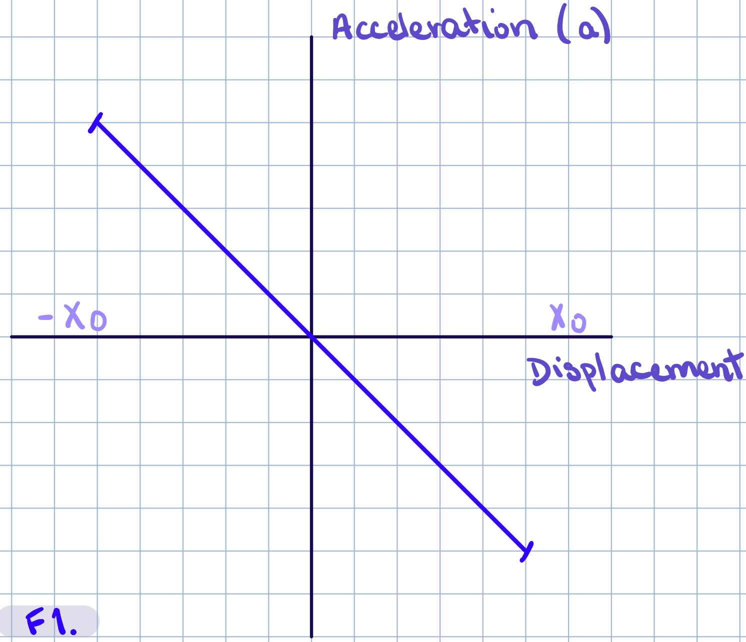
Simple Harmonic Motion



F1.

FIGURE 1: the defining graph of our SHM equation

EQUATION: $a = -kx$

→ "a" = acceleration
→ "x" = displacement from equilibrium position
→ "k" = ?

CONCEPT: SHM is a linear (1D) projection of circular motion!

→ this is the simple harmonic motion (SHM) theory ☺

BECAUSE: 1 oscillation = 2π rad

→ constant "k" = square of angular frequency (ω^2)

THEREFORE: $a = -\omega^2 x$

RECALL: from circular motion ⇒ $\omega = 2\pi f$ rad·s^{-1}

→ $a = -\omega^2 x$

$a = -(2\pi f)^2 x$

$a = -4x \cdot (\pi f)^2$

GRAPH: sinusoidal, where x_0 = amplitude

→ $x \propto \sin(\omega t)$ or $x = x_0 \sin(\omega t)$

WHAT ARE OUR SHM EQUATIONS?

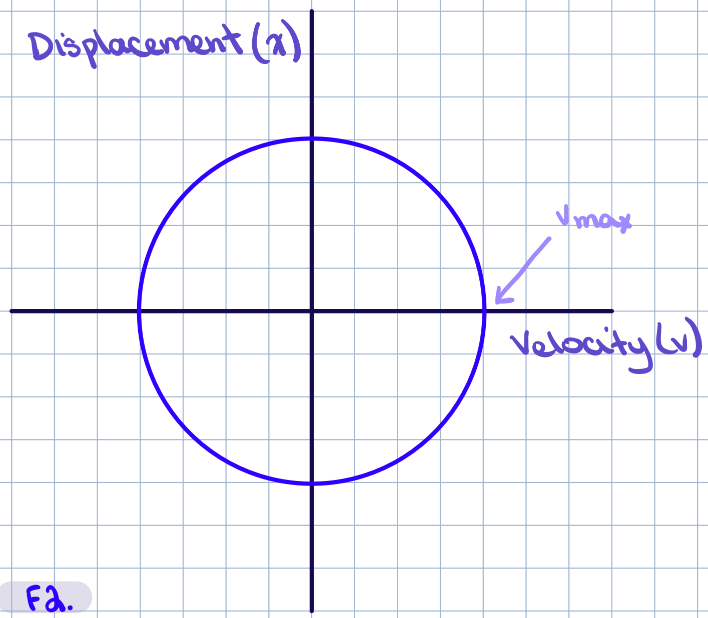
① $x = x_0 \sin(\omega t)$ → dt graph

② $v = \omega x_0 \cos(\omega t)$ → vt graph, gradient of dt graph

③ $a = -\omega^2 x_0 \sin(\omega t)$ → at graph, gradient of vt graph

AFTER REARRANGING:

④ $v = \pm\omega\sqrt{(x_0^2 - x^2)}$



F2.

FIGURE 2: a dv graph

PERIOD: T, one complete oscillation

→ $\omega T = 2\pi$

$T = \frac{2\pi}{\omega}$

FREQUENCY: $f = \frac{1}{T}$

MAXIMUMS: $v_{max} = \omega x_0$

→ $a_{max} = \omega^2 x_0$

2 MOST COMMON SHM EXAMPLES: ① mass on spring & ② pendulum

SPRING

HOOKE'S LAW: the restoring force on a spring is proportional to its extension.

→ $F = kx$

→ F is force pulling the spring, x is extension past rest, and k is the spring constant.

SPRING CONSTANT: force per unit length needed to extend spring

→ units: $N \cdot m^{-1}$

ACCELERATION: always in the opposite direction to displacement

→ derivation from Newton's:

$F = ma \rightarrow a = \frac{F}{m}$

$a = -\frac{kx}{m}$ (minus = opposite dir)

→ comparing to SHM: $a = -\omega^2 x$

$\omega = \sqrt{k/m}$

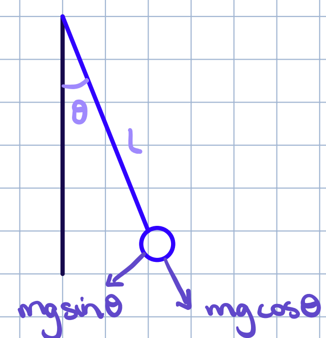
PENDULUM

NOTE: SHM is technically only valid for small angles (<10°) of pendulum displacement

FEATURES: when dispacement constant!

→ more gravity = more acceleration

→ shorter length = more acceleration



RESTORING FORCE: $mg\sin\theta$

→ $F = ma$
$= mg\sin\theta$

→ $\sin\theta \approx \theta = \frac{x}{L}$

NOTE: force is in opposite direction to the displacement

→ $ma = -mg\left(\frac{x}{L}\right)$ minus sign!

ACCELERATION: $a = -\left(\frac{g}{L}\right)x$

→ g = gravitational field

→ compare with SHM: $a = -\omega^2 x$

ANGULAR FREQUENCY: "ω" is the stiffness of the system

→ if ω is large, frequency is high

TIME PERIOD: $T = \frac{2\pi}{\omega}$

→ derived: $T = 2\pi\sqrt{\frac{m}{k}}$

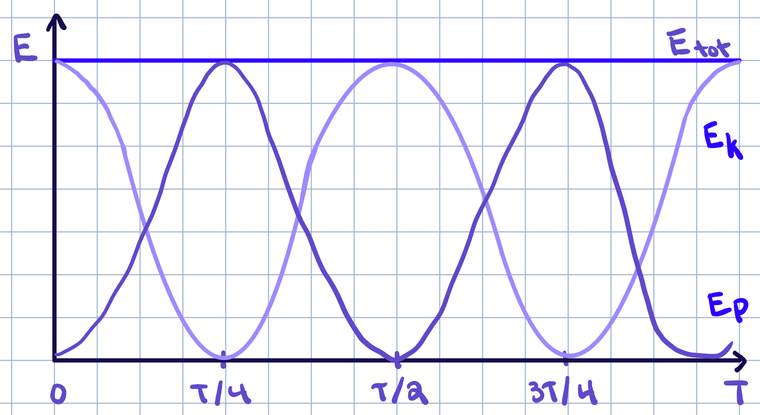
ANGULAR FREQUENCY: "ω"

→ $\omega = \sqrt{\frac{g}{L}}$

TIME PERIOD: of one oscillation of the pendulum!

→ $T = 2\pi\sqrt{\frac{L}{g}}$

WHAT ABOUT ENERGY??



F4. Energy Exchanges in System!

MAXIMUMS: potential & kinetic!

→ max E_p when the pendulum is at max displacement

↳ restorative force also MAX!

→ max E_k at equilibrium and also max velocity

→ total E = either MAX E_k or E_p