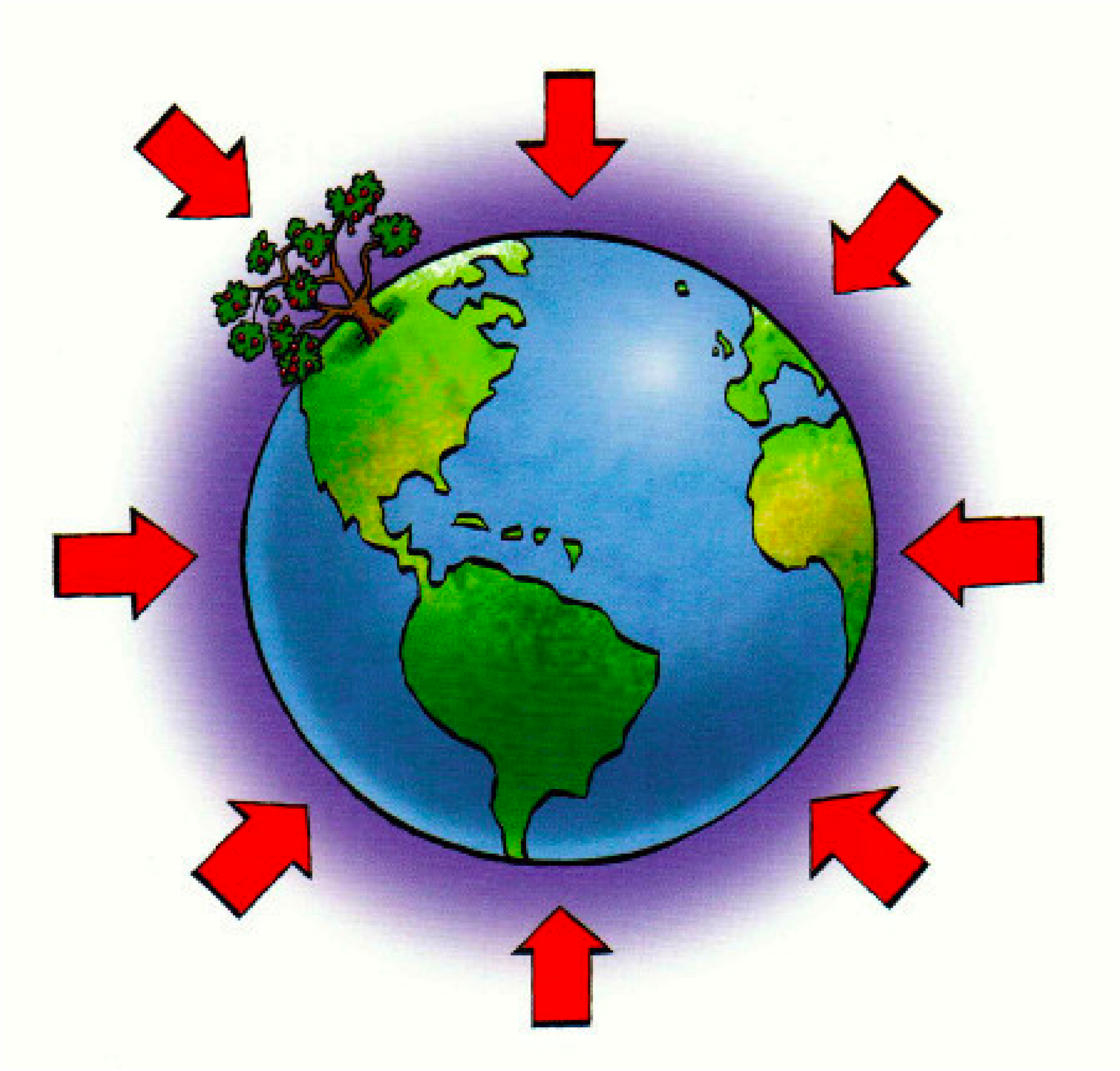
9702 C13 Gravitational Fields



C13

A gravitational field strength is a region of space where a mass experiences a force due to the gravitational attraction of another mass

→ gravitational field is always towards the centre of mass

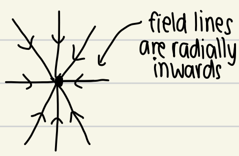
$$\boxed{g = \frac{F_g}{m}}$$

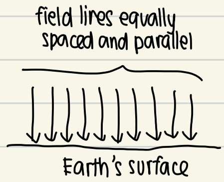
g = gravitational field strength ($N kg^{-1}$)
F_g = force due to gravity (or weight) (N)
m = mass (kg)

gravitational field = force per unit mass
↳ e.g. of a field of force

(2 Types)

Representing Gravitational Fields:

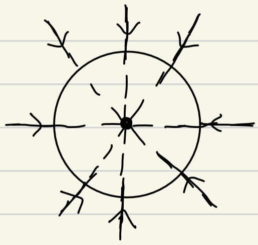




Radial fields = non-uniform fields
→ g depends on how far you are from the centre
→ towards the centre of the sphere or point charge

Parallel fields = uniform field
→ g is the same throughout
→ towards the surface of the object. (e.g. Earth)

For a point outside a uniform sphere, the mass of the sphere may be considered to be a point mass at its centre.
↳ mass is distributed evenly

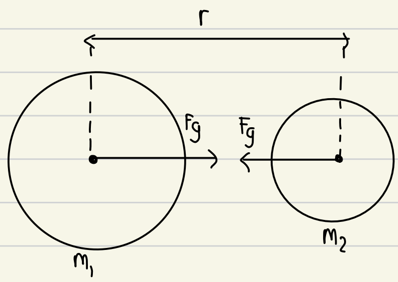


radial field lines evenly spaced + field lines towards centre of the mass

uniform sphere (a planet)

Newton's Law of Gravitational states that: The gravitational force between two point masses is proportional to the product of the masses and inversely proportional to the square their seperation

$$\boxed{F_G = \frac{G m_1 m_2}{r^2}}$$



$\frac{1}{r^2}$ relation is called the 'inverse square law'

F_G = gravitational force between two masses (N)
G = Newton's gravitational constant
m_1, m_2 = two point masses (kg)
r = distance between the centre of the two masses (m)

Consider a satellite with mass m orbiting Earth with mass M at a distance r from the centre travelling with a linear speed v:

$v = \frac{2\pi r}{T}$
v = linear speed (ms^{-1})
T = time period (s)
r = radius (m)

$F_G = F_{centripetal}$

$$\boxed{\frac{GMm}{r^2}} = \frac{mv^2}{r}$$

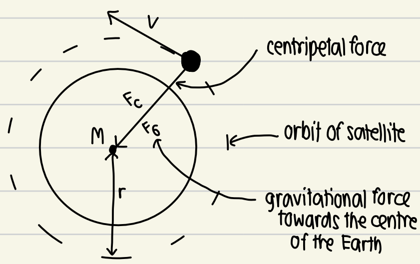
$$v^2 = \frac{GM}{r}$$

$$v^2 = \frac{GM}{r} = \left(\frac{2\pi r}{T}\right)^2$$

$$\boxed{\therefore T^2 = \frac{4\pi^2 r^3}{GM}}$$

Kepler's Third Law of Plenatary motion.

→ $T^2 \propto r^3$



used for telecommunication transmission (e.g. radio, TV)

Geostationary orbit is a type of orbit in which the satellite:
- remains directly above the equator ($\therefore$ orbits at the same point above the Earth's surface)
- moves from west to east (same direction as the Earth spins)
- has an orbital time period equal to Earth's rotational period of 24hr

Deriving Gravitational Field Strength (g)

$F_G = \frac{GMm}{r^2}$

$g = \frac{F}{m} = \frac{\frac{GMm}{r^2}}{m} \Rightarrow \boxed{g = \frac{GM}{r^2}}$ $\rightarrow$ $\boxed{g \propto \frac{1}{r^2}}$

g = gravitational field strength (Nkg^{-1}) (i.e. acceleration due to gravity)

G : Newton's Gravitational Constant

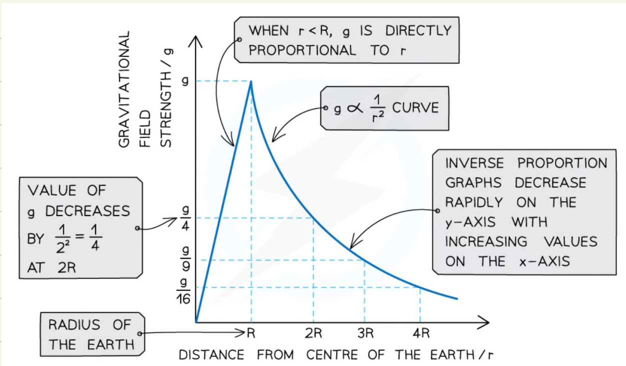
M = mass of the body producing the gravitational field (kg)

r = distance from the mass where you are calculating the field strength (m)

→ g_{Earth} = 9.81 Nkg^{-1} (constant value)

→ $g_{outside\ Earth}$ = not constant = g↓ as r↑ by a factor of $\frac{1}{r^2}$

'inverse square law' relationship with distance



g_{Earth} is approximately constant for small changes in height near the Earth's surface because of the inverse square law relationship, as the r_{Earth} = 6400km the small change in height wouldn't cause any significance difference

$\therefore g_{Earth} = \frac{GM}{(r+h)^2} \simeq \frac{GM}{r^2}$

Gravitational Potential:

$\Delta E_p = mg\Delta h$

ΔE_p on the surface of the Earth = 0J

$\boxed{\Delta E_p = -\frac{GMm}{r}}$

gravitational potential (Φ) = work done per unit mass in bringing a small test mass from infinity to the point.

$\boxed{\Phi = -\frac{GM}{r}}$

⊖ because the potential when r is infinity is defined as 0

Φ = gravitational potential (Jkg^{-1})

G = Newton's gravitational constant

M = mass of the body producing the gravitational field (kg)

r = distance from the centre of the mass to the point mass (m)

→ gravitational forces are always attractive so as r↓, positive work is done by the mass when moving from infinity to that point

gravitational potential energy = the work done in bringing a mass from infinity to that point

The change in E_p from for an object of mass m at a distance r_1 from the centre of mass M, to a distance of r_2 further away is:

$$\Delta E_p = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right) \boxed{GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

$$\Delta\Phi = -\frac{GM}{r_2} - \left(-\frac{GM}{r_1}\right) \boxed{GM\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$