3 A binary star consists of two stars that orbit about a fixed point C, as shown in Fig. 3.1.

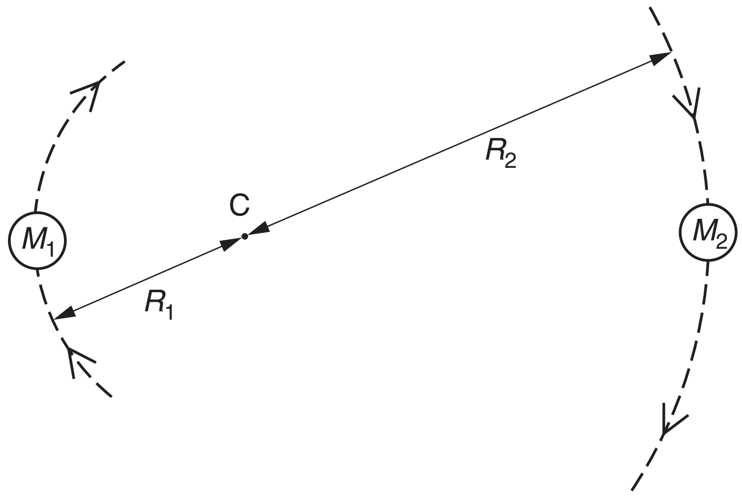

Fig. 3.1

The star of mass M_1 has a circular orbit of radius R_1 and the star of mass M_2 has a circular orbit of radius R_2. Both stars have the same angular speed ω, about C.

(a) State the formula, in terms of G, M_1, M_2, R_1, R_2 and ω for

(i) the gravitational force between the two stars,

$\dfrac{GM_1M_2}{(R_1+R_2)^2}$ ....

(ii) the centripetal force on the star of mass M_1.

$M_1R_1\omega^2$ ....

[2]

(b) The stars orbit each other in a time of 1.26×10^8 s (4.0 years). Calculate the angular speed ω for each star.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.26\times10^8} = 4.98\times10^{-8}$$

angular speed = 4.98×10^{-8} rad s^{-1} [2]

For Examiner's Use

(c) **(i)** Show that the ratio of the masses of the stars is given by the expression

$$\frac{M_1}{M_2} = \frac{R_2}{R_1}.$$

$F_{CA} = F_{CB}$

$M_1 R_1 \cancel{w^2} = M_2 R_2 \cancel{w^2}$

$\frac{M_1}{M_2} = \frac{R_2}{R_2}$ SHOWN!

[2]

(ii) The ratio $\frac{M_1}{M_2}$ is equal to 3.0 and the separation of the stars is 3.2×10^{11} m.

Calculate the radii R_1 and R_2.

$3 = \frac{R_2}{R_1}$

$R_1 = \frac{R_2}{3}$

$R_1 + R_2 = 3.2 \times 10^{11}$

$\frac{R_2}{3} + R_2 = 3.2 \times 10^{11}$

$\frac{4R_2}{3} = 3.2 \times 10^{11}$

$R_2 = 3/4 \times 3.2 \times 10^{11}$

$R_1 = 3.2 \times 10^{11} - R_2$

R_1 = 8×10^{10} m

R_2 = 2.4×10^{11} m

[2]

(d) **(i)** By equating the expressions you have given in **(a)** and using the data calculated in **(b)** and **(c)**, determine the mass of one of the stars.

$F_{CA} = F_{GB}$

$\cancel{M_1} R_1 w^2 = \frac{G \cancel{M_1} M_2}{(R_1 + R_2)^2}$

$M_2 = \frac{R_1 w^2 (R_1 + R_2)^2}{G}$

$M_2 = \frac{8 \times 10^{10} \cdot \{4.98 \times 10^{-8}\}^2 \cdot (3.2 \times 10^{11})^2}{6.67 \times 10^{-11}}$

mass of star = 3.0×10^{29} kg

(ii) State whether the answer in **(i)** is for the more massive or for the less massive star.

LESS MASSIVE STAR

[4]

$3 = \frac{M_1}{3 \times 10^{29}} \rightarrow M_1 = 9 \times 10^{29}$ kg

Answer **all** the questions in the spaces provided.

1 A binary star consists of two stars A and B that orbit one another, as illustrated in Fig. 1.1.

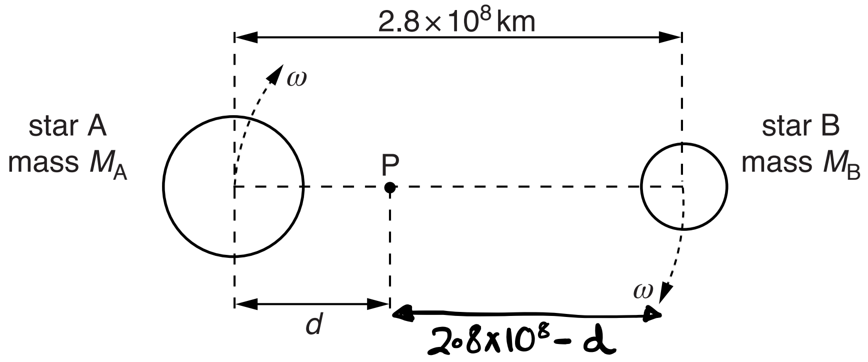

Fig. 1.1

The stars are in circular orbits with the centres of both orbits at point P, a distance d from the centre of star A.

(a) (i) Explain why the centripetal force acting on both stars has the same magnitude.

The gravitational force is the centripetal force according to Newton's 3rd law, force is same.

.... [2]

(ii) The period of the orbit of the stars about point P is 4.0 years.

Calculate the angular speed ω of the stars.

$$\omega = \frac{2\pi}{T} \rightarrow \frac{2\pi}{(4 \cdot 365 \cdot 24 \cdot 3600)} = 4.98 \times 10^{-8}$$

ω = 5.0×10^{-8} rad s^{-1} [2]

(b) The separation of the centres of the stars is 2.8×10^8 km.
The mass of star A is M_A. The mass of star B is M_B.
The ratio $\dfrac{M_A}{M_B}$ is 3.0.

$$\frac{M_A}{M_B} = \frac{R_B}{R_A}$$

(i) Determine the distance *d*.

$$3 = \frac{(2.8 \times 10^8 - d)}{d}$$

$$3d = 2.8 \times 10^8 - d$$

$$4d = 2.8 \times 10^8$$

$$d = \frac{2.8 \times 10^8}{4}$$

d = 7×10^7 km [3]

(ii) Use your answers in **(a)(ii)** and **(b)(i)** to determine the mass M_B of star B.
Explain your working.

$$F_{CA} = F_{GB}$$

$$\cancel{M_A} d\omega^2 = \frac{G \cancel{M_A} M_B}{(d + (2.8 \times 10^{11} - d))^2}$$

$$M_B = \frac{d\omega^2 (d + (2.8 \times 10^{11} - d))}{G}$$

M_B = 2.0×10^{29} kg [3]

[Total: 10]

$$\frac{7 \times 10^{10} \cdot \{5.0 \times 10^{-8}\}^2 \cdot (2.8 \times 10^{11})^2}{6.67 \times 10^{-11}}$$

Answer **all** the questions in the spaces provided.

1 **(a)** State what is meant by a *gravitational force*.

Force on a mass in a gravitational field

.... [1]

(b) A binary star system consists of two stars S_1 and S_2, each in a circular orbit.

The orbit of each star in the system has a period of rotation T.

Observations of the binary star from Earth are represented in Fig. 1.1.

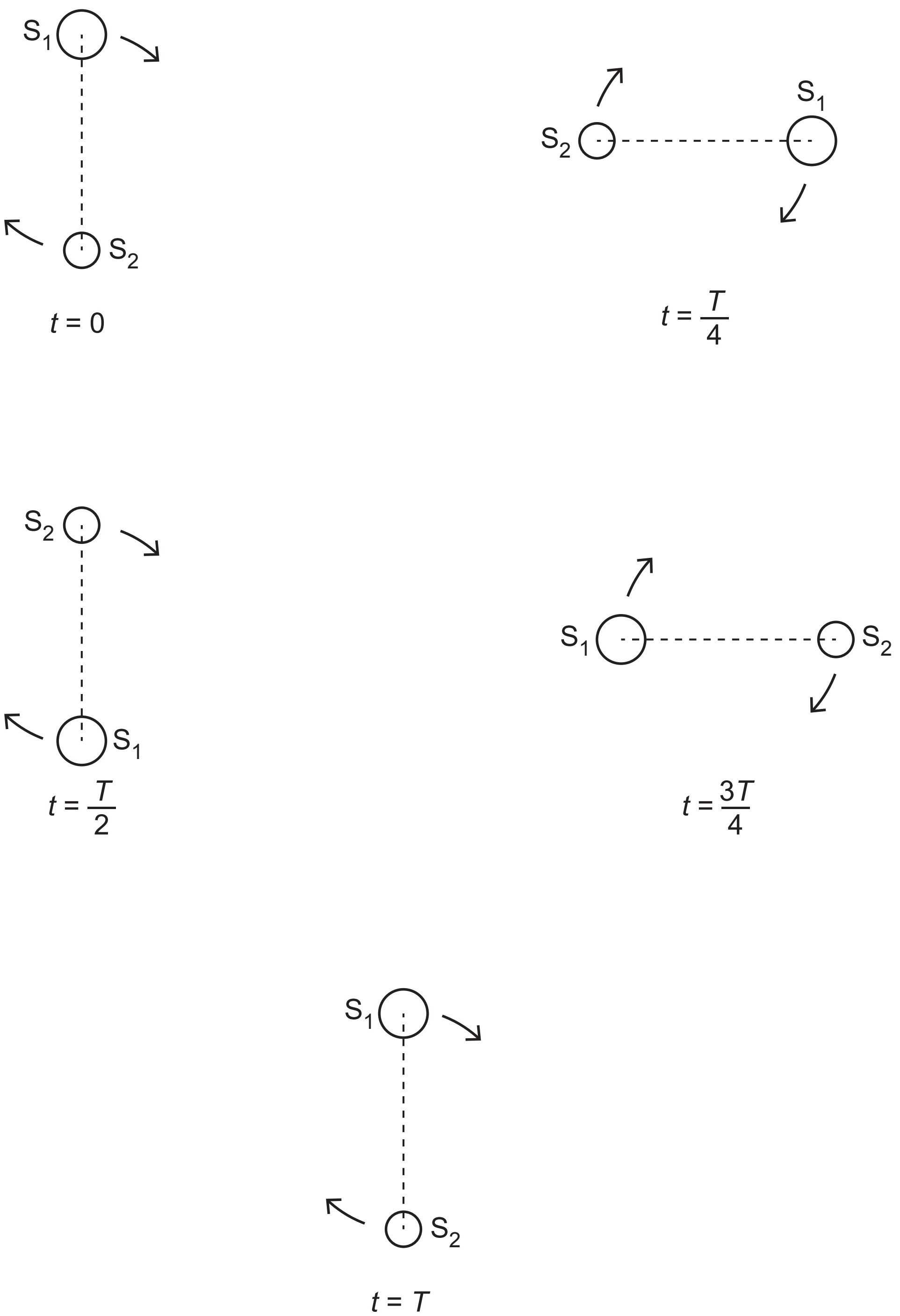

Fig. 1.1 (not to scale)

Observed from Earth, the angular separation of the centres of S_1 and S_2 is 1.2×10^{-5} rad. The distance of the binary star system from Earth is 1.5×10^{17} m.

Show that the separation d of the centres of S_1 and S_2 is 1.8×10^{12} m.

$S = r\theta$

$S = 1.5 \times 10^{17} \cdot 1.2 \times 10^{-5}$

$S = 1.8 \times 10^{12}$ m SHOWN!

[1]

(c) The stars S_1 and S_2 rotate with the same angular velocity ω about a point P, as illustrated in Fig. 1.2.

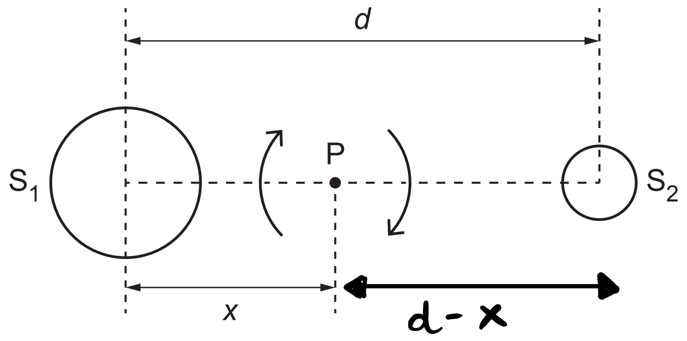

Fig. 1.2 (not to scale)

Point P is at a distance x from the centre of star S_1.
The period of rotation of the stars is 44.2 years.

(i) Calculate the angular velocity ω.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(44.2 \times 365 \times 24 \times 3600)}$$

ω = 4.5×10^{-9} rad s^{-1} [2]

(ii) By considering the forces acting on the two stars, show that the ratio of the masses of the stars is given by

$$\frac{\text{mass of } S_1}{\text{mass of } S_2} = \frac{d - x}{x}.$$

$$F_c S_1 = F_c S_2$$

$$M d \cancel{\omega^2} = M(d-x)\cancel{\omega^2}$$

$$\frac{M_{S_1}}{M_{S_2}} = \frac{d-x}{x}$$

SHOWN!

[2]

(iii) The mass M_1 of star S_1 is given by the expression

$$GM_1 = d^2(d - x)\omega^2$$

where G is the gravitational constant.

The ratio in **(ii)** is found to be 1.5.

Use data from **(b)** and your answer in **(c)(i)** to determine the mass M_1.

$d = 1.8 \times 10^{12}$ m

$$1.5 = \frac{d-x}{x}$$

$$1.5x = d - x$$

$$2.5x = d$$

$$x = 2/5 d$$

$$x = 7.2 \times 10^{11}$$

$$GM_1 = d^2(d-x)\omega^2$$

$$M_1 = \frac{d^2(d-x)\omega^2}{G}$$

M_1 = 1.1×10^{30} kg [3]

[Total: 9]

$$\frac{(1.8\times10^{12})^2(1.8\times10^{12} - 7.2\times10^{11})(4.5\times10^{-9})^2}{6.67\times10^{-11}}$$