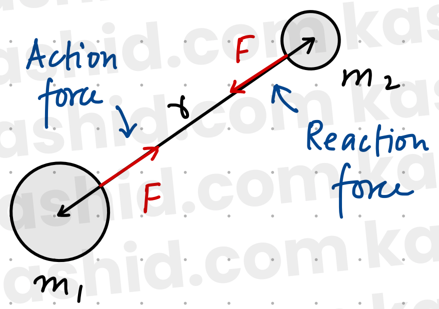
Gravitation

Newton's Law of Universal Gravitation

The force of attraction between two point masses is directly proportional to their product and inversely proportional to the square of their seperation.



$F \propto m_1 m_2$ $\quad$ $F \propto \frac{1}{r^2}$

$$F \propto \frac{m_1 m_2}{r^2}$$

$$\boxed{F = G\frac{m_1 m_2}{r^2}}$$

m_1 m_2: masses
r: center to center distance b/w bodies
F: force

G: Universal Gravitational Constant

$$\boxed{G = 6.67 \times 10^{-11}\ Nm^2kg^{-2}}$$

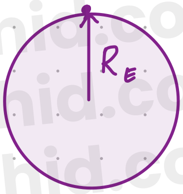
$m_1 = m_2 = 1kg$
$r = 1\,m$

$F = G\frac{(1)(1)}{(1)^2}$

$F = 6.67 \times 10^{-11}\ N$
$(67\,pN)$

$m_1 = 60\ kg$
$m_E = 5.99 \times 10^{24}\ kg$
$R_E = 6.4 \times 10^6\ m$

$F = ??$



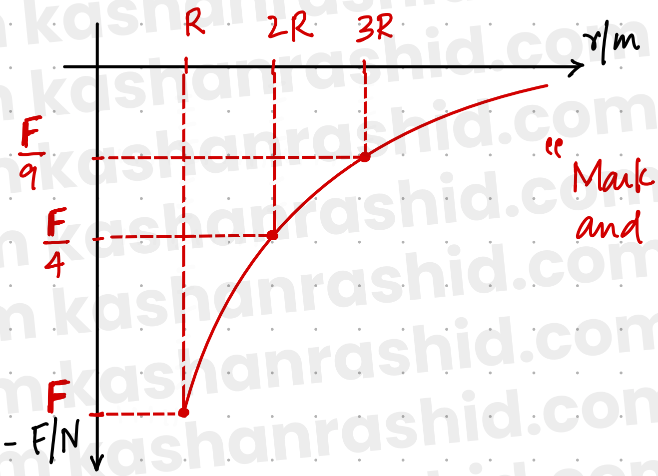
$$F = \frac{6.67\times10^{-11} \times 60 \times 5.99 \times 10^{24}}{(6.4\times10^6)^2}$$

$F = 585\ N$
Weight!

$F = G\frac{m_1 m_2}{r^2}$

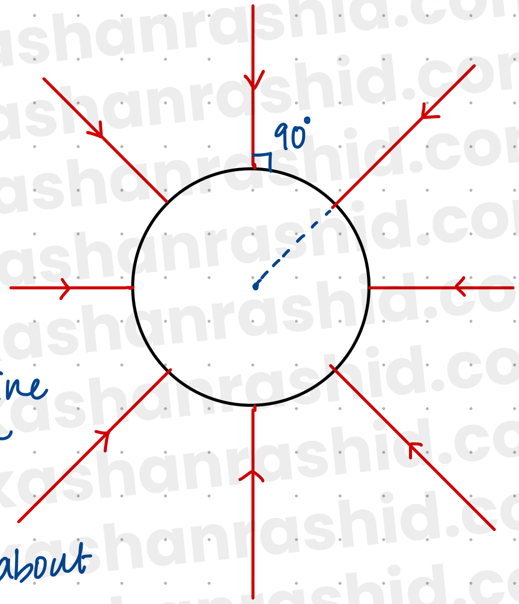
$F \propto \frac{1}{r^2}$

(inverse square law)



"Mark the co-ordinates first and then draw the curve."

Gravitational Field



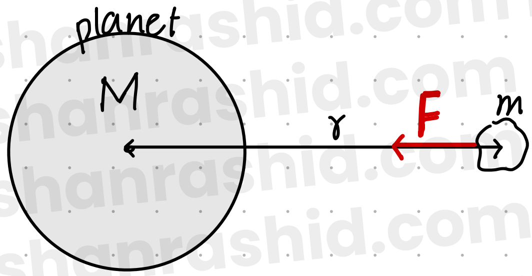
A region in space around a mass where another mass experiences gravitational force.

- ☑ The direction of grav. field line tells the direction of force on a mass.
- ☑ Gap between field lines tell about the strength of field.

less gap → strong field

more gap → weak field

Gravitational Field Strength (g)



Force per unit mass at a point in a gravitational field.

$g = \frac{F}{m}$ so $g = \frac{GMm}{r^2 \times m}$

$$\boxed{g = \frac{GM}{r^2}}$$

SI Unit: $N kg^{-1}$

Vector quantity

$g \propto M$

Mass inc.
- → Moon : $g = 1.6$ N/kg
- → Earth $g = 9.8$ N/kg
- → Jupiter $g = 22$ N/kg

$g \propto \frac{1}{r^2}$

→ The further we move away, the weaker the field gets and hence "g" decreases.

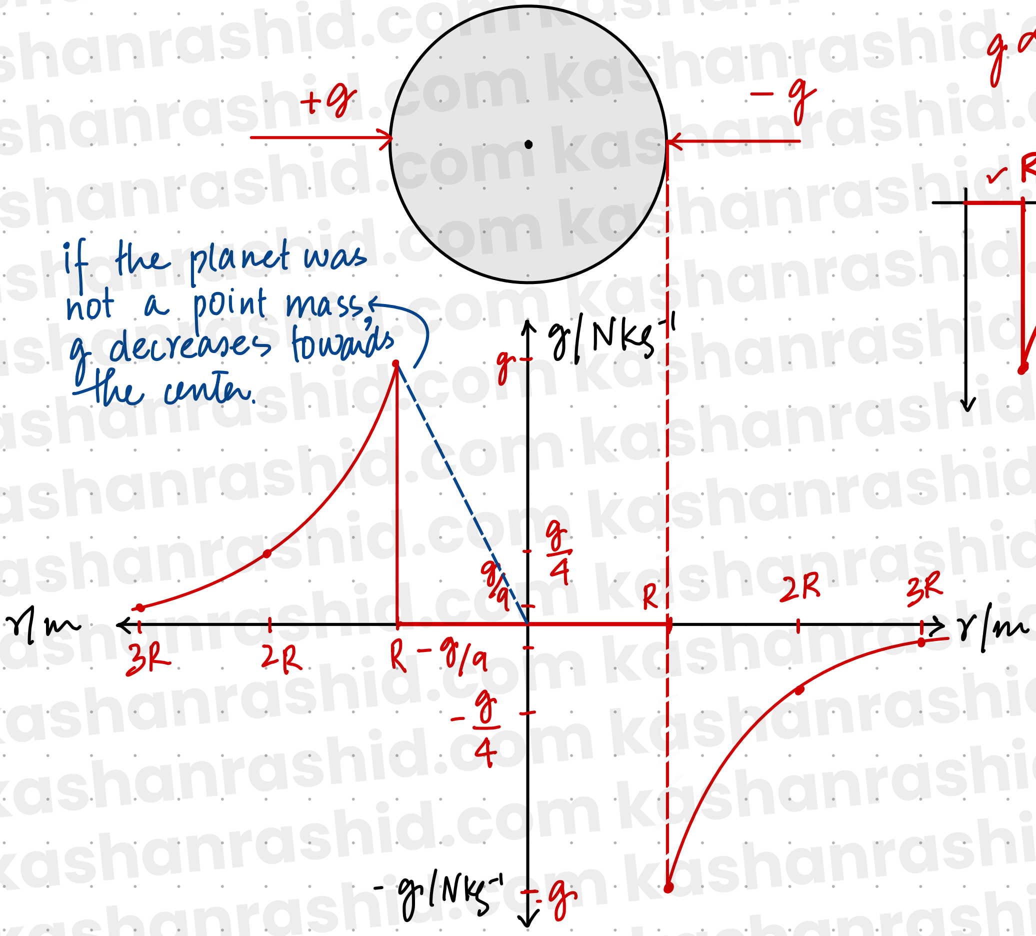
Graph of g-r for a point mass

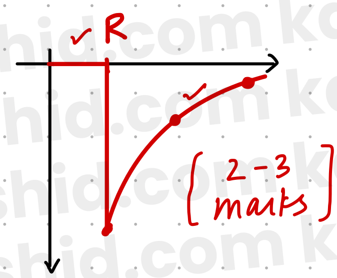
What is a point mass?

- Planets and all other objects are treated as a small point as if all of their mass is concentrated in the middle.

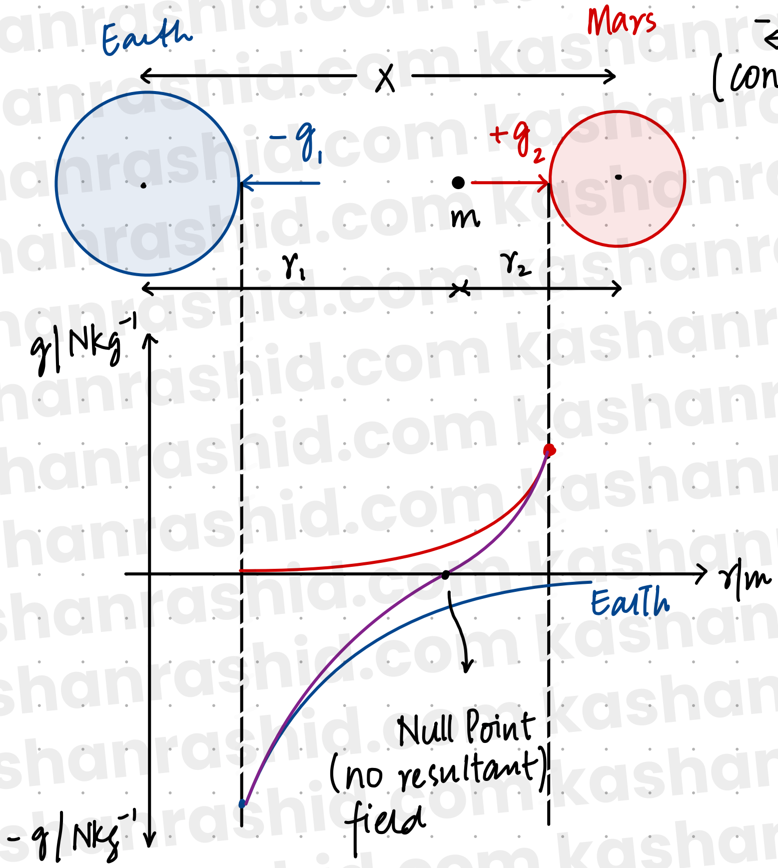
It is justified, as the distance between bodies is very large compared to their sizes.

"There is no gravitational field or force inside the point mass."





Gravitational field strength between two point masses



$\overset{-}{\leftarrow} \overset{+}{\rightarrow}$ (convention)

$$g = \frac{GM}{r^2}$$

$$\left(g \propto \frac{1}{r^2}\right)$$

resultant

$$g = g_2 - g_1$$

$$g = \frac{GM_2}{r_2^2} - \frac{GM_1}{r_1^2}$$

$$0 = \frac{GM_2}{r_2^2} - \frac{GM_1}{r_1^2}$$

$$\frac{\cancel{G}M_1}{r_1^2} = \frac{\cancel{G}M_2}{r_2^2}$$

$$\boxed{\frac{M_1}{M_2} = \left(\frac{r_1}{r_2}\right)^2}$$

* Null point exists near the body with weaker gravitational field strength.

Section A

Answer **all** the questions in the spaces provided.

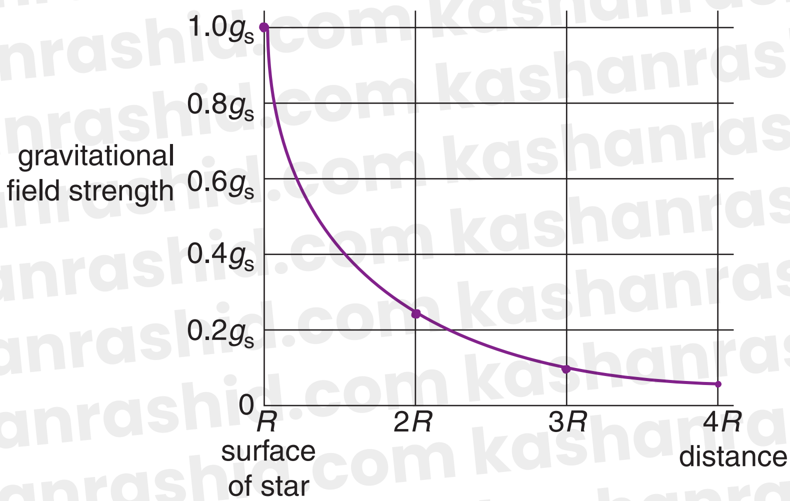
For Examiner's Use

1 (a) Define *gravitational field strength.*

Gravitational force per unit mass at a point in a gravitational field. [1]

(b) An isolated star has radius R. The mass of the star may be considered to be a point mass at the centre of the star.
The gravitational field strength at the surface of the star is g_s.

On Fig. 1.1, sketch a graph to show the variation of the gravitational field strength of the star with distance from its centre. You should consider distances in the range R to $4R$.



$\frac{g}{4} = 0.25g$

$\frac{g}{9} = 0.11g$

$\frac{g}{16} = 0.0625$

Fig. 1.1

[2]

(c) The Earth and the Moon may be considered to be spheres that are isolated in space with their masses concentrated at their centres.
The masses of the Earth and the Moon are 6.00×10^{24} kg and 7.40×10^{22} kg respectively.
The radius of the Earth is R_E and the separation of the centres of the Earth and the Moon is $60\,R_E$, as illustrated in Fig. 1.2.

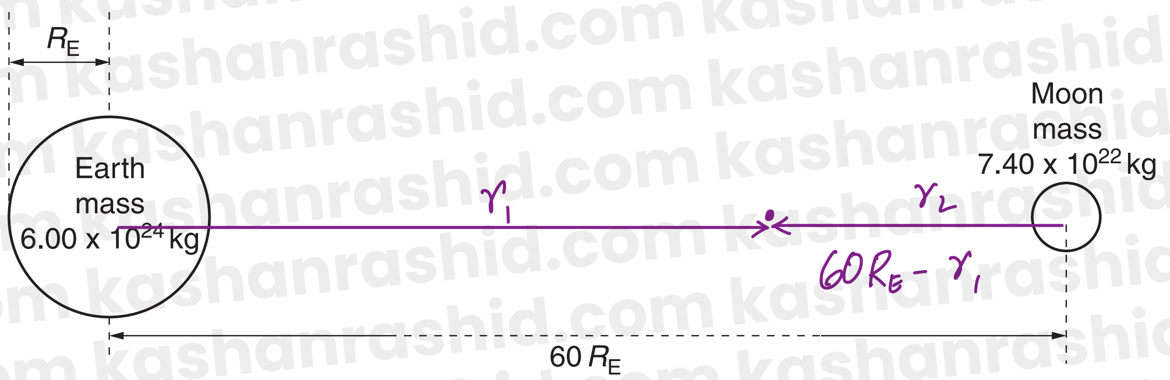


Fig. 1.2 (not to scale)

(i) Explain why there is a point between the Earth and the Moon at which the gravitational field strength is zero.

At a point between them gravitational field strengths of both are equal and opposite because $g \propto \frac{1}{r^2}$. [2]

(ii) Determine the distance, in terms of R_E, from the centre of the Earth at which the gravitational field strength is zero.

at resultant $g = 0$, $g_1 = g_2$

$g_1 = g_2$

$\frac{GM_1}{r_1^2} = \frac{GM_2}{r_2^2}$

$\frac{6 \times 10^{24}}{r_1^2} = \frac{7.4 \times 10^{22}}{(60R_E - r_1)^2}$

$\sqrt{\frac{(60R_E - r_1)^2}{r_1^2}} = \sqrt{\frac{7.4 \times 10^{22}}{6.0 \times 10^{24}}}$

$\frac{60R_E - r_1}{r_1} = 0.111$

$60R_E = 0.111r_1 + r_1$

$r_1 = 54R_E$

distance = ... R_E [3]

(iii) On the axes of Fig. 1.3, sketch a graph to show the variation of the gravitational field strength with position between the surface of the Earth and the surface of the Moon.

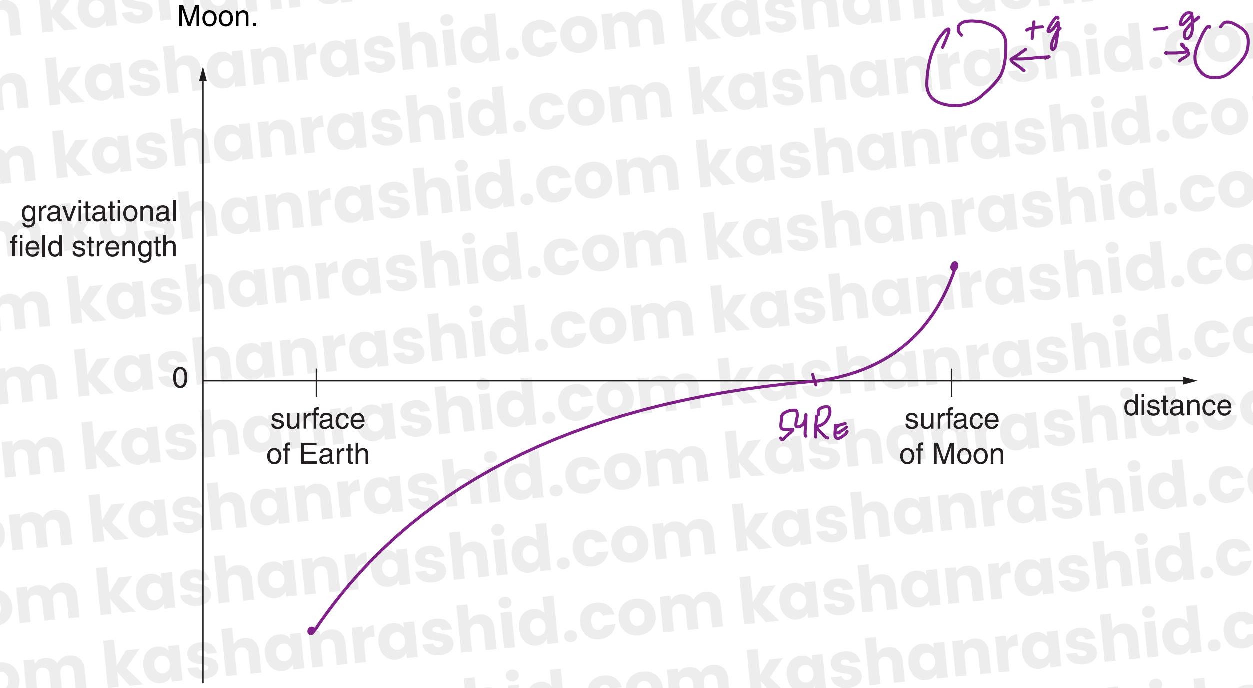
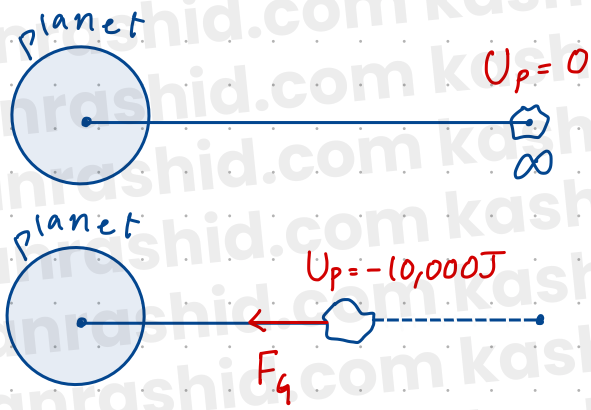


Fig. 1.3

[3]

Gravitational Potential Energy (U_p)



- Workdone in moving an object from infinity to a point in a gravitational field.

- Due to gravitational force, the rock travels towards the planet and covers distance parallel to it. Hence work is done by the rock as it gets closer to the planet.

"Potential energy is not the amount of energy a body has. It is the amount of energy a body needs to escape gravitational field and reach infinity."

$$\boxed{U_p = -\frac{GMm}{r}}$$

Gravitational Potential (ϕ)

"It is the workdone per unit mass to move an object from infinity to a point in a gravitational field."

$$\phi = \frac{U_p}{m} \text{ so}$$

$$\phi = -\frac{GM\cancel{m}}{r \times \cancel{m}} \text{ hence}$$

It is the energy required per kg to escape from the planet's gravitational field from that point.

$$\boxed{\phi = -\frac{GM}{r}}$$

$$\boxed{U_p = \phi \times m}$$

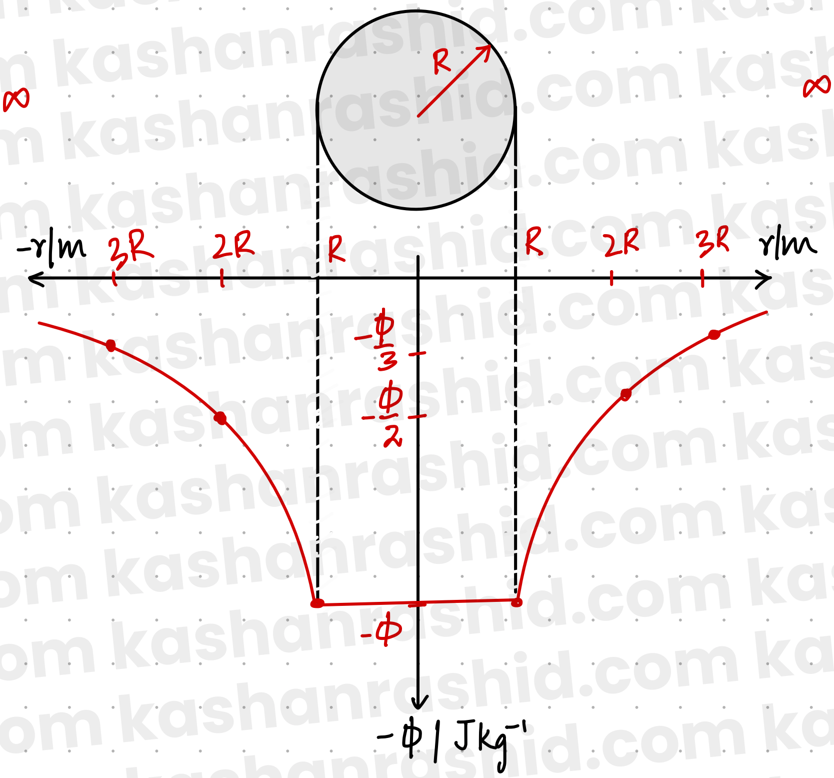
e.g. $\phi = -100,000 \, J/kg$ means 1 kg object needs 100,000J to escape gravitational field.

$$\boxed{\Delta U_p = \Delta \phi \times m}$$

Why is gravitational potential always negative?

- ☑ Gravitational potential is zero at infinity
- ☑ Due to attractive force, work is done by the mass as object moves towards the planet.
- ☑ Therefore gravitational potential decreases and is thus negative.

$\phi - r$ graph for a point mass



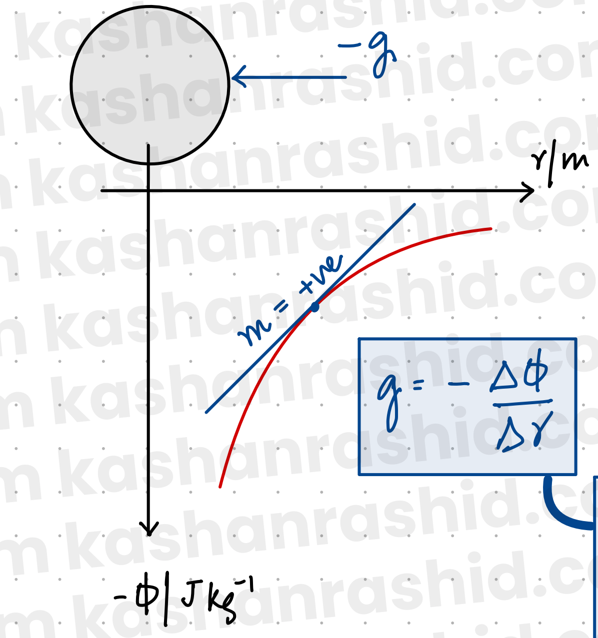
$$\phi = -\frac{GM}{r}$$

$$\phi \propto \frac{1}{r}$$

r/m	ϕ / J kg^{-1}
R	ϕ
2R	$\frac{\phi}{2}$
3R	$\frac{\phi}{3}$

Why is ϕ constant inside the point mass?

- ☐ Inside a point mass, $g = 0$. Hence $F = 0$.
- ☐ As Workdone = $F \times s$ and $F = 0$ so no more work is done and there is no more loss in GPE.
- ☐ Hence it stays constant.



$$grad = \frac{\Delta y}{\Delta x} = \frac{\Delta \phi}{\Delta r} = -g$$

$$\phi = -\frac{GM}{r}$$

$$\boxed{g = -\frac{\Delta \phi}{\Delta r}}$$

$$\frac{\phi}{r} = \frac{-GM}{r \times r} = -\frac{GM}{r^2}$$

Gravitational Field Strength is equal to the negative of gravitational potential gradient.

ϕ_1 (less negative) $-100{,}000\,J/kg$

ϕ_2 (more negative) $-150{,}000\,J/kg$

loss in GPE = gain in KE

$$\Delta\phi \times m = \frac{1}{2}m(v^2 - u^2)$$

$$\boxed{\Delta\phi = \frac{1}{2}(v^2 - u^2)}$$

$$\boxed{\phi_2 - \phi_1 = \frac{1}{2}(v^2 - u^2)}$$

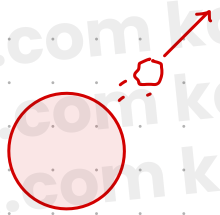
$$GPE = -\frac{GMm}{r}$$

$$\phi = -\frac{GM}{r}$$

$$\phi \times m = GPE$$

$$\boxed{\Delta\phi \times m = \Delta GPE}$$

change in GPE = change in KE



loss in GPE = gain in K.E

$$-\Delta\phi \times m = \frac{1}{2}m(v^2 - u^2)$$

gain in GPE = loss in K.E

$$\Delta\phi \times m = -\frac{1}{2}m(v^2 - u^2)$$

(b) The Earth may be considered to be an isolated sphere of radius R with its mass concentrated at its centre.

The variation of the gravitational potential ϕ with distance x from the centre of the Earth is shown in Fig. 1.1.

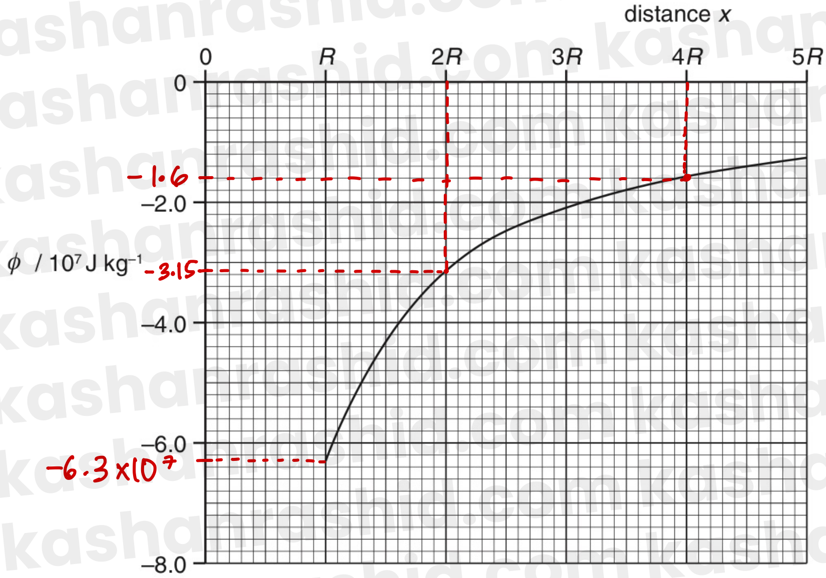


Fig. 1.1

The radius R of the Earth is 6.4×10^6 m.

$G = 6.67\times10^{-11}$ Nm^{2}kg^{-2}

a) Using the ϕ at the surface of Earth, find out the mass of Earth.

b) Calculate the speed of rock travelling from $4R$ to $2R$, at $2R$.

(Given: speed at $4R$) $v = 100$ ms^{-1}

c) Minimum speed needed to reach $4R$ from the surface of Earth

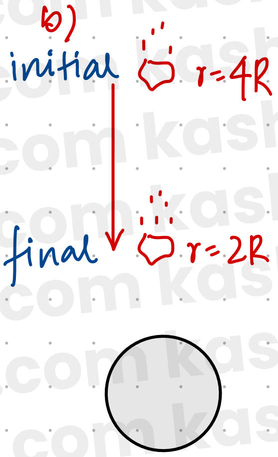
d) The minimum speed required to escape Earth's grav. field from the surface.

a) $\phi = -\dfrac{GM}{r}$

$$-6.3\times10^7 = -\frac{6.67\times10^{-11}\times M}{6.4\times10^6}$$

$M = 6.04\times10^{24}$ kg

b)



loss in GPE = gain in K.E

$-\Delta\phi \times m = \frac{1}{2}m(v^2-u^2)$

$-(\phi_f - \phi_i) = \frac{1}{2}(v^2-u^2)$

$-(-3.15\times10^7 - (-1.6\times10^7)) = \frac{1}{2}(v^2 - 100^2)$

$v = 5568.66$ m/s
(5600 m/s)

c)

gain in GPE = loss in KE

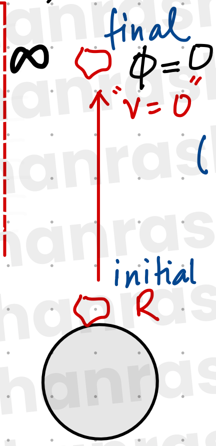
$\Delta\phi \times m = -\frac{1}{2}m(v^2-u^2)$

$(\phi_f - \phi_i) = -\frac{1}{2}(v^2-u^2)$, with $v = 0$

$(-1.6\times10^7 - (-6.3\times10^7)) = -\frac{1}{2}(-u^2)$

$u = 9695.36$ m/s
(9700 m/s)

d)



$\Delta\phi \times m = -\frac{1}{2}m(v^2-u^2)$

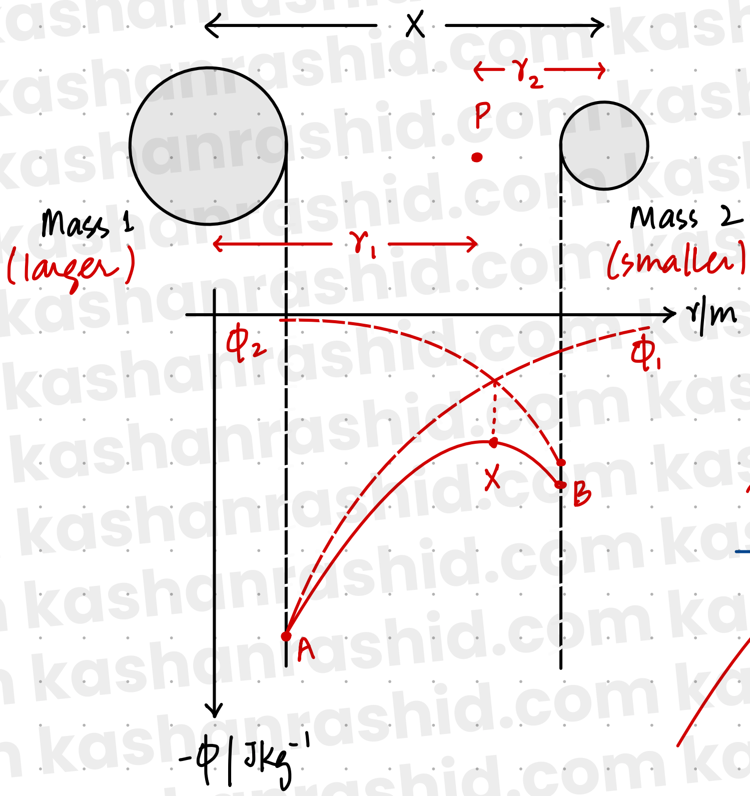
$(\phi_f - \phi_i) = -\frac{1}{2}(v^2-u^2)$

$0-(-6.3\times10^7) = -\frac{1}{2}(0-u^2)$

$u = 11224.76$ m/s

11 km/s

ϕ-r graph between two point masses



$\phi = -\frac{GM}{r}$ $\left(\phi \propto \frac{1}{r}\right)$

"ϕ" is a scalar quantity

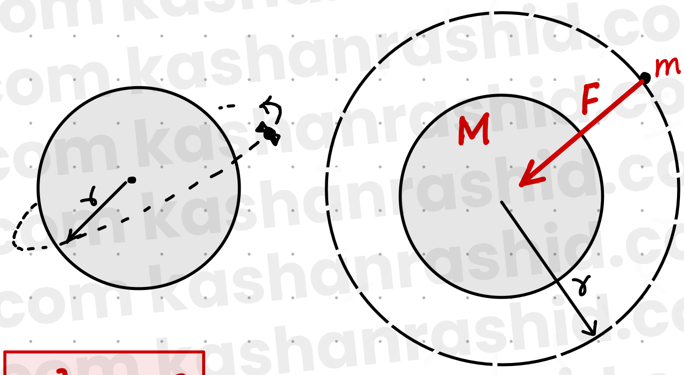
$$\boxed{\phi_{res} = \phi_1 + \phi_2}$$

$$\phi_{res} = -\frac{GM_1}{r_1} + \left(-\frac{GM_2}{r_2}\right)$$

where X: Null point

Orbital Motion

Gravitational force acts as a centripetal force.



$$F_G = F_c$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}r\omega^2$$

$$GM = r^3\omega^2$$

$$GM = r^3\left(\frac{2\pi}{T}\right)^2$$

$$GM = r^3 \times \frac{4\pi^2}{T^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{GM}r^3}$$

$$\boxed{T^2 \propto r^3}$$

T: time period

r: radius of orbit

Answer **all** the questions in the spaces provided.

1 (a) By reference to the definition of gravitational potential, explain why gravitational potential is a negative quantity.

Gravitational potential is zero at infinity. Due to attractive nature of force, work is done by the mass in moving from infinity to any point. ϕ decreases & is therefore negative. [2]

(b) Two stars A and B have their surfaces separated by a distance of 1.4×10^{12} m, as illustrated in Fig. 1.1.

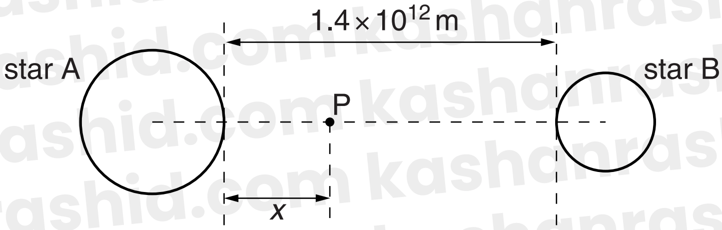


Fig. 1.1

Point P lies on the line joining the centres of the two stars. The distance x of point P from the surface of star A may be varied.

The variation with distance x of the gravitational potential ϕ at point P is shown in Fig. 1.2.

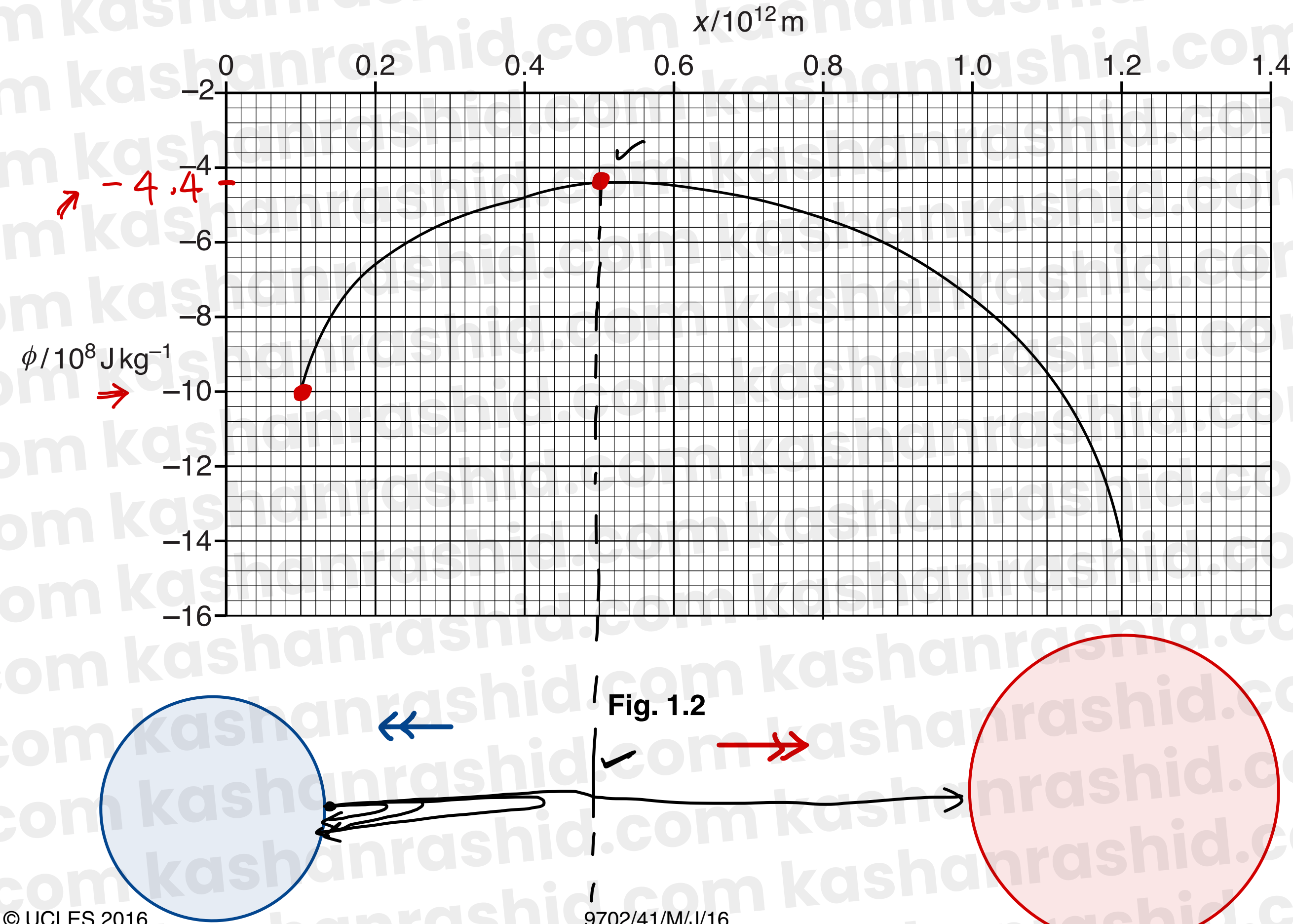


Fig. 1.2

A rock of mass 180 kg moves along the line joining the centres of the two stars, from star A towards star B.

(i) Use data from Fig. 1.2 to calculate the change in kinetic energy of the rock when it moves from the point where $x = 0.1 \times 10^{12}$ m to the point where $x = 1.2 \times 10^{12}$ m.
State whether this change is an increase or a decrease.

gain in KE = loss in PE

gain in KE $= -\Delta\phi \times m$

$= -\left(-14\times10^8 - (-10\times10^8)\right) \times 180$

$= 7.2 \times 10^{10}$ J

change = 7.2×10^{10} J

increase

[3]

(ii) At a point where $x = 0.1 \times 10^{12}$ m, the speed of the rock is v.

Determine the minimum speed v such that the rock reaches the point where $x = 1.2 \times 10^{12}$ m.

gain in GPE = loss in KE

$\Delta\phi \times m = -\frac{1}{2} m (v^2 - u^2)$

$(\phi_f - \phi_i) = -\frac{1}{2}(v^2 - u^2)$

$-4.4\times10^8 - (-10\times10^8) = -\frac{1}{2}(0 - u^2)$

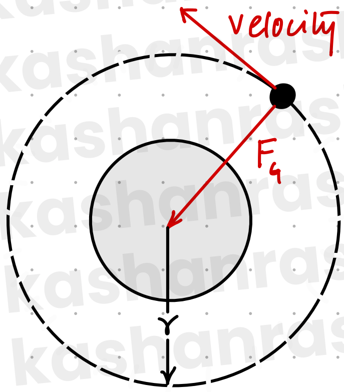
$u = 3.3 \times 10^4\ ms^{-1}$

minimum speed = ... $m s^{-1}$ [3]

[Total: 8]

Orbital Motion

During orbital motion, gravitational force acts as a centripetal force.



$$F_g = F_c$$

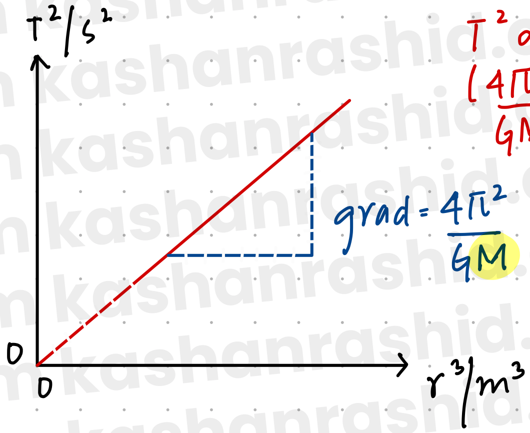
$$\frac{GMm}{r^2} = mr\omega^2$$

$$\frac{GM}{r^3} = \omega^2$$

$$GM = r^3\left(\frac{2\pi}{T}\right)^2$$

$$GM = r^3 \cdot \frac{4\pi^2}{T^2}$$

$$\boxed{T^2 = \frac{4\pi^2}{GM} r^3}$$



grad = $\frac{4\pi^2}{GM}$

$T^2 \propto r^3$

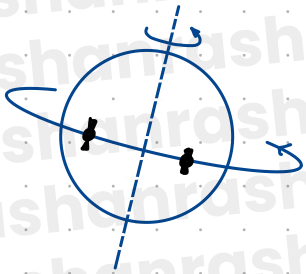
$\left(\frac{4\pi^2}{GM} : \text{constant}\right)$

$$\underbrace{T^2}_{y} = \underbrace{\frac{4\pi^2}{GM}}_{m} \underbrace{r^3}_{x}$$

$y = mx + c$

Geostationary Satellite

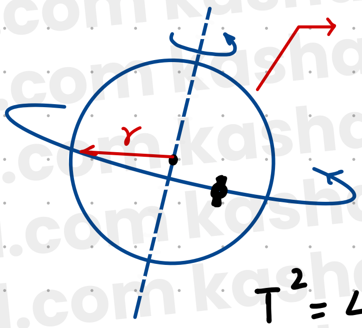
- Satellites in a geostationary orbit
- They have the same angular speed as that of the Earth
- And they move in the same direction as Earth's rotation. i.e. West to East.



→ As they move with the Earth, they appear to be stationary above the Earth's surface.

3 marks !!

Geostationary Orbit
1. It is an equitorial orbit (i.e. above equator)
2. Satellite in this orbit has the same angular speed as of Earth
3. It moves in the same direction as rotation of Earth i.e. West to East.



Earth
mass : 5.99×10^{24} kg
G : 6.67×10^{-11}

Time Period of orbit : 24 hours (86400 seconds)

$$T^2 = \frac{4\pi^2}{GM} r^3$$

$$(24 \times 3600)^2 = \frac{4\pi^2 \times r^3}{6.67 \times 10^{-11} \times 5.99 \times 10^{24}}$$

so $\boxed{r = 4.23 \times 10^7 \text{ m}}$

linear speed of a geostationary satellite..

$$v = r\omega$$

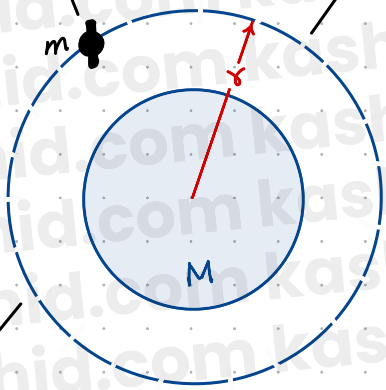
$$v = (4.23\times10^{7})\left\{\frac{2\pi}{24\times3600}\right\}$$

$$v = 3076.1 \simeq 3100\,m/s$$

Energy in a Satellite

Gravitational Potential energy

$$\boxed{U_p = -\frac{GMm}{r}}$$



Kinetic Energy

$$E_K = \frac{1}{2}mv^2$$

As grav. force acts as a centripetal force.

$$F_g = F_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$\frac{GM}{r} = v^2$$

$$E_K = \frac{1}{2}mv^2$$

$$E_K = \frac{1}{2}m\left(\frac{GM}{r}\right)$$

or

$$\boxed{E_K = \frac{GMm}{2r}}$$

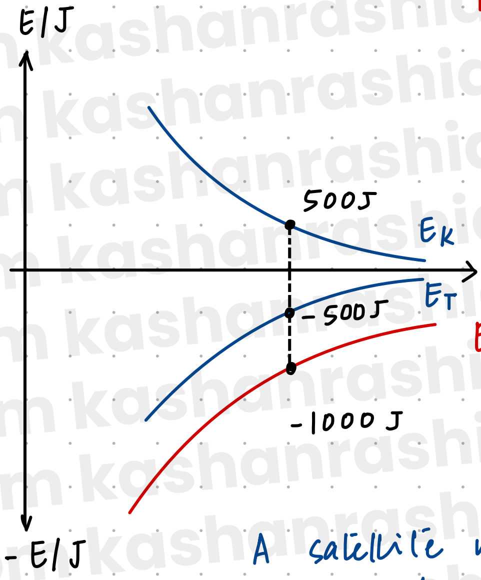
Total Energy

$$E_T = E_K + E_p$$

$$= \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

$$= \frac{GMm}{r}\left(\frac{1}{2} - 1\right)$$

$$\boxed{E_T = -\frac{GMm}{2r}}$$



A satellite with a smaller radius of orbit possesses more K.E and hence orbits faster.