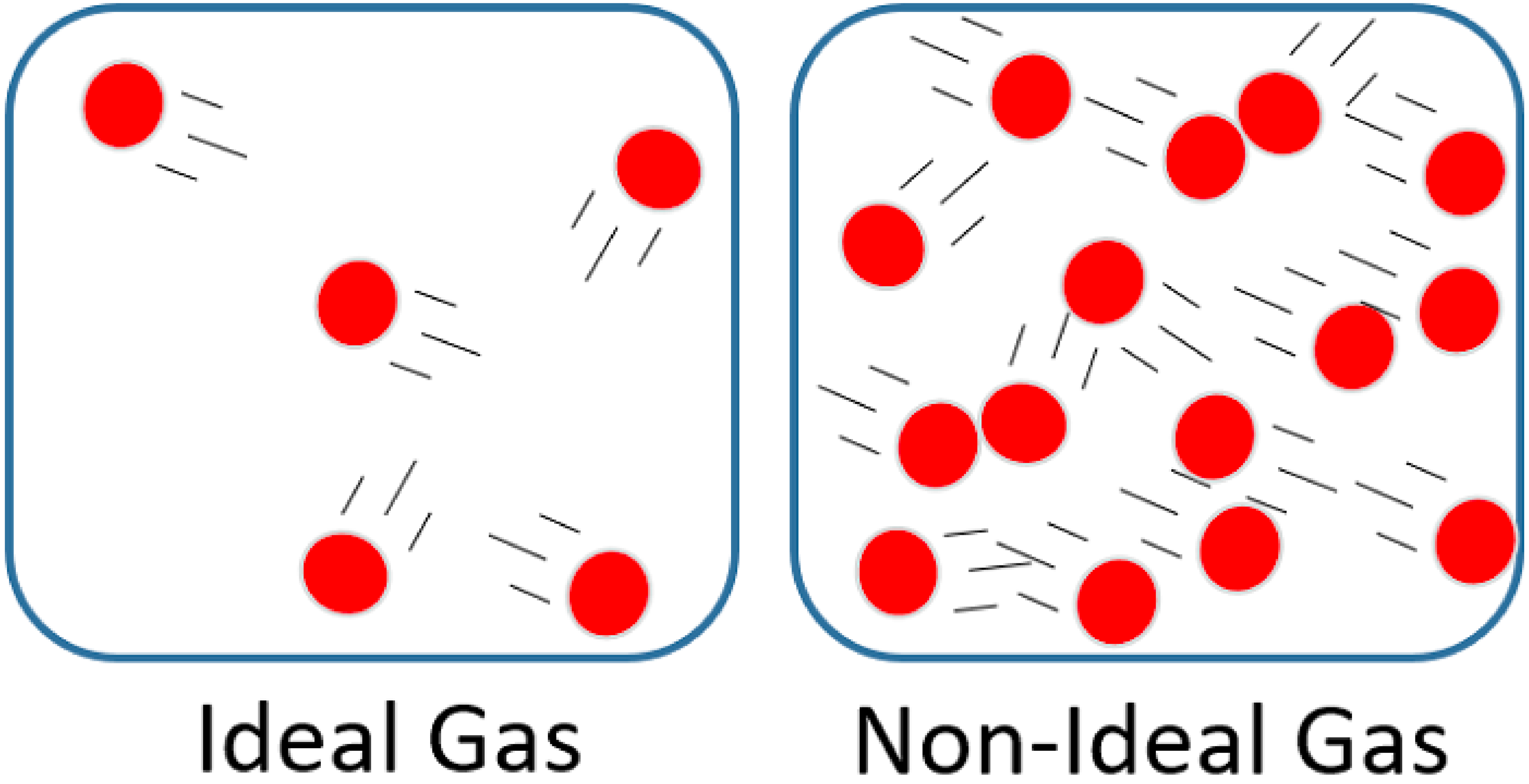
9702 C15 Ideal gases



What is mole?

-Mole is SI base unit of an ‘amount of substance’

-where one mole of any substance contains the same number of atoms/molecules/ions as there are in 12g of carbon-12
-->this number is known as **Avogadro’s number and is approximately 6.02x10^23** atoms/molecules/ions

- The number of moles, *n* therefore can also be calculated from the equation

$$n = \frac{m}{M_r}$$

- Where:
  - m = mass of the substance (g)
  - M_r = molar mass of the substance (g mol^{-1})

- The number of moles, *n* can be calculated using the equation

$$n = \frac{N}{N_A}$$

- Where:
  - N = number of molecules
  - N_A = Avogadro's constant

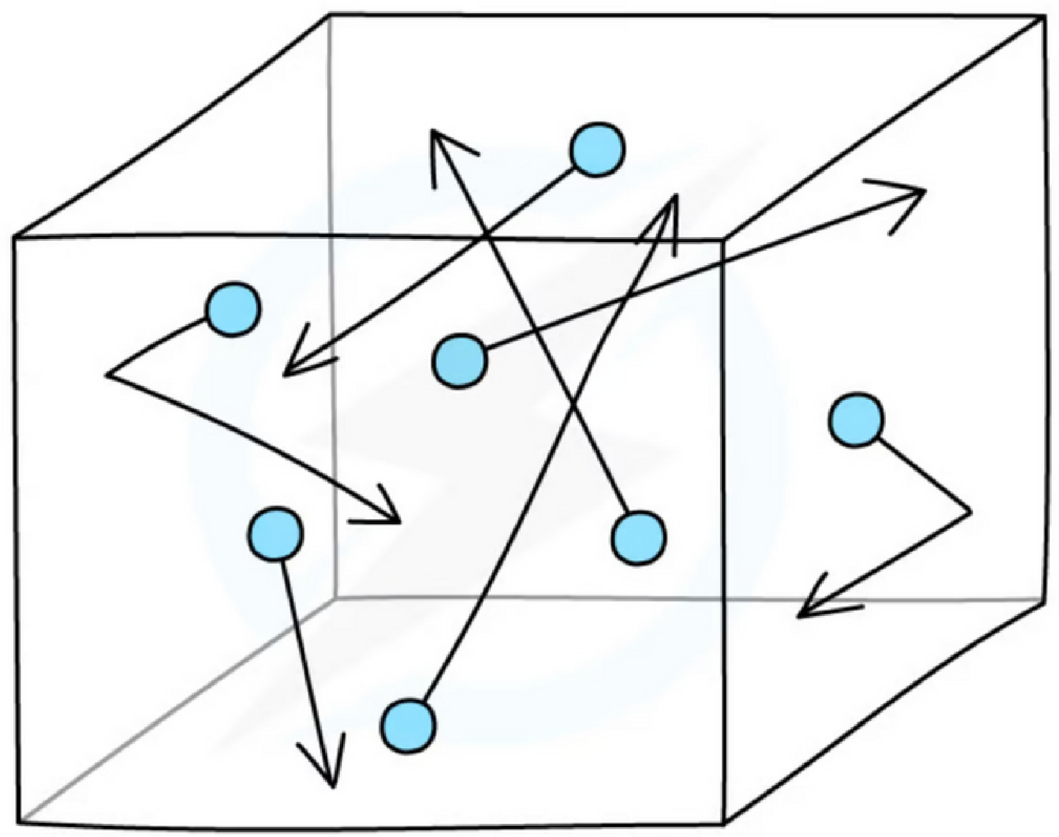
Ideal Gases

An **ideal gas** is one which obeys the relation:

$$pV \propto T$$

Where:

- p = pressure of the gas (Pa)
- V = volume of the gas (m^3)
- T = thermodynamic temperature (K)



Gas molecules move about randomly at high speeds

Temp↑ = Velocity of molecules↑ = molecules collide with the surface of walls↑ = force of molecules collide the the surface of walls↑(due to force=rate of change of momentum)

As pressure=force per unit area
-->temp↑ pressure↑

3 Gases Laws

Boyle's Law

- If the temperature T is constant, then **Boyle's Law** is given by:

$$p \propto \frac{1}{V}$$

- This leads to the relationship between the pressure and volume for a fixed mass of gas at constant temperature:

$$P_1V_1 = P_2V_2$$

Charles's Law

- If the pressure P is constant, then **Charles's law** is given by:

$$V \propto T$$

- This leads to the relationship between the volume and thermodynamic temperature for a fixed mass of gas at constant pressure:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Pressure Law

- If the volume V is constant, the the **Pressure law** is given by:

$$P \propto T$$

- This leads to the relationship between the pressure and thermodynamic temperature for a fixed mass of gas at constant volume:

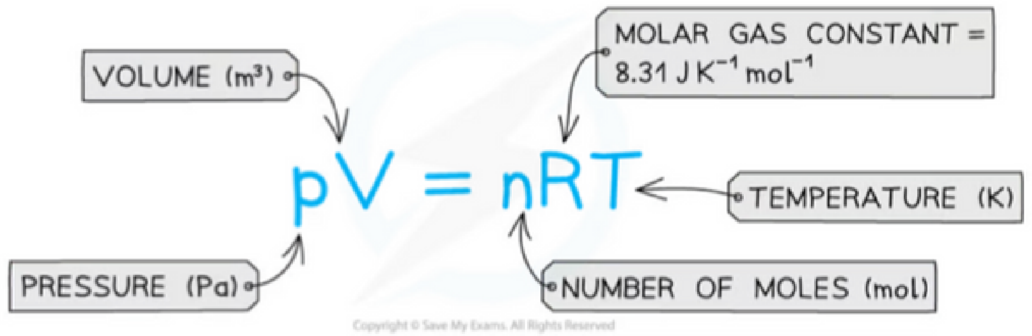
$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Ideal Gas Equation:

By combing all 3 gases laws we can get an Ideal Gas Equation

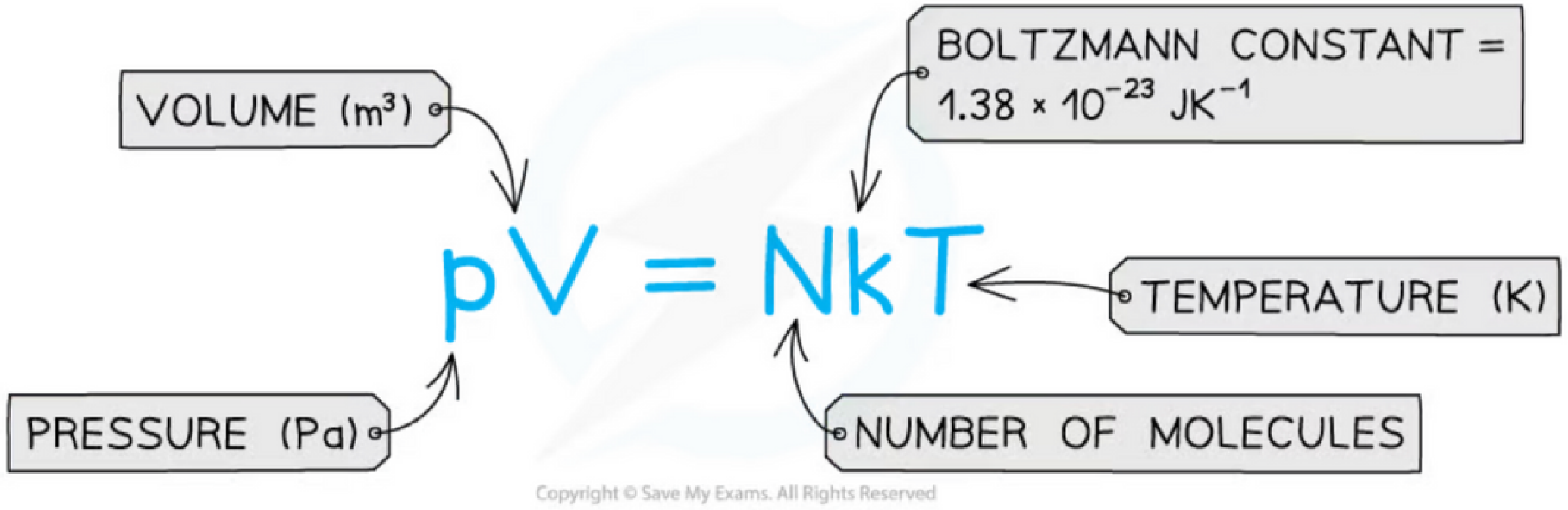
- The equation of state for an ideal gas (or the ideal gas equation) can be expressed as:

$$pV = nRT$$



- The ideal gas equation can also be written in the form:

$$pV = NkT$$



Ideal gas = a gas which obeys the equation of state pV=nRT at all pressures, volumes and thermodynamic temperatures(in Kelvin)

The Boltzmann Constant:

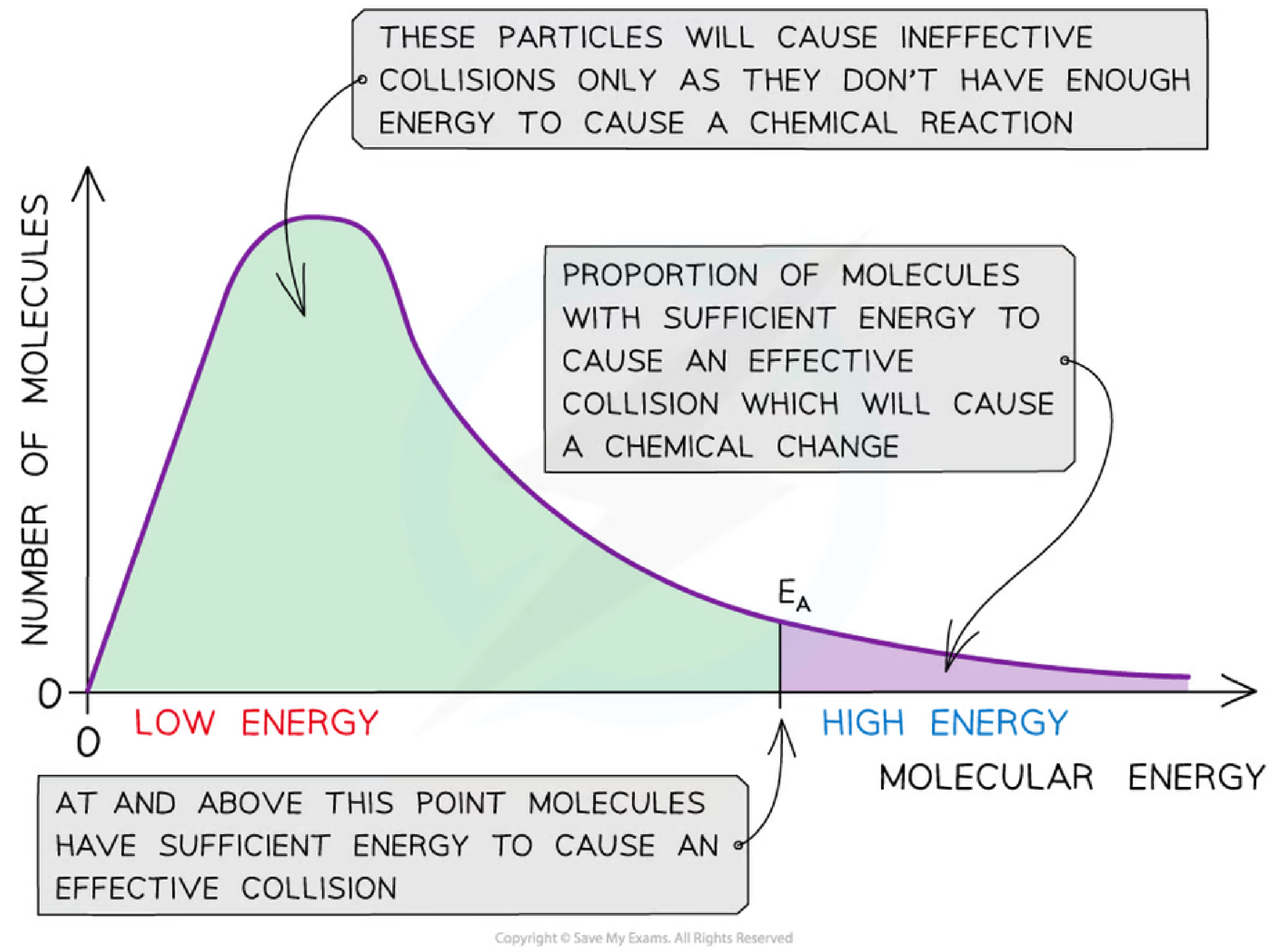
- The Boltzmann constant *k* is used in the ideal gas equation and is defined by the equation:

$$k = \frac{R}{N_A}$$

- Where:
  - R = molar gas constant
  - N_A = Avogadro's constant
- Boltzmann's constant therefore has a value of

$$k = \frac{8.31}{6.02 \times 10^{23}} = \mathbf{1.38 \times 10^{-23}\ J\ K^{-1}}$$

the value k is very small because the increase in Ek of a molecule is very small for every incremental increse in temperature



Assumptions of the Kinetic Theory of Gases:

The kinetic theory of gases models the thermodynamic behaviour of gases by linking the microscopic properties of particles(mass and speed) + macroscopic properties of particles(pressure and volume)

1.Molecules of gas behave as identical, hard, perfectly elastic spheres
2.The volume of the molecule is negligible compared to the volume of the container
3.The time of a collision is negligible compared to the time between collisions
4.There are no forces of attraction or repulsion between the molecules
5.The molecules are in continuous random motion

Root-Mean-Square Speed

Since particles travel in all directions in 3D space and **velocity is a vector**, some particles will have a negative direction and others a positive direction
-->When we consider a large number of particles: the total positibe and negative velocity values will cancel out, giving a net zero value overall

To solve this problem in order to the pressure of gas, the velocities must be squared (as every number squared will always be positive)

- To calculate the **average speed** of the particles in a gas, take the square root of the mean square speed:

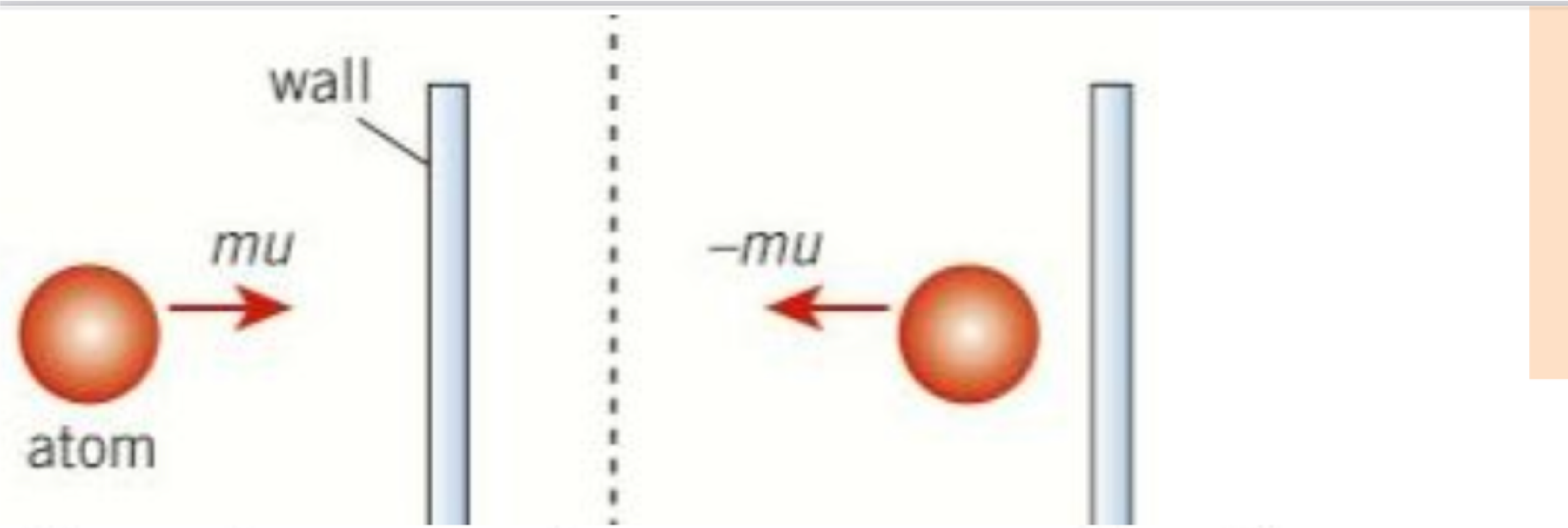
$$\sqrt{< c^2 >} = c_{r.m.s}$$

- $c_{r.m.s}$ is known as the **root-mean-square** speed and still has the units of **m s^{-1}**
- The mean square speed is **not** the same as the mean speed

Derivation of the Kinetic Theory of Gases Equation

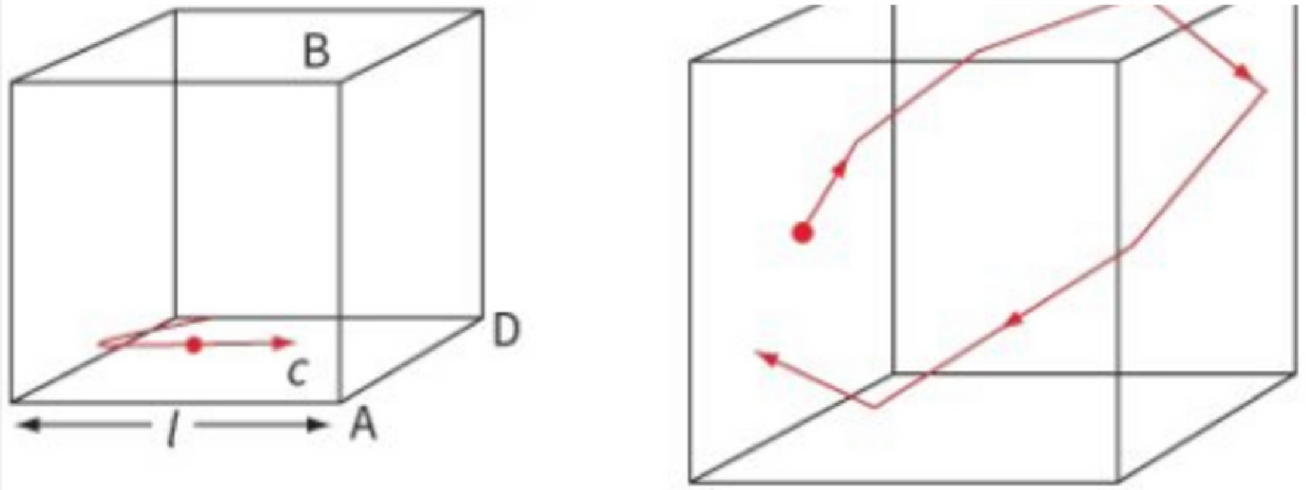
5 Step Derivation (c is speed, not the speed of light in this derivation)

1.Find the change in momentum as a single molecule hits a wall perpendicularly
-assume that molecules rebound elastically (no Ek lost in the collision)



$$\text{change in momentum} = -mc - (+mc)$$
$$= -mc - mc = -2mc$$

2.Calculate the number of collisions per second by the molecule on a wall



time between collisions with side ABCD $= \frac{2l}{c}$

3.Find the change in momemtum per second(aka force) using Newton 2nd Law

$$\text{Force = rate of change of momentum} = \frac{\Delta p}{\Delta t} = \frac{2mc}{\frac{2l}{c}} = \frac{mc^2}{l}$$

4.Calculate the total pressure from N molecules using pressure=force/area

- The area of one wall is l^2
- The pressure is defined using the force and area:

$$\text{Pressure p} = \frac{Force}{Area} = \frac{\frac{mc^2}{l}}{l^2} = \frac{mc^2}{l^3}$$

- This is the pressure **exerted from one molecule**
- To account for the large number of *N* molecules, the pressure can now be written as:

$$p = \frac{Nmc^2}{l^3}$$

- Each molecule has a different velocity and they all contribute to the pressure
- The mean squared speed of c^2 is written with left and right-angled brackets $<c^2>$
- The pressure is now defined as:

$$p = \frac{Nm<c^2>}{l^3}$$

5.Consider the effect of the molecule moving in 3D space

(pressure equation assumes all the molecules are travelling in the same direction and colliding with the same pair of opposite faces of the cube)

-->But in reality, all molecules will be moving in 3D equally

-->We can split the velocity into 3 components and by using pythagorus' theorm in 3D:

$$c^2 = c_x^2 + c_y^2 + c_z^2$$

Since there is nothing special about any particular direction, it can be determined that:

$$\langle c_x^2 \rangle = \langle c_y^2 \rangle = \langle c_z^2 \rangle$$

Simplifying, $\langle c_x^2 \rangle = \frac{1}{3} \langle c^2 \rangle$

6.Rewrite the pressure equation

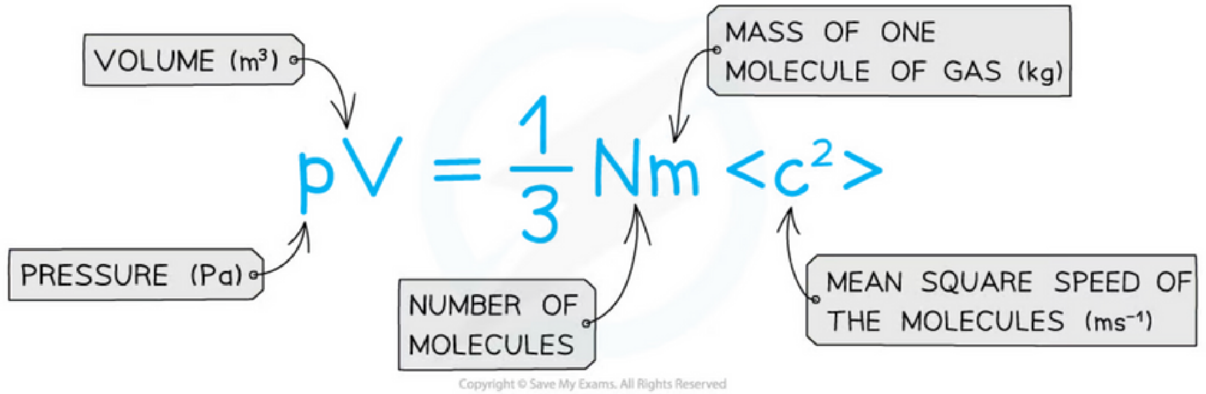
The equation for pressure worked out above involved just the component of the velocity in the x-direction and if c is the actual speed of the particle then we need to divide by 3 to find the pressure exerted.

$$p = \frac{1}{3}\left(\frac{Nm<c^2>}{l^3}\right)$$

Here, l^3 is equal to the volume V of the cube, so we can write:

$$p = \frac{1}{3}\left(\frac{Nm}{V}\right) < c^2 > \quad \text{or} \quad pV = \frac{1}{3}Nm < c^2 >$$

- This is known as the **Kinetic Theory of Gases equation**



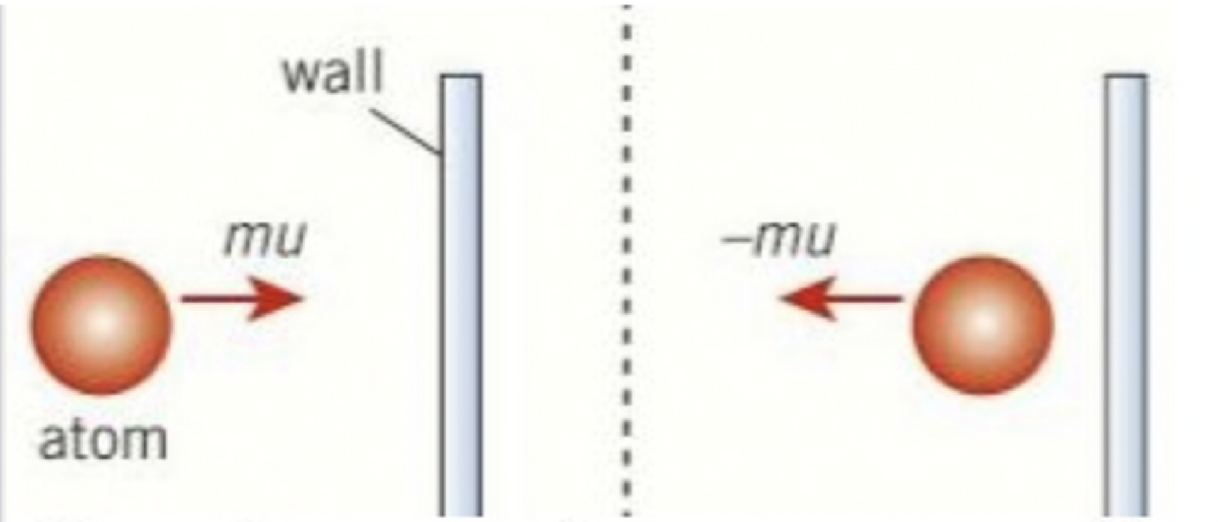
- This can also be written using the density ρ of the gas:

$$\rho = \frac{mass}{volume} = \frac{Nm}{V}$$

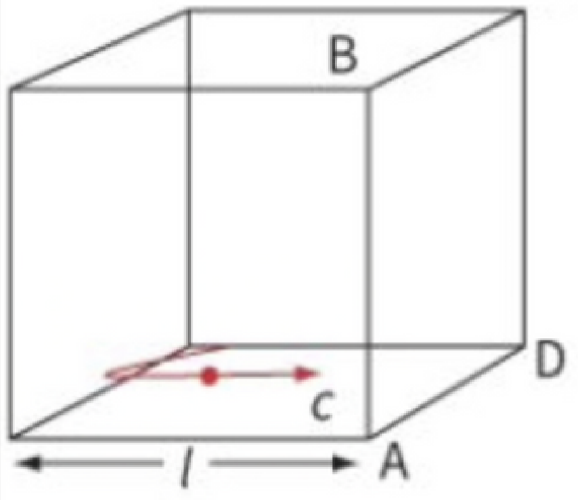
- Rearranging the pressure equation for p and substituting the density ρ:

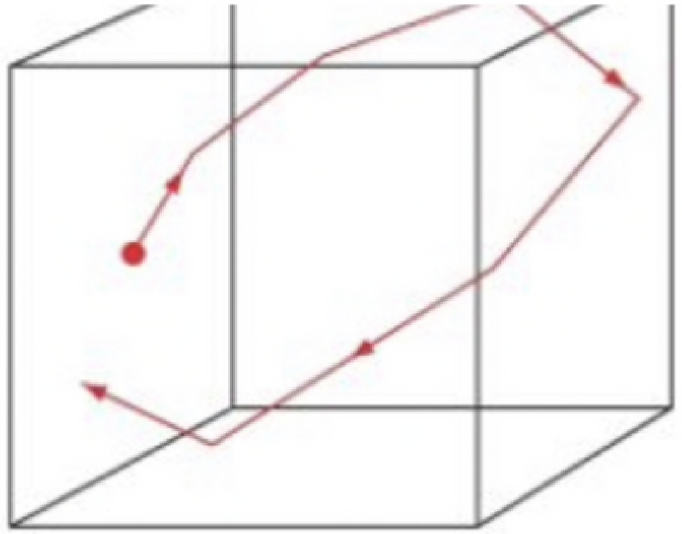
$$p = \frac{1}{3}\rho<c^2>$$

Summary of Derivation of the Kinetic Theory of Gases Equation



$$\text{change in momentum} = -mc - (+mc)$$
$$= -mc - mc = -2mc$$





$$\text{time between collisions with side ABCD} = \frac{2l}{c}$$

$$\text{force} = \frac{\text{change in momentum}}{\text{time taken}}$$
$$\text{Pressure} = \frac{\text{force}}{\text{area}}$$

$$\text{force} = \frac{\text{change in momentum}}{\text{time taken}}$$
$$= \frac{2mc}{\left(\frac{2l}{c}\right)}$$
$$= \frac{mc^2}{l}$$

(one molecule)

$$\text{pressure } p = \frac{\text{force}}{\text{area}}$$
$$= \frac{\left(\frac{mc^2}{l}\right)}{l^2}$$
$$= \frac{mc^2}{l^3}$$

$\rightarrow p = \frac{Nm\langle c^2 \rangle}{l^3}$

$p = \frac{1}{3}\left(\frac{Nm\langle c^2 \rangle}{l^3}\right)$

$\therefore pV = \frac{1}{3} Nm\langle c^2 \rangle$ #

Average translational kinetic energy equation:

From Ideal gas law equation and kinetic theory of gases equation we can derived Average(translational) kinetic energy equation:

$$pV = NkT \qquad pV = \frac{1}{3}Nm\langle c^2\rangle$$

$$NkT = \frac{1}{3}Nm\langle c^2\rangle$$

$$m\langle c^2\rangle = 3kT$$

$$E_k = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT \qquad \left(E_k = \frac{1}{2}mv^2\right)$$

$$\rightarrow E_k \propto T$$

This is the average Ek for only one gas molecule

Translational kinetic energy = the energy a molecule has as it moves from one point to another

-->monatomic molecule only has translational energy, whilst a diatomic molecule has both translational and rotational energy