IDEAL GASES (A2 2022 -23 Syllabus)

Ideal gas:

Meaning: A gas which obeys ideal gas equation $PV = nRT$ at all values of pressure, volume and temperature.

Real gases like oxygen, Nitrogen, CO_2, CO etc behave like an Ideal gas at high temperature and low pressure.

Proof of $PV = nRT$:-

Boyle's law: $P \propto \frac{1}{V}$, no. of moles (amount of gas) and Temperature are constants.

$$PV = \text{Constant}$$

Charle's law: $V \propto T$, no. of moles (amount of gas) and Pressure are constants

$$\frac{V}{T} = \text{Constant}.$$

Pressure law: $P \propto T$, no. of moles (amount of gas) and Volume are constants

$$\frac{P}{T} = \text{Constant}$$

Combining all above laws.

$$\frac{PV}{T} = \text{Constant}.$$

The value of constant is same for one mole of a gas and is called molar gas constant, denoted by R. Its value in S.I is $8.31\,J\,mol^{-1}K^{-1}$

$$\frac{PV_m}{T} = R \quad \text{------------} (1)$$

Here,

Volume of one mole of a gas $= V_m$

" " n- moles " " " $= V$

$$V = nV_m \Rightarrow V_m = \frac{V}{n}$$

Therefore eq. (1) is modified as

$$\frac{P}{T}\left(\frac{V}{n}\right) = R$$

$$\boxed{PV = nRT} \quad \text{------------} (2)$$

2nd form:

no. of particles in one mole = Avogadro's no = N_A

" " " " n moles = N

$$N = nN_A$$

$$n = \frac{N}{N_A}$$

Now eq. (2) is modified as

$$PV = \left(\frac{N}{N_A}\right)RT \Rightarrow PV = N\left(\frac{R}{N_A}\right)T$$

$$\boxed{PV = NKT}$$

Here K is the Boltzmann's constant

It's value in S.I. is

$$K = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}\ J\,mol^{-1}K^{-1}$$

Q) Calculate the volume of one mole of an ideal gas at room temperature ($20^\circ C$) and pressure (1.013×10^5 Pa).

Given: $P = 1.013 \times 10^5$ Pa

$n = 1.0$ mol

$T = 20 + 273.15 = 293K$

Formula: $PV = nRT$

Sol. $(1.013 \times 10^5)(V) = (1.0)(8.31)(293)$

$V = 0.0240\ m^3 = 2.40 \times 10^{-2}\ m^3$

or $\boxed{V = 24.0\ dm^3}$

Note: $1\ dm = 0.1\ m$

Q) A car tyre contains $0.020\ m^3$ of air at $27^\circ C$ and at a pressure of 3.0×10^5 Pa. Calculate the mass of air in the tyre. (Molar mass of air $= 28.8\ g\,mol^{-1}$).

Sol. No. of moles of air inside tyre:

$$PV = nRT \Rightarrow n = \frac{PV}{RT} \Rightarrow n = \frac{(3.0 \times 10^5)(0.020)}{(8.31)(27 + 273.15)} = 2.41\ mol$$

Mass of air inside tyre:

Mass = (number of moles)(molar mass)

$= (2.41)(28.8) = 69.4g$

Assumptions of Kinetic theory of gases:-

1- A gas contains a large number of particles (atoms or molecules)

2- The particles are in a state of perpetual (continuous without any delay) random motion (straight line with an abrupt change of path/direction)

3- The particles collide with each other and with the walls of container and these collisions are elastic in nature, so Newton's laws are applicable.

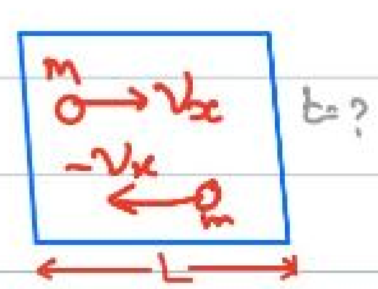
4- The volume of the particles is negligible compared to the volume occupied by the gas.

5 - There are negligible intermolecular forces except during collision.

6- The average kinetic energy of gas particles increases with the rise of temperature.

7 - The time of collision by a particle with the container walls is negligible compared with the time between collisions. i.e

The time of collision with a wall is equal to the time taken to travel a distance of 2L with uniform speed.



$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$v = \frac{L+L}{t}$$

$$t = \frac{2L}{v}$$

PRESSURE OF AN IDEAL GAS:

Proof of $P = \frac{1}{3V} Nm\langle c^2 \rangle$:

Here P - Pressure of gas particles
V - Volume of gas
N - No. of particles
m - mass of a particle
$\langle c^2 \rangle$ - Mean value of squared speed of particles

Steps to be followed:-

1- Find the change in momentum as a single molecule hits a wall at 90°.

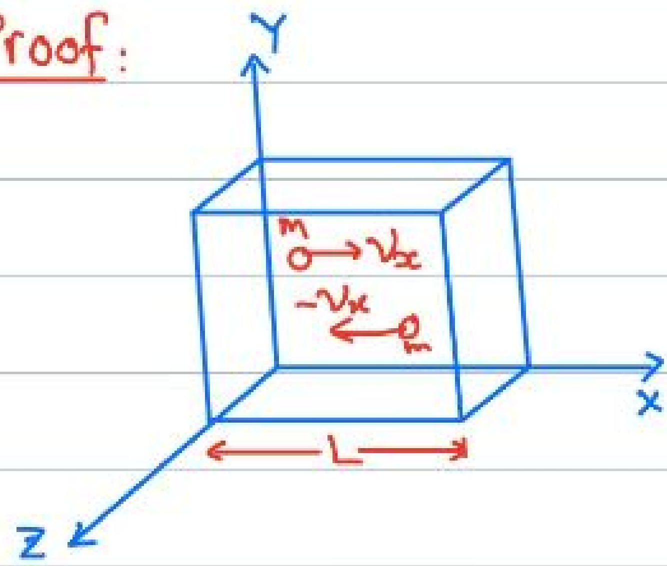
2- Find the change in momentum per second to get force acting on a wall by single molecule.

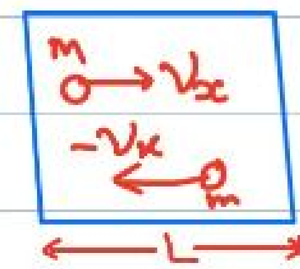
3- Find the pressure on a wall.

4- Find the total pressure of number of molecules acting on a wall.

5- Consider the effect of having three directions in which molecules can move due to their random motions.

Proof:





Initial momentum of a particle $= mv_x$

Final momentum of a particle $= m(-v_x)$

change of momentum $= m(-v_x) - mv_x = -2mv_x$

→ Time of collision with a wall is equal to the time taken to travel of distance of 2L with uniform speed as per assumption. $\{2L = (v_x)(t)\}$

$$F = \frac{\Delta P}{\Delta t}$$

$$F = \frac{-\cancel{2}mv_x}{\frac{\cancel{2}L}{v_x}}$$

$$F = -\frac{mv_x^2}{L}$$

→ -ve sign shows that Force is excerted by right side of container on particle. So by Newton's third law of motion, force excerted to cause pressure at right side by particle is +ve

$$F = \frac{m v_x^2}{L}$$

Pressure, $P = \frac{F}{A} = \frac{\frac{m v_x^2}{L}}{L^2} = \frac{m v_x^2}{L^3}$

$$P = \frac{m v_x^2}{V}$$

→ Since a gas consists of large number of particles/molecules. So the total pressure at right side due to all particle is

$$P = \frac{m v_{1x}^2}{V} + \frac{m v_{2x}^2}{V} + \frac{m v_{3x}^2}{V} + \cdots + \frac{m v_{Nx}^2}{V}$$

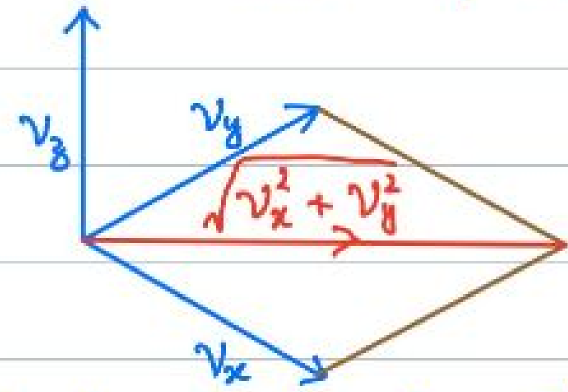
$$P = \frac{m}{V}\left\{ v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots + v_{Nx}^2 \right\}$$

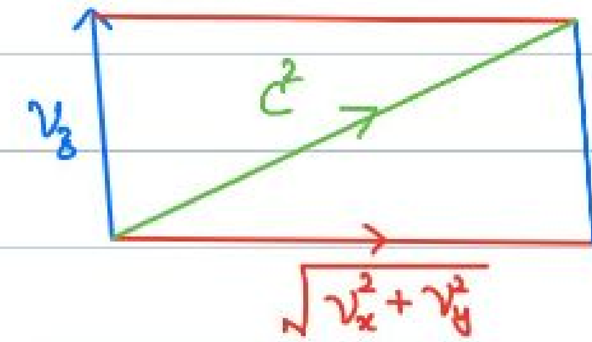
Multiple and divide right hand sides by N - no. of particles

$$P = \frac{Nm}{V}\left\{ \frac{v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \cdots + v_{Nx}^2}{N} \right\}$$

$$P = \frac{Nm}{V} \langle v_x^2 \rangle \quad \text{-------------------} \quad (1)$$

→ But gas particles move randomly through out the available space as shown.





Resultant velocity of a particle through out the available space be

$$c^2 = \left(\sqrt{v_x^2 + v_y^2}\right)^2 + v_z^2$$

$$c^2 = v_x^2 + v_y^2 + v_z^2$$

or $$\langle c^2 \rangle = \langle v_x^2 \rangle + \langle v_y^2 \rangle + \langle v_z^2 \rangle \text{ .... } (2)$$

→ At constant temperature, average kinetic energy or mean squared speed of particles in all directions is also same i.e

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

Therefore, eq. (2) becomes as

$$\langle c^2 \rangle = \langle v_x^2 \rangle + \langle v_x^2 \rangle + \langle v_x^2 \rangle$$

$$\langle c^2 \rangle = 3\langle v_x^2 \rangle$$

$$\langle v_x^2 \rangle = \frac{\langle c^2 \rangle}{3} \text{ .... } (3)$$

$$\boxed{P = \frac{Nm}{V}\left(\frac{\langle c^2 \rangle}{3}\right)}$$

Since $\frac{Nm}{V} = \rho$ (Density)

$$\boxed{P = \frac{\rho}{3}\langle c^2 \rangle}$$

Relationship between average Kinetic energy and temperature :-

Since, $pV = NKT$ .... (1)

Also $p = \frac{Nm}{3V} <c^2>$

$pV = \frac{Nm}{3} <c^2>$ .... (2)

Comparing (1) and (2)

$$\frac{\cancel{N}m}{3} <c^2> = \cancel{N}KT$$

$$m <c^2> = 3KT$$

Divide both sides by 2

$$\frac{1}{2} m <c^2> = \frac{3}{2} KT$$

$$<E_K> = \frac{3}{2} KT$$

$$<E_K> = (\text{Constant}) T$$

$$<E_K> \propto T$$

Note: At Absolute zero, (0 Kelvin) particles have no Kinetic energy. So particles can not move at zero Kelvin.

Q) Calculate the mean translational (linear) Kinetic energy of atoms of an Ideal gas at 27°C.

$$\langle E_k \rangle = \frac{3}{2} KT$$

$$\langle E_k \rangle = \frac{3}{2}(1.38\times10^{-23})(27+273.15)$$

$$\langle E_k \rangle = 6.21\times10^{-21}\ J$$

Q) The mean translational kinetic energy is 5.0×10^{-21} J. Calculate the temperature in Kelvin (K) and in centigrade (°C).

$$\langle E_k \rangle = \frac{3}{2} KT$$

$$5.0\times10^{-21} = \frac{3}{2}(1.38\times10^{-23})T$$

$$T = 242\ K$$

Also $T/K = \theta/°C + 273.15$

$$242 = \theta/°C + 273.15$$

$$\theta/°C = -31.2\ °C$$

Avogadro Constant:-

Def. Number of atoms/nuclei/particles in 12g (0.012kg) of Carbon-12 Isotope.

Symbol: N_A

Value: $6.02\times10^{23}\ mol^{-1}$ {given at data page}

Note: Avogadro constant represents number of particles in one mole of any substance ($6.02\times10^{23}\ mol^{-1}$)

Mole:- — Base unit of amount of a substance

Def. Amount of substance containing same number of particles/atoms/molecules as there are atoms in 12g (0.012kg) of Carbon-12.

OR

Amount of substance containing 6.02×10^{23} particles/molecules/atoms.

Symbol: n

Boltzmann Constant (K):-

$$K = \frac{R}{N_A} = \frac{8.31}{6.02 \times 10^{23}} = 1.38 \times 10^{-23}\ J\,K^{-1}$$

Boltzmann constant tells us how a property of microscopic particles i.e. kinetic energy of gas particles is related to a bulk/macroscopic property of the gas at its absolute temperature. That is why its units are $J\,K^{-1}$.

It's value is very small ($1.38 \times 10^{-23}\ J\,K^{-1}$) because the Increase in kinetic energy in Joule of a molecule is very small for each Kelvin increase in temperature.

Note: If mass number is provided and examiner N-21/42 asks to calculate the no. of nuclei then use:

$$N = \left(\frac{\text{mass of Sample}}{\text{nucleon number}}\right)(\text{Avogadro constant})$$

2 In a sample of gas at room temperature, five atoms have the following speeds:

$1.32 \times 10^3\,\mathrm{m\,s^{-1}}$
$1.50 \times 10^3\,\mathrm{m\,s^{-1}}$
$1.46 \times 10^3\,\mathrm{m\,s^{-1}}$
$1.28 \times 10^3\,\mathrm{m\,s^{-1}}$
$1.64 \times 10^3\,\mathrm{m\,s^{-1}}$.

For these five atoms, calculate, to three significant figures,

(a) the mean speed,

$$\text{Mean speed} = \frac{(1.32 + 1.50 + 1.46 + 1.28 + 1.64)10^3}{5}$$

$$\langle v \rangle = 1.44 \times 10^3\,\mathrm{m s^{-1}}$$

mean speed = ... $\mathrm{m\,s^{-1}}$ [1]

(b) the mean-square speed,

$$\langle v^2 \rangle = \frac{[(1.32)^2 + (1.50)^2 + (1.46)^2 + (1.28)^2 + (1.64)^2]10^6}{5}$$

$$= 2.09 \times 10^6\,\mathrm{m^2 s^{-2}}$$

mean-square speed = ... $\mathrm{m^2\,s^{-2}}$ [2]

(c) the root-mean-square speed.

$$V_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{2.09 \times 10^6}$$

$$= 1.45 \times 10^3\,\mathrm{m s^{-1}}$$

root-mean-square speed = ... $\mathrm{m\,s^{-1}}$ [1]

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IDEAL GASES

1. The pressure p of an ideal gas is given by the expression

$$p = \frac{1}{3}\frac{Nm}{V}<c^2>.$$

(a) Explain the meaning of the symbol $<c^2>$.

Mean/average value of squared speed of particles of an Ideal gas .... [2]

(b) The ideal gas has a density of 2.4 kgm^{-3} at a pressure of 2.0×10^5 Pa and a temperature of 300 K.

(i) Determine the root-mean-square (r.m.s.) speed of the gas atoms at 300 K.

$$P = \frac{\rho}{3}<c^2>$$

$$2.0\times10^5 = \frac{2.4}{3}<c^2> \Rightarrow <c^2> = \frac{6.0\times10^5}{2.4}$$

$$C_{rms} = \sqrt{<c^2>} = \sqrt{\frac{6.0\times10^5}{2.4}} = 500\ ms^{-1}$$

r.m.s. speed = ... m s^{-1} [3]

(ii) Calculate the temperature of the gas for the atoms to have an r.m.s. speed that is twice that calculated in **(i)**.

$$C_{2rms} = 2C_{1rms} \Rightarrow C_{2rms} = 2(500) = 1000$$

$$\frac{1}{2}m<c^2> = \frac{3}{2}KT$$

$$\frac{\cancel{\frac{1}{2}}\cancel{m}<c_2^2>}{\cancel{\frac{1}{2}}\cancel{m}<c_1^2>} = \frac{\frac{3}{\cancel{2}}\cancel{K}T_2}{\frac{3}{\cancel{2}}\cancel{K}T_1} \Rightarrow \frac{<c_2^2>}{<c_1^2>} = \frac{T_2}{T_1}$$

$$\frac{(1000)^2}{(500)^2} = \frac{T_2}{300}$$

$T_2 =$ ______ K temperature = ... K [3]

2. (a) (i) The kinetic theory of gases leads to the equation

$$\tfrac{1}{2}m<c^2> = \tfrac{3}{2}kT.$$

Explain the significance of the quantity $\tfrac{1}{2}m<c^2>$.

Mean/Average Kinetic energy of particles of an Ideal gas.

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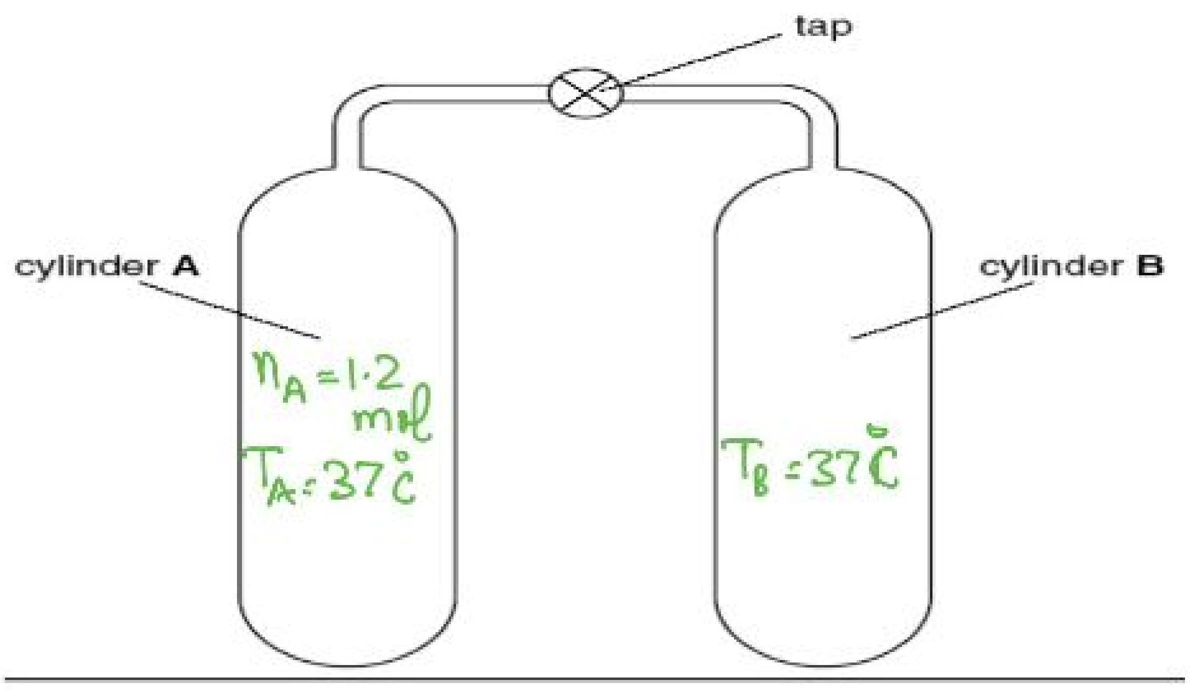
(ii) Use the equation to suggest what is meant by the absolute zero of temperature.

Particles have no kinetic energy, so no motion is observed at Absolute zero. (0 K) [3]

(b) Two insulated gas cylinders **A** and **B** are connected by a tube of negligible volume, as shown in Figure.



Each cylinder has an internal volume of $2.0 \times 10^{-2} m^3$. Initially, the tap is closed and cylinder **A** contains 1.2 mol of an ideal gas at a temperature of 37 °C. Cylinder **B** contains the same ideal gas at pressure 1.2×10^5 Pa and temperature 37 °C.

(i) Calculate the amount, in mol, of the gas in cylinder **B**.

$$P_B V_B = n_B R T_B$$

$$(1.2\times10^5)(2.0\times10^{-2}) = (n_B)(8.31)(37+273.15)$$

$$n_B = 0.93$$

amount = ... mol

(ii) The tap is opened and some gas flows from cylinder **A** to cylinder **B**. Using the fact that the total amount of gas is constant, determine the final pressure of the gas in the cylinders.

$$P_T V_T = n_T R T$$

$$P_T(V_A + V_B) = (n_A + n_B) R T$$

$$P_T(2.0\times10^{-2} + 2.0\times10^{-2}) = (1.2+0.93)(8.31)(37+273.15)$$

$$P_T = 1.37\times10^5 \text{ Pa}$$

pressure = ... Pa
[6]

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3. If an object is projected vertically upwards from the surface of a planet at a fast enough speed, it can escape the planet's gravitational field. This means that the object can arrive at infinity where it has zero kinetic energy. The speed that is just enough for this to happen is known as the escape speed.

(a) (i) By equating the kinetic energy of the object at the planet's surface to its total gain of potential energy in going to infinity, show that the escape speed v is given by

$$v^2 = \frac{2GM}{R},$$

where R is the radius of the planet and M is its mass.

Loss of E_K = Gain in E_P

$$\frac{1}{2}\cancel{m}v^2 = -\frac{GM\cancel{m}}{R}$$

$$v^2 = -\frac{2GM}{R}$$

−ve sign with G.E_P is neglected

$$v^2 = \frac{2GM}{R}$$

(ii) Hence show that

$$v^2 = 2Rg,$$

where g is the acceleration of free fall at the planet's surface.

Since $g = \frac{GM}{R^2} \Rightarrow \frac{GM}{R} = gR$

So $v^2 = 2gR$

[3]

(b) The mean kinetic energy Ek of an atom of an ideal gas is given by

$$E_k = \frac{3}{2}kT,$$

where k is the Boltzmann constant and T is the thermodynamic temperature.
Using the equation in **(a)(ii)**, estimate the temperature at the Earth's surface such that helium atoms of mass 6.6×10^{-27} kg could escape to infinity.
You may assume that helium gas behaves as an ideal gas and that the radius of Earth is 6.4×10^6m.

$$\frac{1}{2}mv^2 = \frac{3}{2}KT$$

$$\frac{1}{\cancel{2}}m(\cancel{2}gR) = \frac{3}{2}KT$$

$$T = \frac{2mgR}{3K}$$

$$T = \frac{2(6.6\times10^{-27})(9.81)(6.4\times10^{6})}{3(1.38\times10^{-23})}$$

$$T = 2.0\times10^{4}\ K$$

temperature = ... K [4]

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4.

The pressure p of an ideal gas is given by both of the following equations.

$$p = \frac{Nm < c^2 >}{3V} \qquad\qquad p = \frac{NkT}{V}$$

(i) Use the equations to show that the average translational kinetic energy of a molecule is proportional to the temperature T.

Comparing given equations

$$\frac{\cancel{N} m < c^2 >}{3\cancel{V}} = \frac{\cancel{N} KT}{\cancel{V}} \Rightarrow m < c^2 > = 3KT$$

$$\frac{1}{2} m < c^2 > = \frac{3}{2} KT \Rightarrow < E_K > = (\text{Constant})\, T$$

$$< E_K > \propto T$$

[3]

(ii) Calculate the average kinetic energy of a molecule of an ideal gas at a temperature of 27 °C.

$$< E_K > = \frac{3}{2} KT$$

$$= \frac{3}{2} (1.38 \times 10^{-23})(27 + 273.15)$$

$$< E_K > = 6.21 \times 10^{-21}\ \text{J}$$

kinetic energy = ... J [2]

(iii) Explain why the answer to **(ii)** is independent of the mass of the gas molecules.

Less mass gas particles move with greater speed to have same Kinetic energy as greater mass particles move with lesser speed. [2]

(iv) A laboratory contains 2600 mol of air at a temperature of 27 °C. Calculate the total kinetic energy of all the molecules of air in the laboratory.

$$\text{Average } E_K = \frac{\text{Total } E_K}{\text{no. of molecules}} \Rightarrow < E_K > = \frac{\text{Total Kinetic energy}}{(n)(N_A)}$$

$$6.21 \times 10^{-21} = \frac{E_K}{(2600)(6.02 \times 10^{23})}$$

$E_K =$ ________ J

kinetic energy = ... J [2]

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IDEAL GASES

5.

(a) The equation

$$pV = constant \times T$$

relates the pressure p and volume V of a gas to its kelvin (thermodynamic) temperature T.

State two conditions for the equation to be valid.

1. Amount of gas / no. of moles of gas is constant.

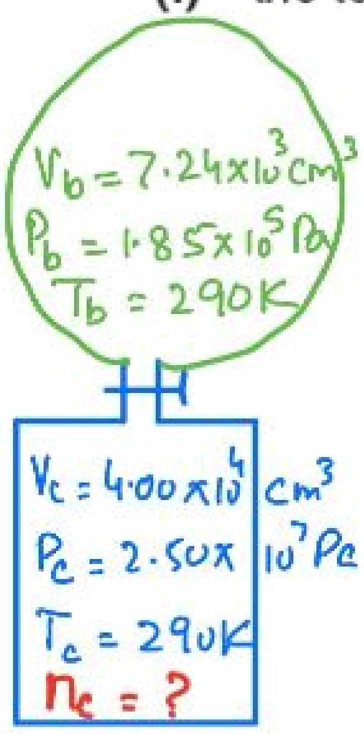
2. Gas is Ideal with no cohesive forces between its particles. [2]

(b) A gas cylinder contains $4.00 \times 10^4 \text{cm}^3$ of hydrogen at a pressure of 2.50×10^7 Pa and a temperature of 290 K.

The cylinder is to be used to fill balloons. Each balloon, when filled, contains $7.24 \times 10^3 \text{cm}^3$ of hydrogen at a pressure of 1.85×10^5 Pa and a temperature of 290 K.

Calculate, assuming that the hydrogen obeys the equation in **(a)**,

(i) the total amount of hydrogen in the cylinder,



$$P_c V_c = n_c R T_c$$

$$(2.50 \times 10^7)(4.00 \times 10^4 \times 10^{-6}) = (n_c)(8.31)(290)$$

$$n_c = 415 \text{ mol}$$

amount = ... mol [3]

(ii) the number of balloons that can be filled from the cylinder.

A* grade Total volume of gas at balloon's pressure.

$$P_b V_T = n_c R T_c$$

$$(1.85 \times 10^5) V_T = (415)(8.31)(290) \Rightarrow V_T = 5.41 \text{ m}^3$$

Total volume of gas = Volume of gas in a cylinder + (no. of balloons to be inflated)(Volume of gas in a balloon)

$$5.41 = 4.00 \times 10^4 \times 10^{-6} + (n)(7.24 \times 10^3)$$

$$n = 741.7$$

number = 741 [3]

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IDEAL GASES

6.

(a) Explain what is meant by the Avogadro constant.

....

....

.... [2]

(b) Argon-40 ($^{40}_{18}Ar$) may be assumed to be an ideal gas.
A mass of 3.2 g of argon-40 has a volume of $210\,cm^3$ at a temperature of 37 °C.

Determine, for this mass of argon-40 gas,

(i) the amount, in mol,

amount = ... mol [1]

(ii) the pressure,

pressure = ... Pa [2]

(iii) the root-mean-square (r.m.s.) speed of an argon atom.

r.m.s. speed = ... $m\,s^{-1}$ [3]

{Q. 2/41 & 42 Variant/ June 2014}

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IDEAL GASES

7.

(a) State what is meant by an *ideal gas*.

....

....

....

.... [3]

(b) Two cylinders A and B are connected by a tube of negligible volume, as shown in Fig. 2.1.

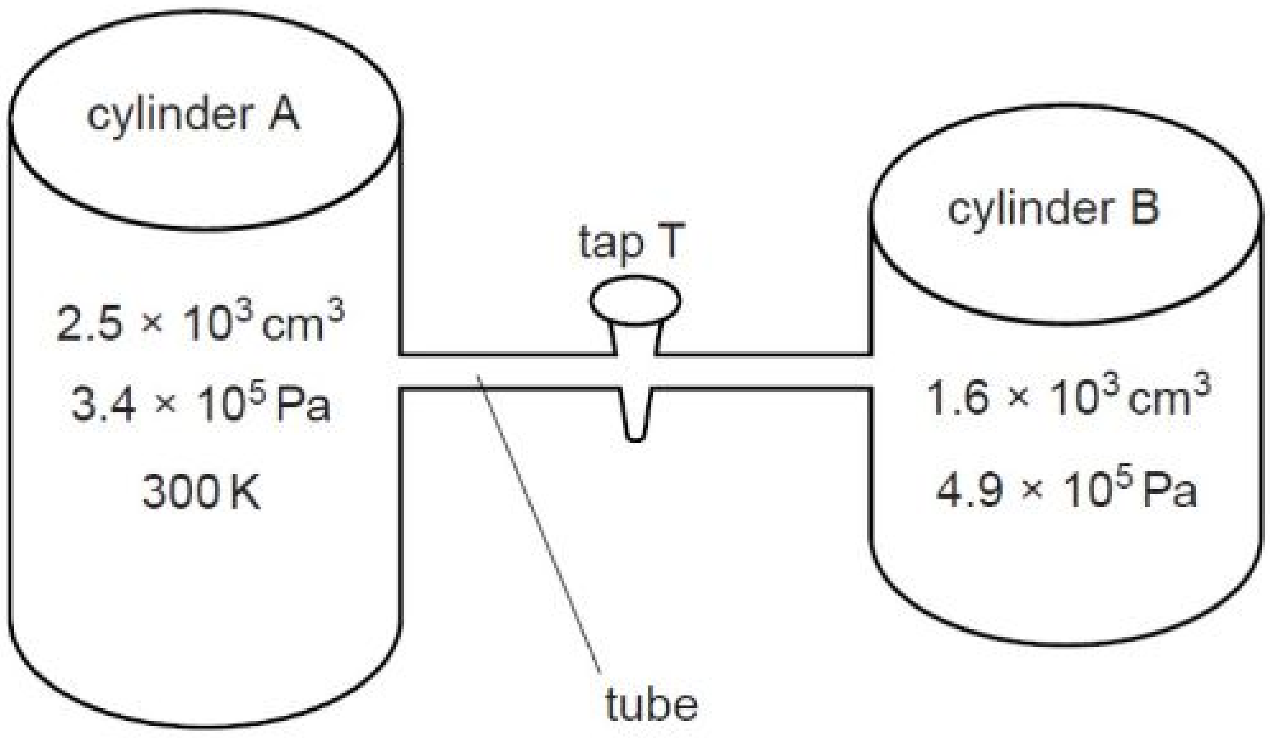

Fig. 2.1

Initially, tap T is closed. The cylinders contain an ideal gas at different pressures.

(i) Cylinder A has a constant volume of $2.5 \times 10^3\,\text{cm}^3$ and contains gas at pressure $3.4 \times 10^5\,\text{Pa}$ and temperature 300 K.

Show that cylinder A contains 0.34 mol of gas.

[1]

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(ii) Cylinder B has a constant volume of $1.6 \times 10^3\,cm^3$ and contains 0.20 mol of gas. When tap T is opened, the pressure of the gas in both cylinders is $3.9 \times 10^5\,Pa$. No thermal energy enters or leaves the gas.

Determine the final temperature of the gas.

temperature = ... K [2]

(c) From Thermal properties of Materials chapter [3]

{Q. 2/41 & 43 Variant/ June 2013}

8.

(a) An ideal gas is assumed to consist of atoms or molecules that behave as hard, identical spheres that are in continuous motion and undergo elastic collisions.

State two further assumptions of the kinetic theory of gases.

1. ....

....

2. ....

....

[2]

(b) Helium-4 (^4_2He) may be assumed to be an ideal gas.

(i) Show that the mass of one atom of helium-4 is $6.6 \times 10^{-24}\,g$.

[1]

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(ii) The mean kinetic energy E_K of an atom of an ideal gas is given by the expression

$$E_K = \frac{3}{2}kT.$$

Calculate the root-mean-square (r.m.s.) speed of a helium-4 atom at a temperature of 27 °C.

r.m.s. speed = ... $m\,s^{-1}$ [3]

{Q. 2/41 & 43 Variant/ June 2016}

9.

(a) State what is meant by

(i) the Avogadro constant N_A,

....

.... [1]

(ii) the mole.

....

.... [2]

(b) A container has a volume of $1.8 \times 10^4\ cm^3$.

The ideal gas in the container has a pressure of 2.0×10^7 Pa at a temperature of 17 °C.

Show that the amount of gas in the cylinder is 150 mol.

[1]

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(c) Gas molecules leak from the container in **(b)** at a constant rate of $1.5 \times 10^{19}\,s^{-1}$.
The temperature remains at 17 °C.
In a time *t*, the amount of gas in the container is found to be reduced by 5.0%.

Calculate

(i) the pressure of the gas after the time *t*,

pressure = ... Pa [2]

(ii) the time *t*.

t = ...s [3]

{Q. 2/42 Variant/ June 2016}

Marking Key

Q.1 to Q. 5) To be practiced in class

Q. 6. *{Ref.: Q. 2/41 & 42 Variant/ June 2014}*

(a) the number of atoms — M1
in 12 g of carbon-12 — A1 [2]

(b) (i) amount = 3.2/40
= 0.080 mol — A1 [1]

(ii) $pV = nRT$
$p \times 210 \times 10^{-6} = 0.080 \times 8.31 \times 310$ — C1
$p = 9.8 \times 10^{5}$ Pa — A1 [2]
(do not credit if T in °C not K)

(iii) *either* $pV = 1/3 \times Nm \langle c^2 \rangle$
$N = 0.080 \times 6.02 \times 10^{23}$ $(= 4.82 \times 10^{22})$
and $m = 40 \times 1.66 \times 10^{-27}$ $(= 6.64 \times 10^{-26})$ — C1
$9.8 \times 10^{5} \times 210 \times 10^{-6} = 1/3 \times 4.82 \times 10^{22} \times 6.64 \times 10^{-26} \times \langle c^2 \rangle$ — C1
$\langle c^2 \rangle = 1.93 \times 10^{5}$
$c_{RMS} = 440$ m s^{-1} — A1 [3]

or $Nm = 3.2 \times 10^{-3}$ — (C1)
$9.8 \times 10^{5} \times 210 \times 10^{-6} = 1/3 \times 3.2 \times 10^{-3} \times \langle c^2 \rangle$ — (C1)
$\langle c^2 \rangle = 1.93 \times 10^{5}$
$c_{RMS} = 440$ m s^{-1} — (A1)

or $1/2\, m\langle c^2 \rangle = 3/2\, kT$ — (C1)
$1/2 \times 40 \times 1.66 \times 10^{-27} \langle c^2 \rangle = 3/2 \times 1.38 \times 10^{-23} \times 310$ — (C1)
$\langle c^2 \rangle = 1.93 \times 10^{5}$
$c_{RMS} = 440$ m s^{-1} — (A1)

*(if T in °C not K award max 1/3, unless already penalised in **(b)(ii)**)*

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IDEAL GASES

7. {Ref.: *Q. 2/41 & 43 Variant/ June 2013*}

(a)	obeys the equation pV = constant × T or $pV = nRT$	M1	
	p, V and T explained	A1	
	at all values of p, V and T/fixed mass/n is constant	A1	[3]
(b) (i)	$3.4 \times 10^5 \times 2.5 \times 10^3 \times 10^{-6} = n \times 8.31 \times 300$	M1	
	n = 0.34 mol	A0	[1]
(ii)	for total mass/amount of gas		
	$3.9 \times 10^5 \times (2.5 + 1.6) \times 10^3 \times 10^{-6} = (0.34 + 0.20) \times 8.31 \times T$	C1	
	T = 360 K	A1	[2]

8. {*Ref.: Q. 2/41 & 43 Variant/ June 2016*}

(a) e.g. time of collisions negligible compared to time between collisions

no intermolecular forces (except during collisions)

random motion (of molecules)

large numbers of molecules

(total) volume of molecules negligible compared to volume of containing vessel
or
average/mean separation large compared with size of molecules

any two B2 [2]

(b) (i) mass = 4.0 / (6.02×10^{23}) = 6.6×10^{-24} g
or
mass = $4.0 \times 1.66 \times 10^{-27} \times 10^3 = 6.6 \times 10^{-24}$ g B1 [1]

(ii) $\frac{3}{2}kT = \frac{1}{2}m\langle c^2\rangle$ C1

$\frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = \frac{1}{2} \times 6.6 \times 10^{-27} \times \langle c^2\rangle$

$\langle c^2\rangle = 1.88 \times 10^6$ ($m^2 s^{-2}$) C1

r.m.s. speed = 1.4×10^3 m s^{-1} A1 [3]

9. {*Ref.: Q. 2/42 Variant/ June 2016*}

(a) (i) number of atoms/nuclei in 12 g of carbon-12 B1 [1]

(ii) amount of substance M1

containing N_A (or 6.02×10^{23}) particles/molecules/atoms
or
which contains the same number of particles/atoms/molecules as there are atoms in 12 g of carbon-12 A1 [2]

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(b) $pV = nRT$

$2.0 \times 10^7 \times 1.8 \times 10^4 \times 10^{-6} = n \times 8.31 \times 290$, so n = 149 mol or 150 mol A1 [1]

(c) (i) V and T constant and so pressure reduced by 5.0%
pressure = $0.95 \times 2.0 \times 10^7$ C1

or

calculation of new n (= 142.5 mol) and correct substitution into $pV = nRT$ (C1)

pressure = 1.9×10^7 Pa A1 [2]