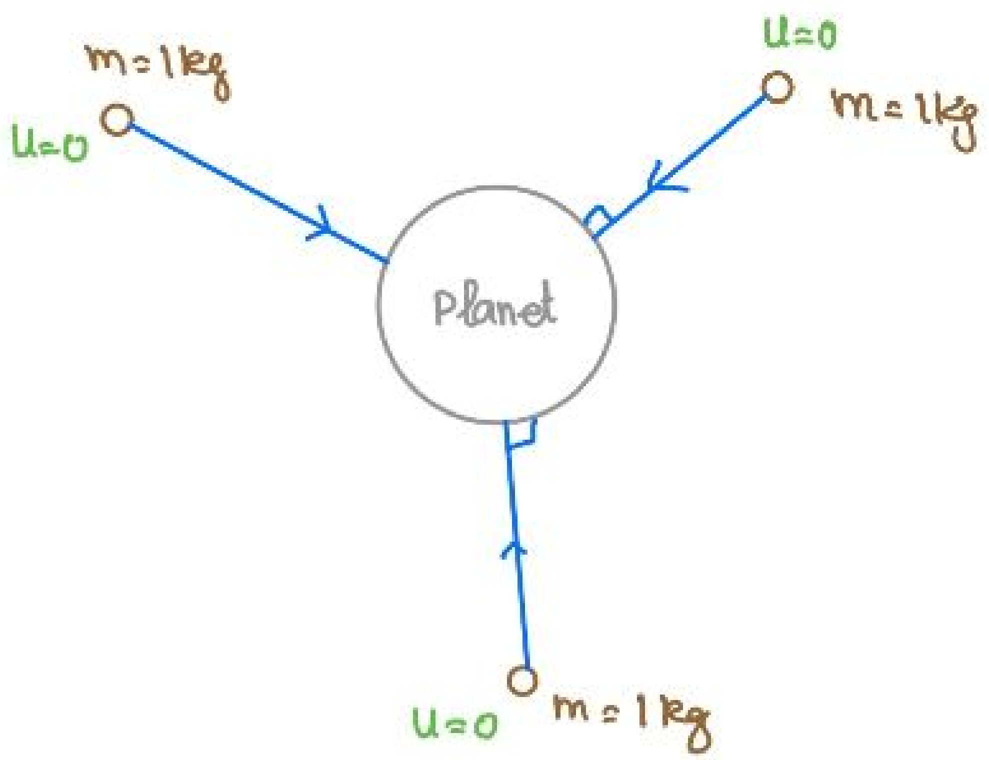
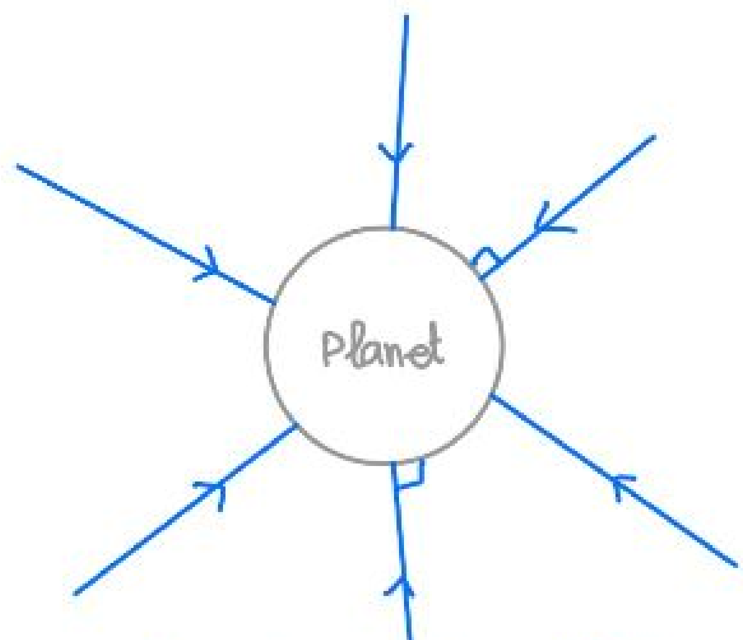
GRAVITATIONAL FIELDS

Gravitational field:-

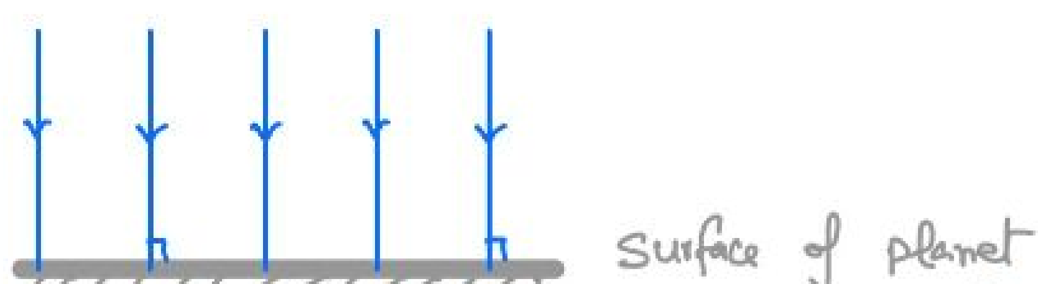
Meaning
* Field: 3D region or space
* Source: Mass of an object
* Test / Identify: Another mass experience attractive force.
* Nature: Always attractive.

Representation:- By field lines i.e each field line represent the force on a unit mass.





Non-uniform field of isolated planet



Uniform field (gap b/w field lines is constant)

Newton's Law of Gravitation:

Concept:

Point objects/masses

Newton's third law → Acts on two object

→ Same magnitude

Force → → opposite direction

→ for same time interval.

$$F_1 = -F_2 = |F| = F$$

Mathematical form:

$$F \propto m_1 m_2 \quad \text{------------} \quad (1)$$

$$F \propto \frac{1}{r^2} \quad \text{------------} \quad (2)$$

Combining (1) and (2)

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = G \frac{m_1 m_2}{r^2}}$$

Here 'G' is the constant of proportionality and is called Gravitational constant. It's value in S.I. is $6.67 \times 10^{-11}\ Nm^2 kg^{-2}$.

Statement: The force of attraction b/w two point objects is directly proportional to the product of their masses and inversely proportional to the square of separation b/w their centres.

Point Objects: If the size of an object is infinitely small as compared to separation b/w two objects. Also one can not identify the shape of an object by standing on another object.

Mass of planet/Earth:- Mass is the measure of Inertia

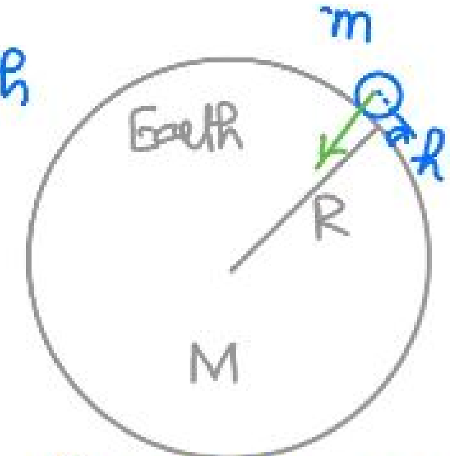
Expression:

weight of an object = Gravitational pull of Earth

$$W = F_G$$

$$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$$

$$\boxed{M = \frac{gR^2}{G}}$$



$h << R$, so 'h' is neglected

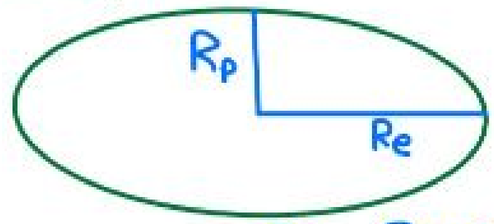
Value: $M = \frac{(9.81)(6.4\times10^6)^2}{6.67\times10^{-11}} = 5.98\times10^{24}$ kg

$$\boxed{M = 6.0\times10^{24}\text{ kg}}$$

Density of planet/Earth:-

$\rho = \frac{M}{V} \longrightarrow V = \frac{4}{3}\pi R^3$

Assumptions:



$R = \frac{R_p + R_e}{2} = 6.4\times10^6$ m

$R_e = R_p + 21$ km

1 - Planet/Earth is a uniform sphere.

2 - Planet/Earth has uniform deposits through out it to keep its density constant.

Expression:

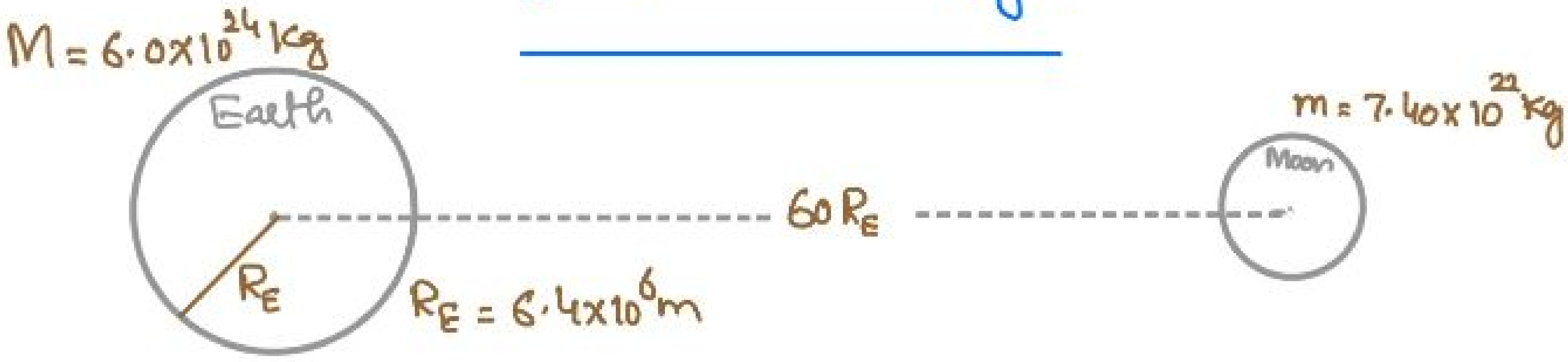
$$\rho = \frac{M}{V} = \frac{\frac{g\cancel{R^2}}{G}}{\frac{4}{3}\pi R^{\cancel{3}}}$$

$$\boxed{\rho = \frac{3g}{4\pi G R}}$$

Value: For Earth

$$\rho = \frac{3(9.81)}{4(3.14)(6.67\times10^{-11})(6.4\times10^{6})}$$

$$\rho = \qquad\qquad kg\,m^{-3}$$



Calculate Gravitational attraction b/w Earth and moon.

$$F_G = \frac{GMm}{(60R_E)^2} = \frac{(6.67\times10^{-11})(6.0\times10^{24})(7.40\times10^{22})}{[60(6.4\times10^{6})]^2}$$

$$F_G = 7.23\times10^{23}\ N$$

Gravitational field strength:- (g)

Def. $W = mg \Rightarrow g = \frac{F_G}{m}$

→ Gravitational force per unit mass [1]

→ Attractive pull of a planet due to its mass per unit mass of an object in its field. [2]

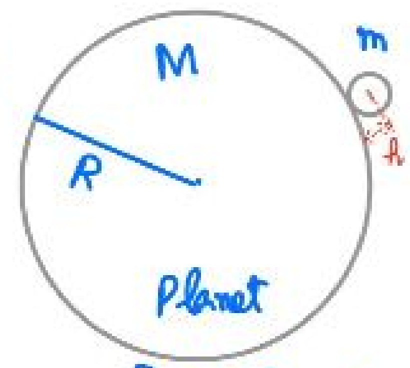
Expression:-

Weight = Gravitational pull of planet

$W = F_G$

$\cancel{m}g = \frac{GM\cancel{m}}{R^2}$

$$\boxed{g = \frac{GM}{R^2}}$$



$h << R$, so h is neglected.

Value: For Earth

$$g = \frac{(6.67\times10^{-11})(6.0\times10^{24})}{(6.4\times10^{6})^2} \Rightarrow g = 9.81\ N\,kg^{-1}$$

Units: $N\,kg^{-1} = (kg\,m\,s^{-2})(kg^{-1}) = m\,s^{-2}$

Note, 'g' does not depend upon mass of object but is dependent upon the mass of planet.

$g_{Earth} = 9.81\ N\,kg^{-1}$, $g_{moon} = 1.6\ N\,kg^{-1}$

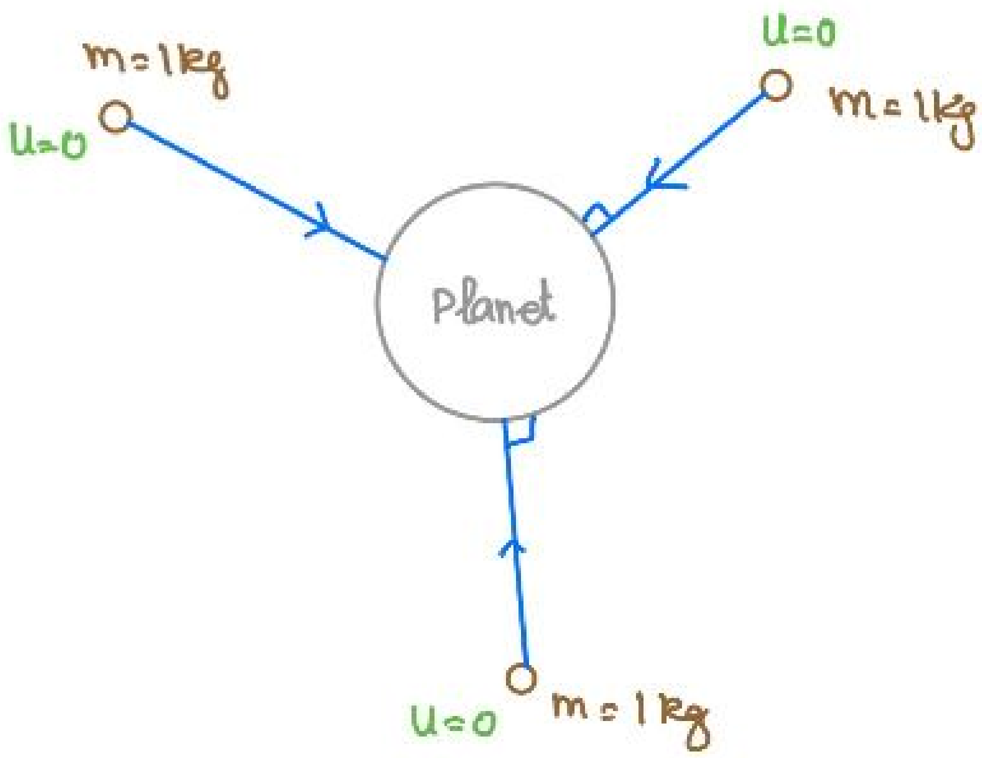
$g_{Mars} = 3.0\ N\,kg^{-1}$

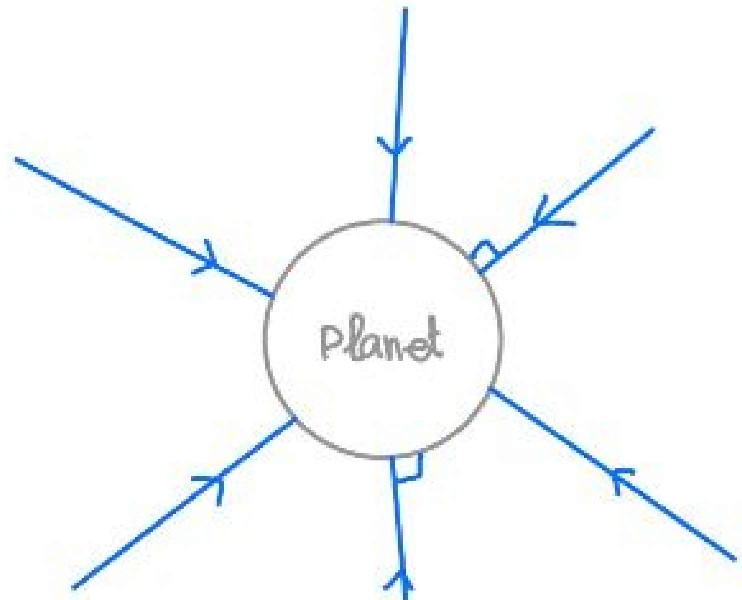
P.S. Vector

Direction: Towards the direction of motion of unit mass from rest

Field pattern:

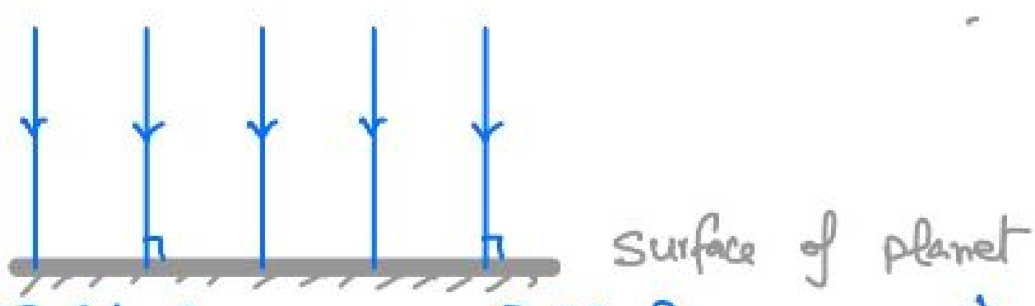
N-2003
Sp. paper June 2016





Non-uniform field of isolated planet

Note: No change in pattern (remain non uniform) but field lines come closer to each other if the mass of planet is kept constant but its volume is reduced (radius decreases)



Uniform field (gap b/w field lines is constant)

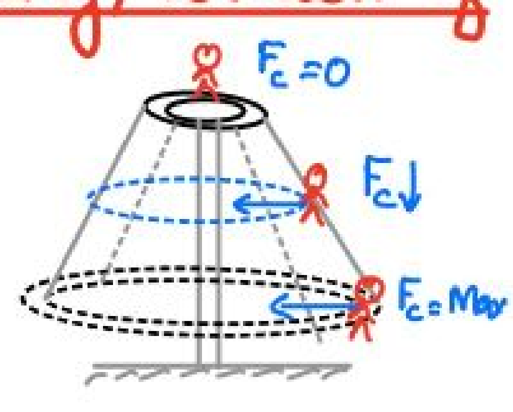
Dependance:

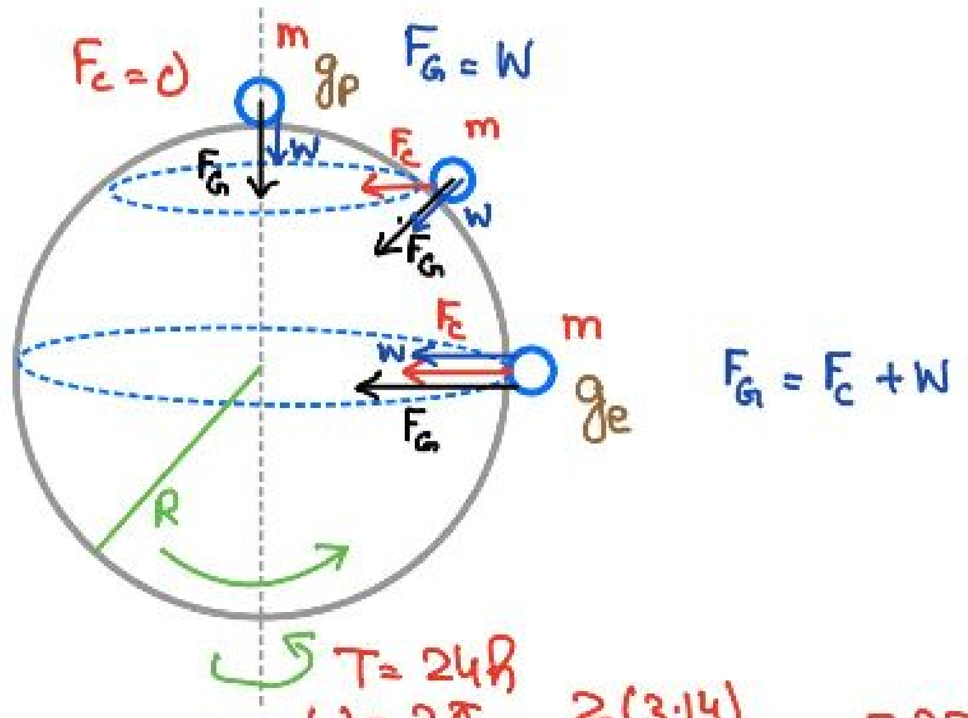
(i) **Variation in 'g' due to spinning/rotation of planet about its axis:-**

v.v. Imp
N-2003
N-2008
Sp. paper J-16

$$F_c = m r \omega^2$$

but m and ω are constant





$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{24 \times 60 \times 60} = 7.27 \times 10^{-5}\ \text{rad s}^{-1}$$

At equator:

$$F_G = W + F_c$$

$$\frac{GM\cancel{m}}{R^2} = \cancel{m}g_e + \cancel{m}R\omega^2$$

$$\frac{GM}{R^2} - R\omega^2 = g_e \quad \text{------------------} (1)$$

At pole:- $F_c = 0$ due to zero distance from the central axis of planet / Earth.

$$F_G = W$$

$$\frac{GM\cancel{m}}{R^2} = \cancel{m}g_P \Rightarrow g_P = \frac{GM}{R^2} \quad \text{--------} (2)$$

Put eq. (2) into (1)

$$g_P - R\omega^2 = g_e$$

or $\boxed{g_P = g_e + R\omega^2}$

Result:- $g_P > g_e$

Q) 1.

A spherical planet has mass M and radius R.
The planet may be assumed to be isolated in space and to have its mass concentrated at its centre.
The planet spins on its axis with angular speed ω, as illustrated in Fig. 1.1.

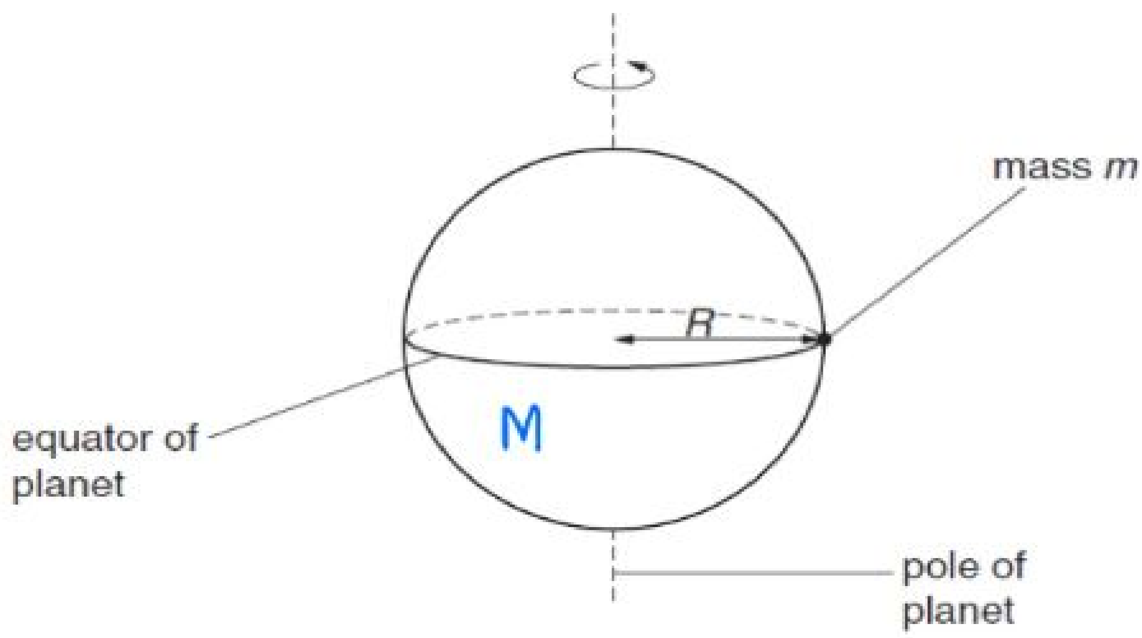


Fig. 1.1

A small object of mass m rests on the equator of the planet. The surface of the planet exerts a normal reaction force on the mass. Normal reaction force = weight

(a) State formulae, in terms of M, m, R and ω, for

(i) the gravitational force between the planet and the object,

$F_G = \dfrac{GMm}{R^2}$ [1]

(ii) the centripetal force required for circular motion of the small mass,

$F_c = mR\omega^2$ [1]

(iii) the normal reaction exerted by the planet on the mass. $F_G = F_c + W$

$\dfrac{GMm}{R^2} = mR\omega^2 + \text{Normal reaction} \Rightarrow \text{Normal reaction} = \dfrac{GMm}{R^2} - mR\omega^2$ [1]

(b) (i) Explain why the normal reaction on the mass will have different values at the equator and at the poles.

Normal reaction $= \frac{GMm}{R^2} - m\omega^2 R$

$= (\text{Constant}) - (\text{Constant}) R$

As R↓ from central axis of planet if move from equator to pole and becomes zero at pole. [2]

(ii) The radius of the planet is 6.4×10^6 m. It completes one revolution in 8.6×10^4 s. Calculate the magnitude of the centripetal acceleration at

1. the equator,

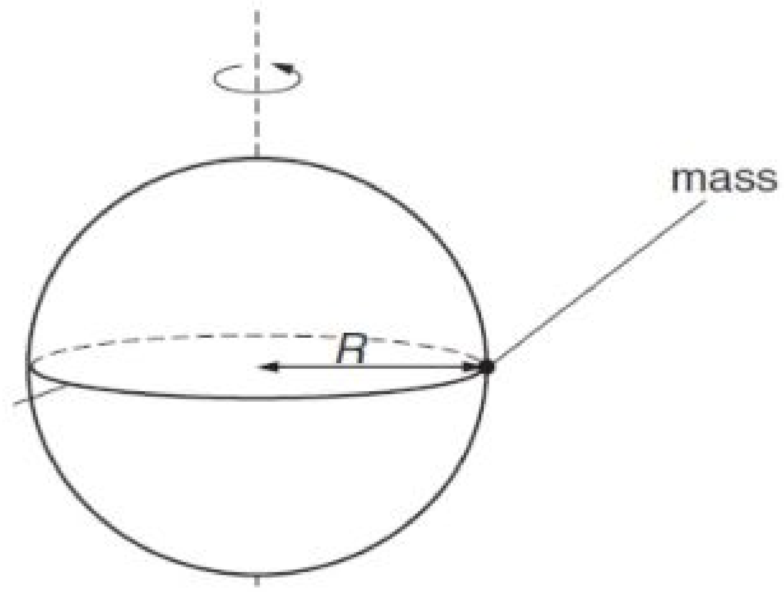
$a = R\omega^2$

$= R\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 R}{T^2}$

$a = \frac{4(3.14)^2(6.4\times10^6)}{(8.6\times10^4)^2}$

acceleration = ... $m\,s^{-2}$ [2]

2. one of the poles.



$a = r\omega^2$

Here radius from central axis of planet becomes zero and so centripetal force = 0

acceleration = ...0........ $m\,s^{-2}$ [1]

(c) Suggest two factors that could, in the case of a real planet, cause variations in the acceleration of free fall at its surface.

1. Planet is not a uniform sphere

2. Planet has variable deposits through out its surface.

2. ....

3. Planet may not be isolated and may be in the field of another planet. [2]

{Nov 08/4/Q.1}

Q.3) Specimen paper J-2016

(a) (i) On Fig. 7.1, draw lines to represent the gravitational field outside an isolated uniform sphere.

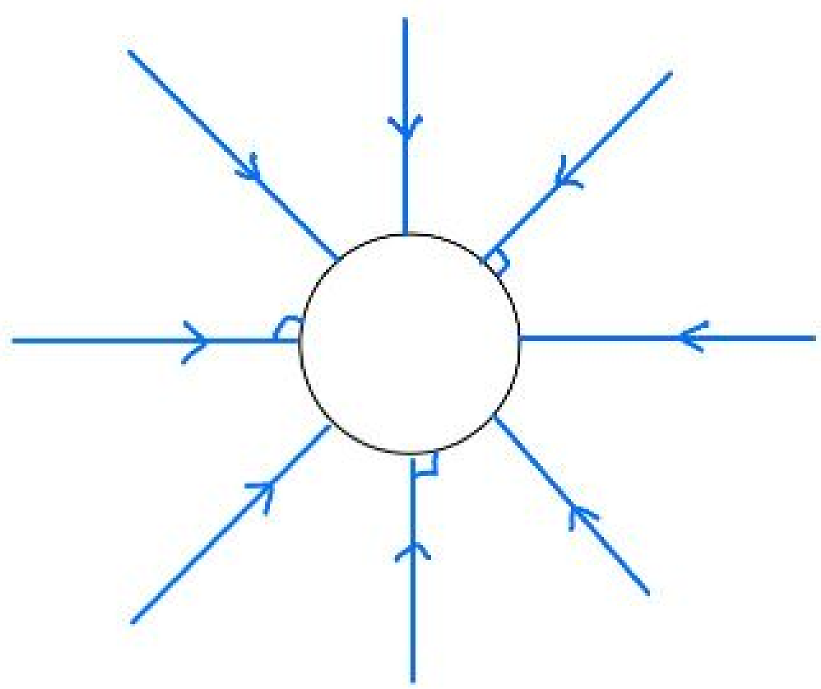


Fig. 7.1

(ii) A second sphere has the same mass but a smaller radius. Suggest what difference, if any, there is between the patterns of field lines for the two spheres.

No change $\frac{GM}{r^2}$ pattern = ~~constant field~~ $\frac{1}{R^2}$ lines $\frac{1}{R^2}$ come closer to each other

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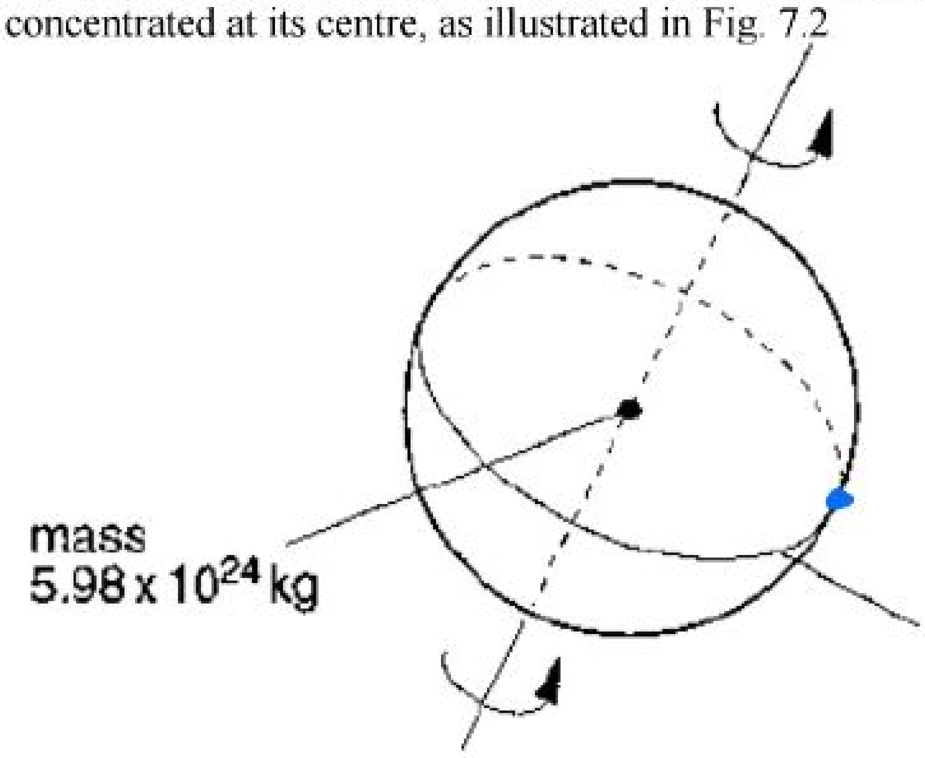
(b) The Earth may be considered to be a uniform sphere of radius 6380 km with its mass of 5.98×10^{24} kg concentrated at its centre, as illustrated in Fig. 7.2



$F_G = F_c + W$

$W = F_G - F_c$

$mg = F_G - F_c$

$g = \frac{9.77}{1.00} = 9.77\,ms^{-2}$

Fig. 7.2

A mass of 1.00 kg on the Equator rotates about the axis of the Earth with a period of 1.00 day (8.64×10^4 s).
Calculate, to three significant figures,

(i) the gravitational force F_G of attraction between the mass and the Earth,

$$F_G = \frac{GMm}{R^2} = \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.00)}{(6380\times10^3)^2}$$

F_G = ……9.80…… N

(ii) the centripetal force F_C on the 1.00 kg mass,

$$F_c = mR\omega^2 = mR\left(\frac{2\pi}{T}\right)^2 = \frac{4\pi^2 mR}{T^2}$$

$$= \frac{4(3.14)^2(1.00)(6380\times10^3)}{(8.64\times10^4)^2}$$

F_C = ……0.0337…… N

(iii) the difference in magnitude of the forces.

$F = F_G - F_c$

$= 9.80 - 0.0337$

$= 9.77$ N

difference = ……9.77…… N

(c) By reference to your answers in (b), suggest, with a reason, a value for the acceleration of free fall at the Equator.

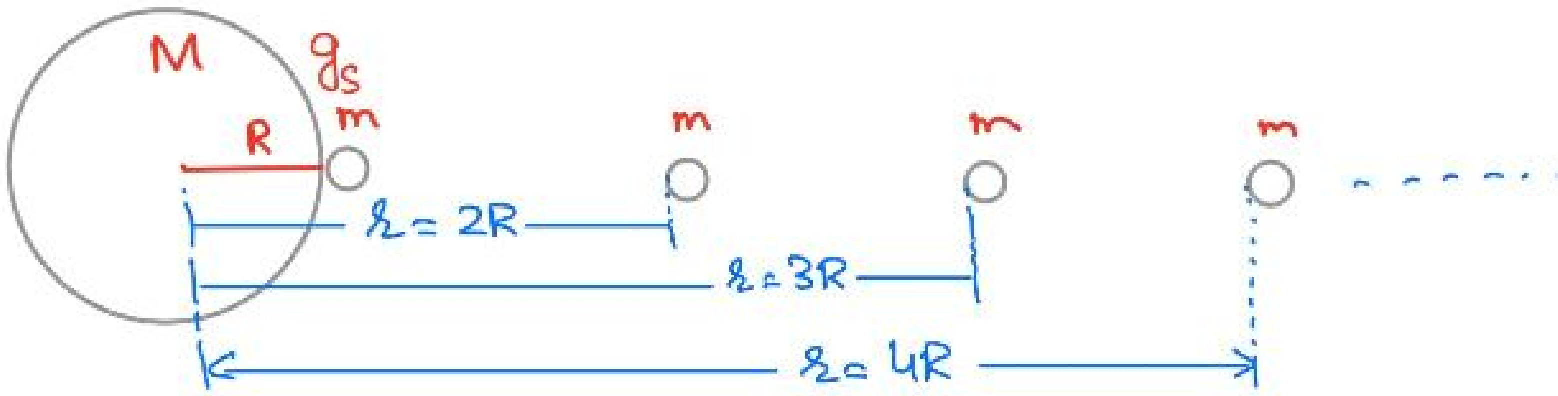
$ma = 9.77 \Rightarrow a = \frac{9.77}{1.00} = 9.77\,ms^{-2}$

…………………………………………………………………………………………

…………………………………………………………………………………………

………………………………………………………………………………………… [2]

(ii) Variation in g due to altitude / height :



At surface of planet (g_s)	At altitude / height (g_R)
$g_s = \frac{GM}{R^2}$ ......(1)	$g_R = \frac{GM}{r^2}$ ....(2)

Divide (2) by (1)

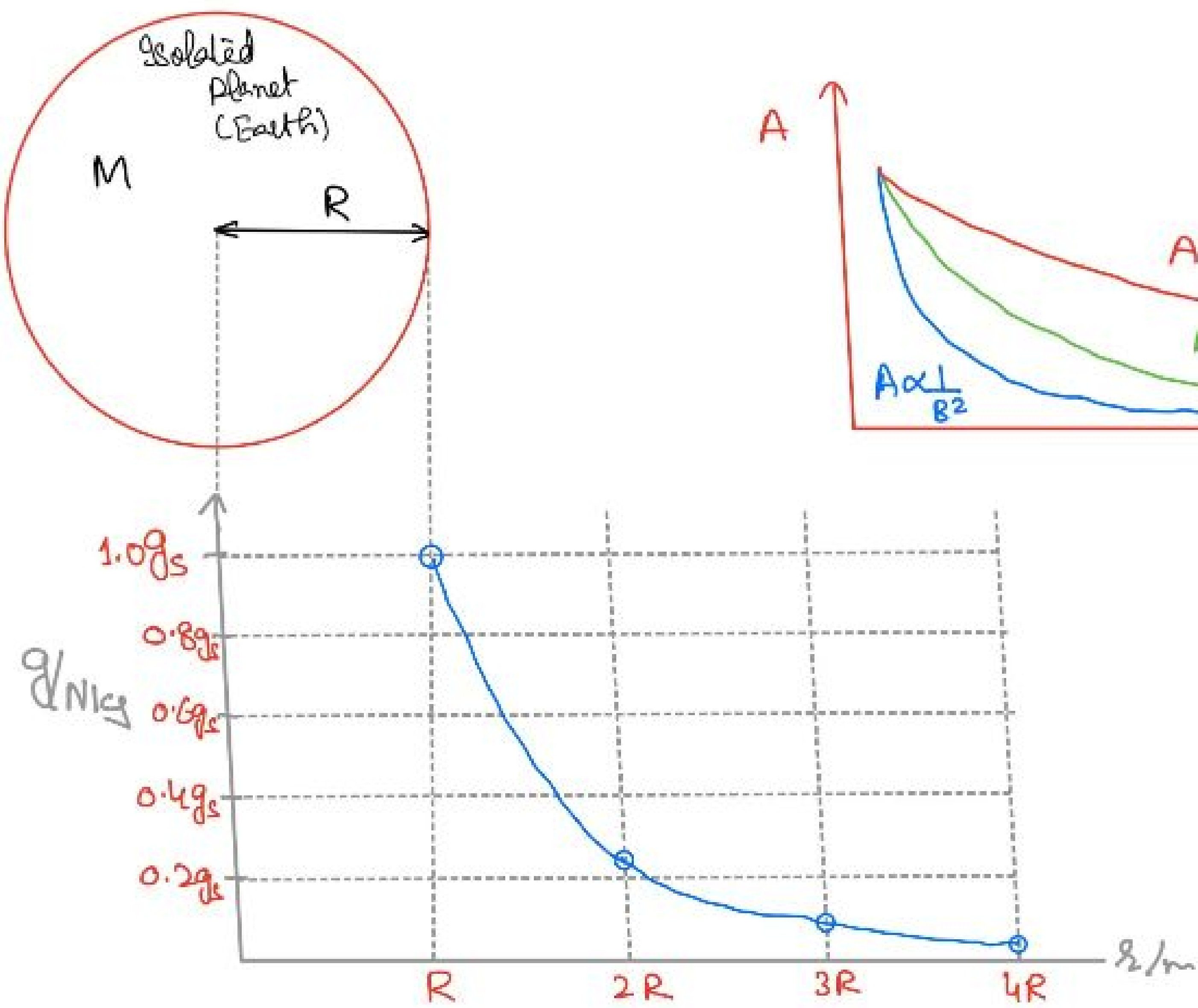
$$\frac{g_R}{g_s} = \frac{\frac{\cancel{GM}}{r^2}}{\frac{\cancel{GM}}{R^2}} \Rightarrow \frac{g_R}{g_s} = \frac{R^2}{r^2}$$

$$g_R = \frac{(g_s)(R^2)}{r^2} \Rightarrow g_R = \frac{\text{Constant}}{r^2} \Rightarrow g_R \propto \frac{1}{r^2}$$

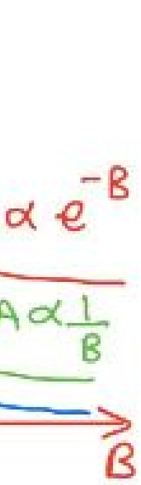
Therefore, value of g decreases by following inverse square relationship when moved away from surface of planet.

$$g_R = \frac{(g_s)(R^2)}{r^2}$$

r	R	2R	3R	4R
g_R	g_s	$0.25g_s$	$0.11g_s$	$0.063g_s$

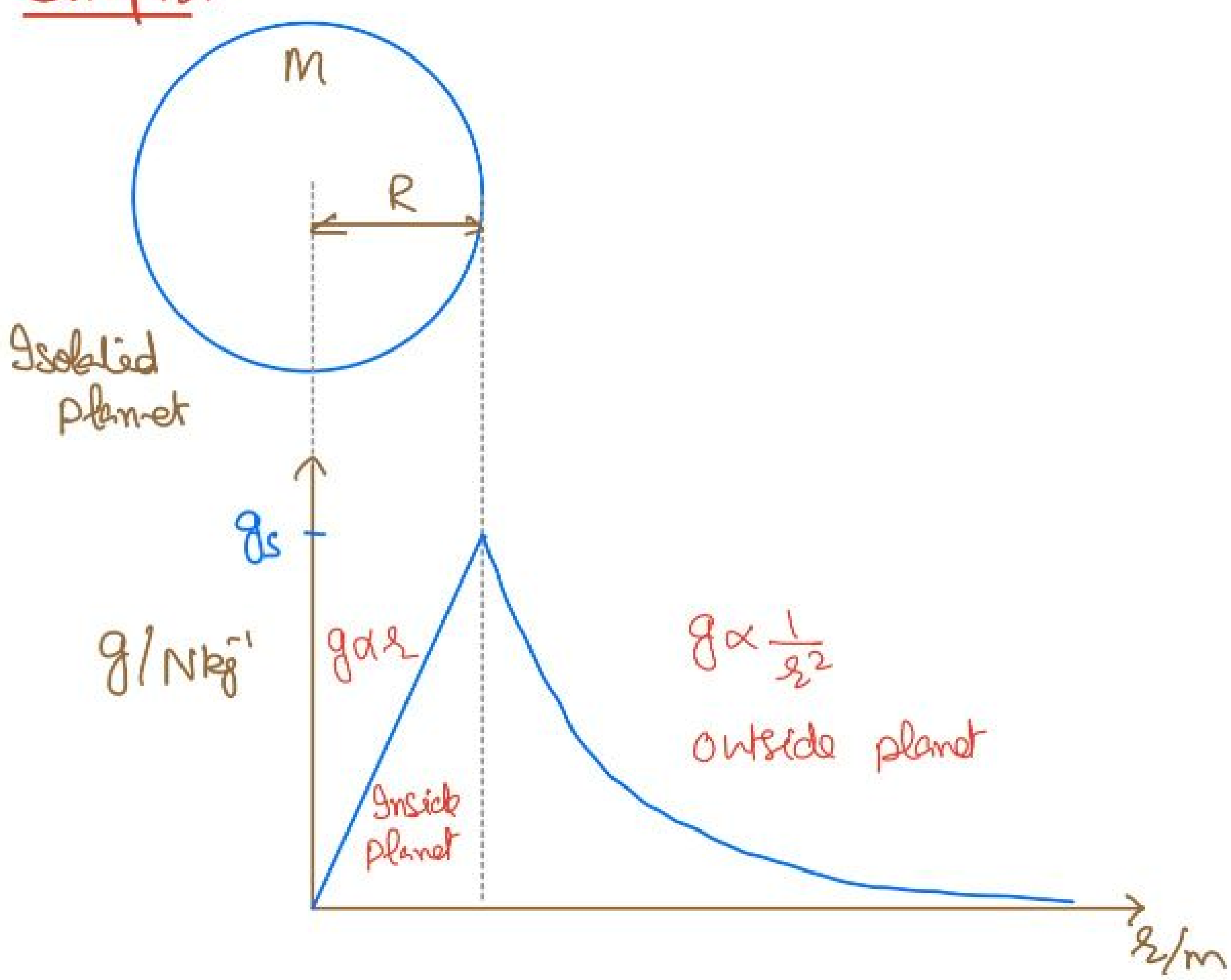


Isolated planet (Earth)
M
R
1.0g_s
0.8g_s
0.6g_s
0.4g_s
0.2g_s
g/Nkg
R
2R
3R
4R
r/m



A
$A \propto e^{-B}$
$A \propto \frac{1}{B}$
$A \propto \frac{1}{B^2}$
B

Graph:



M
R
Isolated planet
g_s
g/Nkg^{-1}
$g \propto r$
Inside planet
$g \propto \frac{1}{r^2}$
outside planet
r/m

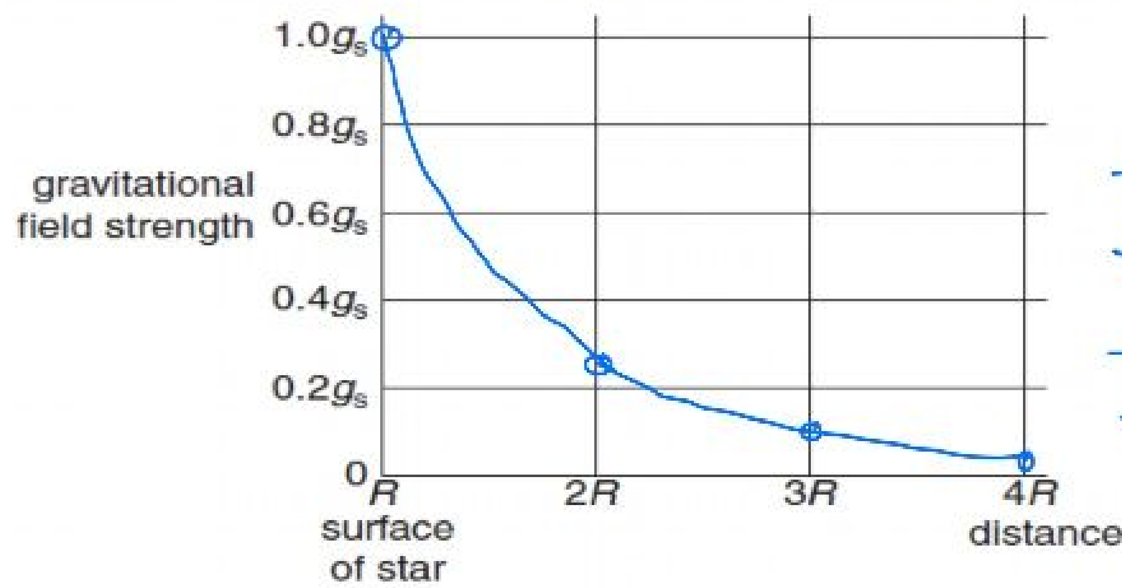
(a) Define *gravitational field strength*.

Gravitational force per unit mass.

.... [1]

(b) An isolated star has radius R. The mass of the star may be considered to be a point mass at the centre of the star.
The gravitational field strength at the surface of the star is g_s.

On Fig. 1.1, sketch a graph to show the variation of the gravitational field strength of the star with distance from its centre. You should consider distances in the range R to $4R$.



$g = \frac{GM}{r^2}$

$\rightarrow g_R = g_s$

$\rightarrow g_{2R} = 0.25$

$\rightarrow g_{3R} = 0.11$

$\rightarrow g_{4R} = 0.06$

Fig. 1.1

[2]

(c) The Earth and the Moon may be considered to be spheres that are isolated in space with their masses concentrated at their centres.
The masses of the Earth and the Moon are 6.00×10^{24} kg and 7.40×10^{22} kg respectively.
The radius of the Earth is R_E and the separation of the centres of the Earth and the Moon is $60 R_E$, as illustrated in Fig. 1.2.

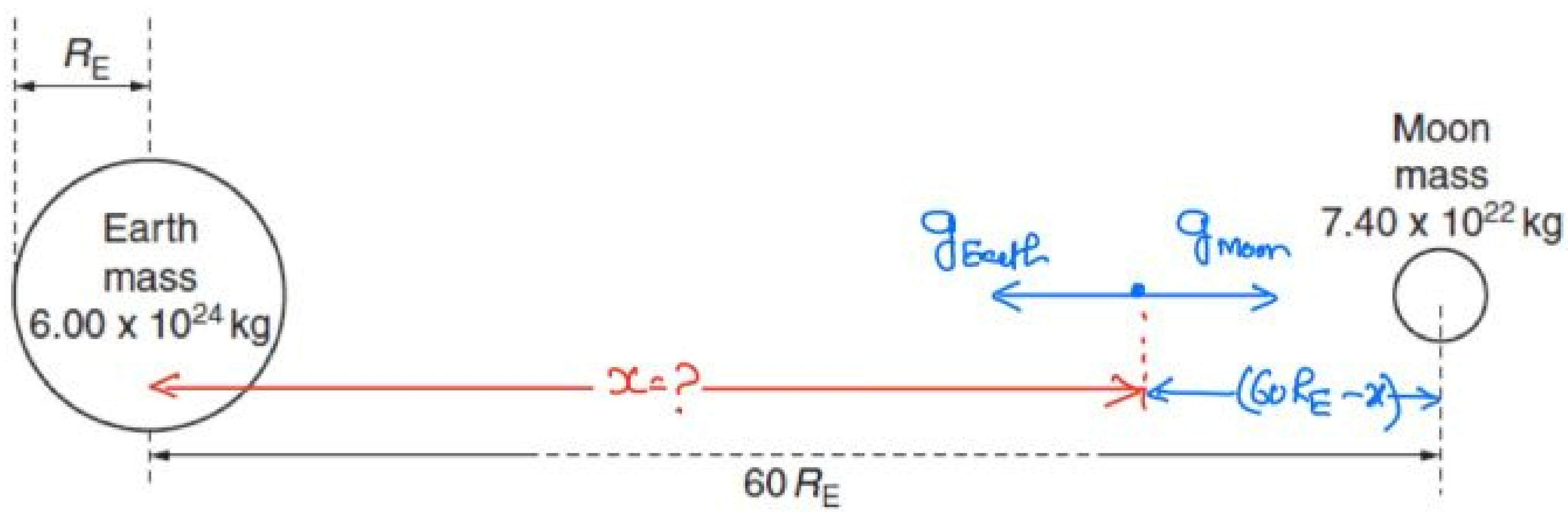


Fig. 1.2 (not to scale)

(i) Explain why there is a point between the Earth and the Moon at which the gravitational field strength is zero.

Since 'g' is a vector quantity, so magnitude of g for Earth and Moon is same but in opposite directions to cancel the effect! [2]

(ii) Determine the distance, in terms of R_E, from the centre of the Earth at which the gravitational field strength is zero.

Let distance from centre of Earth = x

$$g_{Earth} = g_{moon}$$

$$\frac{\cancel{G}M}{x^2} = \frac{\cancel{G}m}{(60R_E - x)^2}$$

$$\left(\frac{60R_E - x}{x}\right) = \sqrt{\frac{7.40\times10^{22}}{6.00\times10^{24}}} \Rightarrow x = 54R_E$$

distance = 54R_E R_E [3]

(iii) On the axes of Fig. 1.3, sketch a graph to show the variation of the gravitational field strength with position between the surface of the Earth and the surface of the Moon.

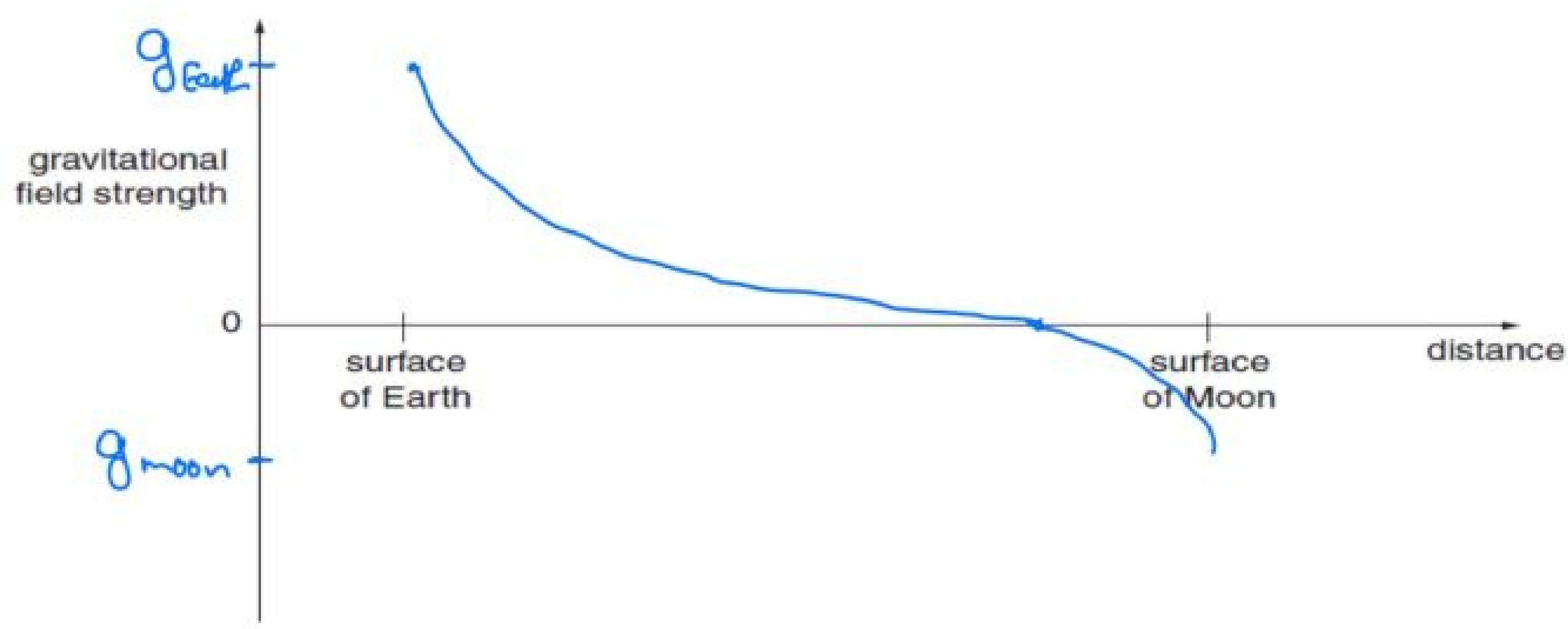


Fig. 1.3

[3]

{Nov 10/41 & 42/Q, 1}

Gravitational potential:-

Attractive field (Gravitational) — Position (potential)

$V = \frac{W}{Q}$ — Electric potential

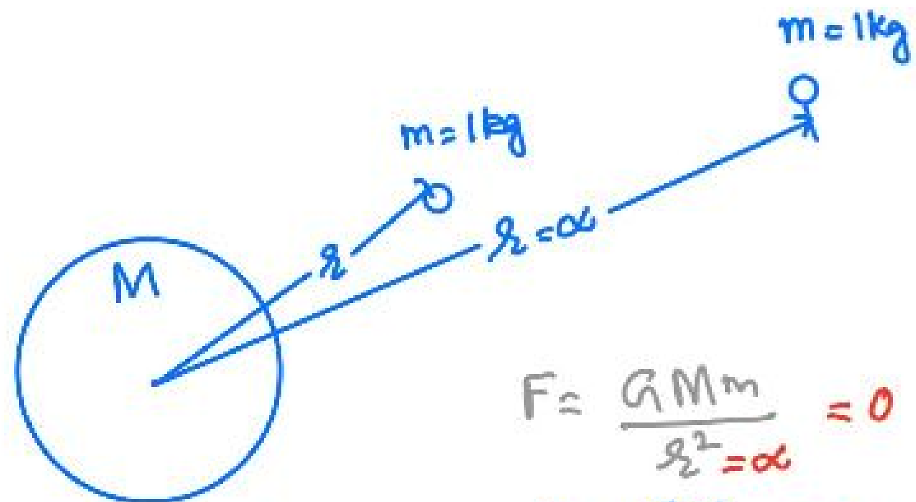
$\phi = \frac{W}{m}$

Def. Amount of work done per unit mass to bring it from infinity upto a point in the Gravitational field.

Symbol: ϕ

Formula: $\phi = \frac{W}{m}$

$$\phi = -\frac{GM}{r}$$



$F = \frac{GMm}{r^2 = \infty} = 0$

$g = \frac{GM}{r^2 = \infty} = 0$

Units: $J\,kg^{-1} = (kg\,m^2 s^{-2})(kg^{-1}) = m^2 s^{-2}$

P.S: Scalar

Significance of –ve sign:-

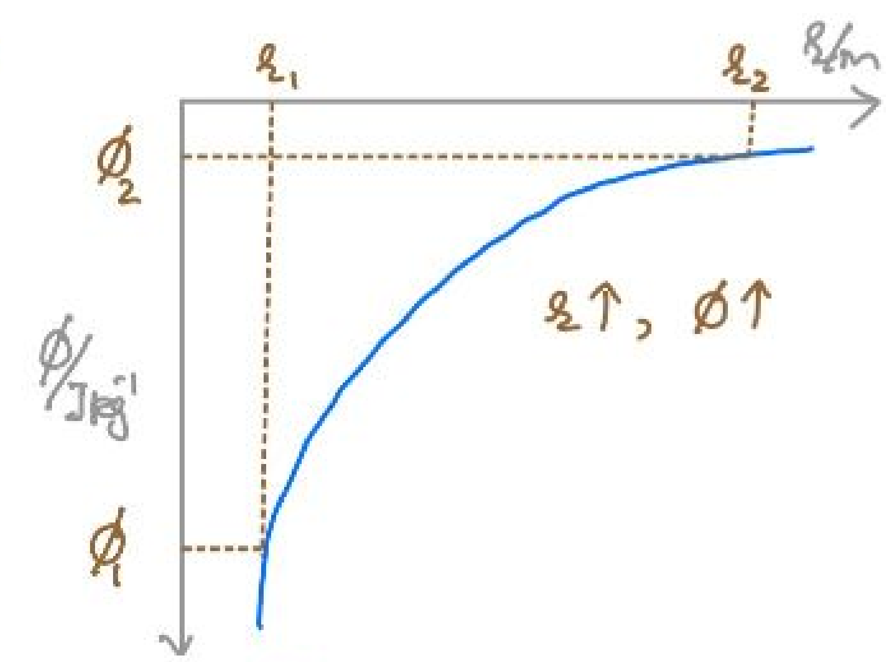
(i) **Conventional significance:-** Since Gravitational field is an attractive field and so is –ve by convention.

(ii) **Mathematical significance:-** $\phi = -\frac{GM}{r}$

* $r\uparrow$, $\left(\frac{GM}{r}\right)\downarrow$, $-\left(\frac{GM}{r}\right)\uparrow$ so $\phi\uparrow$
* $r = \infty$, $\phi = 0$ (Max)

Maximum value of Gravitational potential is zero at infinity and this value reduces from zero and becomes in –ve when unit mass is brought closer to planet.

Graph: $\phi = -\dfrac{GM}{r}$



Change of Gravitational potential:-

Since $\phi_2 > \phi_1$

$$\Delta\phi = \phi_2 - \phi_1$$
$$= -\frac{GM}{r_2} - \left(-\frac{GM}{r_1}\right)$$
$$\boxed{\Delta\phi = GM\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

Gravitational potential energy:-

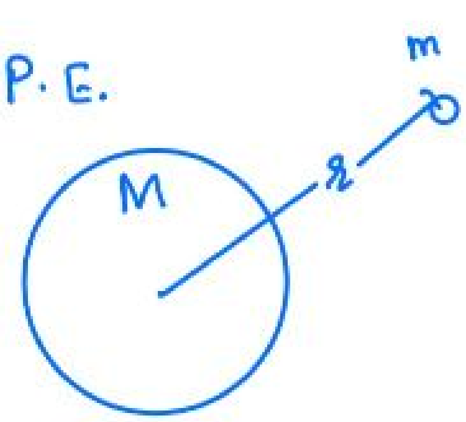
Gravitational → Attractive field; potential → Position; energy → Ability to do work

Def Ability of a mass to do work due to change of its position in an attractive field of another planet.

Symbol: E_p , GE_p , G.P.E. , P.E.

Formula: $\phi = \dfrac{W}{m}$

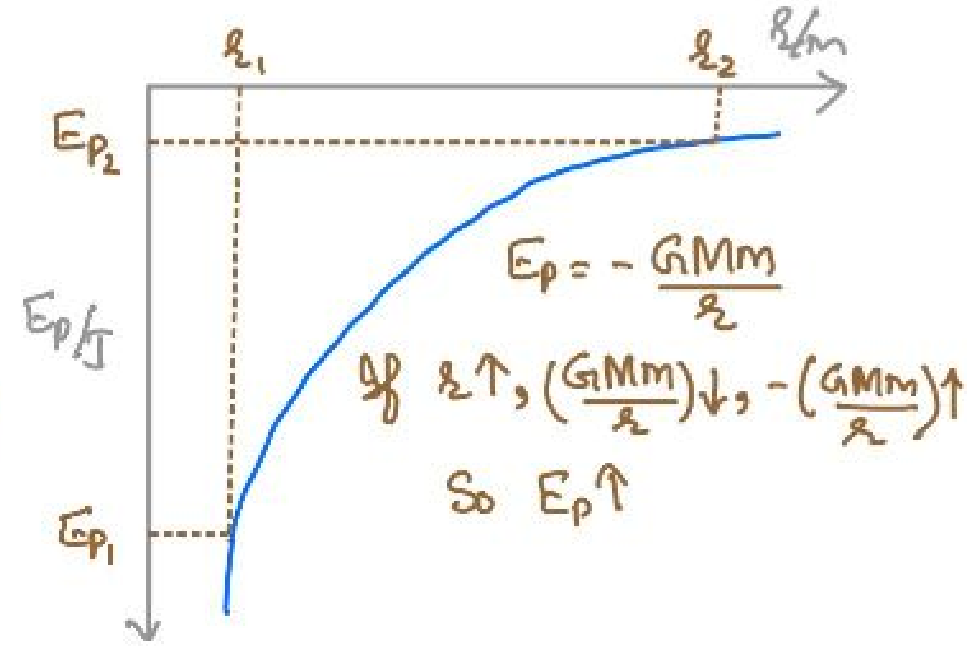
$$-\frac{GM}{r} = \frac{E_p}{m}$$



$$\boxed{E_p = -\frac{GMm}{r}}$$

Graph:

$E_p \uparrow$ as work is done against an attractive force to increase distance b/w masses.



Change of G.Ep:- Here $E_{p_1} < E_{p_2}$

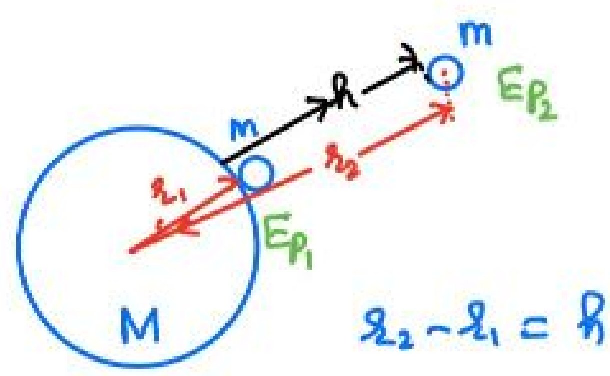
$$\Delta E_p = E_{p_2} - E_{p_1}$$
$$= -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$

$$\boxed{\Delta E_p = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]}$$

Proof of $\Delta E_p = mgh$ from $\Delta E_p = GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$:

Given: * Value of 'g' is constant at r_1 and at r_2.

$$\Delta E_p = E_{p_2} - E_{p_1}$$
$$= \frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right)$$



$$= GMm\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$
$$= GMm\left[\frac{r_2 - r_1}{r_1 r_2}\right] \Rightarrow \Delta E_p = GMm\left[\frac{h}{r_1 r_2}\right]$$

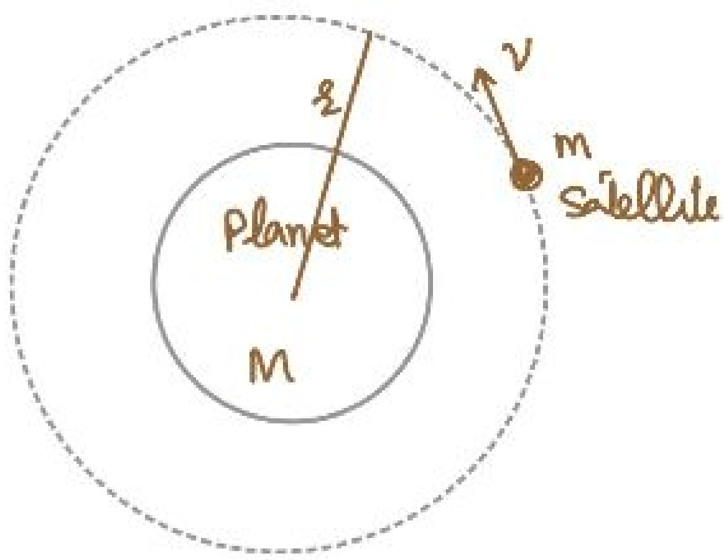
Since as per given condition, value of 'g' is constant at r_1 and at r_2 i.e $r_1 \approx r_2$

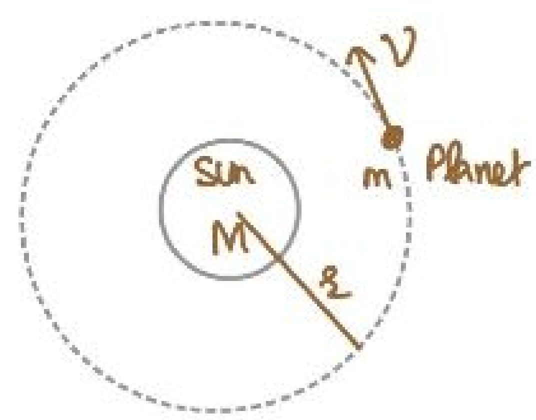
$$\Delta E_p = GMm\left[\frac{h}{r_1 r_1}\right]$$

$$= GMm\left[\frac{h}{r_1^2}\right] = m\left(\frac{GM}{r_1^2}\right)h$$

$$\boxed{\Delta E_p = mgh}$$

Orbiting speed of a Satellite (or planet in solar system)





Gravitational force provides centripetal force

$$F_G = F_c$$

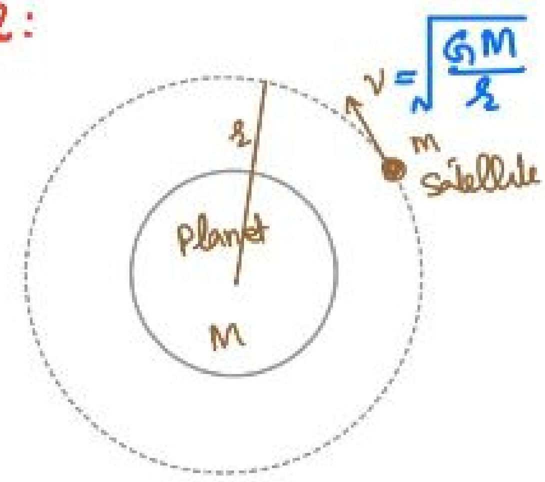
$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}} \Rightarrow \frac{GM}{r} = v^2$$

$$\boxed{v = \sqrt{\frac{GM}{r}}}$$

Also, $g = \frac{GM}{r^2} \Rightarrow gr = \frac{GM}{r}$

$$\boxed{v = \sqrt{gr}}$$

Kinetic energy of a satellite:



$$E_K = \frac{1}{2} m v^2$$

$$E_K = \frac{1}{2} m \left[\sqrt{\frac{GM}{r}}\right]^2$$

$$\boxed{E_K = \frac{GMm}{2r}}$$

Total energy of a satellite:

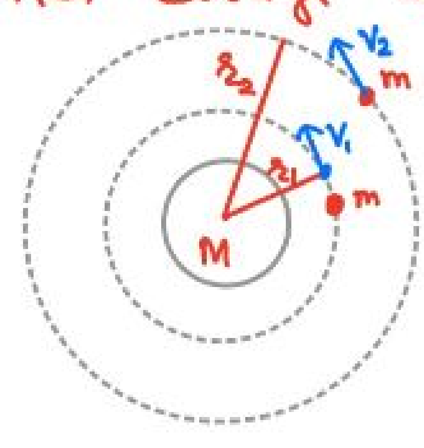
$$E_T = E_K + E_P$$

$$= \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

$$= \frac{GMm}{r}\left[\frac{1}{2} - 1\right]$$

$$\boxed{E_T = -\frac{GMm}{2r}}$$

Note: A satellite is moved away from Earth from lower to higher orbit. What change occur in its



i) velocity $v = \sqrt{\frac{GM}{r}}$

If $r \uparrow$, $v \downarrow$

(ii) Total energy.

$$E_T = -\frac{GMm}{2r}$$

If $r \uparrow$, $\left(\frac{GMm}{2r}\right) \downarrow$, $\left[-\frac{GMm}{2r}\right] \uparrow$ so $E_T \uparrow$

Escape speed of a Satellite:

The minimum speed with which a satellite is projected so that it is no longer under the Gravitational pull of planet.

Expression:

Initial E_K for projection = Final $G \cdot E_p$

$$\frac{1}{2}\cancel{m}v^2 = -\frac{GM\cancel{m}}{R}$$

$$v^2 = -\frac{2GM}{R}$$

-ve sign with $G \cdot E_p$ is neglected

$$\boxed{V = \sqrt{\frac{2GM}{R}}}$$

Also $g = \frac{GM}{R^2} \Rightarrow gR = \frac{GM}{R}$

$$\boxed{V = \sqrt{2gR}}$$

Value: From surface of Earth.

$$v = \sqrt{\frac{2GM}{R}}$$

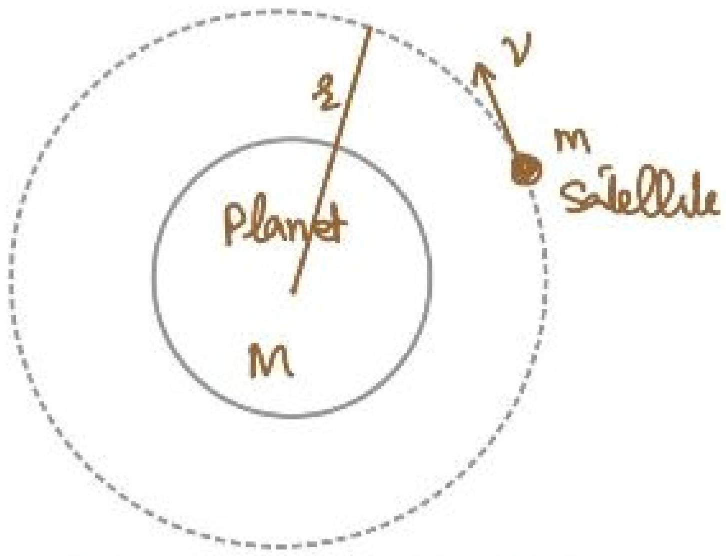
$$v = \sqrt{\frac{2(6.67\times10^{-11})(6.0\times10^{24})}{6.4\times10^{6}}}$$

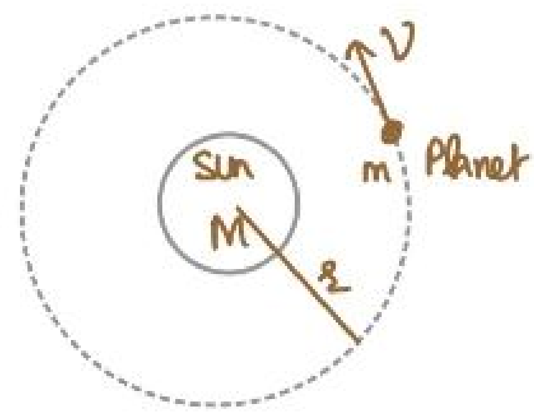
$$v = \sqrt{2gR}$$

$$v = \sqrt{2(9.81)(6.4\times10^{6})}$$

$V = 11200\,ms^{-1} \Rightarrow v = 11.2\,km\,s^{-1}$

Relationship b/w time period and radius of path traced by a satellite:- (Kepler's third law)





Gravitational force provides centripetal force

$$F_G = F_C$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m} r \omega^2$$

$$GM = r^3 \left(\frac{2\pi}{T}\right)^2$$

$$GM = r^3 \left(\frac{4\pi^2}{T^2}\right)$$

$$r^3 = \left(\frac{GM}{4\pi^2}\right) T^2$$

$$r^3 = (\text{Constant})\, T^2$$

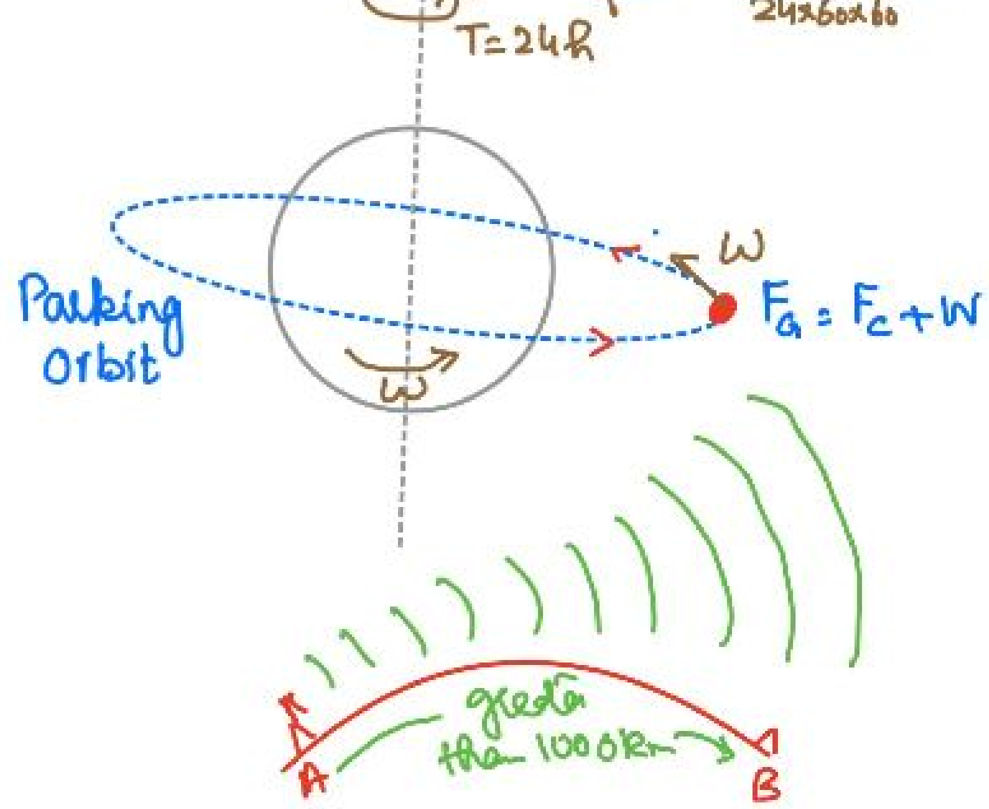
$$r^3 \propto T^2$$

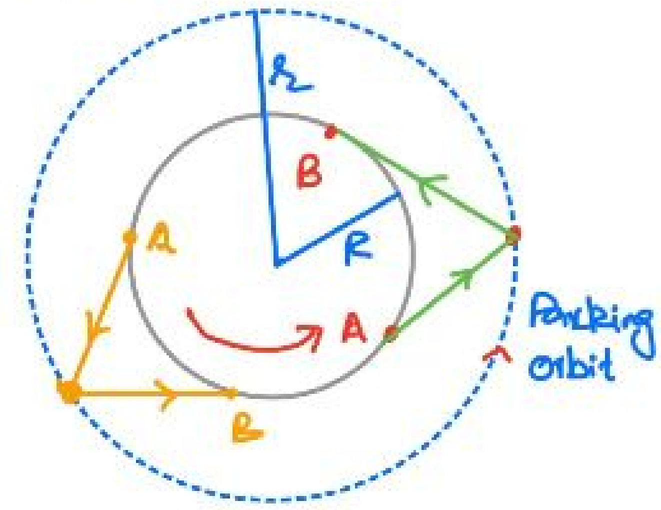
i.e (Cube of radius of circular path) $\propto$ (Square of time period)

$\underbrace{\text{Geo}}_{\text{Earth}}\underbrace{\text{stationary}}_{\text{Rest}}$ satellite :-

Satellite which revolves around Earth and look stationary relative to a position at the surface of Earth.

$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{24\times 60\times 60} = 7.27\times10^{-5}\ rad\ s^{-1}$$





$v = r\omega$

$v \propto r$ (constant)

$v_{satellite} > v_{Earth}$

<u>Note</u>:

1- The time period ($T = 24h$) and angular velocity ($\omega = \frac{2\pi}{T} = 7.27\times10^{-5}\ rad\ s^{-1}$) of Geostationary satellite and a point at the surface of Earth are same.

2. The Geostationary is always projected from equator in the direction of rotation of Earth.

   (i) from equator $\longrightarrow$ to get centripetal force $F_G = F_c + W$

   (ii) in the direction of rotation of Earth $\longrightarrow$ to remain stationary relative to Earth

3. The linear velocity of Geostationary satellite is greater than a point at the surface of Earth due to greater radius by $v = r\omega$ i.e $v \propto r$

4. Geostationary satellite is used for 24 hours live transmission without tracking antenna.

Parking orbit: Equtorial path around Earth at which Geostationary satellite move to provide 24 hours Line transmission.

$$F_G = F_c$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m} r \omega^2$$

$$GM = r^3 \left(\frac{2\pi}{T}\right)^2$$

$$r = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3}$$

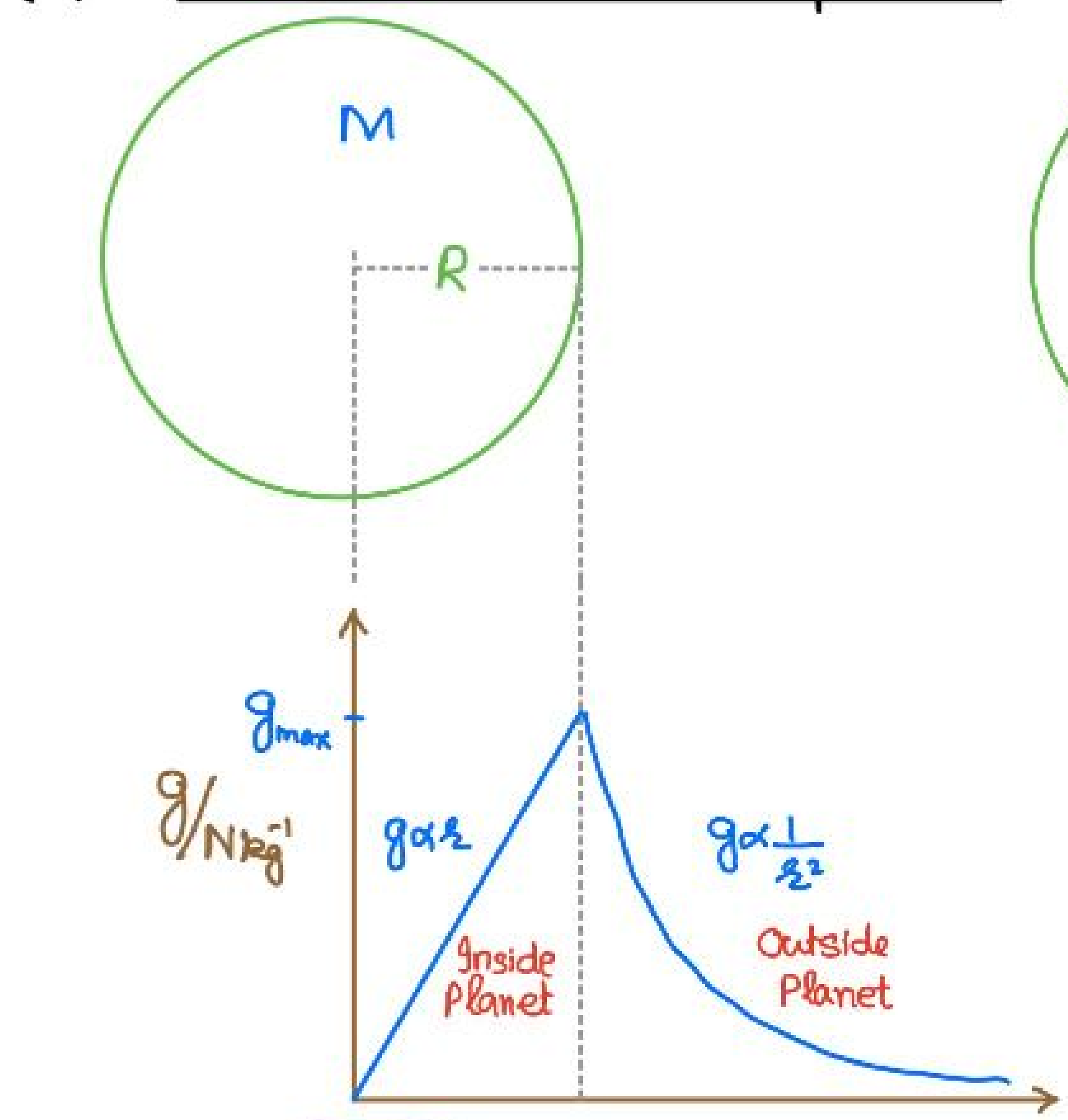
$$r = \left[\frac{(6.67\times10^{-11})(6.0\times10^{24})(24\times60\times60)^2}{4(3.14)^2}\right]^{1/3}$$

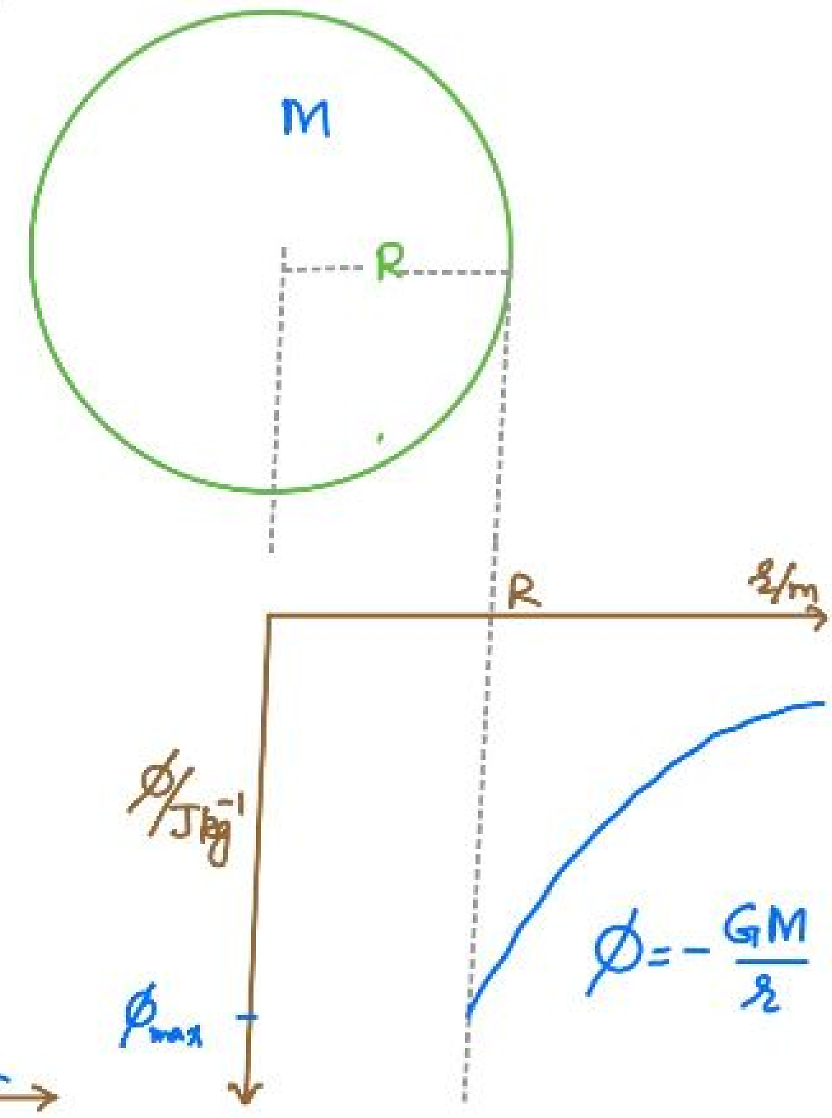
$$r = 4.23\times10^7 m$$

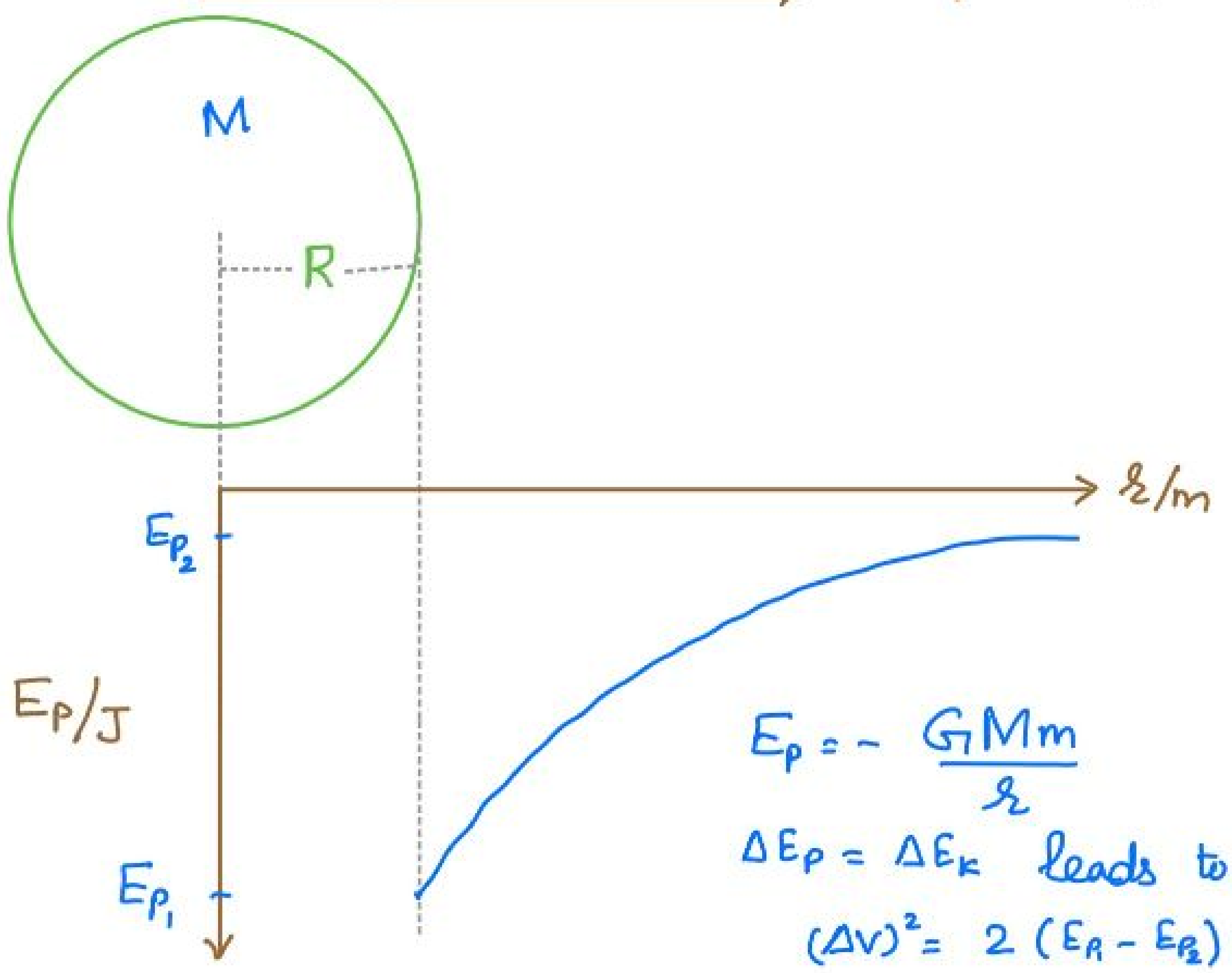
$r = 6.6R$ i.e 6.6 (Radius of Earth)

Graph: Summary

(a) For an isolated planet:-





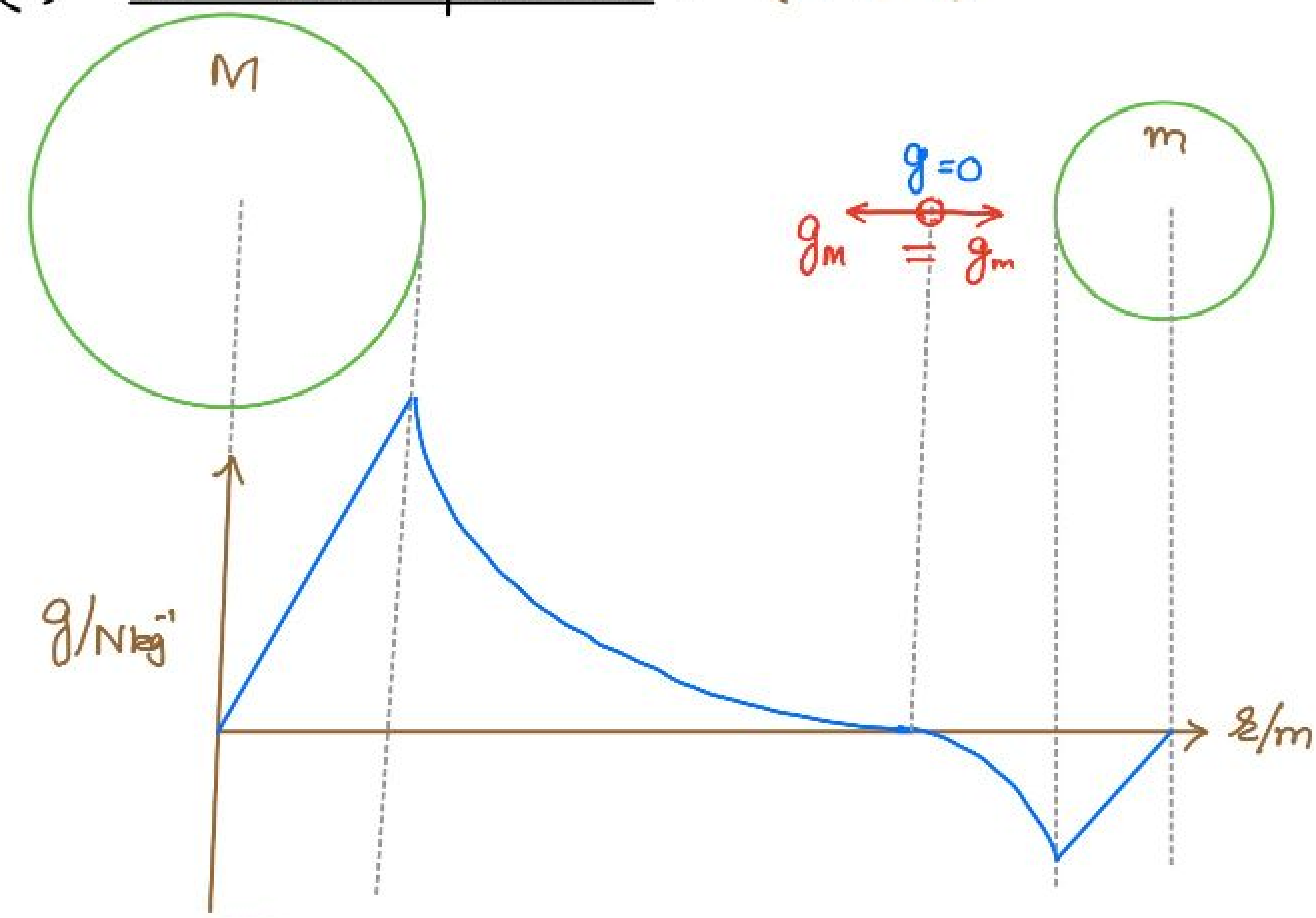


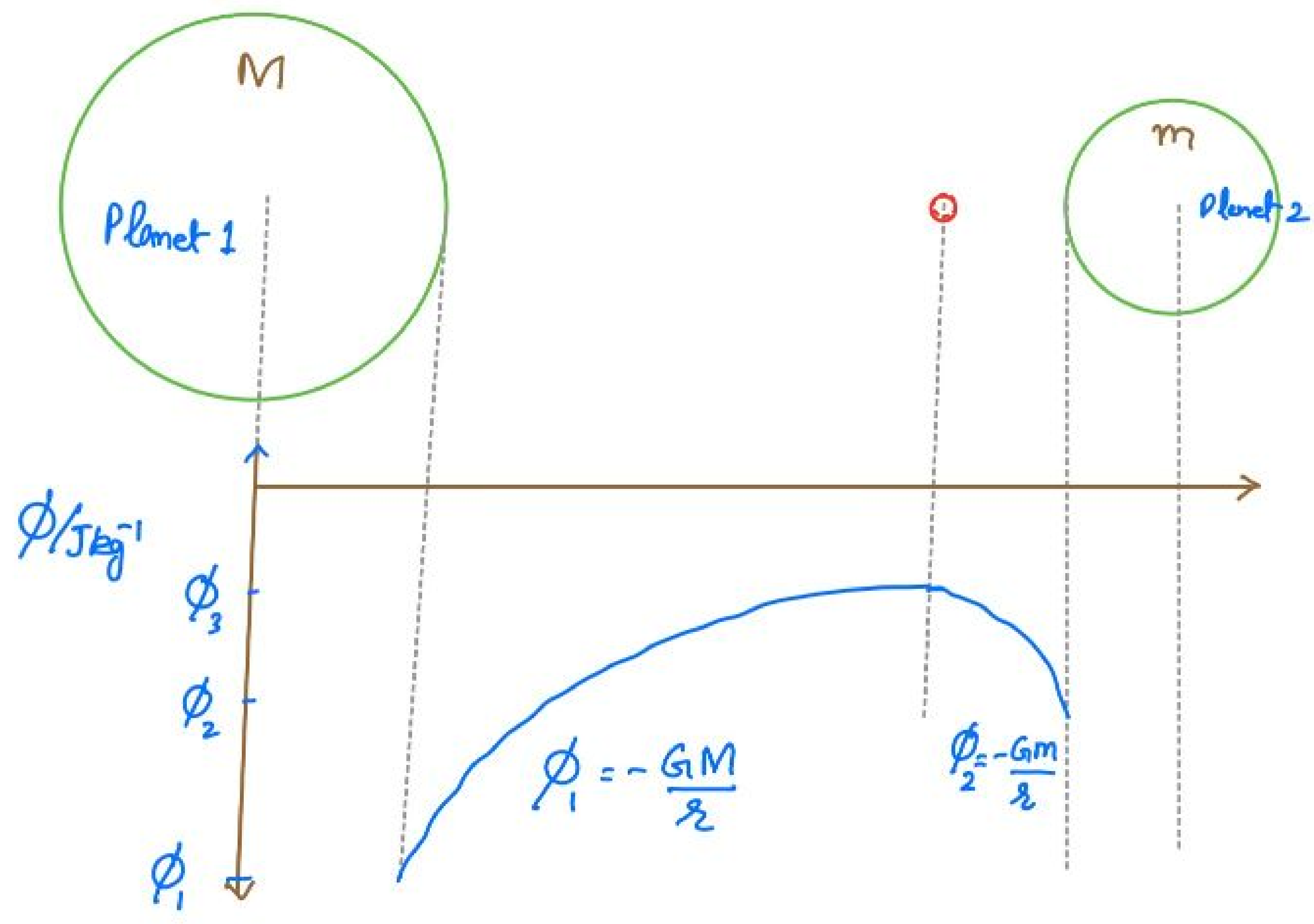
$E_p = -\frac{GMm}{r}$

$\Delta E_P = \Delta E_K$ leads to

$(\Delta v)^2 = 2(E_{P_1} - E_{P_2})$

(b) For two planets :- (M > m)





Change in speed between planet 1 and 2

$$\phi_2 - \phi_1 = \frac{1}{2}(\Delta v)^2$$

$$\Delta v = \sqrt{2(\phi_2 - \phi_1)}$$

Minimum speed:

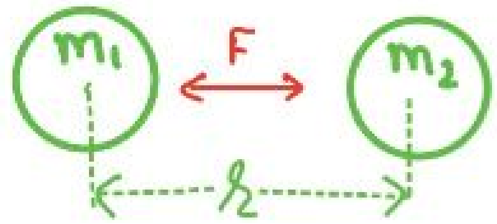
$$v_{min} = \sqrt{2(\phi_3 - \phi_2)}$$

(1) Gravitational field: 3D region around a mass in which a unit mass esperience an attractive force.

(2) Representation of G-field: By field lines which represent the direction along which a unit mass esperience a force.

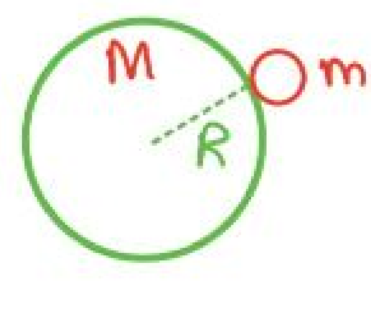
(3) Newton's Law of Gravitation:-

$$F = \frac{G m_1 m_2}{r^2}$$



$$G = 6.67 \times 10^{-11}\ Nm^2 kg^{-2}$$

(4) Mass of Earth:



$$F_G = W$$

$$M = \frac{gR^2}{G}$$

$$M = 5.98 \times 10^{24}\ kg$$

(6) Gravitational field strength:

$$g = \frac{GM}{R^2} \implies g_{Earth} = 9.81\,N\,kg^{-1}$$

* Outside Earth: $g \propto \frac{1}{r^2}$

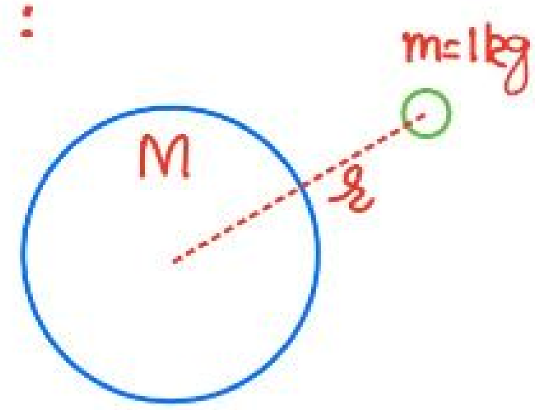
* At equator: $F_G = W + F_c$
* At poles: $F_c = 0$

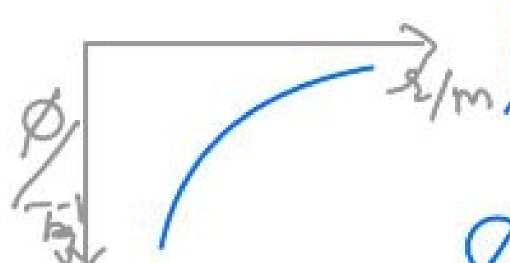
$$F_G = W$$

So $g_{Pole} > g_{equator}$ due to spinning of Earth

(7) Gravitational potential:

$$\phi = \frac{W}{m}$$



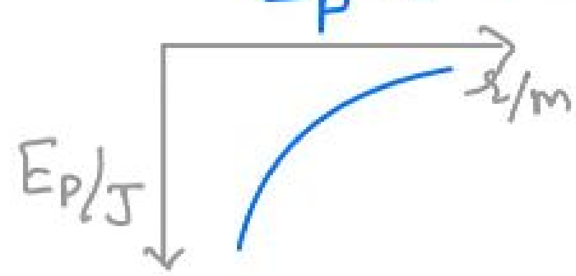


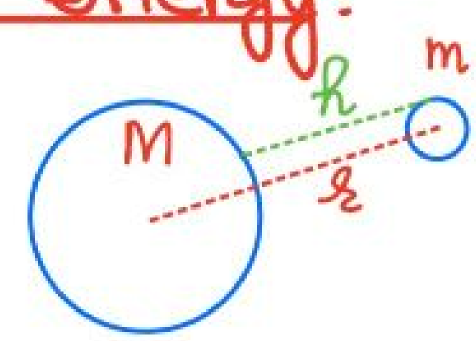
$$\phi = -\frac{GM}{r}$$

$\phi \uparrow$ as $r \uparrow$ as work is done against an attractive force.

(8) Gravitational potential energy:

$$E_p = -\frac{GMm}{r} = mgh$$





(9) Orbiting speed of a Satellite:

$$F_G = F_c$$
$$v = \sqrt{\frac{GM}{r}}$$

(10) Kinetic energy of a Satellite:

$$E_K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

(11) Total energy of a satellite:

$$E_T = E_K + E_P$$
$$E_T = -\frac{GMm}{2r}$$

(12) Escape speed of a satellite:

$$E_K = E_P \Rightarrow v = \sqrt{\frac{2GM}{R}} = \sqrt{2gR}$$

(13) Relation between radius and time period of a satellite:

$$F_G = F_c$$
$$r^3 = \frac{GMT^2}{4\pi^2}$$
$$r^3 \propto T^2$$