PROSPERITY ACADEMY

A2 PHYSCIS 9702

Crash Course

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GRAVITATION

COMPLETE NOTES

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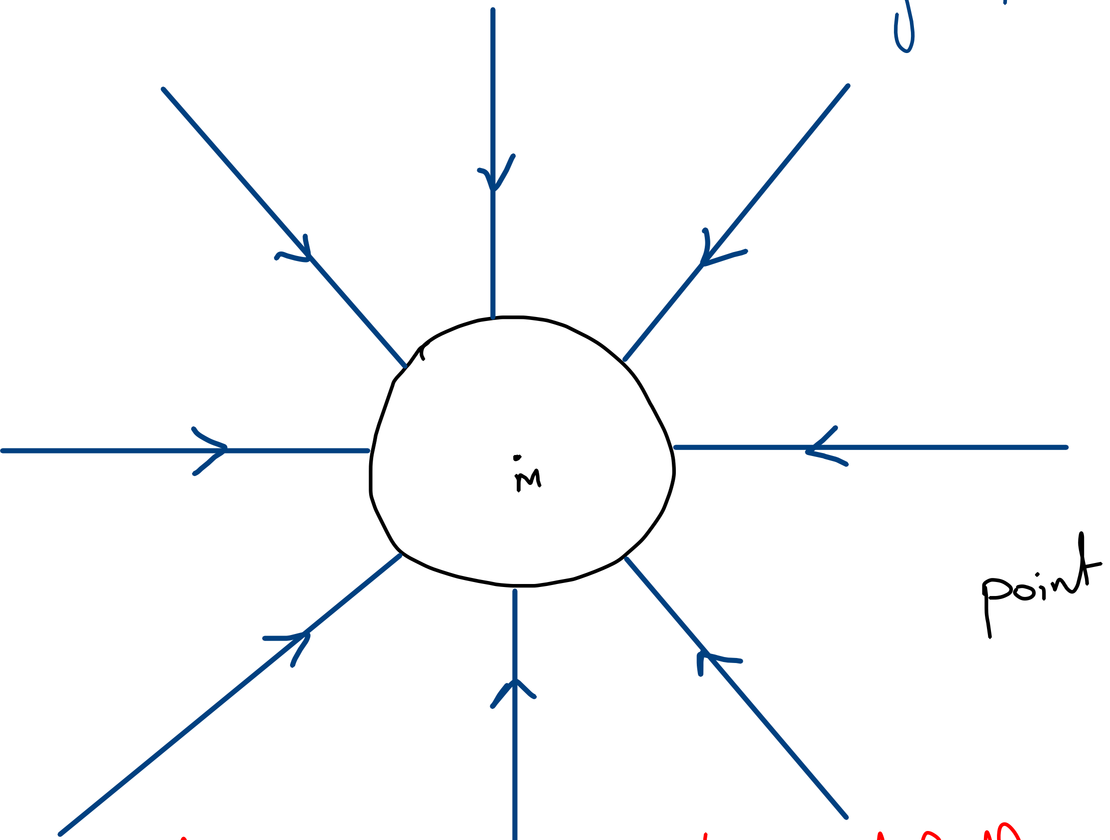
Gravitation:-

Gravitational field:- It is a region or space where a mass experiences the attractive force of gravity.

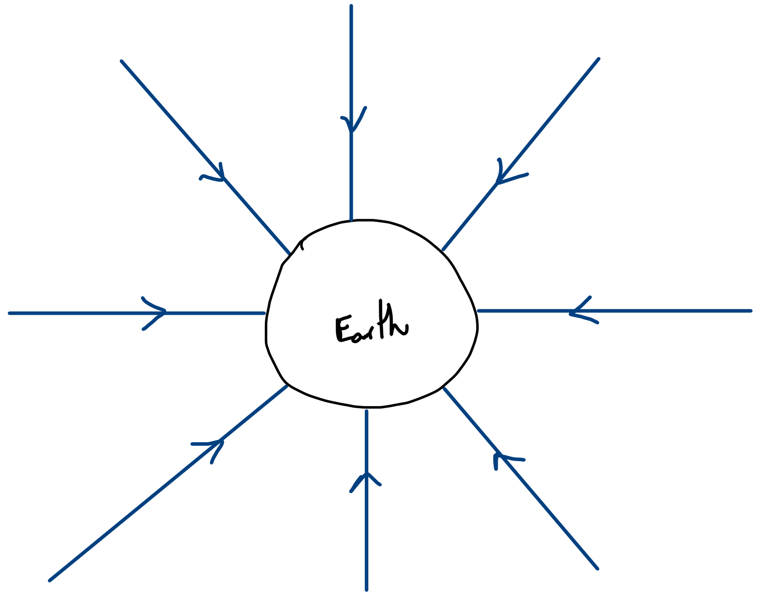
Gravitational force:- It is the force of attraction a mass experiences when placed in the gravitational field of another mass.

Gravitational field line:- It is a hypothetical line of force that shows the force on a unit mass when placed in a gravitational field

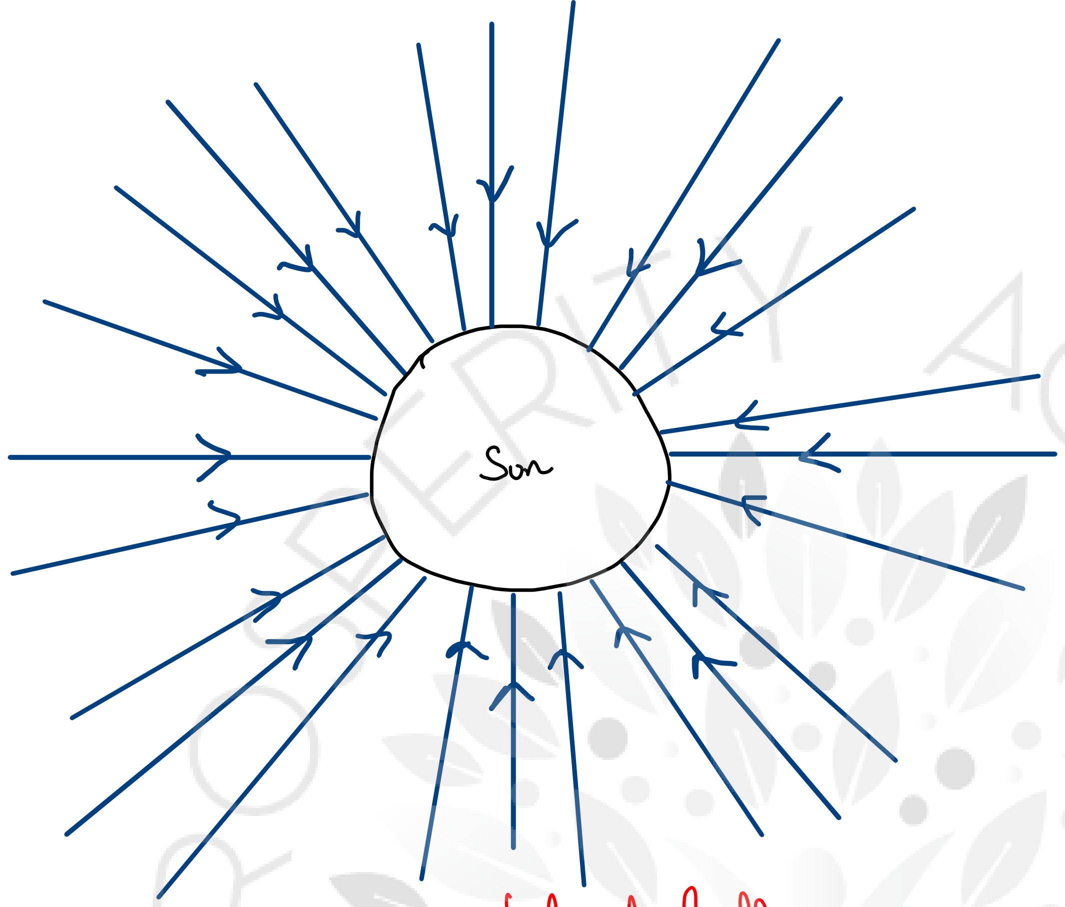


Point masses have gravitational field lines that are radial and pointing inwards

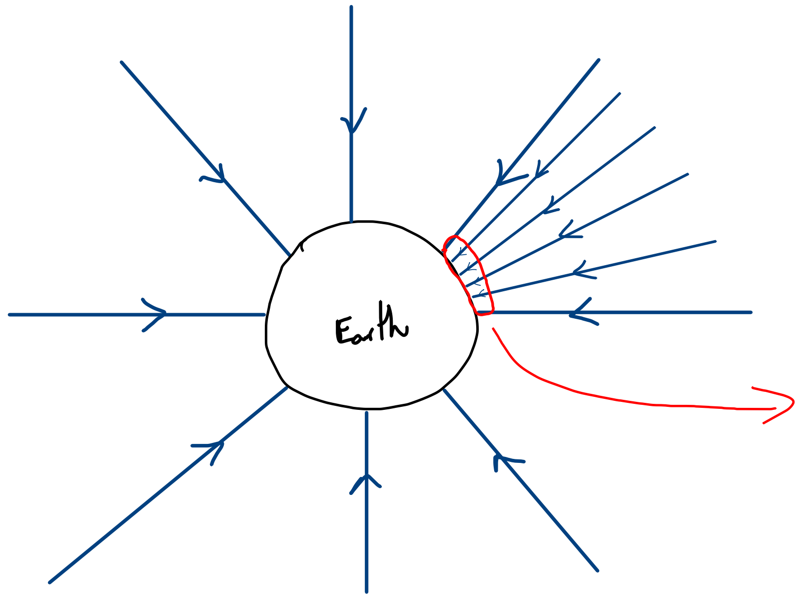
point mass:- Mostly spheres with a uniform density. We consider their total mass to act from their centre at a point.

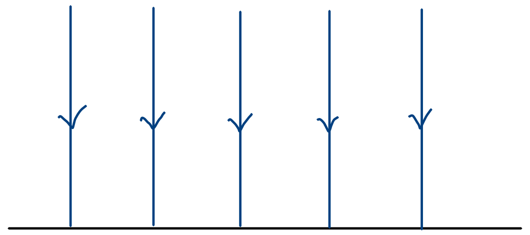


weaker gravitational field



stronger gravitational field.





equally spaced parallel lines show a uniform gravitational field

Uniform Gravitational field:- It is a region or space where the force experienced per unit mass is a constant ($g = 9.81$ const)

Gravitational field strength:- It is the force experienced per unit mass for a mass placed in a gravitational field.

$$g = \frac{F_G}{m}$$

Newton's law of Gravitation (3 marks) :- It states that the force of gravity is directly proportional to the product of the masses but inversely proportional to the square of the distances between their centres

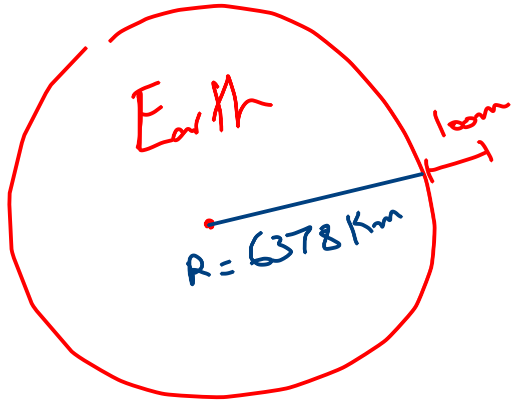
$$F_G \propto \frac{Mm}{r^2}$$

$$F_G = \frac{GMm}{r^2}$$

G is the gravitational constant $(G = 6.67 \times 10^{-11})$

If $g = \dfrac{F_G}{m} = \dfrac{\frac{GM\cancel{m}}{r^2}}{\cancel{m}} \Rightarrow \dfrac{GM}{r^2} = g \Rightarrow \dfrac{(6.67 \times 10^{-11})(6.0 \times 10^{24})}{(6378 \times 10^{3} + 100)^2}$

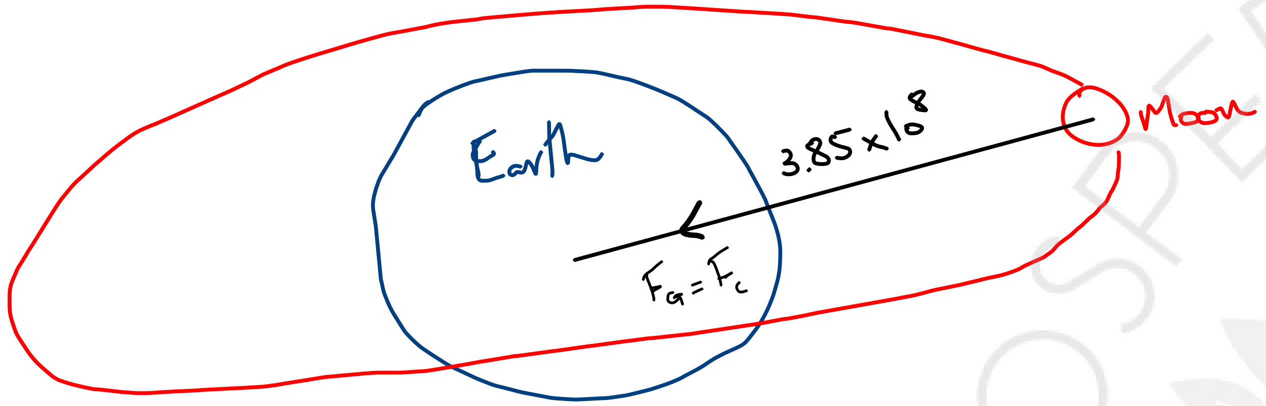
$= 9.84$



$M = 6.0 \times 10^{24}$ Kg

for small h :- $g = \dfrac{GM}{(R+h)^2} \simeq \dfrac{GM}{R^2}$

Find out the time period of the moon's rotation:- $\omega = \frac{2\pi}{T}$



Mass of the Earth :- 6.0×10^{24}
Mass of the moon :- 7.35×10^{22}

$$F_G = F_c$$

$$\frac{GM_m}{r^2} = m r \omega^2$$

$$\frac{GMm}{r^2} = m r \left(\frac{2\pi}{T}\right)^2$$

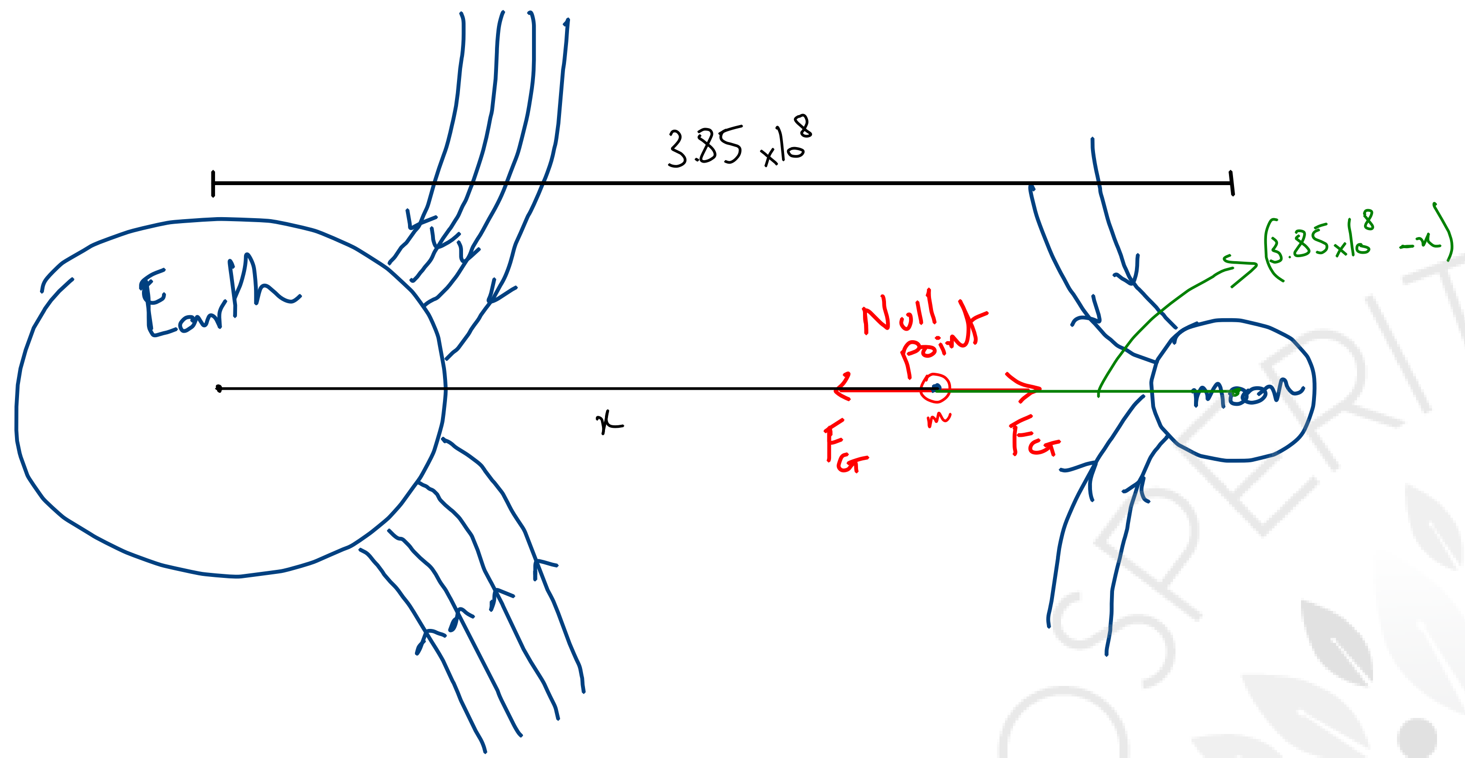
$$\frac{GM\cancel{m}}{r^2} = \cancel{m} r \left(\frac{4\pi^2}{T^2}\right)$$

$$\sqrt{T^2} = \sqrt{\frac{r^3 \times 4\pi^2}{GM}}$$

$$T = \sqrt{\frac{(3.85 \times 10^8)^3 \times 4\pi^2}{(6.67 \times 10^{-11})(6.0 \times 10^{24})}}$$

$$T = 2.37 \times 10^6 \text{ s}$$

$$2.37 \times 10^6 \div (60 \times 60 \times 24) = 27.4 \text{ days}$$



Mass of the Earth :- 6.0×10^{24}
Mass of the moon :- 7.35×10^{22}
distance :- 3.85×10^{8}

M :- Mass due to which you are considering F_G
m :- Mass on which you are considering F_G

Null point :- The resultant force of gravity at this point is zero.

$$F_{G_e} = F_{G_m}$$

$$\frac{\cancel{G} M_e \cancel{m}}{r^2} = \frac{\cancel{G} M_m \cancel{m}}{r^2}$$

$$\frac{(6.0 \times 10^{24})}{x^2} = \frac{(7.35 \times 10^{22})}{(3.85 \times 10^{8} - x)^2}$$

$$\sqrt{\frac{(3.85 \times 10^{8} - x)^{\cancel{2}}}{x^{\cancel{2}}}} = \sqrt{\frac{7.35 \times 10^{22}}{6.0 \times 10^{24}}}$$

$$\frac{3.85 \times 10^{8} - x}{x} = 0.11608$$

$$3.85 \times 10^{8} - x = 0.11608x$$

$$3.85 \times 10^{8} = 1.11608x$$

$$346634582 = x$$

$$3.47 \times 10^{8} = x$$

distance from moon : $(3.85 - 3.47) \times 10^{8} = 3.8 \times 10^{7}$ m

Basics

1 (a) (i) On Fig. 1.1, draw lines to represent the gravitational field outside an isolated uniform sphere.

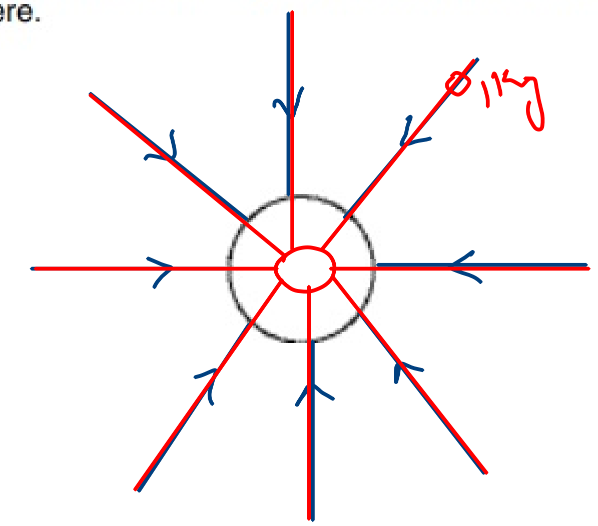


Fig. 1.1

$$F_G = \frac{GMm}{r^2}$$

(ii) A second sphere has the same mass but a smaller radius. Suggest what difference, if any, there is between the patterns of field lines for the two spheres.

No difference

[3]

(b) The Earth may be considered to be a uniform sphere of radius 6380 km with its mass of 5.98×10^{24} kg concentrated at its centre, as illustrated in Fig. 1.2.

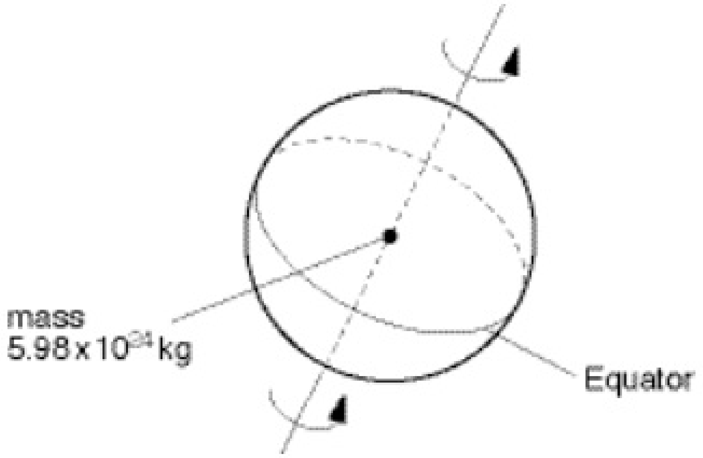


Fig. 1.2

A mass of 1.00 kg on the Equator rotates about the axis of the Earth with a period of 1.00 day (8.64×10^4 s).

Calculate, to three significant figures,

(i) the gravitational force F_G of attraction between the mass and the Earth,

$$F_G = \frac{GMm}{r^2}$$

$$= \frac{(6.67\times10^{-11})\times(5.98\times10^{24})(1)}{(6380\times10^{3})^2} = 9.799$$

F_G = 9.80 N

(ii) the centripetal force F_C on the 1.00 kg mass,

$$F_c = mr\omega^2$$

$$= (1)(6380\times10^{3})\left(\frac{2\pi}{8.64\times10^{4}}\right)^2$$

$$= 0.0337\ \text{N}$$

F_C = 0.0337 N

(iii) the difference in magnitude of the forces.

9.7663

$$R = F_G - F_c$$

difference = 9.77 N

[6]

(c) By reference to your answers in **(b)**, suggest, with a reason, a value for the acceleration of free fall at the Equator.

acceleration of free fall at the equator is 9.77N as it is defined as the resultant force per unit mass.

[2]

w17 qp41 q3

3 (a) Define *gravitational field strength*.

Force experienced per unit mass for a mass in a gravitational field. $\left(g = \frac{F_G}{m}\right)$ [1]

(b) Explain why, for changes in vertical position of a point mass near the Earth's surface, the gravitational field strength may be considered to be constant.

The changes in height are much less than the radius of the Earth so the field lines become almost parallel.

$g = \frac{GM}{R^2} \simeq \frac{GM}{(R+h)^2}$ [2]

(c) The orbit of the Earth about the Sun is approximately circular with a radius of 1.5×10^8 km. The time period of the orbit is 365 days. $365 \times 24 \times 60 \times 60 = 31536000$

Determine a value for the mass *M* of the Sun. Explain your working.

The force of gravity provides the necessary centripetal force

$$F_G = F_c$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}r\left(\frac{2\pi}{T}\right)^2$$

$$M = \frac{r^3 \times 4\pi^2}{G \times T^2}$$

$$M = \frac{(1.5\times10^8\times10^3)^3 \times 4\pi^2}{(6.67\times10^{-11}) \times (31536000)^2} = 2.00\times10^{30}$$

M = 2.0×10^{30} kg [5]

[Total: 8]

(ii) By reference to lines of gravitational force near to the surface of the Earth, explain why the gravitational field strength *g* close to the Earth's surface is approximately constant.

The changes in height are very small compared to the radius of the earth so the gravitational field lines become almost parallel.

Furthermore as h is small

$g = \frac{GM}{R^2} \approx \frac{GM}{(R+h)^2}$ [3]

w16 qp42 q1

Answer **all** the questions in the spaces provided.

1 (a) Define *gravitational field strength*.

Force experienced per unit mass by a mass placed in a gravitational field. [1]

(b) The nearest star to the Sun is Proxima Centauri.
This star has a mass of 2.5×10^{29} kg and is a distance of 4.0×10^{13} km from the Sun.
The Sun has a mass of 2.0×10^{30} kg.

(i) State why Proxima Centauri may be assumed to be a point mass when viewed from the Sun.

The radius of the sun compared to the seperation is negligible so Proxima Centauri can be assumed to be a point mass / Proxima Centauri is a uniform sphere [1]

(ii) Calculate

1. the gravitational field strength due to Proxima Centauri at a distance of 4.0×10^{13} km,

$$g = \frac{GM}{r^2} = \frac{(6.67\times10^{-11})(2.5\times10^{29})}{(4.0\times10^{13}\times10^{3})^2}$$

$$= 1.042\times10^{-14}$$

field strength = 1.0×10^{-14} N kg^{-1} [2]

2. the gravitational force of attraction between the Sun and Proxima Centauri.

g

$$F_G = \frac{GM}{r^2}m \Rightarrow g\times m$$

$$= 1.042\times10^{-14} \times 2.0\times10^{30}$$

$$= 2.084\times10^{16}$$

force = 2.0×10^{16} N [2]

(c) Suggest quantitatively why it may be assumed that the Sun is isolated in space from other stars.

Due to the distance a small force acts on the sun. The sun having a huge mass, only experiences a very small acceleration. [2]

[Total: 8]

$$F = ma$$

$$2.0\times10^{16} = 2.0\times10^{30} \times a$$

$$a = 1\times10^{-14}$$

1 (a) State Newton's law of gravitation.

It states that the force of gravity between 2 masses is directly proportional to the product of the masses but inversely to the square of the distances between their centres. [2]

(b) The planet Jupiter and one of its moons, Io, may be considered to be uniform spheres that are isolated in space.
Jupiter has radius R and mean density ρ.
Io has mass m and is in a circular orbit about Jupiter with radius nR, as illustrated in Fig. 1.1.

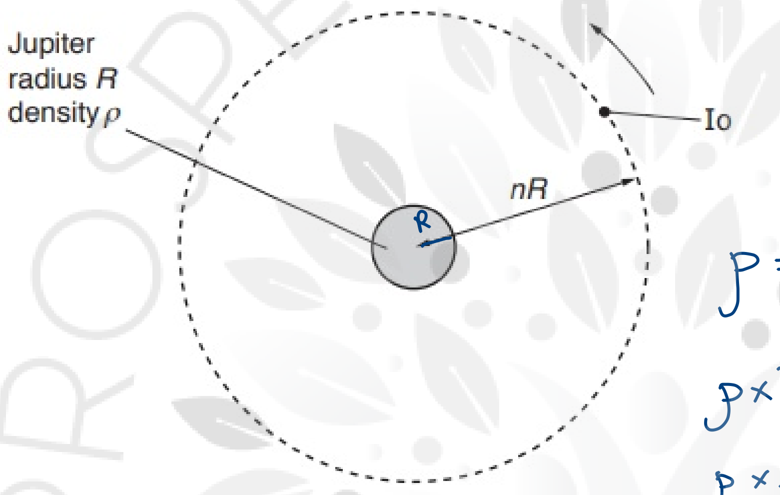


Fig. 1.1

$\rho = \frac{M}{V}$

$\rho \times V = M$

$\rho \times \frac{4}{3}\pi R^3 = M$

The time for Io to complete one orbit of Jupiter is T.

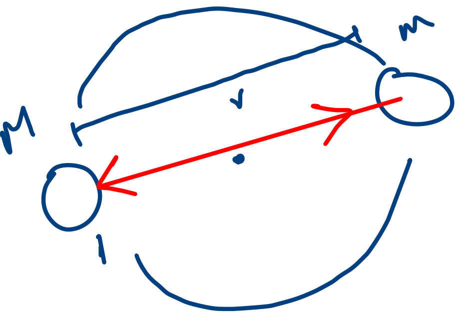
Show that the time T is related to the mean density ρ of Jupiter by the expression

$$\rho T^2 = \frac{3\pi n^3}{G}$$

where G is the gravitational constant.

$F_{c_1} = F_{c_2}$ $F_{G_1} = F_{G_2}$

$\omega_1 = \omega_2$



$F_{c_1} = \frac{GMm}{r^2}$

$F_{c_2} = \frac{GMm}{r^2}$

$F_G = F_c$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}r\omega^2 = \frac{G \times \rho \times \frac{\cancel{4}}{3}\cancel{\pi}R^3}{(nR)^2} = (nR) \times \frac{\cancel{4}\pi^{\cancel{2}}}{T^2}$$

$$\rho T^2 = \frac{3 \times n^3 \cancel{R^3}}{G\cancel{R^3}} \times \pi$$

$$\rho T^2 = \frac{3\pi n^3}{G}$$

(c) (i) The radius R of Jupiter is 7.15×10^4 km and the distance between the centres of Jupiter and Io is 4.32×10^5 km. nR
The period T of the orbit of Io is 42.5 hours.

Calculate the mean density ρ of Jupiter.

$nR = 4.32 \times 10^5$ Km

$R = 7.15 \times 10^4$ Km

$n(7.15 \times 10^4) = 4.32 \times 10^5$

$n = 6.04$

$$\rho T^2 = \frac{3\pi n^3}{G}$$

$$\rho(42.5 \times 60 \times 60)^2 = \frac{3\pi(6.04)^3}{(6.67 \times 10^{-11})} = 1330.07$$

ρ = 1330 (1.33×10^3) kg m^{-3} [3]

(ii) The Earth has a mean density of 5.5×10^3 kg m^{-3}. It is said to be a planet made of rock. By reference to your answer in **(i)**, comment on the possible composition of Jupiter.

Jupiter is most probably liquid (gas ✓) [1]

[Total: 10]

s17 qp42 q1

Answer **all** the questions in the spaces provided.

1 **(a)** Define *gravitational field strength*.

Force experienced per unit mass by a mass placed in a gravitational field. [1]

(b) The mass of a spherical comet of radius 3.6 km is approximately 1.0×10^{13} kg.

(i) Assuming that the comet has constant density, calculate the gravitational field strength on the surface of the comet.

$$g = \frac{GM}{R^2} = \frac{(6.67\times10^{-11})(1.0\times10^{13})}{(3.6\times10^3)^2}$$

$$= 5.1466\times10^{-5}$$

field strength = 5.1×10^{-5} N kg^{-1} [2]

(ii) A probe having a weight of 960 N on Earth lands on the comet.
Using your answer in **(i)**, determine the weight of the probe on the surface of the comet.

$$W = mg$$

$$960 = m \times 9.81$$

$$m = \frac{960}{9.81}$$

$$W = m \times g$$

$$W = \frac{960}{9.81} \times (5.1\times10^{-5})$$

$$= 4.9908\times10^{-3}$$

weight = 5.0×10^{-3} N [2]

(c) A second comet has a length of approximately 4.5 km and a width of approximately 2.6 km. Its outline is illustrated in Fig. 1.1.

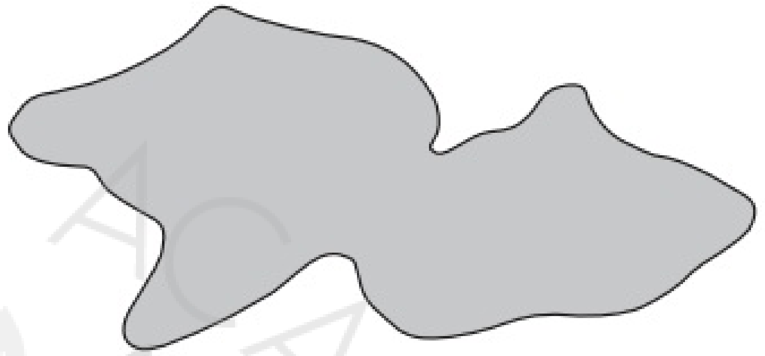


Fig. 1.1

Suggest one similarity and one difference between the gravitational fields at the surface of this comet and at the surface of the comet in **(b)**.

similarity: The gravitational field lines will be attractive/ pointing inwards for both.

difference: The spherical comet will have radial field lines while the comet in (b) will have non radial field lines

[2]

[Total: 7]

s19 qp42 q1

Answer **all** the questions in the spaces provided.

1 **(a)** Two point masses are separated by a distance x in a vacuum.
State an expression for the force F between the two masses M and m.
State the name of any other symbol used.

$F = \frac{GMm}{x^2}$. G is the gravitational constant ($G = 6.67 \times 10^{-11}$)

....[1]

(b) A small sphere S is attached to one end of a rod, as shown in Fig. 1.1.

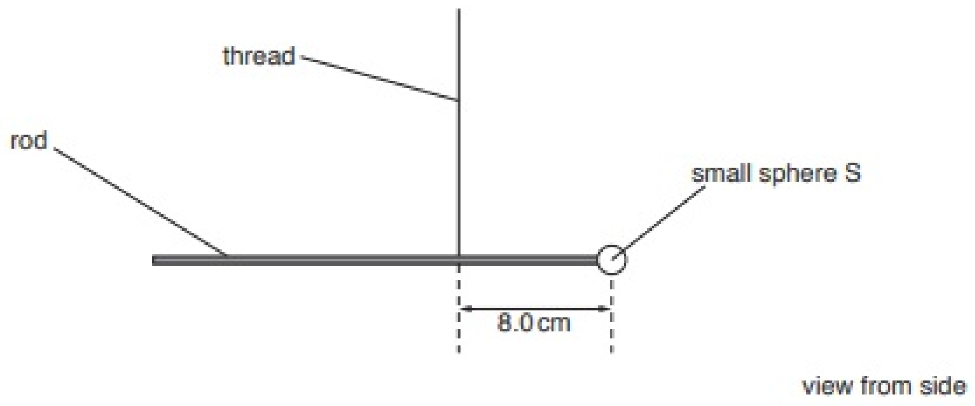
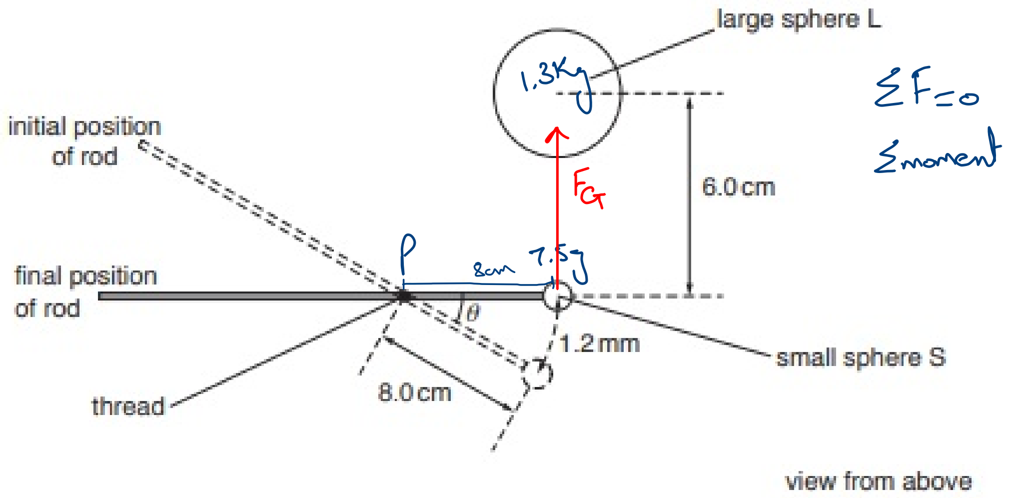


Fig. 1.1 (not to scale)

The rod hangs from a vertical thread and is horizontal.
The distance from the centre of sphere S to the thread is 8.0 cm.

A large sphere L is placed near to sphere S, as shown in Fig. 1.2.



$\Sigma F = 0$

$\Sigma moment = 0$

Fig. 1.2 (not to scale)

There is a force of attraction between spheres S and L, causing sphere S to move through a distance of 1.2 mm.
The line joining the centres of S and L is normal to the rod.

(i) Show that the angle θ through which the rod rotates is 1.5×10^{-2} rad.

$S = r\theta$

$(1.2 \times 10^{-3}) = (8 \times 10^{-2})\,\theta$

$\theta = 1.5 \times 10^{-2}$

[1]

(ii) The rotation of the rod causes the thread to twist.
The torque T (in N m) required to twist the thread through an angle β (in rad) is given by

$$T = 9.3 \times 10^{-10} \times \beta.$$

Calculate the torque in the thread when sphere L is positioned as shown in Fig. 1.2.

$T = 9.3 \times 10^{-10} \times 1.5 \times 10^{-2}$

$= 1.395 \times 10^{-11}$

torque = 1.4×10^{-11} N m [1]

(c) The distance between the centres of spheres S and L is 6.0 cm.
The mass of sphere S is 7.5 g and the mass of sphere L is 1.3 kg.

(i) By equating the torque in **(b)(ii)** to the moment about the thread produced by gravitational attraction between the spheres, calculate a value for the gravitational constant.

$F_G \times d = T$

$\frac{GMm}{r^2} \times d = T$

$G = \frac{T \times r^2}{Mm \times d} = \frac{(1.4 \times 10^{-11}) \times (6 \times 10^{-2})^2}{(1.3)(7.5 \times 10^{-3}) \times (8 \times 10^{-2})}$

$= 6.46 \times 10^{-11}$

gravitational constant = 6.5×10^{-11} N m^2 kg^{-2} [3]

(ii) Suggest why the total force between the spheres may not be equal to the force calculated using Newton's law of gravitation.

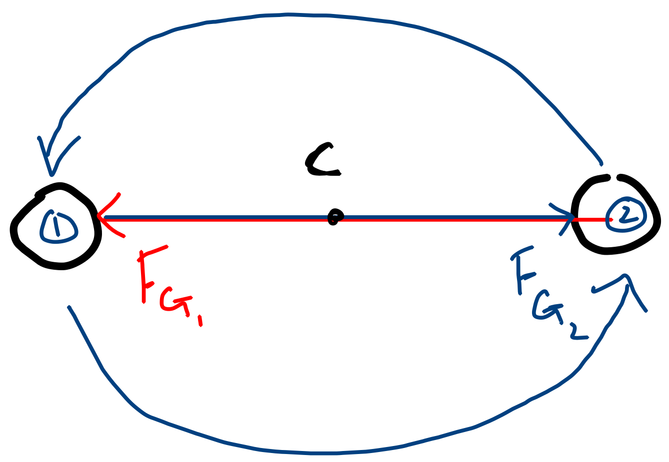
Maybe the spheres are charged.

....[1]

[Total: 7]

- maybe the spheres are not point masses
  1) maybe they are not uniform
  2) maybe the seperation is not large enough
- maybe there is some force between the rod and the small sphere
- maybe the masses are not isolat

Binary Star System:-



① $F_G = F_c$ The gravitational force provides the necessary centripetal force.

② $F_{G_1} = F_{G_2}$ The force on Star 1 is equal to the force on star 2 (Newton's 3rd law)

$F_{c_1} = F_{c_2}$ Therefore, the centripetal forces are also equal.

③ $\omega_1 = \omega_2$ The angular speeds of the 2 stars are also equal as they have the same time period of rotation.

3 A binary star consists of two stars that orbit about a fixed point C, as shown in Fig. 3.1.

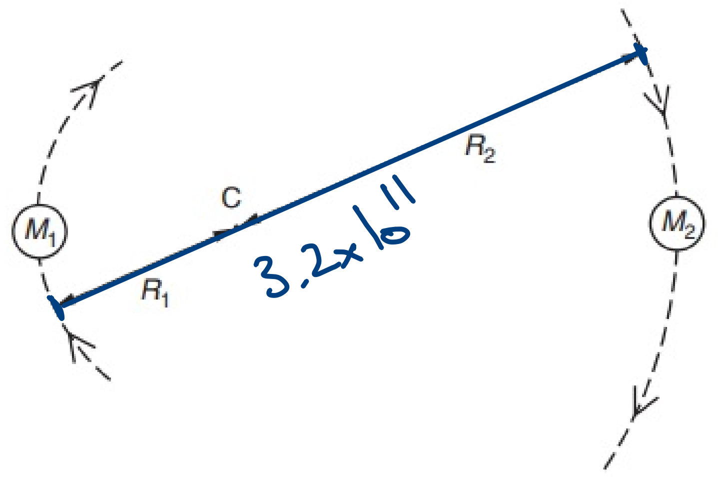


Fig. 3.1

The star of mass M_1 has a circular orbit of radius R_1 and the star of mass M_2 has a circular orbit of radius R_2. Both stars have the same angular speed ω, about C.

(a) State the formula, in terms of G, M_1, M_2, R_1, R_2 and ω for

(i) the gravitational force between the two stars,

$GM_1M_2/(R_1+R_2)^2$

(ii) the centripetal force on the star of mass M_1.

$M_1R_1\omega^2$

[2]

(b) The stars orbit each other in a time of 1.26×10^8 s (4.0 years). Calculate the angular speed ω for each star.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(1.26\times10^8)}$$

$$= 4.986\times10^{-8}$$

angular speed = 4.99×10^{-8} rad s^{-1} [2]

(c) (i) Show that the ratio of the masses of the stars is given by the expression

$$\frac{M_1}{M_2} = \frac{R_2}{R_1}.$$

$$F_{c_1} = F_{c_2}$$

$$M_1R_1\cancel{\omega^2} = M_2R_2\cancel{\omega^2}$$

$$\frac{M_1}{M_2} = \frac{R_2}{R_1}$$

[2]

(ii) The ratio $\frac{M_1}{M_2}$ is equal to 3.0 and the separation of the stars is 3.2×10^{11} m.

Calculate the radii R_1 and R_2.

$$3 = \frac{R_2}{R_1}$$

$$3R_1 = 3.2\times10^{11} - R_1$$

$$4R_1 = 3.2\times10^{11}$$

$$R_1 = 8\times10^{10}$$

$$R_1 + R_2 = 3.2\times10^{11}$$

$$R_2 = 3.2\times10^{11} - R_1$$

$$R_2 = 3.2\times10^{11} - 8\times10^{10}$$

$$= 24\times10^{10} \Rightarrow 2.4\times10^{11}$$

R_1 = 8×10^{10} m

R_2 = 2.4×10^{11} m

[2]

(d) (i) By equating the expressions you have given in **(a)** and using the data calculated in **(b)** and **(c)**, determine the mass of one of the stars.

$$\frac{G\cancel{M_1}M_2}{(R_1+R_2)^2} = \cancel{M_1}R_1\omega^2$$

$$M_2 = \frac{R_1\times\omega^2\times(R_1+R_2)^2}{G} \Rightarrow \frac{(8\times10^{10})\times(4.99\times10^{-8})^2\times(3.2\times10^{11})^2}{(6.67\times10^{-11})}$$

$$M_2 = 3.058\times10^{29}$$

mass of star = 3.06×10^{29} kg

(ii) State whether the answer in **(i)** is for the more massive or for the less massive star.

The star is the less massive star

[4]

$$\frac{M_1}{M_2 \text{ ->smaller}} = 3.0$$

$$\frac{6}{3} = 2$$

$$\frac{3}{6} = \frac{1}{2}$$

Answer **all** the questions in the spaces provided.

1 **(a)** State what is meant by a *gravitational force*.

It is the force of attraction experienced by a mass placed in the gravitational field of another mass. [1]

(b) A binary star system consists of two stars S_1 and S_2, each in a circular orbit.

The orbit of each star in the system has a period of rotation T.

Observations of the binary star from Earth are represented in Fig. 1.1.

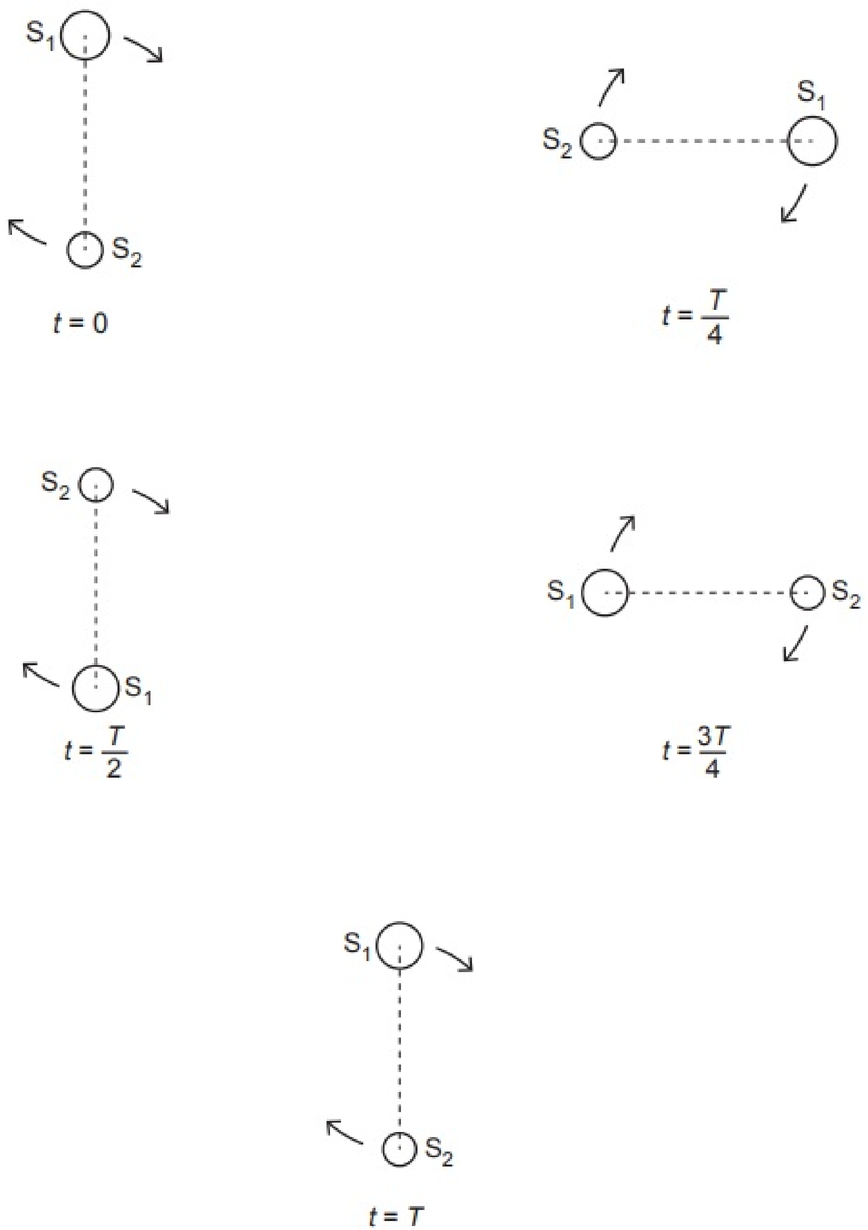


Fig. 1.1 (not to scale)

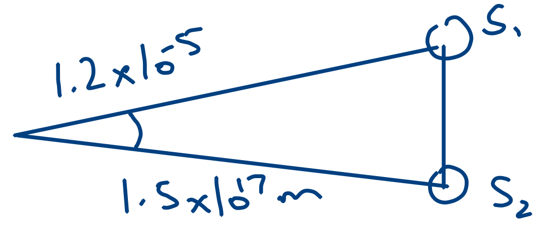
Observed from Earth, the angular separation of the centres of S_1 and S_2 is 1.2×10^{-5} rad. The distance of the binary star system from Earth is 1.5×10^{17} m.

Show that the separation d of the centres of S_1 and S_2 is 1.8×10^{12} m.

$s = r\theta$

$s = (1.5 \times 10^{17})(1.2 \times 10^{-5})$

$= 1.8 \times 10^{12}$



[1]

(c) The stars S_1 and S_2 rotate with the same angular velocity ω about a point P, as illustrated in Fig. 1.2.

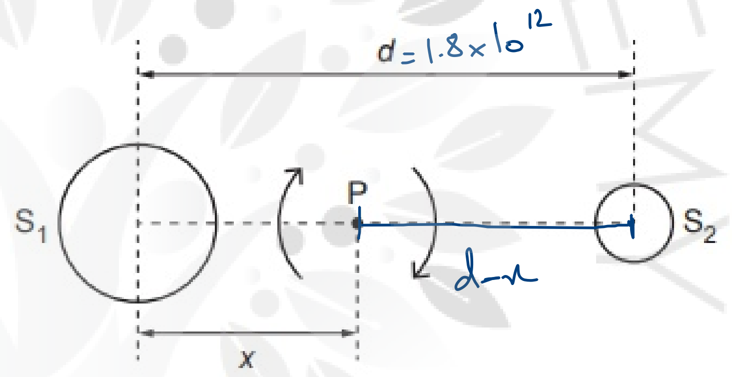


Fig. 1.2 (not to scale)

Point P is at a distance x from the centre of star S_1. The period of rotation of the stars is 44.2 years.

(i) Calculate the angular velocity ω.

$\omega = \frac{2\pi}{T} \Rightarrow \frac{2\pi}{(44.2 \times 365 \times 24 \times 60 \times 60)}$

$= 4.507 \times 10^{-9}$

$\omega =$ 4.51×10^{-9} rad s^{-1} [2]

(ii) By considering the forces acting on the two stars, show that the ratio of the masses of the stars is given by

$$\frac{\text{mass of } S_1}{\text{mass of } S_2} = \frac{d-x}{x}.$$

$$F_{c_1} = F_{c_2}$$

$$M_1 R_1 \cancel{\omega^2} = M_2 R_2 \cancel{\omega^2}$$

$$\frac{M_1}{M_2} = \frac{R_2}{R_1} \Rightarrow \frac{M_1}{M_2} = \frac{d-x}{x}$$

[2]

(iii) The mass M_1 of star S_1 is given by the expression

$$GM_1 = d^2(d-x)\omega^2$$

where G is the gravitational constant.

The ratio in **(ii)** is found to be 1.5.

Use data from **(b)** and your answer in **(c)(i)** to determine the mass M_1.

$$\frac{d-x}{x} = 1.5$$

$$d - x = 1.5x$$

$$d = 2.5x$$

$$\frac{1.8 \times 10^{12}}{2.5} = x$$

$$7.2 \times 10^{11}$$

$$M_1 = \frac{(1.8\times10^{12})^2(1.8\times10^{12} - 7.2\times10^{11})\times(4.51\times10^{-9})^2}{6.67\times10^{-11}}$$

$$M_1 = 1.1 \times 10^{30} \text{ kg}$$

M_1 = ...1.1×10^{30}........ kg [3]

Gravitational field strength :- It is the force experienced per unit mass for a mass placed in a gravitational field.

$$g = \frac{F_G}{m}$$

Vector measured in ms^{-2}.

$$g = \frac{\frac{GM\cancel{m}}{r^2}}{\cancel{m}} = \frac{GM}{r^2}$$

$$g = \frac{GM}{r^2} \quad \text{const}$$

$$g \propto \frac{1}{r^2} \qquad g \propto \frac{1}{x^2}$$

Determining g inside of Earth :-

① Earth has uniform density

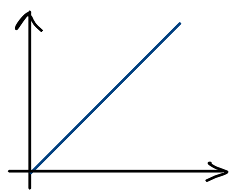
$$g = \frac{GM}{r^2} \qquad \rho = \frac{M}{V}$$

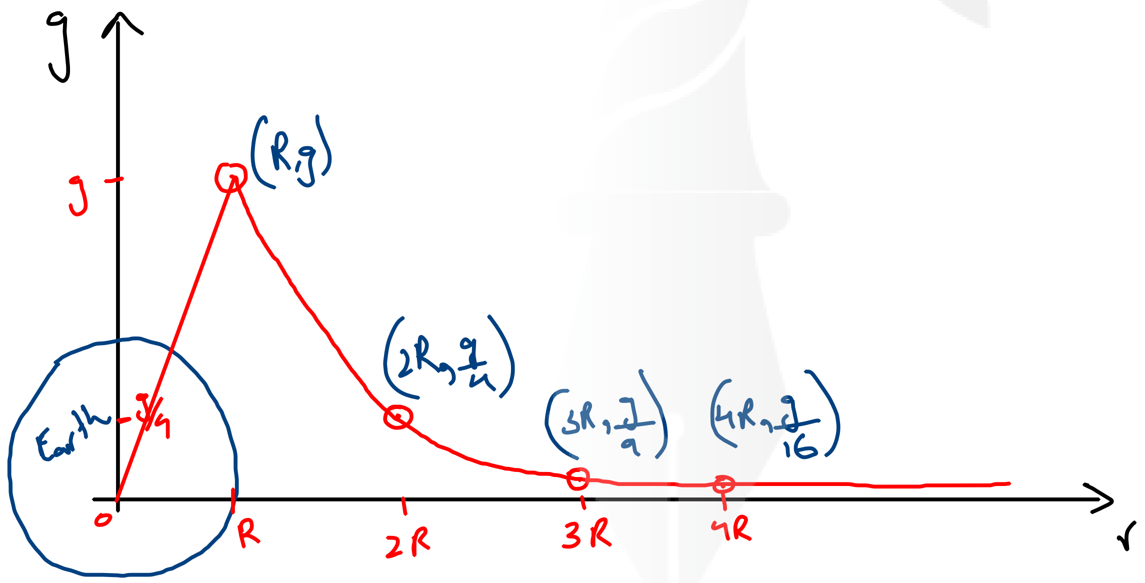
$$g = \frac{G \times \frac{4}{3}\pi \cancel{r^3}^{\,r} \rho}{\cancel{r^2}} \qquad \rho = \frac{M}{\frac{4}{3}\pi r^3}$$

$$g = \underbrace{\left(\frac{4}{3}\pi \rho G\right)}_{\text{const}} \times r \qquad M = \frac{4}{3}\pi r^3 \rho$$

$$g \propto r$$

$$g \propto x$$





$$g_1 = \frac{K}{r_1^2}$$

$$g_1 r_1^2 = K = g_2 r_2^2$$

$$g R^2 = g_2 (2R)^2$$

$$g \cancel{R^2} = g_2 \times 4 \cancel{R^2}$$

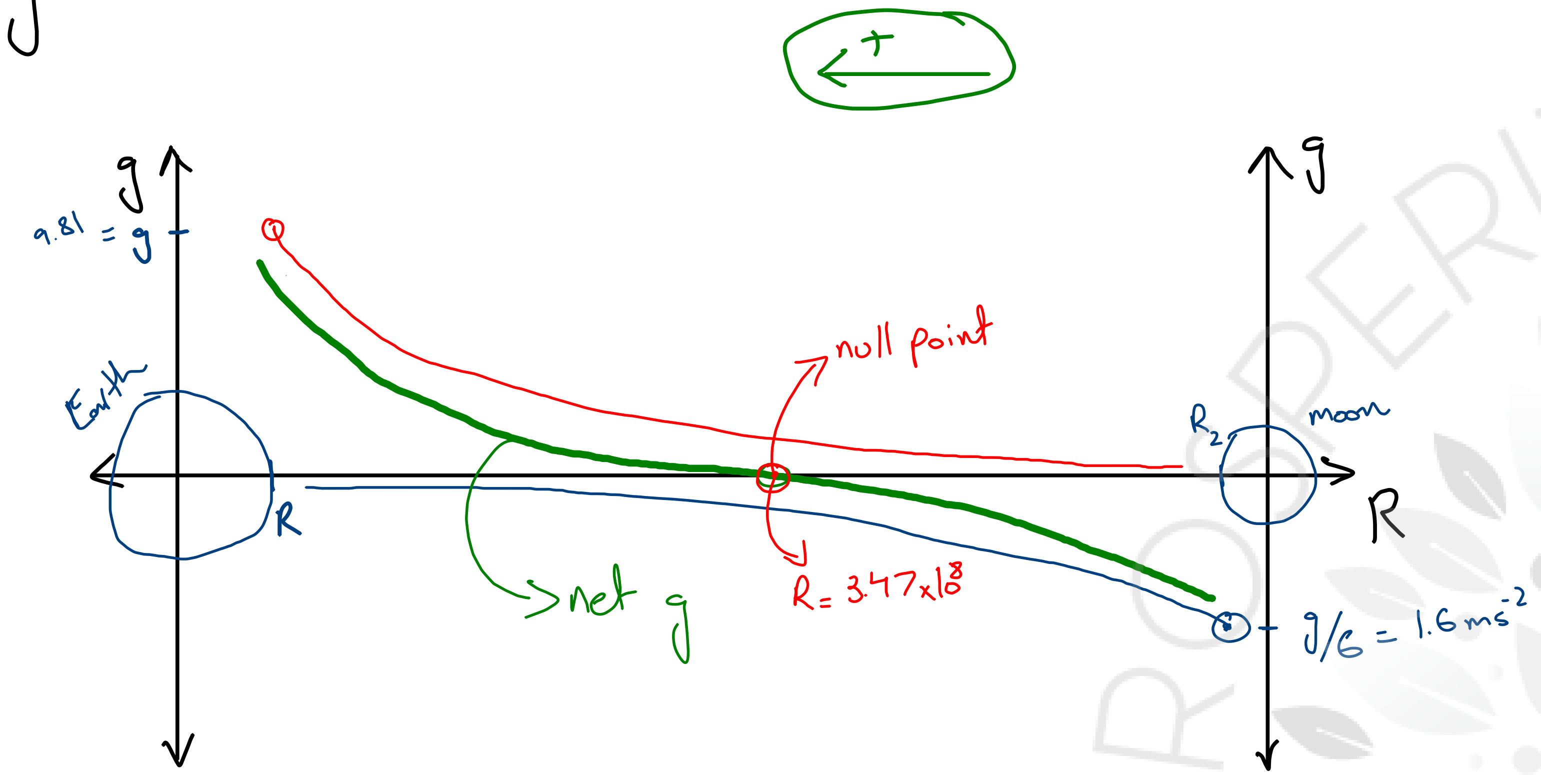
$$g_2 = \frac{g}{4}$$

$$g R^2 = g_3 (3R)^2$$

$$g \cancel{R^2} = g_3 \, 9 \cancel{R^2}$$

$$g_3 = \frac{g}{9}$$

g between Earth and moon:-



Gravitational Field Strength

w18 qp42 q1

Answer **all** the questions in the spaces provided.

1 (a) (i) State what is meant by *gravitational field strength*.

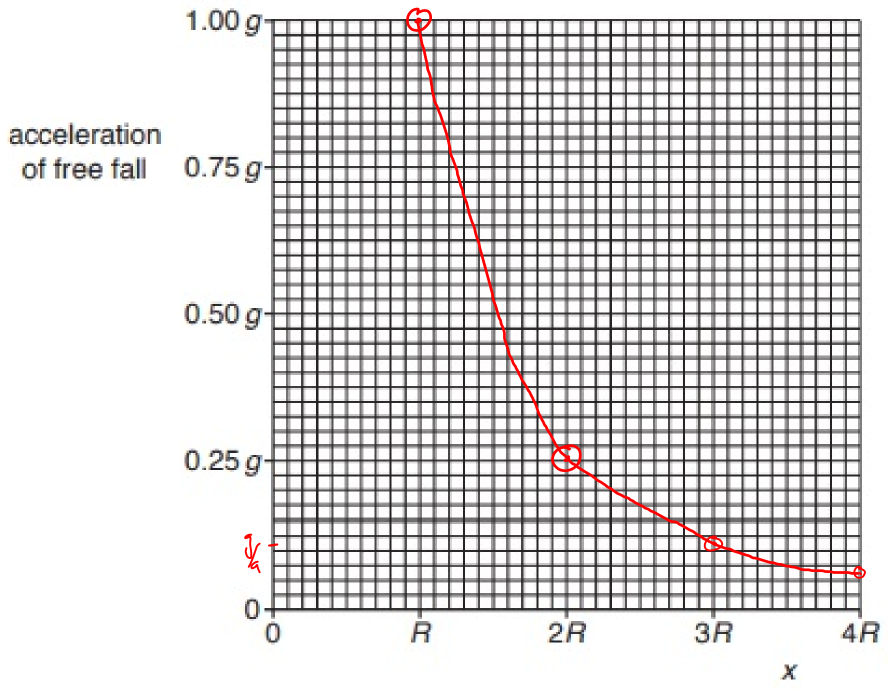
It is the force experienced per unit mass by a mass placed in a gravitational field. It is a vector measured in ms^{-2}. [1]

(ii) Explain why, at the surface of a planet, gravitational field strength is numerically equal to the acceleration of free fall.

Very close to the surface of the planet, the gravitational force (F_G) of the planet becomes dominant over other gravitational forces and as $g = F_G/m$ and $a = F/m$, they become equal. [1]

(b) An isolated uniform spherical planet has radius *R*.
The acceleration of free fall at the surface of the planet is *g*.

On Fig. 1.1, sketch a graph to show the variation of the acceleration of free fall with distance *x* from the centre of the planet for values of *x* in the range *x* = *R* to *x* = 4*R*.



(c) The planet in **(b)** has radius *R* equal to 3.4×10^3 km and mean density $4.0 \times 10^3\,kg\,m^{-3}$.

Calculate the acceleration of free fall at a height *R* above its surface.

$g = \frac{GM}{r^2}$ $\quad \rho = 4.0 \times 10^3 = \frac{M}{V}$

$4.0 \times 10^3 \times \frac{4}{3}\pi(3.4 \times 10^6)^3 = M$

$$g = \frac{(6.67 \times 10^{-11}) \times (4.0 \times 10^3) \times \frac{4}{3}\pi(3.4 \times 10^6)^3}{(2 \times 3.4 \times 10^6)^2} = 0.9499$$

acceleration of free fall = 0.95 ms^{-2} [3]

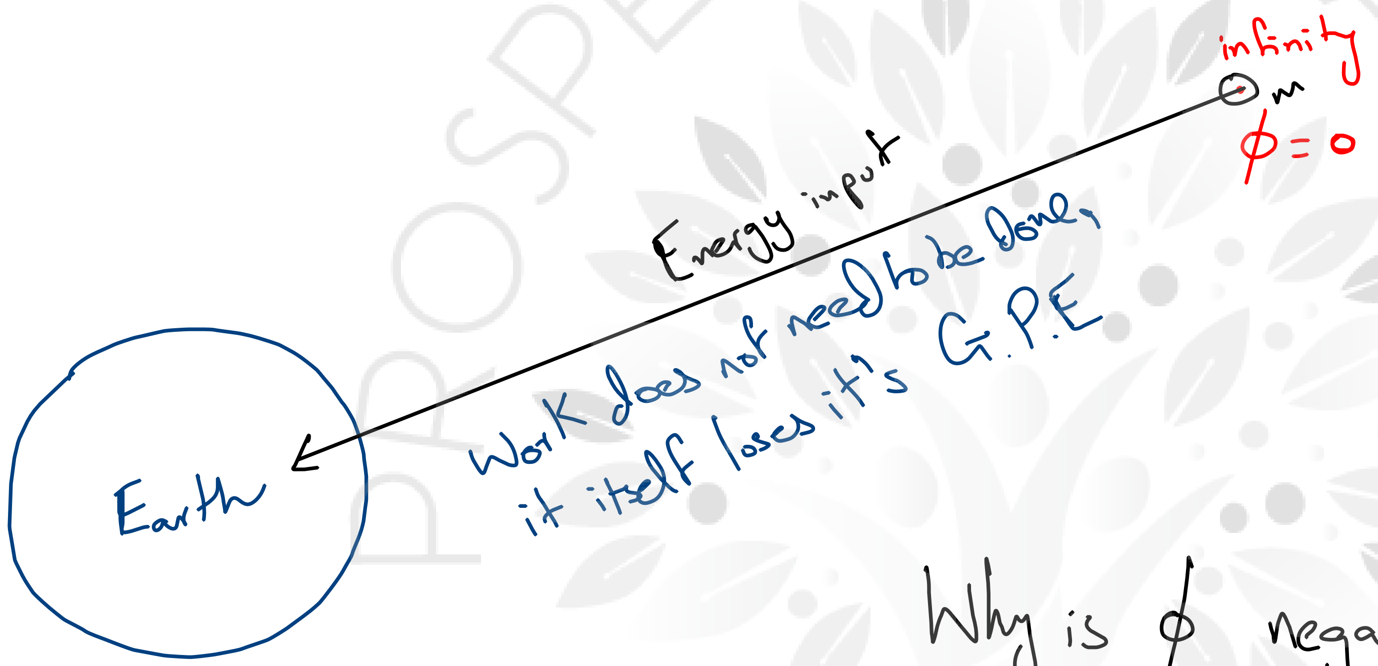
[Total: 8]

Gravitational potential (ϕ):- It is the work done per unit mass in bringing that mass from infinity to a point within a gravitational field (Scalar)

$\phi = 0$

(a point so far away that you escape the gravitational field of any mass)

$$\boxed{\phi = -\frac{W}{m}}$$



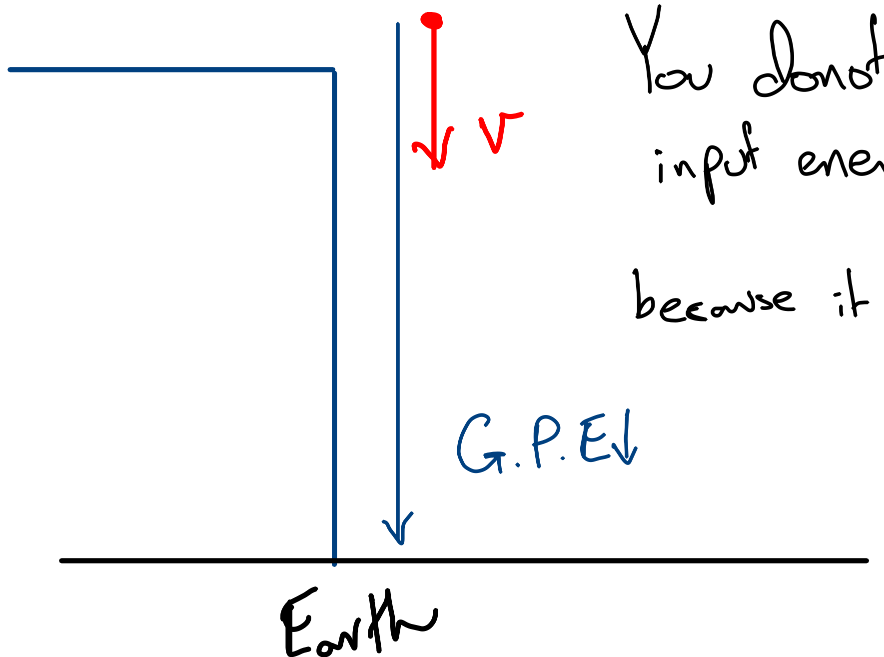
Why is ϕ negative?

At infinity ϕ is zero. It is defined as the work done per unit mass in bringing a mass from infinity to a point within a gravitational field. As gravitational fields are always attractive, work gets out of the system.

* $\Delta G.P.E = m \times \Delta\phi$

$= m \times (\phi_f - \phi_i)$

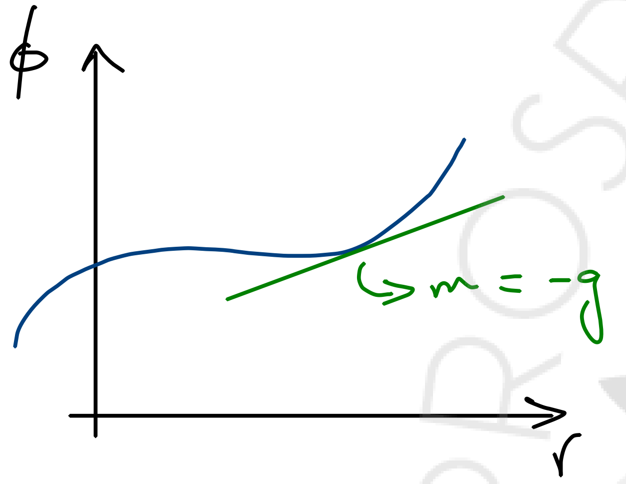
Recall

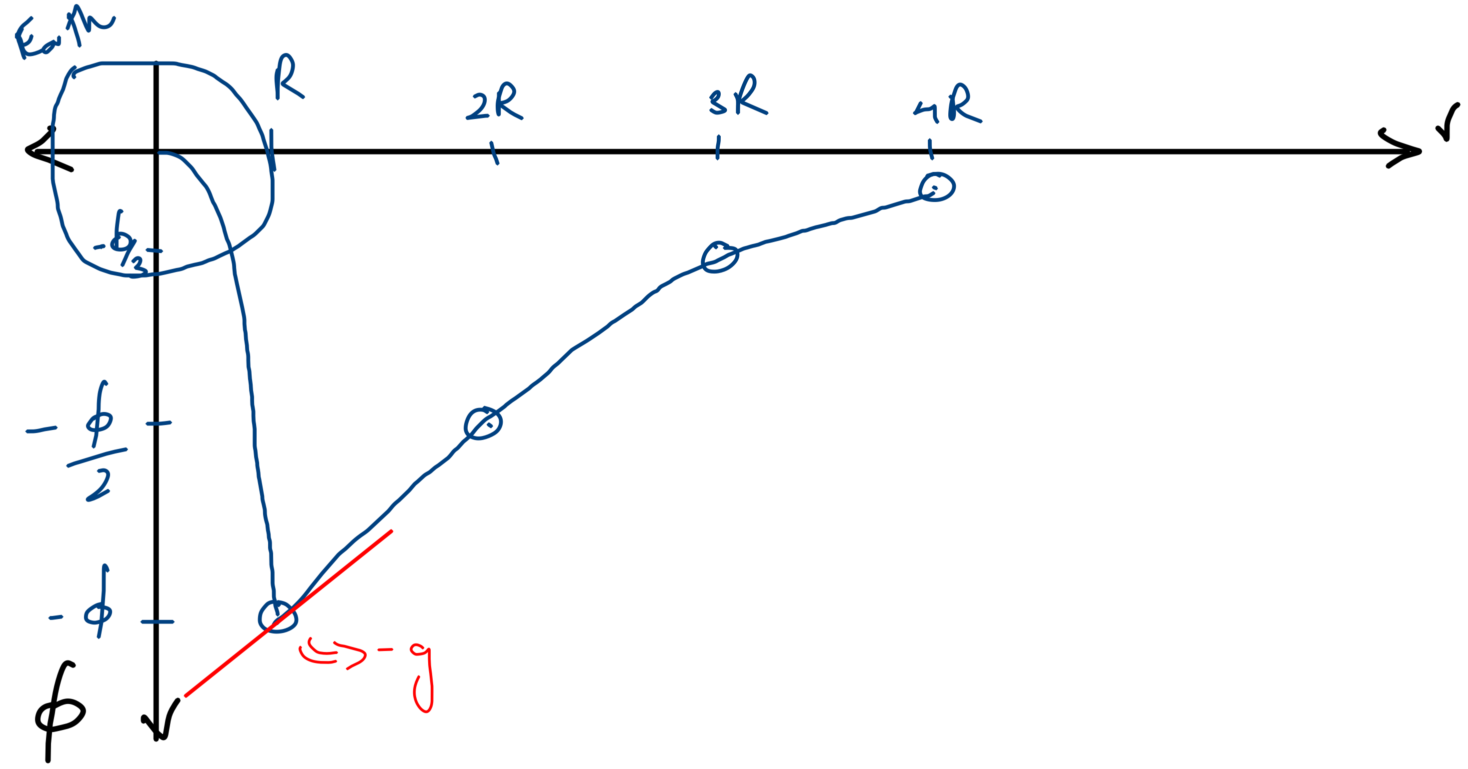


You donot need to input energy because it already has G.P.E

$$\phi = -\frac{W}{m} = \frac{-F_G \times r}{m} = -\frac{\frac{GM\cancel{m}}{r^{\cancel{2}}} \times \cancel{r}}{\cancel{m}} = \frac{-GM}{r}$$

$$* \quad g = -\frac{d\phi}{dr}$$





$$\phi = -\frac{\overset{\text{const}}{GM}}{r} \Rightarrow \phi \propto -\frac{1}{r} \quad \left(y \propto -\frac{1}{x}\right)$$

$$\phi = \frac{K}{r}$$

$$\phi_1 r_1 = K = \phi_2 r_2$$

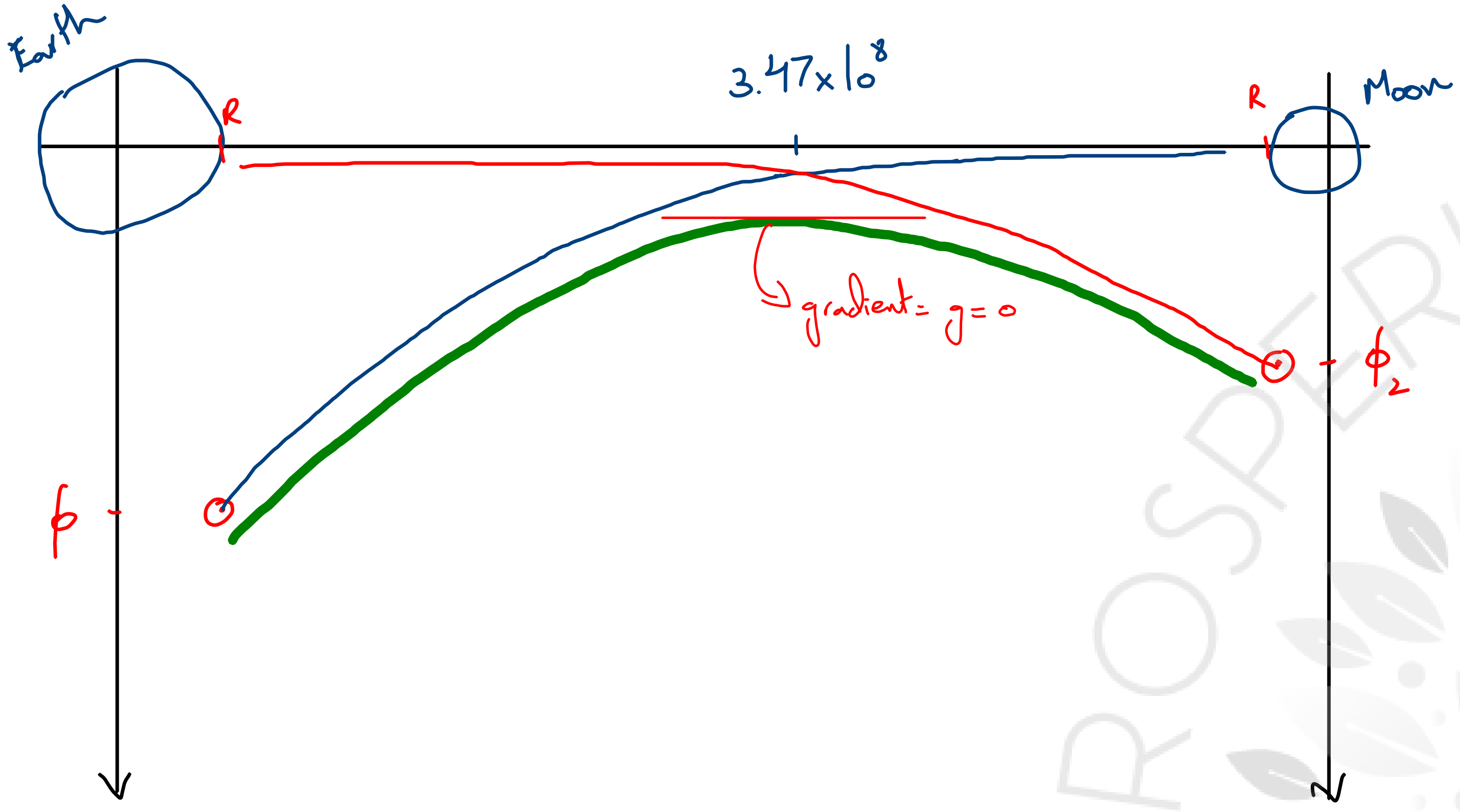
$$\phi_1 r_1 = \phi_2 r_2$$

$$\phi \cancel{R} = \phi_2 (2\cancel{R})$$

$$\phi_2 = \frac{\phi}{2}$$

$$\phi \cancel{R} = \phi_3 \times 3\cancel{R}$$

$$\phi_3 = \frac{\phi}{3}$$



Q Calculate ϕ at the Earth's surface?

$$\phi = \frac{-GM}{R} = \frac{-(6.67\times10^{-11})(6.0\times10^{24})}{6.4\times10^6}$$

$$= -6.25 \times 10^7 \ J\,Kg^{-1}$$

- A unit mass when travelling from infinity to the Earth's surface will lose 6.25×10^7 J of G.P.E
- A unit mass requires 6.25×10^7 J to escape Earth's gravitational field

Gravitational potential energy:-

$$\phi = \frac{W}{m} \Rightarrow W = \phi \times m$$

$$G.P.E = -\frac{GM}{r} \times m$$

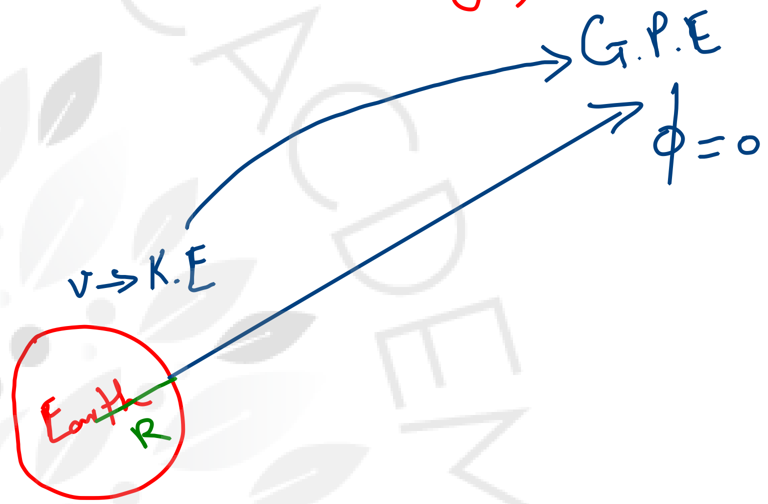
$$G.P.E = -\frac{GMm}{r}$$

$$\Delta G.P.E = m \times \Delta\phi \Rightarrow m \times (\phi_f - \phi_i) \Rightarrow m \times \left(-\frac{GM}{r_f} - \left(-\frac{GM}{r_i}\right)\right) \Rightarrow m \times \left(-\frac{GM}{r_f} + \frac{GM}{r_i}\right)$$

$$\Delta G.P.E = GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

Escape velocity:-

Minimum velocity to escape a mass' gravitational field ($\phi = 0$ [infinity])



$$\Delta K.E = \Delta G.P.E$$

$$\frac{1}{2}\cancel{m}v^2 = \Delta\phi \times \cancel{m}$$

$$\frac{1}{2}v^2 = \phi_f - \phi_i$$

$$\frac{1}{2}v^2 = 0 - \left(-\frac{GM}{R}\right)$$

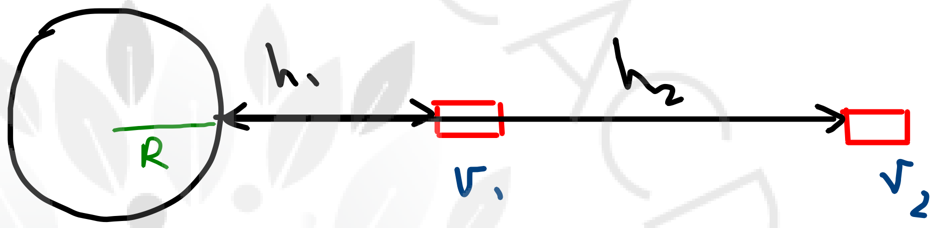
$$v = \sqrt{\frac{2GM}{R}}$$

4 A rocket is launched from the surface of the Earth.

Fig. 4.1 gives data for the speed of the rocket at two heights above the Earth's surface, after the rocket engine has been switched off.

height / m	speed / m s^{-1}
$h_1 = 19.9 \times 10^6$	$v_1 = 5370$
$h_2 = 22.7 \times 10^6$	$v_2 = 5090$

Fig. 4.1



The Earth may be assumed to be a uniform sphere of radius $R = 6.38 \times 10^6$ m, with its mass M concentrated at its centre. The rocket, after the engine has been switched off, has mass m.

(a) Write down an expression in terms of

(i) G, M, m, h_1, h_2 and R for the change in gravitational potential energy of the rocket,

$GMm\left[(R+h_1)^{-1} - (R+h_2)^{-1}\right]$ .... [1]

(ii) m, v_1 and v_2 for the change in kinetic energy of the rocket.

$\frac{1}{2}m(v_2^2 - v_1^2)$ .... [1]

(b) Using the expressions in **(a)**, determine a value for the mass M of the Earth.

$\Delta G.P.E = \Delta\phi \times m$

$(\phi_f - \phi_i) \times m$

$\left(\frac{-GM}{r_f} - \left(-\frac{GM}{r_i}\right)\right) \times m$

$\left[\frac{-GM}{(R+h_2)} + \frac{GM}{(R+h_1)}\right] \times m$

$GMm\left(\frac{1}{(R+h_1)} - \frac{1}{(R+h_2)}\right)$

$\Delta K.E = \Delta G.P.E$

$\frac{1}{2}\cancel{m}(5090^2 - 5370^2) = (6.67\times10^{-11})M\cancel{m}\left[\frac{1}{26.28\times10^6} - \frac{1}{29.08\times10^6}\right]$

$-1464400 = (2.44379\times10^{-19})M$

ignore

$5.99\times10^{24} = M$

s20 qp42 q1

Answer **all** the questions in the spaces provided.

1 **(a)** Define *gravitational potential* at a point.

Work done per unit mass in bringing that mass from infinity to a point within a gravitational field. [2]

(b) An isolated solid sphere of radius r may be assumed to have its mass M concentrated at its centre. The magnitude of the gravitational potential at the surface of the sphere is ϕ.

On Fig. 1.1, show the variation of the gravitational potential with distance d from the centre of the sphere for values of d from $d = r$ to $d = 4r$.

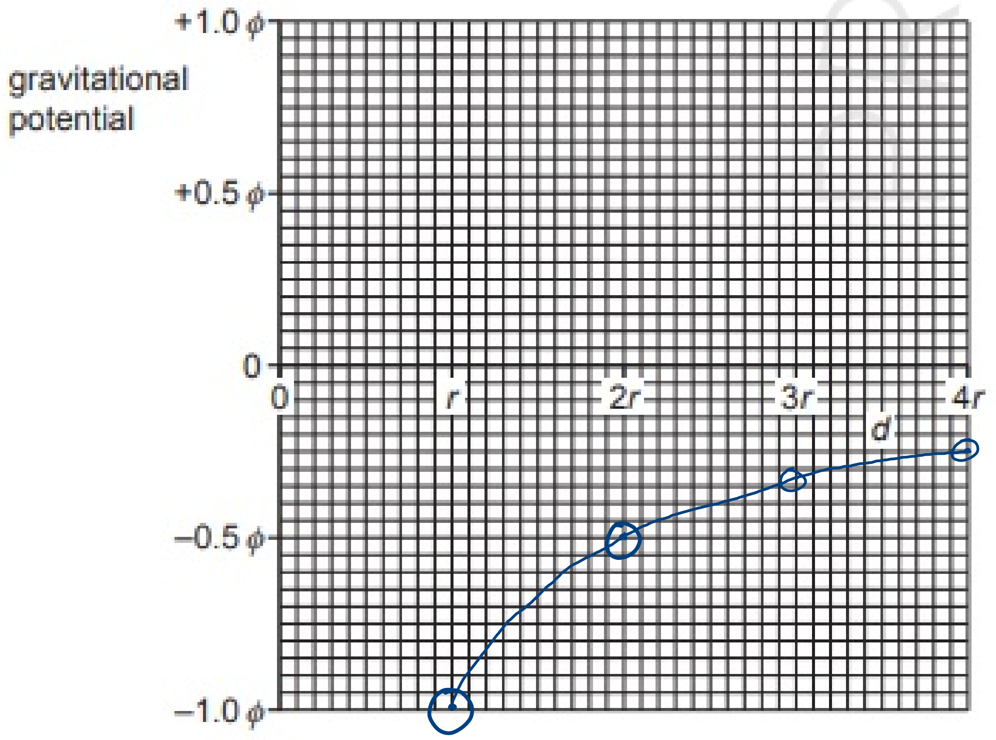


Fig. 1.1

[3]

(c) The sphere in **(b)** is a planet with radius r of 6.4×10^6 m and mass M of 6.0×10^{24} kg. The planet has no atmosphere.

A rock of mass 3.4×10^3 kg moves directly towards the planet. Its distance from the centre of the planet changes from $4r$ to $3r$.

r_i r_f

(i) Calculate the change in gravitational potential energy of the rock.

$$\Delta G.P.E = (\phi_f - \phi_i) \times m \Rightarrow \left[\frac{-GM}{r_f} - \left(\frac{-GM}{r_i}\right)\right] \times m$$

$$\Delta G.P.E = GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

$$= (6.67 \times 10^{-11})(6 \times 10^{24})(3.4 \times 10^3) \times \left[\frac{1}{25.6 \times 10^6} - \frac{1}{19.2 \times 10^6}\right]$$

$$= -1.77 \times 10^{10}$$

change = -1.8×10^{10} J [3]

(ii) Explain whether the rock's speed increases, decreases or stays the same.

The gravitational potential energy decreased so the kinetic energy must have increased so the rock's speed increases. [2]

[Total: 10]

w21 qp42 q2

2 (a) State the relationship between gravitational potential and gravitational field strength.

gravitational field strength is the negative rate change of gravitational potential with respect to distance $(g = -\frac{d\phi}{dr})$ [2]

(b) A moon of mass M and radius R orbits a planet of mass $3M$ and radius $2R$. At a particular time, the distance between their centres is D, as shown in Fig. 2.1.

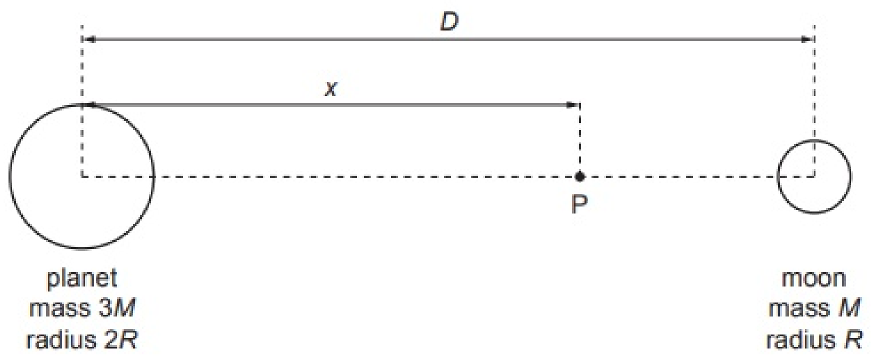


Fig. 2.1

Point P is a point along the line between the centres of the planet and the moon, at a variable distance x from the centre of the planet.

The variation with x of the gravitational potential ϕ at point P, for points between the planet and the moon, is shown in Fig. 2.2.

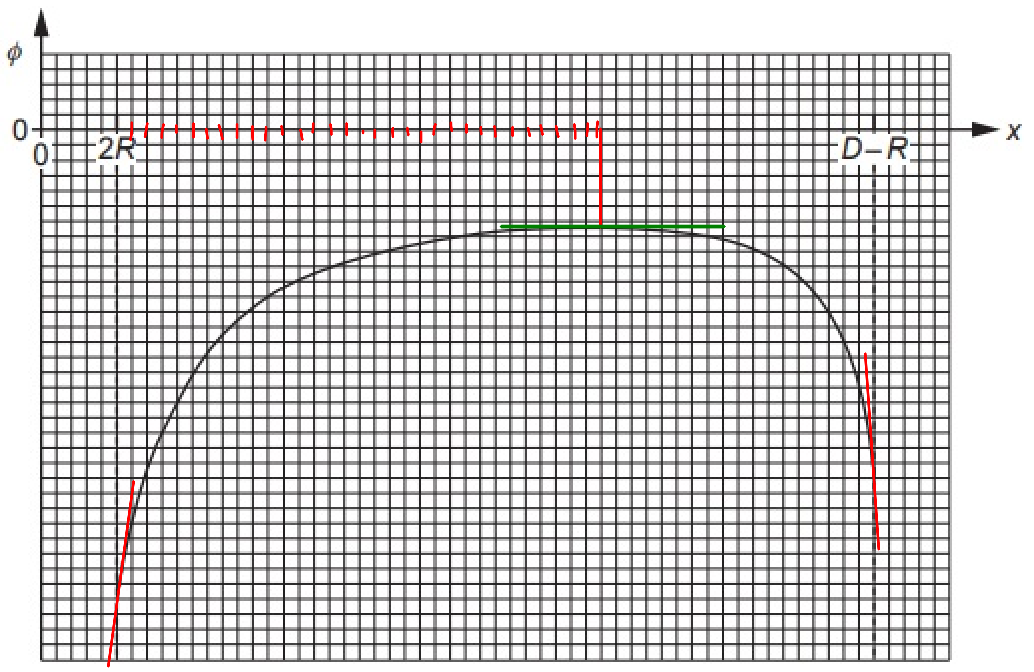


Fig. 2.2

$g = -\frac{d\phi}{dr}$

(i) Explain why ϕ is negative throughout the entire range $x = 2R$ to $x = D - R$.

ϕ is zero at infinity. It is defined as the work done per unit mass in bringing from infinity to a point within a gravitational field. As gravitational fields are always attractive, work gets out therefore ϕ is negative [3]

(ii) One of the features of Fig. 2.2 is that ϕ is negative throughout.

Describe **two** other features of Fig. 2.2.

1. The potential graph has a maximum point which is closer to the moon.

2. Potential is more negative at the surface of the planet.

[2]

(iii) On Fig. 2.3, sketch the variation with x of the gravitational field strength g at point P between $x = 2R$ and $x = D - R$.

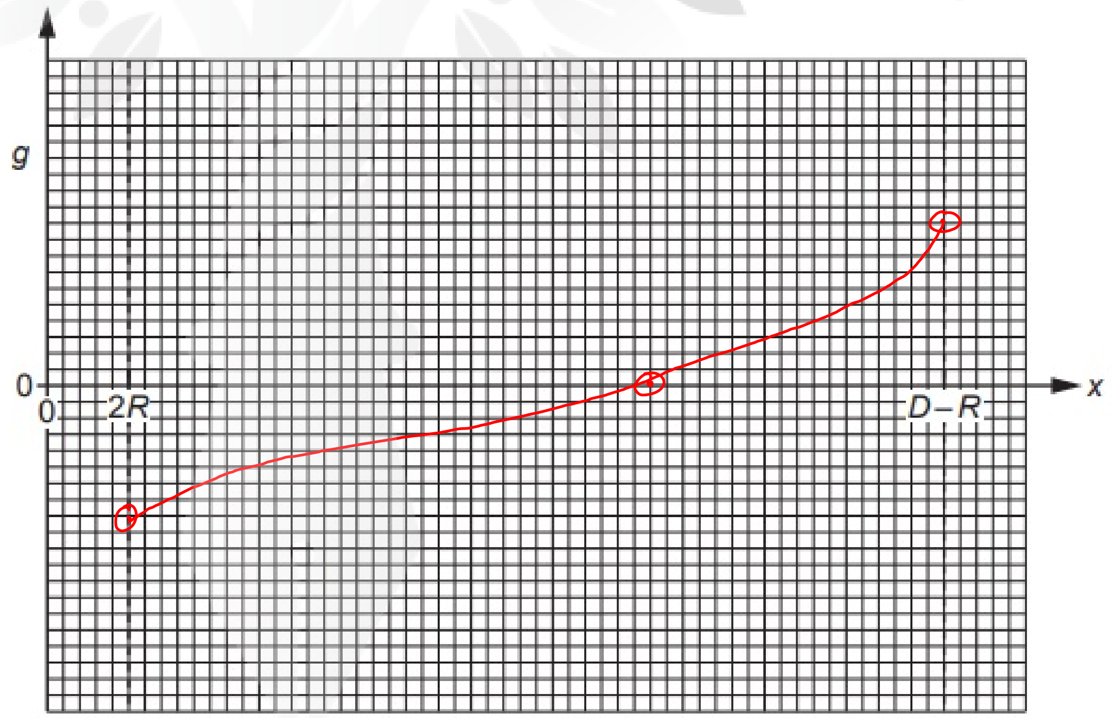


Fig. 2.3

[3]

[Total: 10]

m17 qp42 q1

Answer **all** the questions in the spaces provided.

1 **(a)** Define *gravitational potential* at a point.

Work done per unit mass in bringing that mass from infinity to a point within a gravitational field

[2]

(b) A rocket is launched from the surface of a planet and moves along a radial path, as shown in Fig. 1.1.

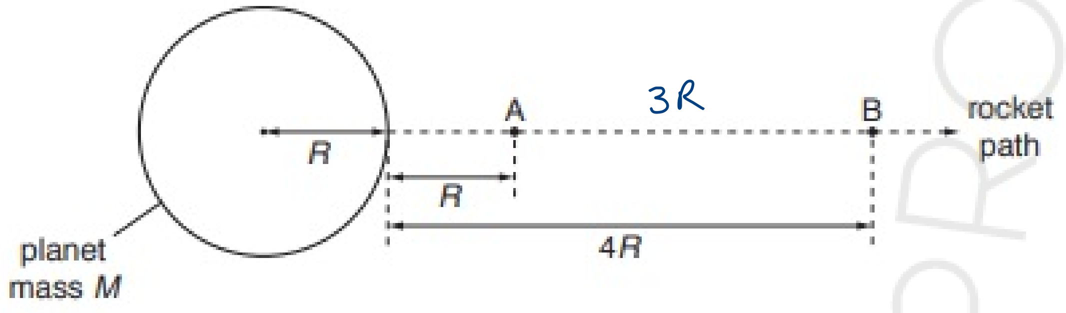


Fig. 1.1

The planet may be considered to be an isolated sphere of radius R with all of its mass M concentrated at its centre. Point A is a distance R from the surface of the planet. Point B is a distance $4R$ from the surface.

(i) Show that the difference in gravitational potential $\Delta\phi$ between points A and B is given by the expression

$$\Delta\phi = \frac{3GM}{10R}$$

where G is the gravitational constant.

$\Delta\phi = \phi_f - \phi_i$

$\frac{-GM}{5R} - \left(-\frac{GM}{2R}\right)$

$\frac{2\times -GM}{2\times 5R} + \frac{GM \times 5}{2R\times 5} \Rightarrow \frac{5GM - 2GM}{10R}$

$= \frac{3GM}{10R}$

[1]

(ii) The rocket motor is switched off at point A. During the journey from A to B, the rocket has a constant mass of 4.7×10^4 kg and its kinetic energy changes from 1.70 TJ to 0.88 TJ.

For the planet, the product GM is $4.0 \times 10^{14}\,\text{N m}^2\,\text{kg}^{-1}$. It may be assumed that resistive forces to the motion of the rocket are negligible.

Use the expression in **(b)(i)** to determine the distance from A to B.

$\Delta K.E = \Delta G.P.E$

$(1.70 - 0.88) \times 10^{12} = \frac{3GMm}{10R}$

$R = \frac{3 \times (4\times10^{14})(4.7\times10^{4})}{10 \times (0.82 \times 10^{12})}$

$R = 6878048$

$AB = 3(6878048)$

$= 2.1 \times 10^{7}\,m$

distance = 2.1×10^{7} m [3]

[Total: 6]

1 **(a)** By reference to the definition of gravitational potential, explain why gravitational potential is a negative quantity.

ϕ is zero at infinity. It is defined as the work done per unit mass n bringing from infinity to a point within a gravitational field. As gravitational fields are always attractive, work gets out therefore ϕ is negative

[2]

(b) Two stars A and B have their surfaces separated by a distance of 1.4×10^{12} m, as illustrated in Fig. 1.1.

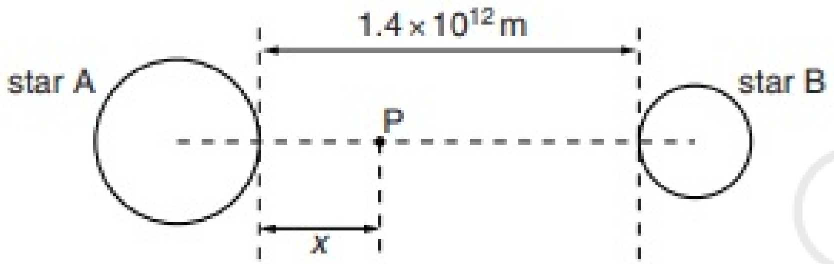


Fig. 1.1

Point P lies on the line joining the centres of the two stars. The distance x of point P from the surface of star A may be varied.

The variation with distance x of the gravitational potential ϕ at point P is shown in Fig. 1.2.

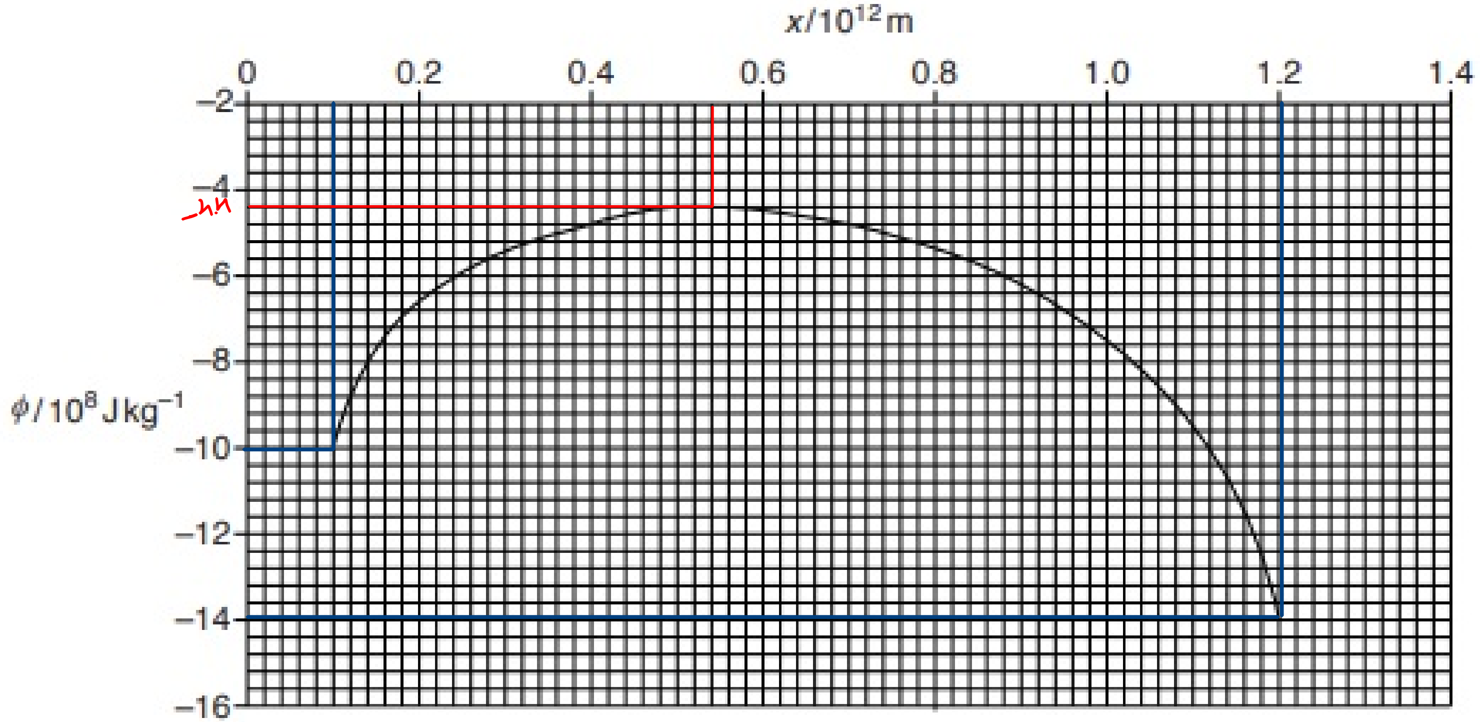


Fig. 1.2

A rock of mass 180 kg moves along the line joining the centres of the two stars, from star A towards star B.

(i) Use data from Fig. 1.2 to calculate the change in kinetic energy of the rock when it moves from the point where $x = 0.1 \times 10^{12}$ m to the point where $x = 1.2 \times 10^{12}$ m. State whether this change is an increase or a decrease.

$\downarrow \Delta G.P.E = \Delta K.E \uparrow$

$(\phi_f - \phi_i) \times m$

$\{[-14 - (-10)] \times 10^8\} \times 180 = \Delta K.E$

$-7.2 \times 10^{10} = \Delta K.E$

change = 7.2×10^{10} J

increases

[3]

(ii) At a point where $x = 0.1 \times 10^{12}$ m, the speed of the rock is v.

Determine the minimum speed v such that the rock reaches the point where $x = 1.2 \times 10^{12}$ m.

$\Delta G.P.E = \Delta K.E$

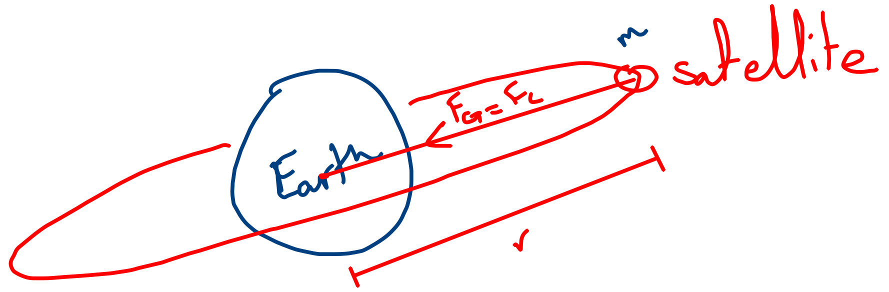
$[(-4.4 + 10) \times 10^8]\cancel{m} = \frac{1}{2}\cancel{m}v^2$

$\sqrt{2[(-4.4 + 10) \times 10^8]} = v = 33466$

minimum speed = 3.3×10^4 m s^{-1} [3]

[Total: 8]

Orbitting System:-



1) Kinetic Energy:-

$$K.E = \frac{1}{2} m v^2$$

$$F_G = F_c$$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m} v^2}{\cancel{r}}$$

$$v^2 = \frac{GM}{r}$$

$$K.E = \frac{1}{2} m \times \frac{GM}{r}$$

$$\boxed{K.E = \frac{GMm}{2r}} \ *$$

2) Gravitational potential energy:-

$$\Delta G.P.E = \Delta\phi \times m$$

$$= (\phi_f - \phi_i) \times m$$

$$= \left[\frac{-GM}{r_f} - \left(\frac{-GM}{r_i}\right)\right] \times m$$

$$\Delta G.P.E = GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

G.P.E (relative to infinity) $= (\phi_f - 0) \times m$

$$= \frac{-GM}{r} \times m$$

$$\boxed{G.P.E = -\frac{GMm}{r}}$$

3) Total energy:- K.E + G.P.E

$$E_T = \frac{GMm}{2r} - \frac{2GMm}{2r} \Rightarrow \boxed{\frac{-GMm}{2r} = E_T}$$

Orbitting Systems

1 (a) State Newton's law of gravitation.

The force of gravity is directly proportional to the product of the masses and inversely proportional to the square of the distance between their centres. [2]

(b) A satellite of mass m is in a circular orbit of radius r about a planet of mass M.
For this planet, the product GM is $4.00 \times 10^{14}\,\mathrm{N\,m^2\,kg^{-1}}$, where G is the gravitational constant.
The planet may be assumed to be isolated in space.

(i) By considering the gravitational force on the satellite and the centripetal force, show that the kinetic energy E_K of the satellite is given by the expression

$$E_K = \frac{GMm}{2r}.$$

$$F_C = F_G$$

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$v^2 = \frac{GM}{r}$$

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times m \times \frac{GM}{r} = \frac{GMm}{2r}$$

[2]

(ii) The satellite has mass 620 kg and is initially in a circular orbit of radius 7.34×10^6 m, as illustrated in Fig. 1.1.

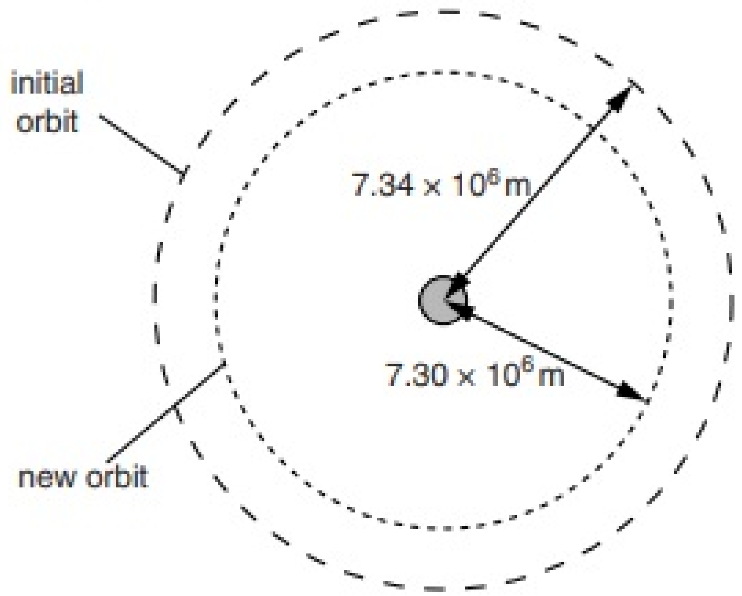


Fig. 1.1 (not to scale)

Resistive forces cause the satellite to move into a new orbit of radius 7.30×10^6 m.

Determine, for the satellite, the change in

1. kinetic energy,

$$\Delta K.E = K.E_f - K.E_i = \frac{GMm}{2r_f} - \frac{GMm}{2r_i}$$

$$\frac{GMm}{2}\left(\frac{1}{r_f} - \frac{1}{r_i}\right)$$

$$= \frac{4\times10^{14} \times 620}{2} \times \left(\frac{1}{7.30\times10^6} - \frac{1}{7.34\times10^6}\right)$$

change in kinetic energy = $+9.26\times10^7$ J [2]

2. gravitational potential energy.

$$\Delta G.P.E = \Delta\phi \times m = (\phi_f - \phi_i) \times m$$

$$= \left[\frac{-GM}{r_f} - \left(\frac{-GM}{r_i}\right)\right] \times m$$

$$= GMm\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

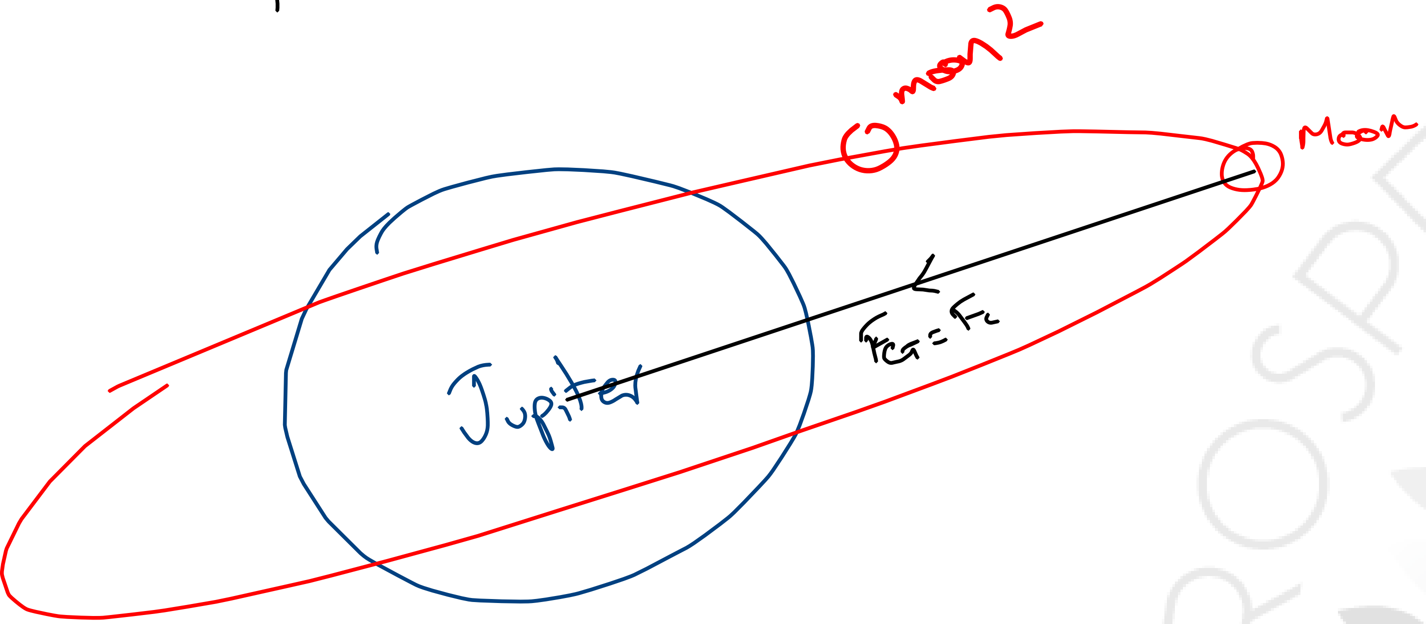
$$= 4\times10^{14} \times 620 \times \left(\frac{1}{7.34\times10^6} - \frac{1}{7.30\times10^6}\right)$$

change in potential energy = -1.85×10^8 J [2]

(iii) Use your answers in (ii) to explain whether the linear speed of the satellite increases, decreases or remains unchanged when the radius of the orbit decreases.

The gravitational potential energy decreased hence, the kinetic energy must have increased so the linear speed of the satellite also increased. [2]

Kepler's law:-



$$F_G = F_c$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}r\omega^2$$

$$\frac{GM}{r^3} = \frac{4\pi^2}{T^2}$$

const

$$T^2 = \frac{4\pi^2}{GM} \times r^3$$

$$\boxed{T^2 \propto r^3}$$ Kepler's law

$$T^2 \propto r^3 \Rightarrow T^2 = Kr^3 \Rightarrow \frac{T_1^2}{r_1^3} = K = \frac{T_2^2}{r_2^3}$$

$$\boxed{\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}}$$

Q. One of jupiter's moons has a time period of 8.64×10^7 s and is at a distance of 9×10^8 m from Jupiter's centre. Given another moon's distance $= 10 \times 10^8$, calculate it's time period.

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$\frac{(8.64 \times 10^7)^2}{(9 \times 10^8)^3} = \frac{T_2^2}{(10 \times 10^8)^3}$$

$$\sqrt{T_2^2} = \sqrt{\frac{(8.64 \times 10^7)^2 (10 \times 10^8)^3}{(9 \times 10^8)^3}}$$

$$T_2 = 1.0 \times 10^8$$

Kepler's Law

1 **(a)** State Newton's law of gravitation.

The force of gravity is directly proportional to the product of the masses and inversely proportional to the square of the distance between their centres [2]

(b) Some of the planets in the Solar System have several moons (satellites) that have circular orbits about the planet.
The planet and each of its moons may be considered to be point masses.

Show that the radius x of a moon's orbit is related to the period T of the orbit by the expression

$$GM = \frac{4\pi^2 x^3}{T^2} \Rightarrow x^3 = \frac{GM}{4\pi^2} \times T^2$$

$$y = m x$$

where G is the gravitational constant and M is the mass of the planet. Explain your working.

[3]

(c) The planet Neptune has eight moons, each in a circular orbit of radius x and period T. The variation with T^2 of x^3 for some of the moons is shown in Fig. 1.1.

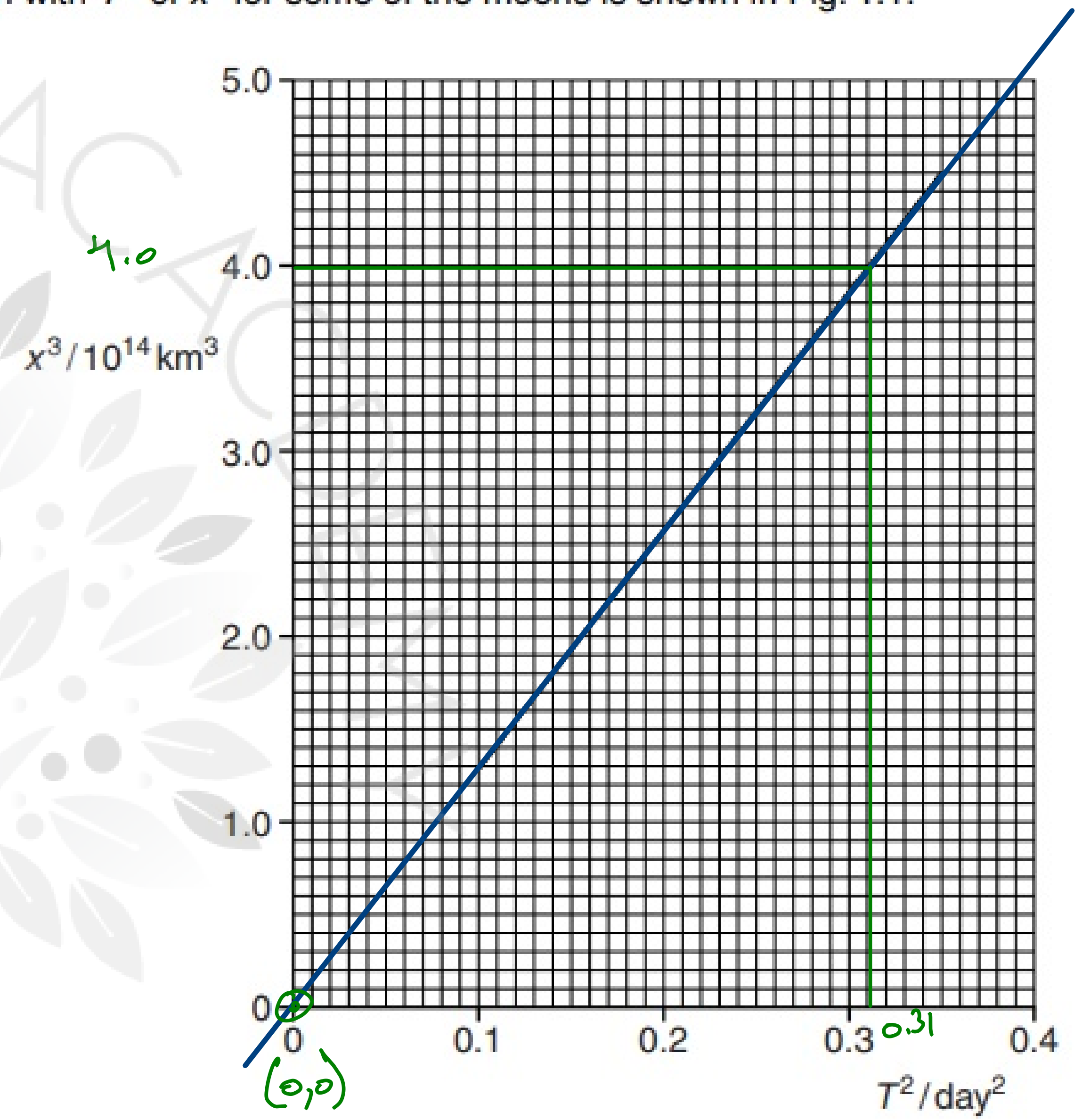


Fig. 1.1

Use Fig. 1.1 and the expression in **(b)** to determine the mass of Neptune.

$$m = \frac{GM}{4\pi^2} \Rightarrow \frac{(4.0 - 0) \times 10^{14} \times (10^3)^3}{(0.31 - 0) \times (60 \times 60 \times 24)^2} = \frac{(6.67 \times 10^{-11}) M}{4\pi^2}$$

or

$$GM = \frac{4\pi^2 x^3}{T^2} \Rightarrow (6.67 \times 10^{-11}) M = \frac{4\pi^2 \times (4 \times 10^{14}) \times (10^3)^3}{(0.31) \times (60 \times 60 \times 24)^2}$$

$$M = 1.02 \times 10^{26}$$

Geostationary Satellite:- (G.P.S satellites)

1. Orbit the earth about the equator
2. Orbits the earth, west to east
3. It takes 24 hours to orbit the earth
4. They orbit at a fixed distance of 4.23×10^{7} m from the Earth's center / It orbits at a fixed distance of 3.58×10^{7} from the Earth's surface.

$$F_G = F_c$$

$$\frac{GM\cancel{m}}{r^2} = \cancel{m}r\,\frac{4\pi^2}{T^2}$$

$$\sqrt[3]{\frac{GM\times T^2}{4\pi^2}} = \sqrt[3]{r^3}$$

$$\sqrt[3]{\frac{(6.67\times10^{-11})(6.0\times10^{24})\times(24\times60\times60)^2}{4\pi^2}} = r = 4.23\times10^{7}$$

m21 qp42 q1

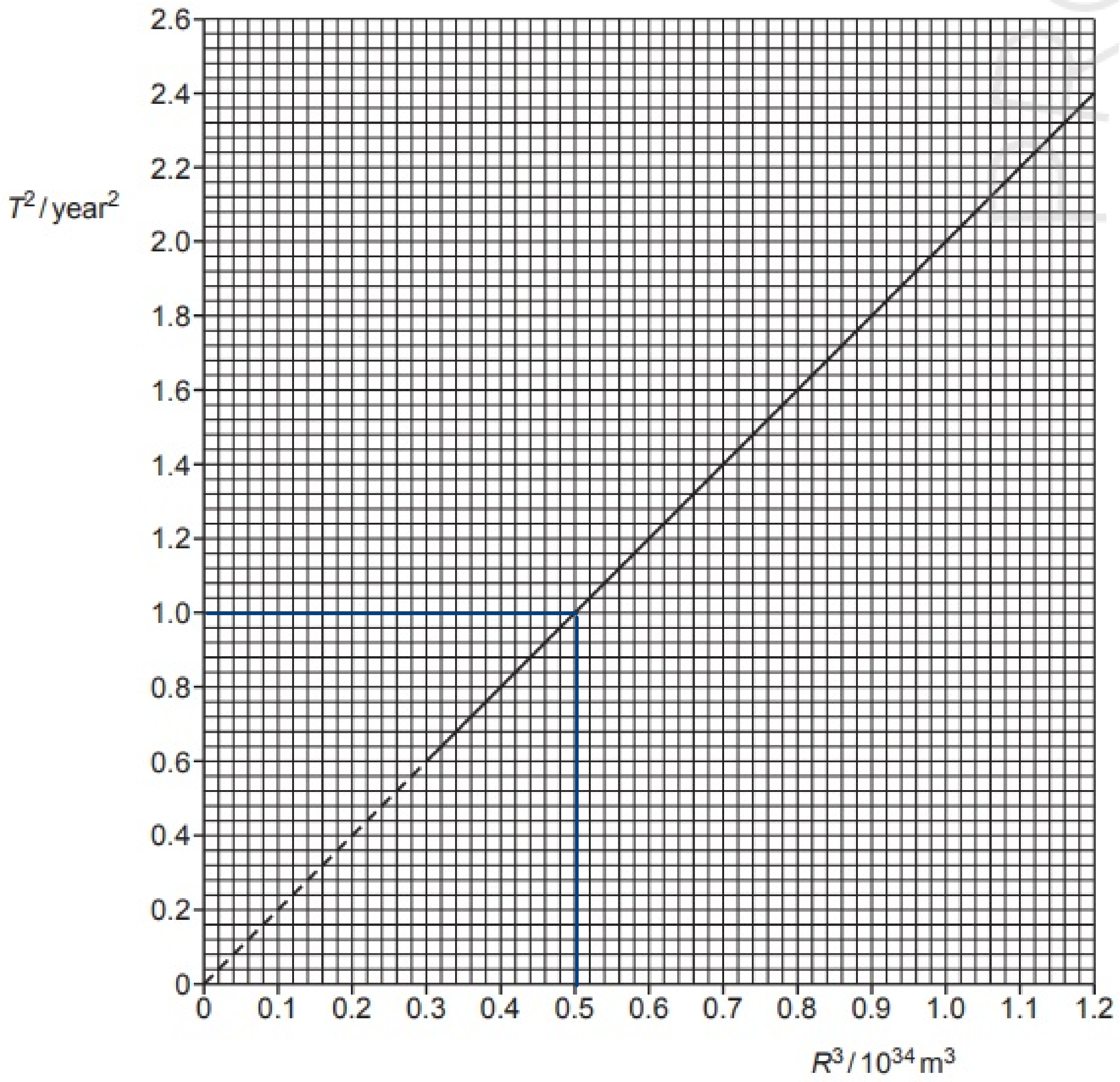
Answer **all** questions in the spaces provided.

1 **(a)** State Newton's law of gravitation.

The force of gravity is directly proportional to the product of the masses and inversely proportional to the square of the distance between their centres [2]

(b) Planets have been observed orbiting a star in another solar system. Measurements are made of the orbital radius r and the time period T of each of these planets.

The variation with R^3 of T^2 is shown in Fig. 1.1.



The relationship between T and R is given by

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

where G is the gravitational constant and M is the mass of the star.

Determine the mass M.

$$(1) \times (365 \times 24 \times 60 \times 60)^2 = \frac{4\pi^2 \times 0.5 \times 10^{34}}{(6.67 \times 10^{-11})\, M}$$

$$M = \frac{4\pi^2 \times 0.5 \times 10^{34}}{(6.67 \times 10^{-11}) \times (365 \times 24 \times 60 \times 60)^2}$$

M = 3.0×10^{30} kg [3]

(c) A rock of mass m is also in orbit around the star in **(b)**. The radius of the orbit is r.

(i) Explain why the gravitational potential energy of the rock is negative.

gravitational potential is zero at infinity as gravitational fields are attractive, work / gravitational potential energy gets out of the system.

[3]

(ii) Show that the kinetic energy E_k of the rock is given by

$$E_k = \frac{GMm}{2r}.$$

$F_c = F_G$

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}} \Rightarrow v^2 = \frac{GM}{r} \qquad E_k = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$ proved

[2]

(iii) Use the expression in **(c)(ii)** to derive an expression for the total energy of the rock.

$E_T = E_K + G.P.E$

$$\frac{GMm}{2r} - \frac{GMm}{r} \Rightarrow E_T = \frac{-GMm}{2r}$$

[2]

[Total: 12]