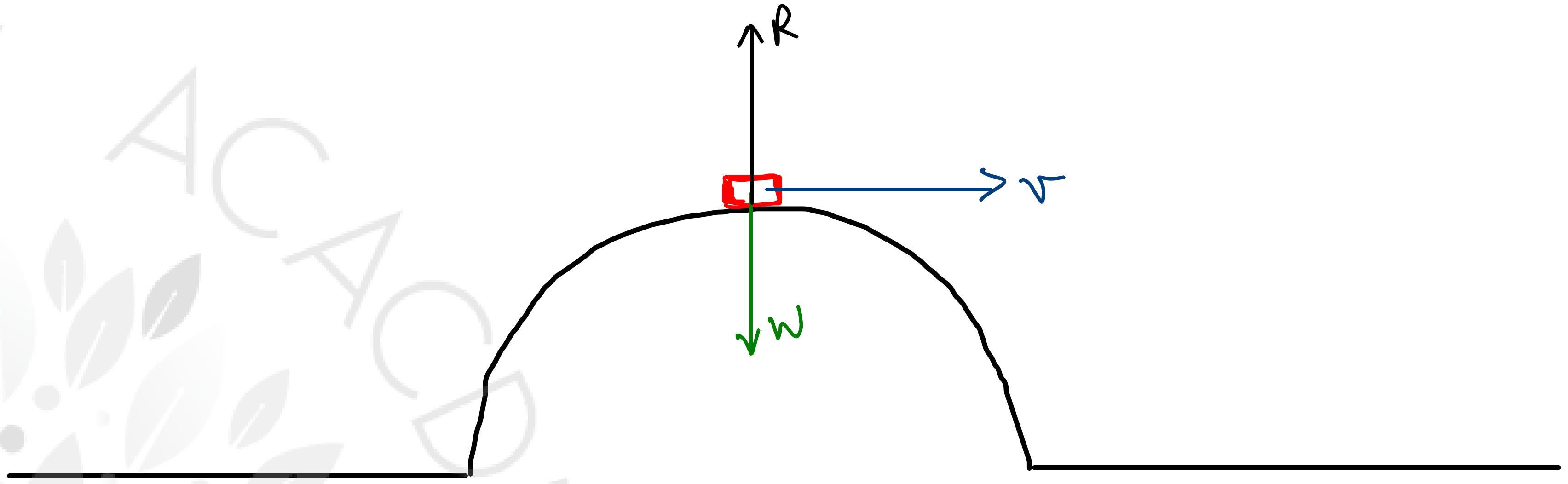


$$F_c = R + W$$

To not lose contact at top most point, $R > 0$

$$R = F_c - W$$
$$F_c - W > 0$$
$$F_c > W$$
$$\frac{\cancel{m}v^2}{r} > \cancel{m}g$$
$$v^2 > gr$$
$$v > \sqrt{gr}$$

Speed breaker :-



$$F_c = W - R$$

To not lose contact at top most point, $R > 0$

$$R = W - F_c$$
$$W - F_c > 0$$
$$W > F_c$$
$$\cancel{m}g > \frac{\cancel{m}v^2}{r}$$
$$v < \sqrt{gr}$$

1 **(a)** Explain

(i) what is meant by a *radian*,

The angle subtended at the center of the circle where arc length = radius

....[2]

(ii) why one complete revolution is equivalent to an angular displacement of 2π rad.

The circumference of a circle is $2\pi r$. Using $s = r\theta$,

$2\pi \cancel{r} = \cancel{r}\theta \Rightarrow 2\pi = \theta$ [1]

(b) An elastic cord has an unextended length of 13.0 cm. One end of the cord is attached to a fixed point C. A small mass of weight 5.0 N is hung from the free end of the cord. The cord extends to a length of 14.8 cm, as shown in Fig. 1.1.

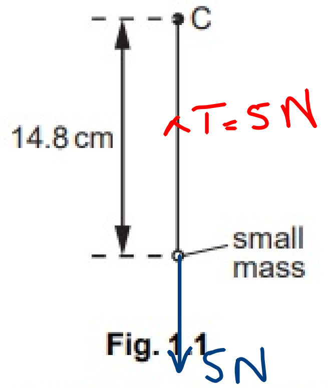


Fig. 1.1

$W = mg$

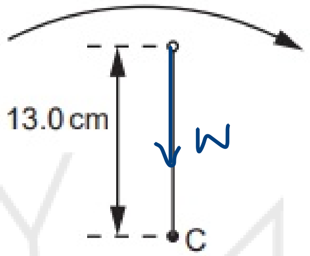
$5 = m(9.81)$

$m = \frac{5}{9.81}$

The cord and mass are now made to rotate at constant angular speed ω in a vertical plane about point C. When the cord is vertical and above C, its length is the unextended length of 13.0 cm, as shown in Fig. 1.2.

$T = K \times e \Rightarrow 5 = K \times [(14.8 - 13) \times 10^{-2}]$

$K = 277.78$



no tension as cord is unextended

Fig. 1.2

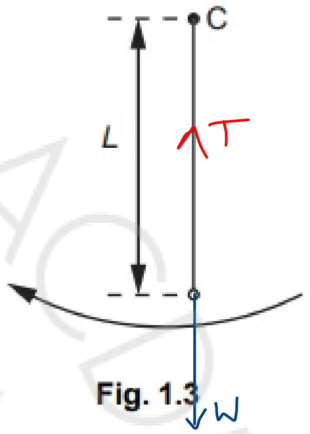


Fig. 1.3

$F_c = T - W$

(i) Show that the angular speed ω of the cord and mass is 8.7 rad s^{-1}.

$F_c = W$

$\cancel{m} r\omega^2 = \cancel{m} g$

$(13 \times 10^{-2})\omega^2 = 9.81$

$\omega = 8.68 \approx 8.7$ rad s^{-1}

[2]

(ii) The cord and mass rotate so that the cord is vertically below C, as shown in Fig. 1.3.

Calculate the length *L* of the cord, assuming it obeys Hooke's law.

$F_c = T - W$

$mr\omega^2 = T - W$

$\left(\frac{5}{9.81}\right)(L \times 10^{-2}) \times (8.7)^2 = (277.78)[(L - 13) \times 10^{-2}] - 5$

$0.3858L = 2.778L - 36.114 - 5$

$-2.3922L = -41.114$

$L = 17.18$

≈ 17.2

L = ~~17~~ / 17.2 cm

w21 qp42 q1

Answer **all** the questions in the spaces provided.

1 **(a)** State what is meant by *centripetal* acceleration.

Net acceleration directed towards the center that Keeps an object in circular motion

[1]

(b) An unpowered toy car moves freely along a smooth track that is initially horizontal. The track contains a vertical circular loop around which the car travels, as shown in Fig. 1.1.

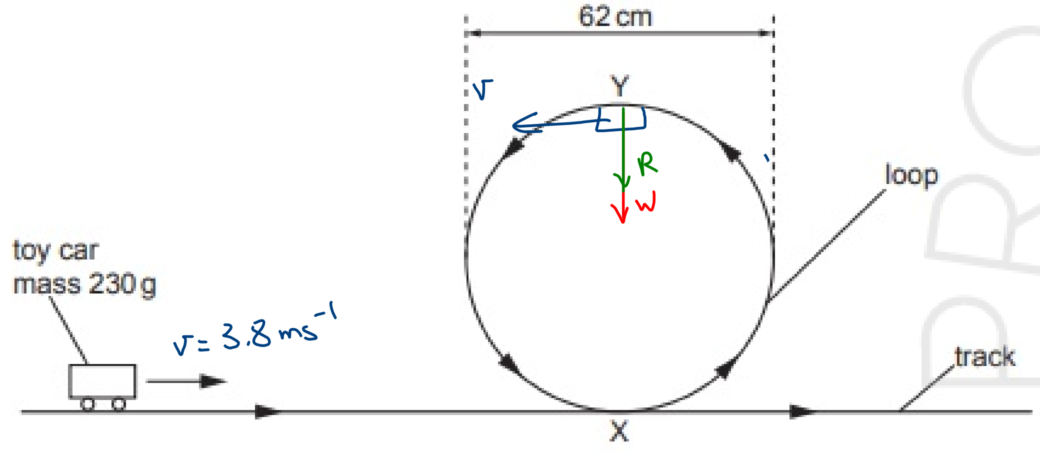


Fig. 1.1

The mass of the car is 230 g and the diameter of the loop is 62 cm. Assume that the resistive forces acting on the car are negligible.

(i) State what happens to the magnitude of the centripetal acceleration of the car as it moves around the loop from X to Y. $K.E\downarrow \Rightarrow v^2\downarrow \Rightarrow \frac{\downarrow v^2}{r} = a_c\downarrow$

Centripetal acceleration will decrease [1]

(ii) Explain, if the car remains in contact with the track, why the centripetal acceleration of the car at point Y must be greater than $9.8\,m\,s^{-2}$.

It must be greater as contact force has to be greater than zero for remaining in contact.

[2]

(c) The initial speed at which the car in **(b)** moves along the track is $3.8\,m\,s^{-1}$.

Determine whether the car is in contact with the track at point Y. Show your working.

At topmost point, $R > 0$

$F_c = W + R$

$R = F_c - W$

$F_c - W > 0$

$F_c > W$

$\frac{\cancel{m}v^2}{r} > \cancel{m}g$

$v > \sqrt{gr}$

$K.E_x = G.P.E_y + K.E_y$

$\frac{1}{2}\cancel{m}(3.8)^2 = \cancel{m}(9.81)(62\times10^{-2}) + \frac{1}{2}\cancel{m}v^2$

$v^2 = \left[\frac{1}{2}(3.8)^2 - (9.81)(62\times10^{-2})\right]\times 2$

$\sqrt{v^2} = \sqrt{2.2756} \Rightarrow v = 1.508$

$1.508 > \sqrt{9.81\times31\times10^{-2}}$

You will lose contact $\Rightarrow$ $1.508 > 1.74$ ✗

[3]

(d) Suggest, with a reason but without calculation, whether your conclusion in **(c)** would be different for a car of mass 460 g moving with the same initial speed.

no, as mass cancels out in equation.

[1]

[Total: 8]