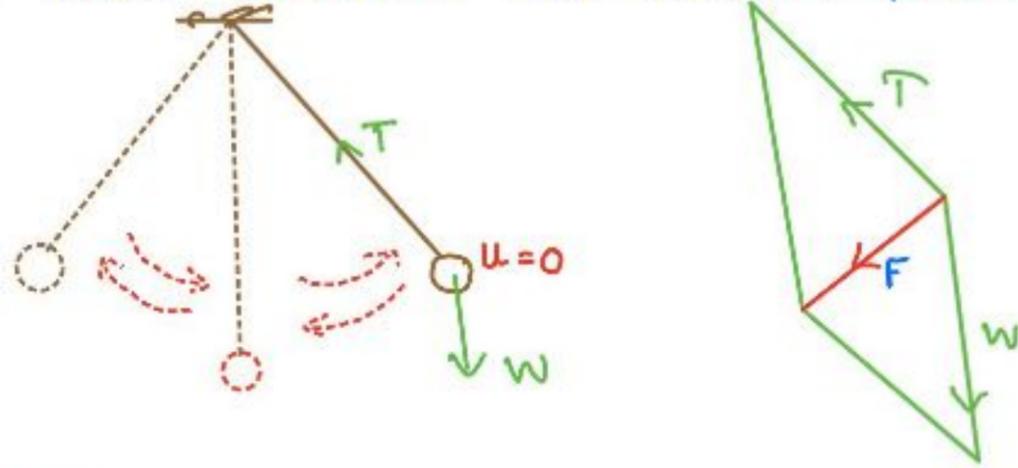


# OSCILLATIONS

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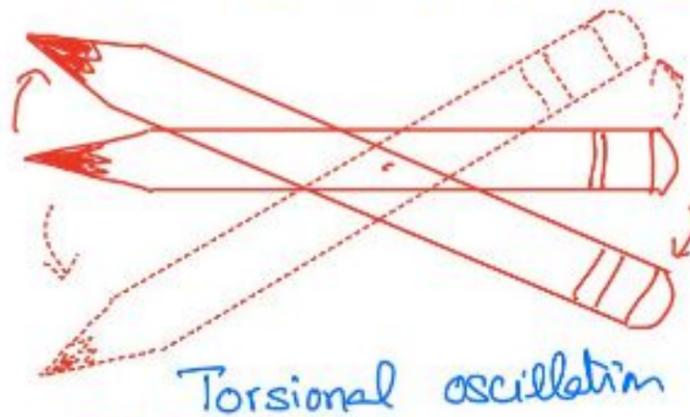
Oscillation :-

**Def.** The repeated motion of an object about its mean position and between two extreme positions.



Types of Oscillation:-

- 1- To and fro motion  $\rightarrow$  Left and right side
- 2- Back and forth  $\rightarrow$  Towards or away from observer
- 3- Top and bottom  $\rightarrow$  upwards and downward.
- 4- Torsional  $\rightarrow$  Motion about centre of Gravity of object such that its distance from a fixed plane remain constant.



Planes of oscillation:-

- 1- Vertical plane  $\rightarrow$  Height or Gravitational potential energy of object changes during motion.
- 2- Horizontal plane  $\rightarrow$  Height or G.P.E of object remain constant during motion.

### Important terms:-

- 1- Time period (T): Time taken to complete one oscillation.
- 2- Frequency (f): No of complete oscillations per unit time.

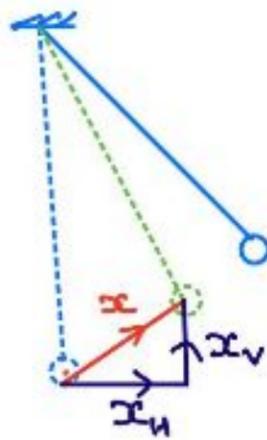
$$f = \frac{n}{t}$$

For one oscillation,  $\frac{n}{t} = \frac{1}{T}$

$$f = \frac{1}{T}$$

Units:  $s^{-1}$  or Hertz (Hz)

- 3- Displacement (x): Straight (shortest) directed distance of an oscillating object from its mean position.



P.S

Vector

Direction

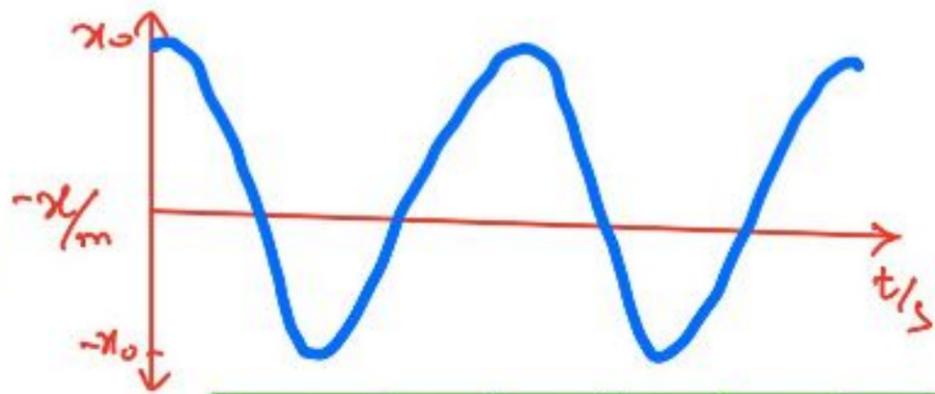
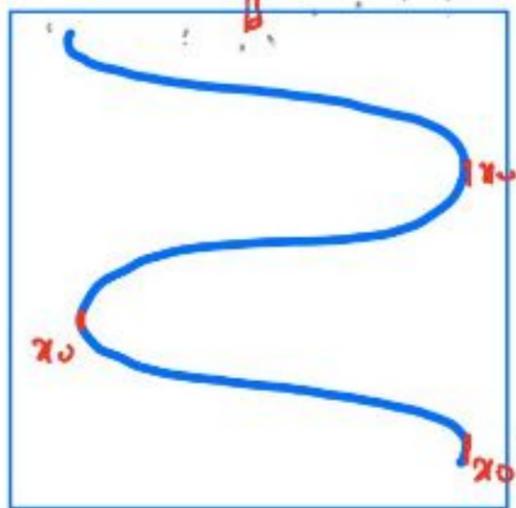
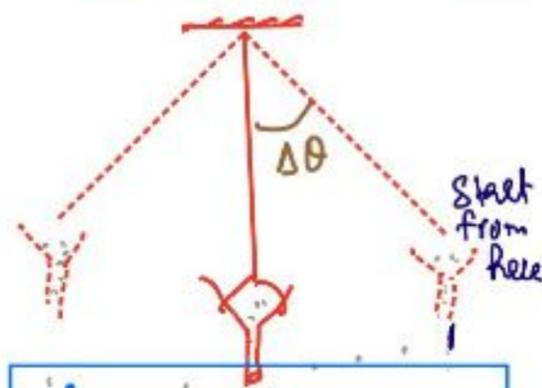
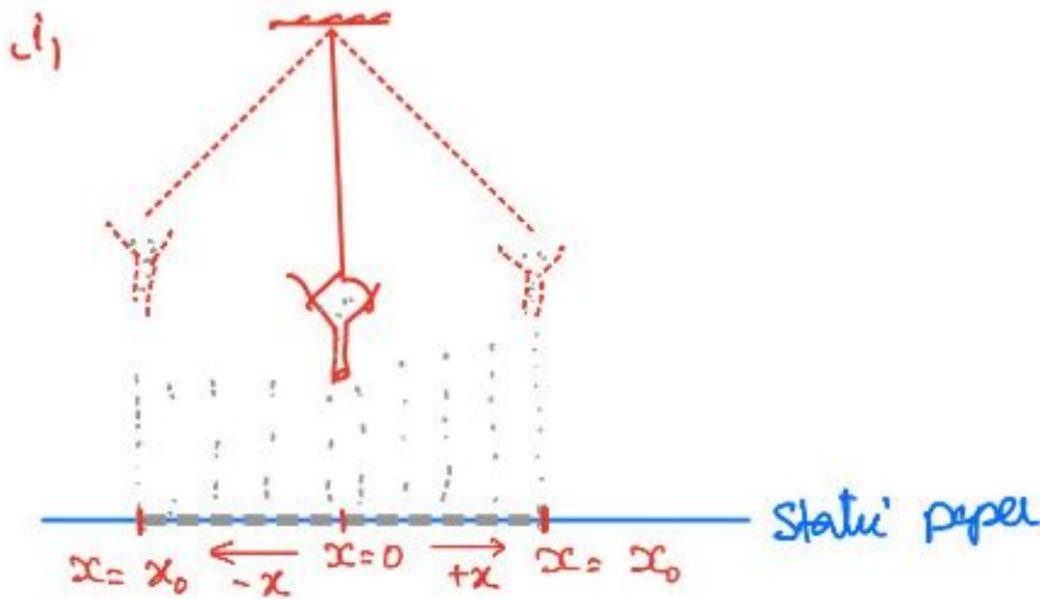
Away from mean position.

- (4) Amplitude : ( $x_0$ ) Maximum value of displacement is amplitude. It is a scalar quantity.

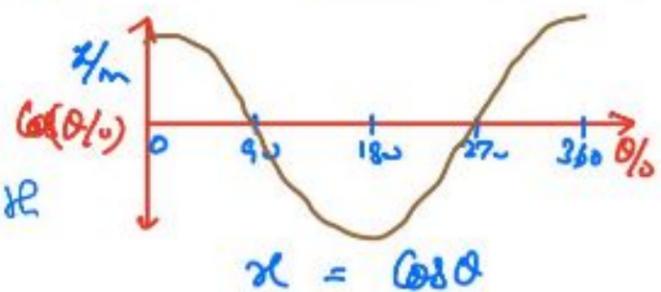
(5) Periodic motion:- Motion which repeats itself in equal interval of time. e.g

- (i) Solar system
- (ii) Heart beat
- (iii) Simple pendulum.
- (iv) Vibrating mass suspended from a spring.

(6) Shape of graph:



$\theta/^\circ$	0	90	180	270	360
$\cos(\theta/^\circ)$	1	0	-1	0	1



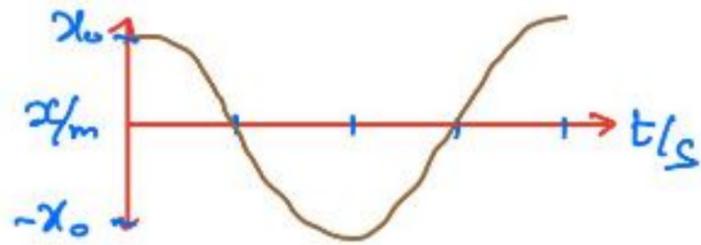
But  $\omega = \frac{\Delta\theta}{\Delta t}$

$\Delta\theta = \omega \Delta t$

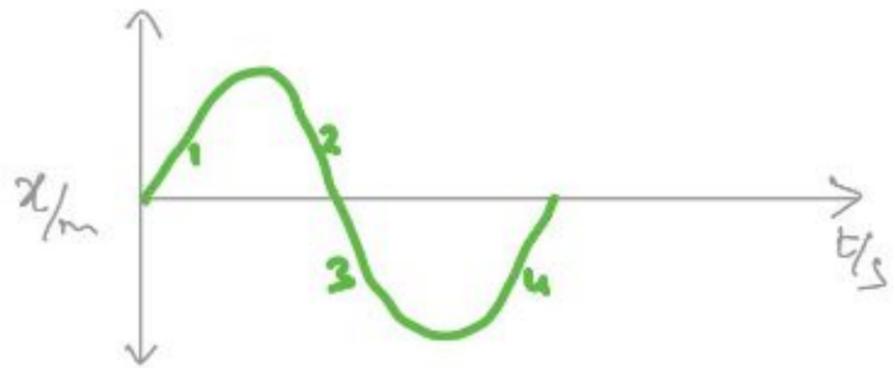
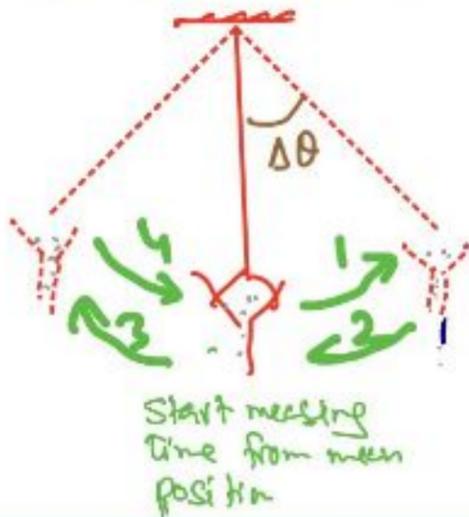
$\Delta\theta = (\text{constant}) \Delta t$

$\Delta\theta \propto \Delta t$

Amplitude  $\nearrow$   
 Periodicity  $\nearrow$   
 $x = x_0 \cos \omega t$   
 Y-axis  $\nwarrow$   
 X-axis  $\nearrow$   
 Trend of graph  $\perp$

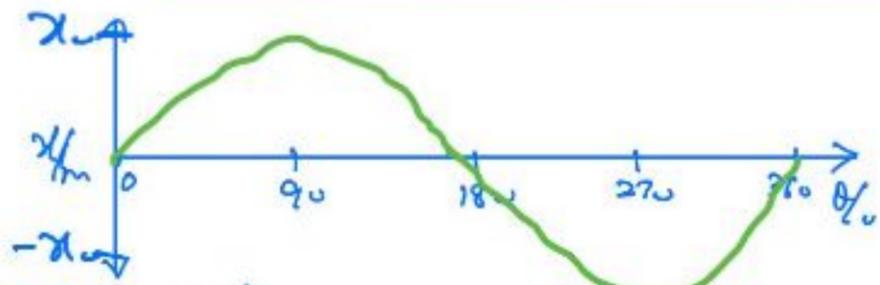
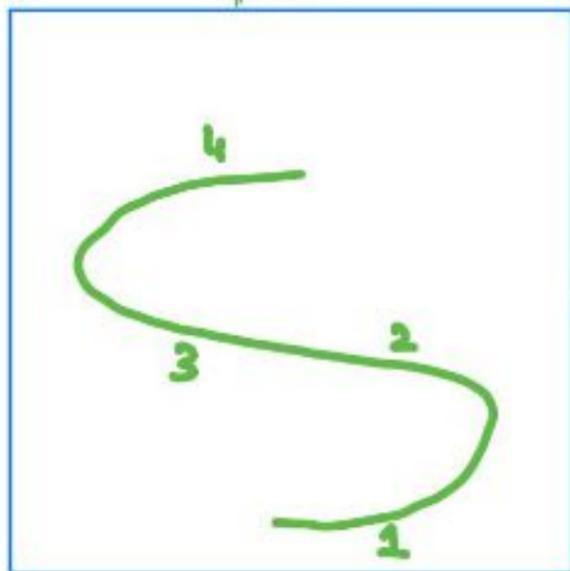


Note: We always set (cos) graph if time is started from extreme position.

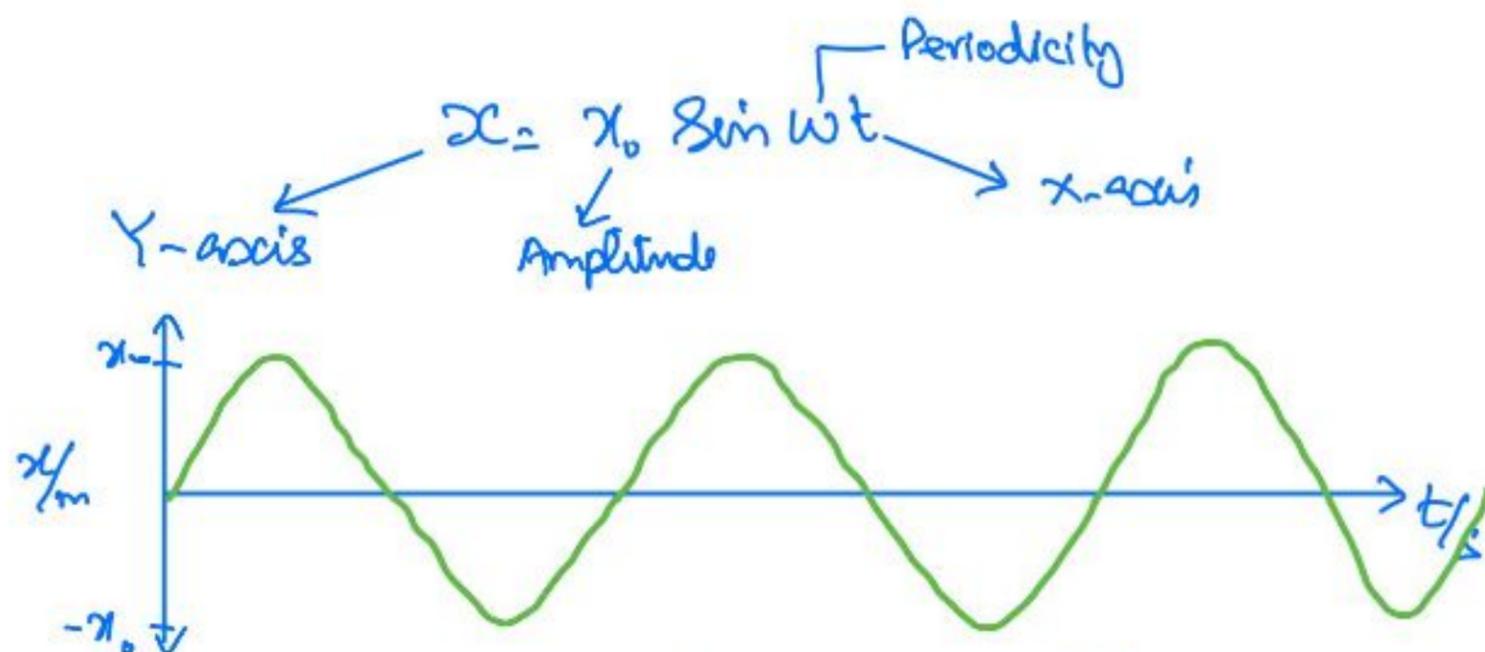


$x = \sin \theta$

$\theta^\circ$	0	90	180	270	360
$\sin(\theta^\circ)$	0	1	0	-1	0



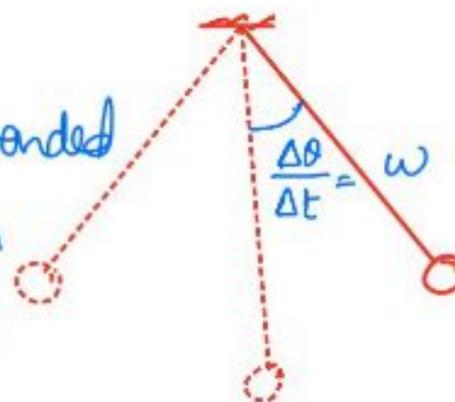
But  $\theta = \omega t$



**Note:** Sine graph is obtained if measurement of time is started from mean position.

## 7. Angular Frequency

Def change of angle subtended by a vibrating object with its mean position per unit time is angular frequency.



Symbol:  $\omega$

Formula  $\omega = \frac{\Delta \theta}{\Delta t}$   $\omega = \frac{2\pi}{T} = 2\pi f$

Units:  $\text{rad s}^{-1}$

P.S.: Scalar

## (8) Simple Harmonic motion:

- Def.
- \* Periodicity of motion
  - \* magnitude of acceleration  $\propto$  displacement
  - \* direction of acceleration  $\rightarrow$  towards mean position

The periodic motion in which magnitude of acceleration is directly proportional to displacement and is directed towards mean position.

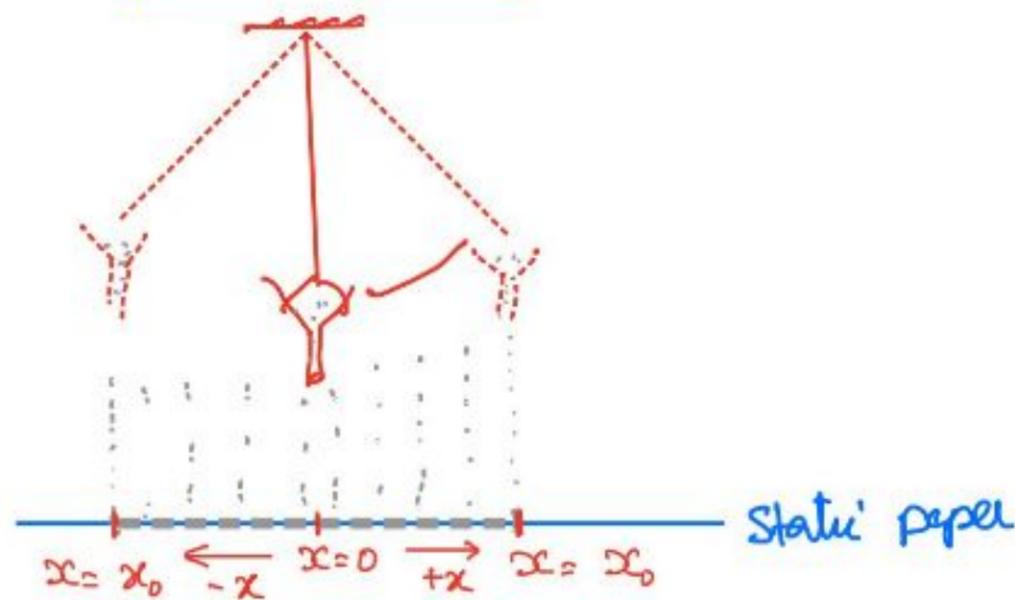
Mathematical form:-

$$a \propto -x$$

$$a = \omega^2 (-x)$$

$$a = -\omega^2 x$$

$$\text{Here } \omega = \frac{2\pi}{T}$$



Significance of -ve sign:-

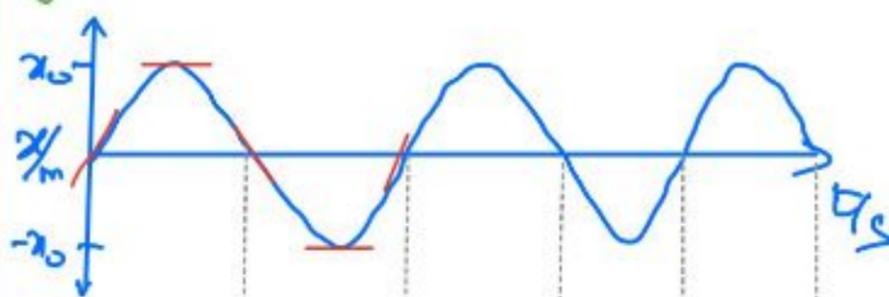
- 1 - Vectorial significance: Both displacement and acceleration are in opposite directions.
- 2 - Conventional significance: Acceleration is directed towards mean position.

(a) Kinematics' graphs:

(i) If stopwatch is started at mean position:-

1- Displacement-time graph:-

$$x = x_0 \sin \omega t$$

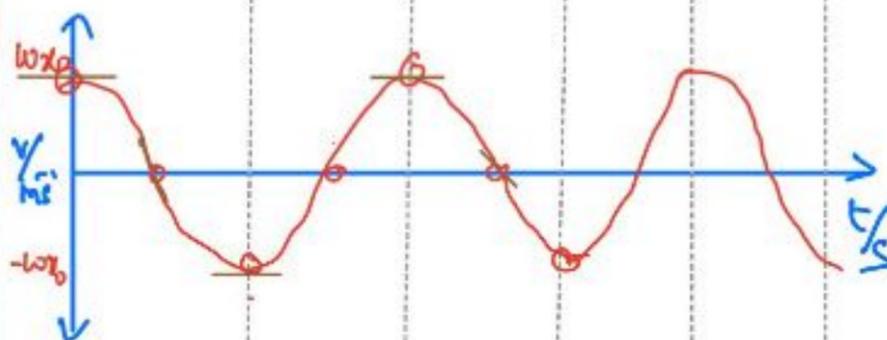


2- Velocity-time graph

$v =$  Gradient of displacement-time graph.

$$v = \underbrace{\omega x_0}_{\text{Amplitude}} \cos \omega t$$

→ Shape of graph



Not in syllabus

$$\begin{aligned}
 v &= \frac{dx}{dt} = \frac{d}{dt}(x_0 \sin \omega t) \\
 &= x_0 \frac{d}{dt} \sin \omega t \\
 &= x_0 \cos \omega t \left( \frac{d}{dt} \omega t \right) \\
 &= x_0 \cos \omega t (\omega \frac{dt}{dt}) \\
 &= \omega x_0 \cos \omega t
 \end{aligned}$$

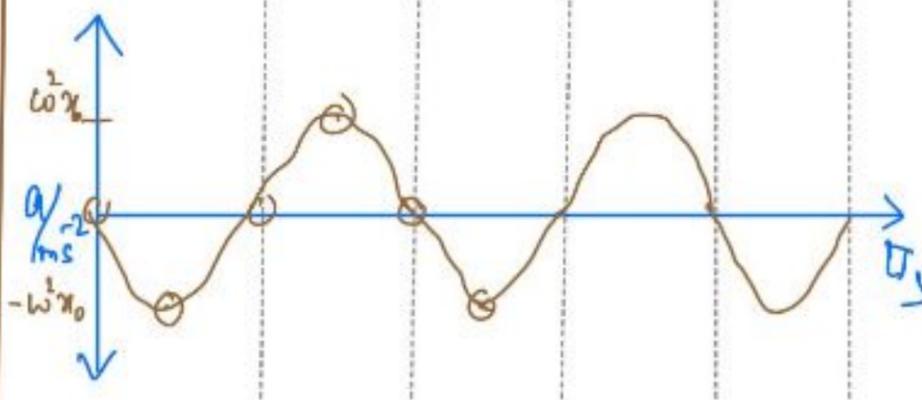
(3) Acceleration-time graph:

$a =$  Gradient of velocity-time graph

$$a = \omega^2 x_0 (-\sin \omega t)$$

$$a = -\omega^2 (x_0 \sin \omega t)$$

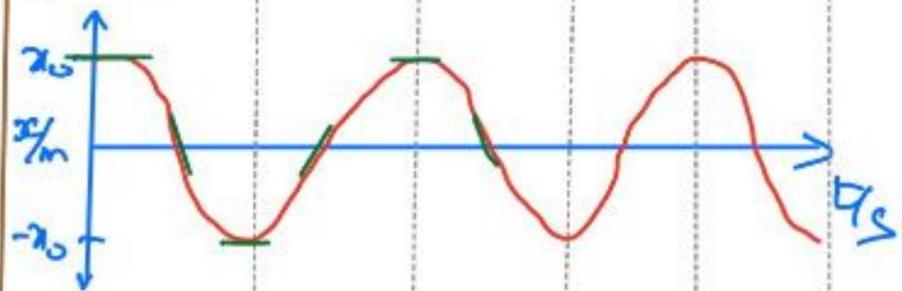
$$a = -\omega^2 x$$



(i) If stopwatch is started at extreme position:-

1. Displacement-time graph:-

$$x = x_0 \cos \omega t$$

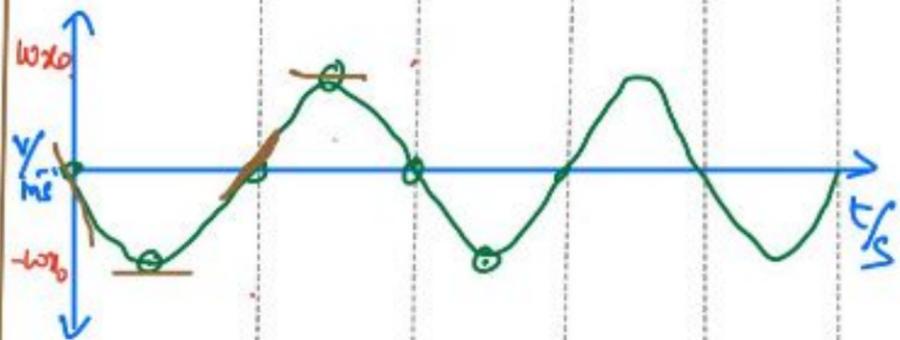


2. Velocity-time graph

$v =$  Gradient of displacement-time graph.

$$v = \frac{\omega x_0}{\text{Amplitude}} (-\sin \omega t)$$

→ Shape of graph

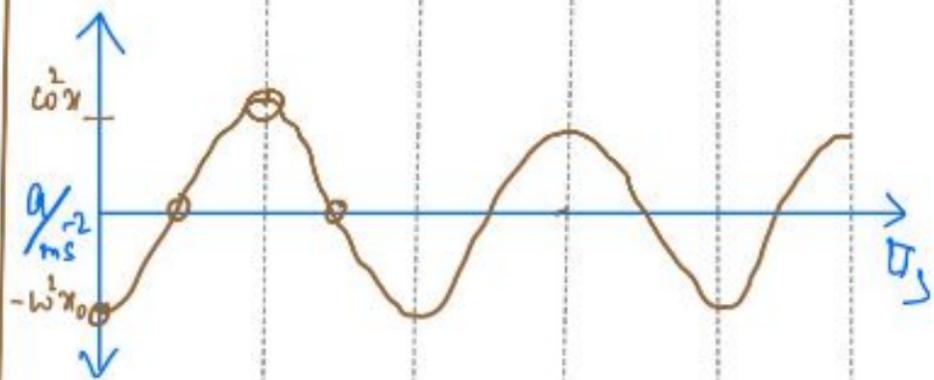


(3) Acceleration-time graph:

$a =$  Gradient of velocity-time graph

$$a = -\omega^2 x_0 \cos \omega t$$

$$a = -\omega^2 x$$



(b) Dynamics graph:

(i) Velocity-displacement graph:-

The equation of velocity-displacement (even given on formula page) is

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

At mean position,  $x=0$

$$v = \pm \omega \sqrt{x_0^2 - (0)^2}$$

$$v = \pm \omega \sqrt{x_0^2}$$

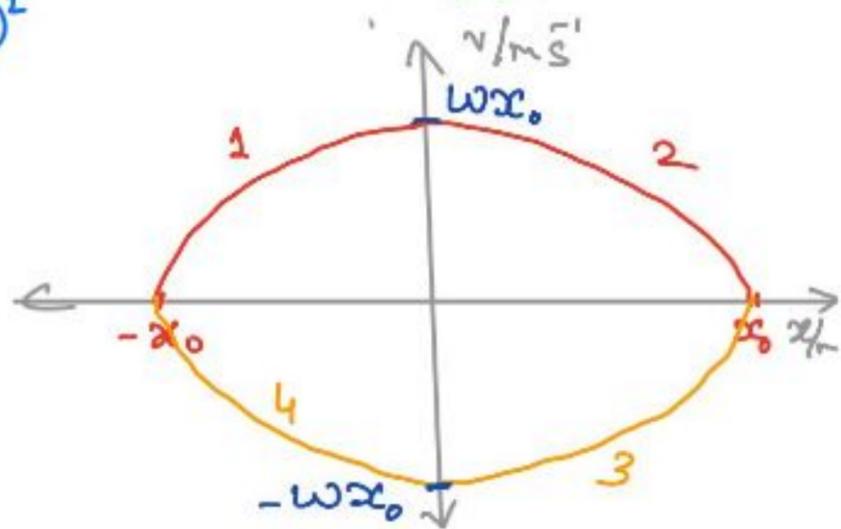
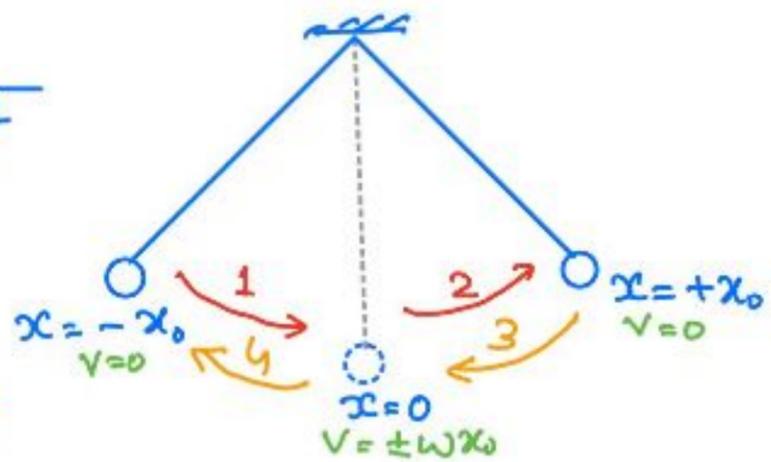
$$v = \pm \omega x_0$$

At extreme positions,  $x = \pm x_0$

$$v = \pm \omega \sqrt{x_0^2 - (\pm x_0)^2}$$

$$= \pm \omega \sqrt{x_0^2 - x_0^2}$$

$$= 0$$



(ii) Acceleration - displacement graph :-

The defining eq. of SHM is

$$a = -\omega^2 x$$

At mean position,  $x=0$

$$a = -\omega^2(0)$$

$$a = 0$$

At extreme position:

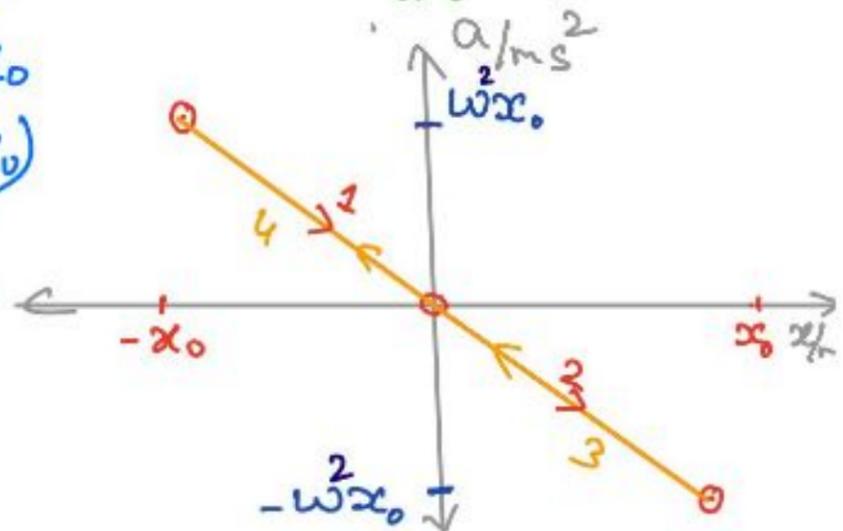
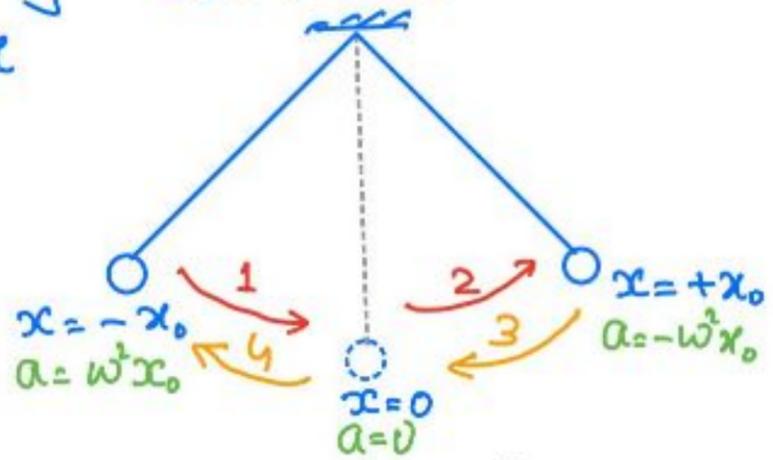
If  $x = +x_0$

$$a = -\omega^2 x_0$$

If  $x = -x_0$

$$a = -\omega^2(-x_0)$$

$$a = \omega^2 x_0$$



(c) Energy displacement graphs:

(i) Kinetic energy-displacement graph:

$$E_k = \frac{1}{2} m v^2$$

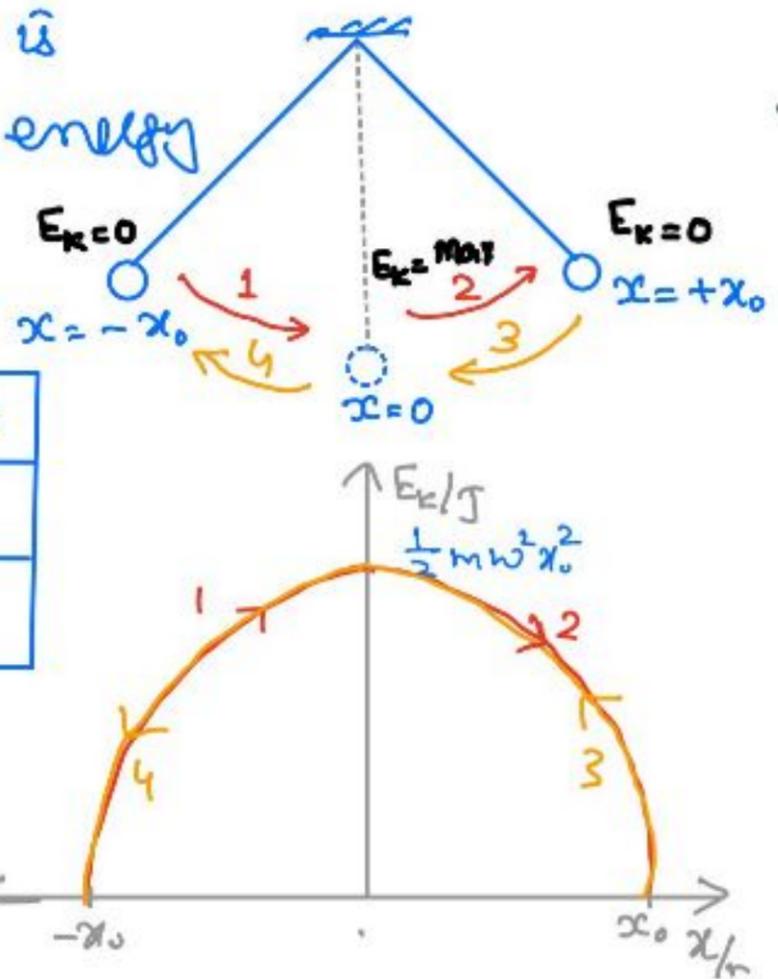
$$\text{But } v = \pm \omega \sqrt{x_0^2 - x^2}$$

$$E_k = \frac{1}{2} m \left[ \pm \omega \sqrt{x_0^2 - x^2} \right]^2$$

$$E_k = \frac{1}{2} m \omega^2 (x_0^2 - x^2)$$

is the general eq. of Kinetic energy-displacement graph

Position	Mean	Extreme
Displacement	0	$\pm x_0$
$E_k/J$	$\frac{1}{2} m \omega^2 x_0^2$	0



(ii) Total energy-displacement graph:-

At any position,

$$E_T = E_k + E_p$$

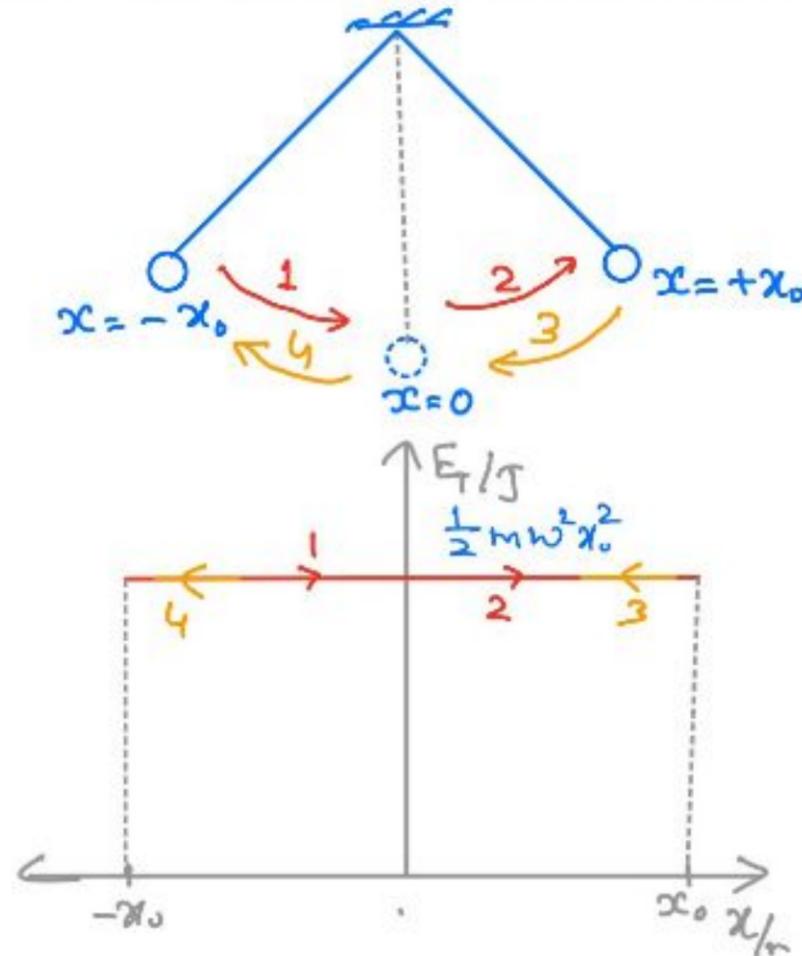
$$E_T = \frac{1}{2} m \omega^2 (x_0^2 - x^2) + E_p$$

At mean position,  $x=0$  and  $E_p \approx 0$

$$E_T = \frac{1}{2} m \omega^2 (x_0^2 - 0^2) + 0$$

$$E_T = \frac{1}{2} m \omega^2 x_0^2$$

Total energy is independent of displacement ( $x$ ) and therefore remain conserved at all positions.



(iii) Potential energy - displacement graph:-

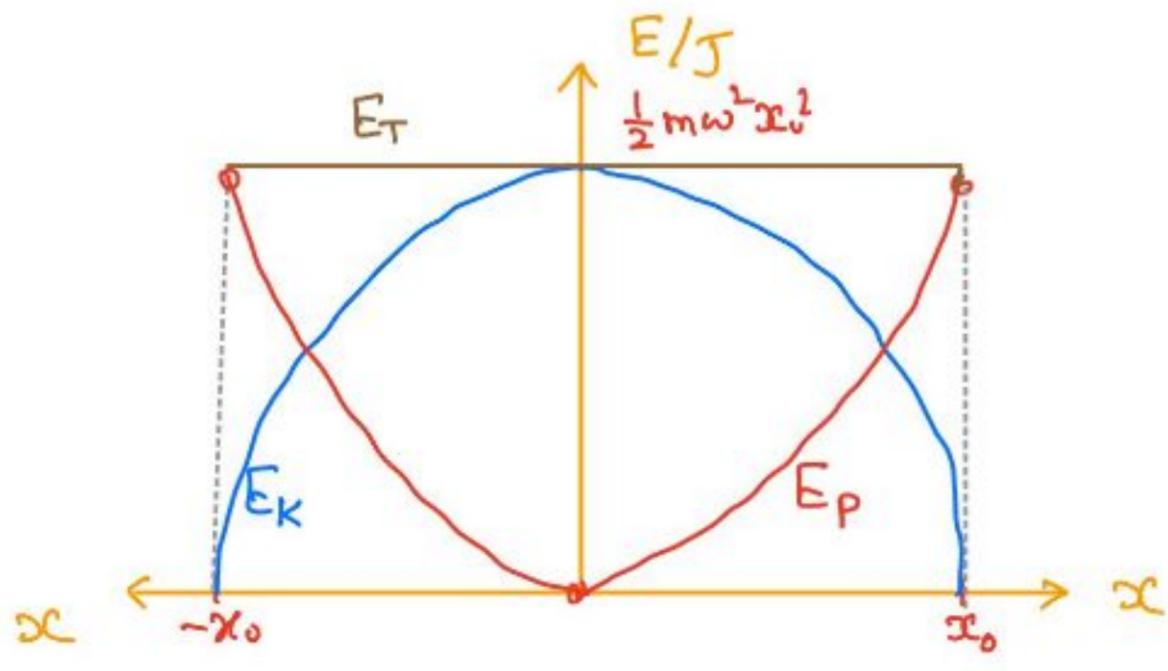
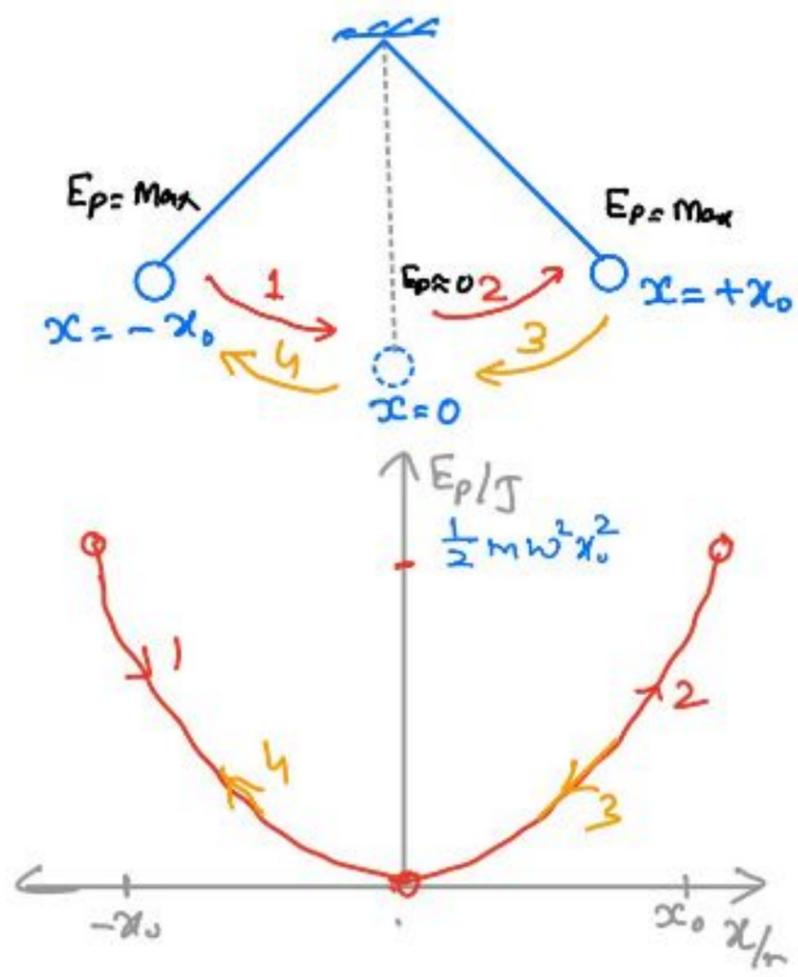
$$E_T = E_K + E_P$$

$$\frac{1}{2} m \omega^2 x_0^2 = \frac{1}{2} m \omega^2 (x_0^2 - x^2) + E_P$$

$$\cancel{\frac{1}{2} m \omega^2 x_0^2} = \cancel{\frac{1}{2} m \omega^2 x_0^2} - \frac{1}{2} m \omega^2 x^2 + E_P$$

$$E_P = \frac{1}{2} m \omega^2 x^2$$

Position	Mean	Extreme
Displacement	0	$\pm x_0$
$E_P/J$	0	$\frac{1}{2} m \omega^2 x_0^2$



## Energy-time graphs:

Case 1: If stopwatch is started at mean position:-

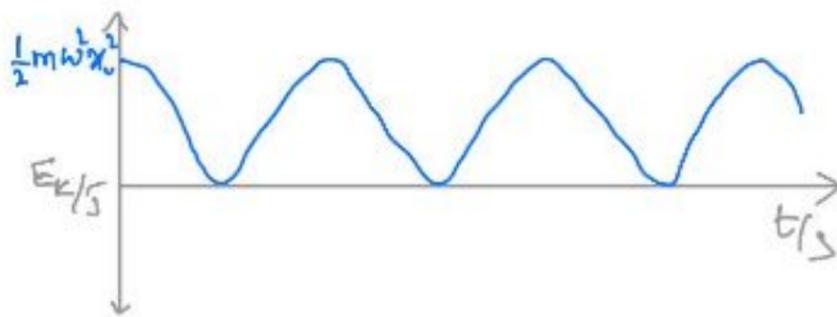
$$E_k = \frac{1}{2} m v^2$$

$$\text{But } v = \omega x_0 \cos \omega t$$

$$E_k = \frac{1}{2} m (\omega x_0 \cos \omega t)^2$$

$$E_k = \frac{1}{2} m \omega^2 x_0^2 \underbrace{\cos^2 \omega t}_{\substack{\text{trend of graph} \\ \text{X-axis}}}$$

Y-axis ← Amplitude position



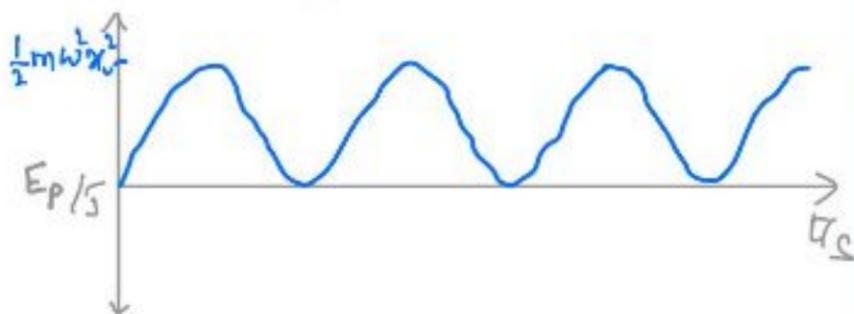
Potential energy-time graph

$$E_p = \frac{1}{2} m \omega^2 x^2$$

$$\text{But } x = x_0 \sin \omega t$$

$$E_p = \frac{1}{2} m \omega^2 (x_0 \sin \omega t)^2$$

$$E_p = \frac{1}{2} m \omega^2 x_0^2 \underbrace{(\sin^2 \omega t)}_{\substack{\text{shape of graph} \\ \text{Amplitude}}}$$



Case 2: If stopwatch is started at extreme position:-

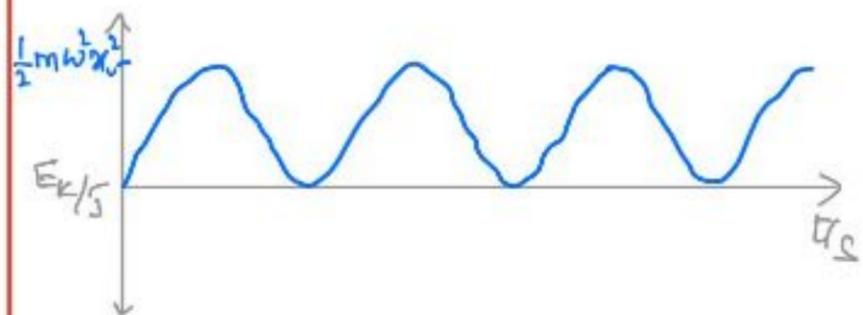
$$E_k = \frac{1}{2} m v^2$$

$$\text{But } v = \omega x_0 (-\sin \omega t)$$

$$E_k = \frac{1}{2} m (\omega x_0 (-\sin \omega t))^2$$

$$E_k = \frac{1}{2} m \omega^2 x_0^2 \underbrace{\sin^2 \omega t}_{\substack{\text{trend of graph} \\ \text{X-axis}}}$$

Y-axis ← Amplitude position



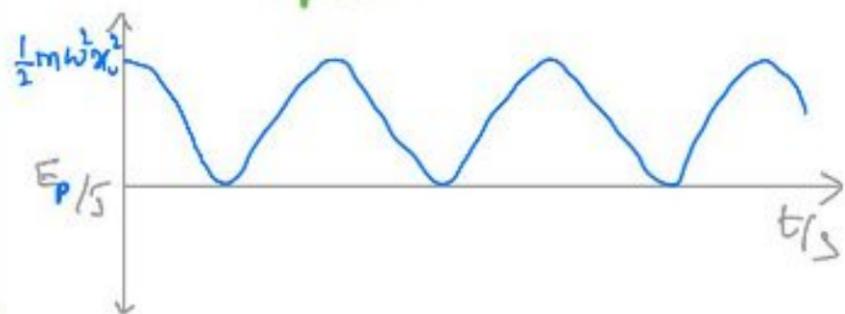
Potential energy-time graph

$$E_p = \frac{1}{2} m \omega^2 x^2$$

$$\text{But } x = x_0 \cos \omega t$$

$$E_p = \frac{1}{2} m \omega^2 (x_0 \cos \omega t)^2$$

$$E_p = \frac{1}{2} m \omega^2 x_0^2 \underbrace{(\cos^2 \omega t)}_{\substack{\text{shape of graph} \\ \text{Amplitude}}}$$



Total energy - time graph:

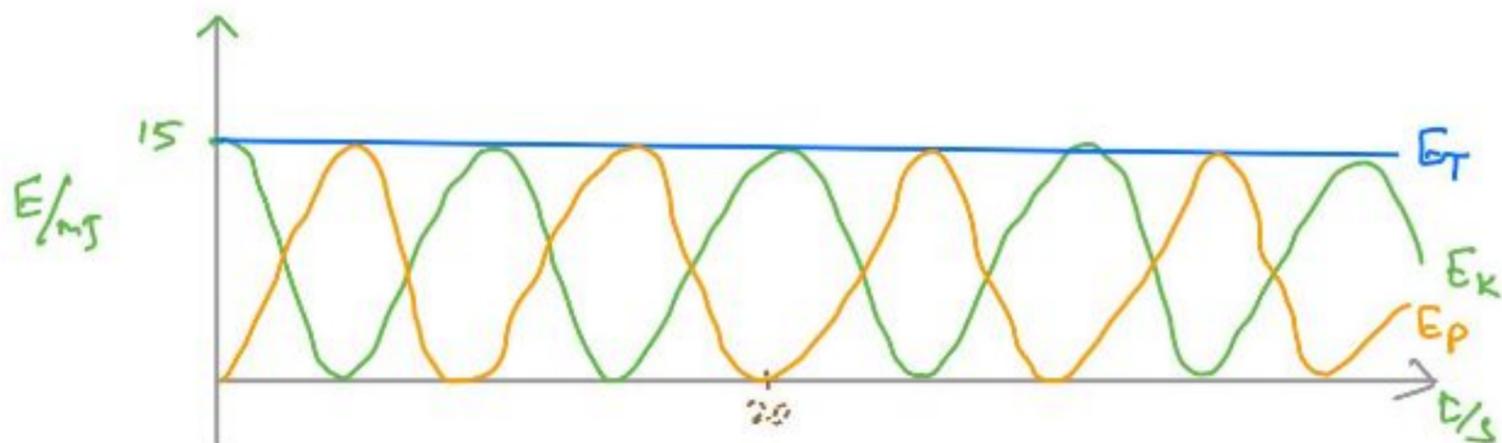
$$E_T = E_K + E_P$$

$$= \frac{1}{2} m \omega^2 x_0^2 \sin^2 \omega t + \frac{1}{2} m \omega^2 x_0^2 \cos^2 \omega t$$

$$E_T = \frac{1}{2} m \omega^2 x_0^2 (\sin^2 \omega t + \cos^2 \omega t)$$

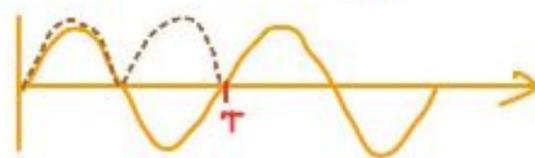
$$E_T = \frac{1}{2} m \omega^2 x_0^2$$

Since total energy is independent of time and remain constant at all times.

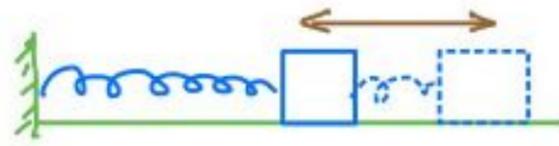
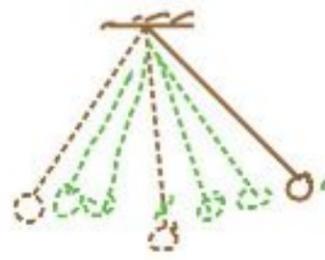


$$E_P = \frac{1}{2} m \omega^2 x_0^2 \sin^2 \omega t$$

$$E_P = (15 \times 10^{-3}) \sin^2 \left( \frac{2\pi}{20} \right) t$$



## Damping:- Concept:



Meaning: The decrease in amplitude of vibrating body due to loss of energy in doing work against friction/viscous forces is damping.

### Reason:

Total energy of vibrating object

$$E_T = \frac{1}{2} m \omega^2 x_0^2$$

constant

$$E_T = (\text{constant}) x_0^2$$

$$E_T \propto x_0^2$$

i.e. (Total energy)  $\propto$  (Amplitude)<sup>2</sup>

Since energy is lost, so energy of vibrating object decreases and so is its amplitude.

### Degree of damping:-

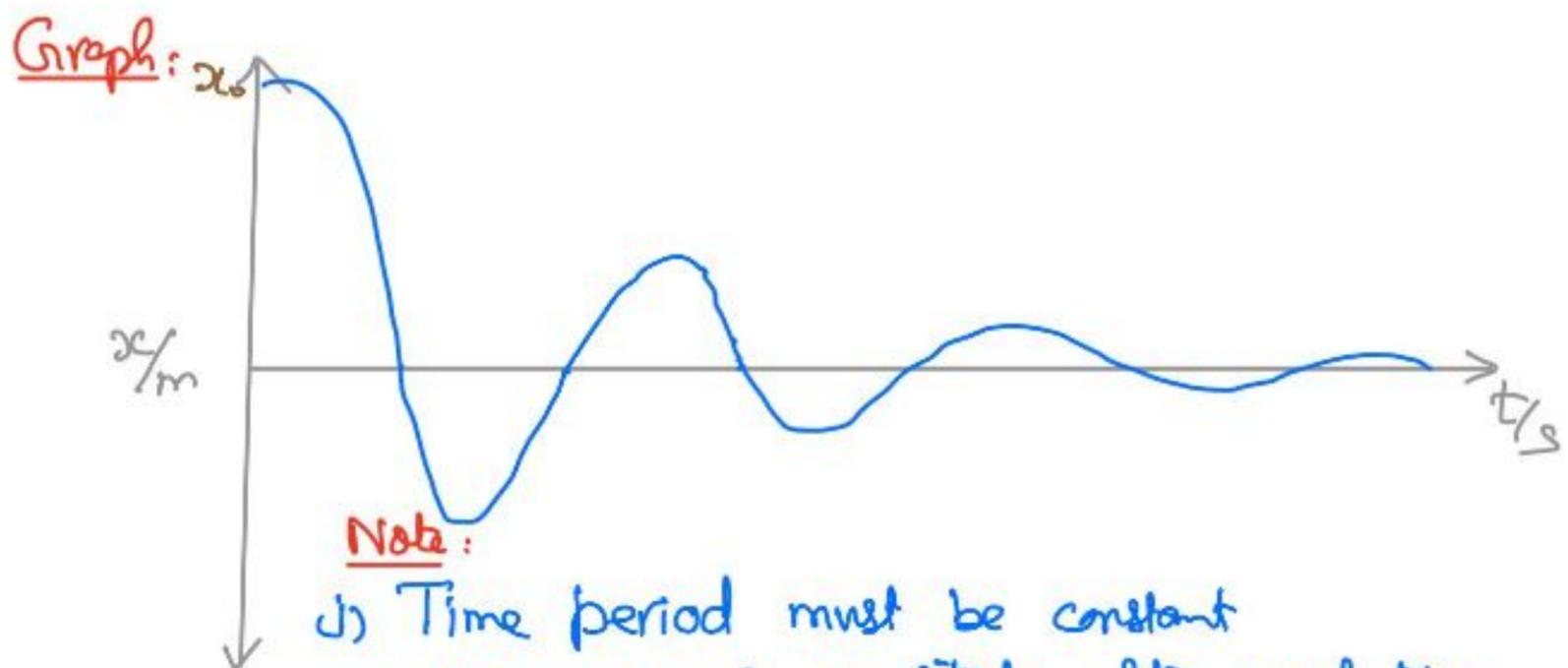
Light damping  $\longrightarrow$  Critical damping  $\longrightarrow$  Heavy damping

#### (i) Light damping:

Identification:- (i) cross equilibrium/mean position a no. of times and

(ii) take a long time to come to rest.

Examples: (i) Simple pendulum in air  
(ii) vibrating mass suspended from a spring in air



Note:

- (i) Time period must be constant
- (ii) Decrease in amplitude after equal time interval is not constant because decrease in amplitude follows an exponential trend/curve.

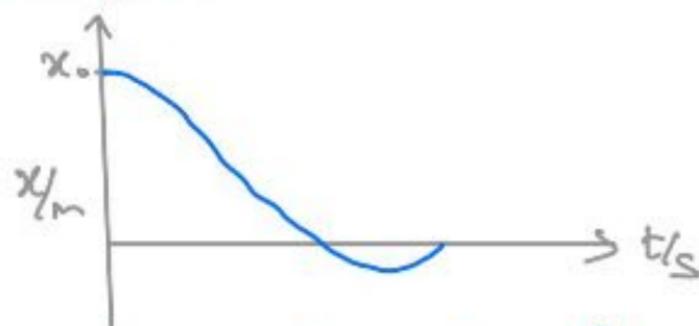
### (ii) Critical damping:

Identification:- Object comes to rest in the least possible time.

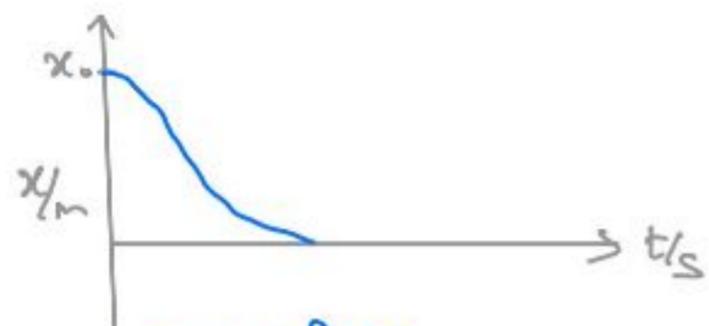
Example:-

- (i) Suspension system of vehicles.
- (ii) Door closer
- (iii) Analogue measuring device

Graphs:



Suspension system of vehicle

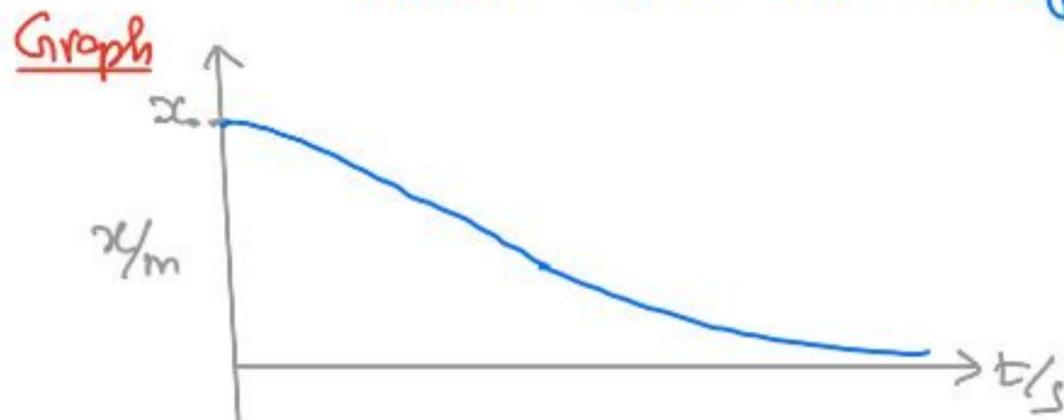


Door closer

## Heavy damping:

Identification:- Object takes an infinite time to come to its equilibrium position.

Example. Simple pendulum or mass suspended from a helical spring ( $k \downarrow$ ) in a viscous fluid such as honey, jam etc.

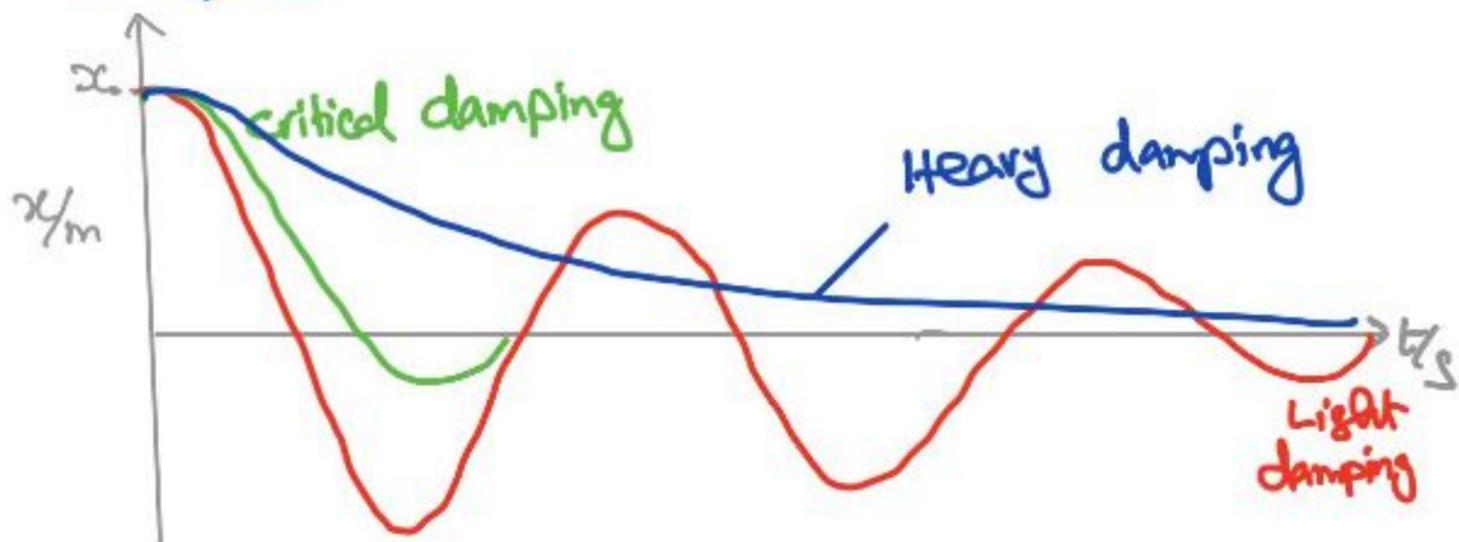


## Summary:

(i) (Amplitude)  $\downarrow$  as  $(E_T) \downarrow$  because energy is lost to surrounding ( $E_T \propto x_0^2$ )

(ii) Degree of damping  
Light  $\rightarrow$  Critical  $\rightarrow$  Heavy

(iii) Graphs:



(iv) Degree of damping can be increased

(a) place the vibrating object in a viscous fluid.

(b) Increase the surface area of vibrating object perpendicular to the plane of vibration.

Working rules for problems involving SHM:-

Step 1: Draw a free force diagram representing all the forces acting on the vibrating object.

Step 2: Find the resultant force

(i) in the direction of increasing displacement  
or

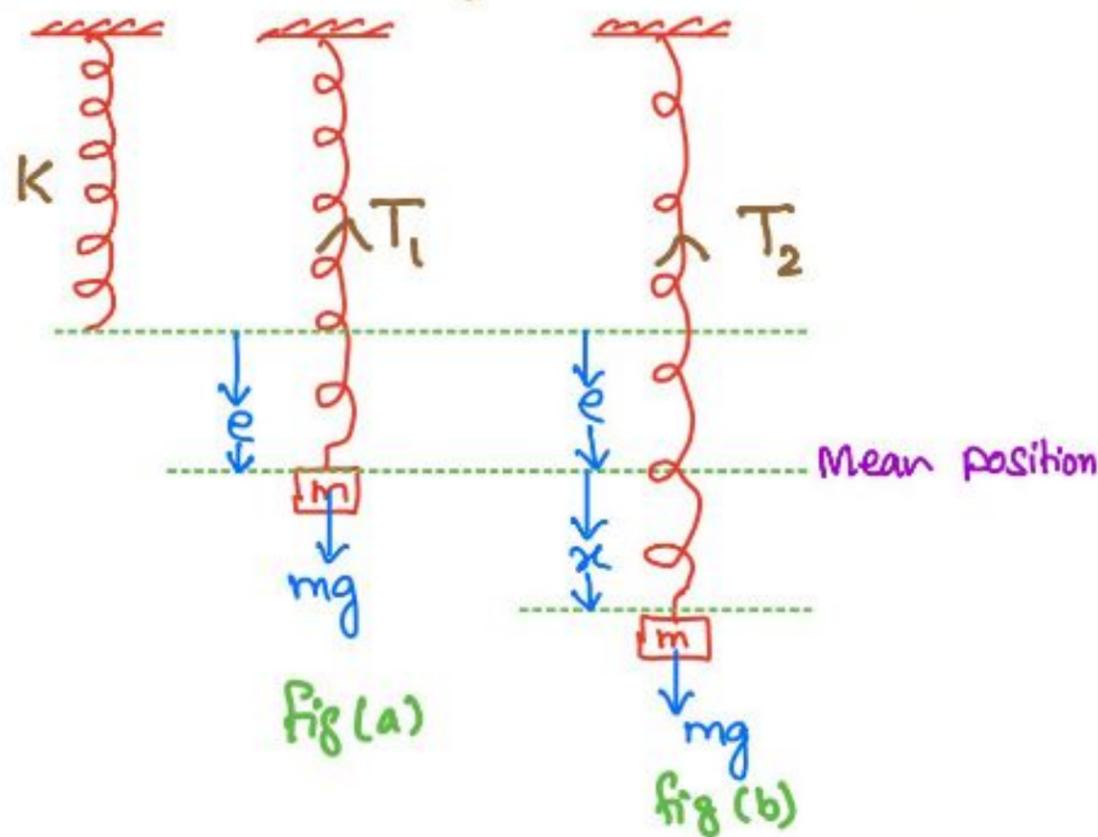
(ii) towards mean position and put -ve sign as per convention.

Step 3: Equate force from step (2) as per Newton's second law of motion to "ma" and get an equation with acceleration 'a' as subject.

Step 4: Compare eq. from step (3) with general equation of SHM to get angular frequency  $\omega$ .

Step 5: Time period and hence frequency is obtained using  $T = \frac{2\pi}{\omega}$  and  $f = \frac{\omega}{2\pi}$  respectively.

Ex. 1: Motion of a mass suspended from a spring



Step 1: From fig (a), In equilibrium state  
 $T_1 = mg$   
 $Ke = mg \dots \dots \dots (1)$

Step 2: From fig (b), Resultant force  
 (i) in the direction of increasing displacement  
 $F = mg - T_2$   
 or

(ii) towards mean position =  $-(T_2 - mg)$   
 $F = mg - T_2$

$$F = mg - K(e+x)$$

$$F = mg - Ke - Kx$$

Step 3: By Newton's second law of motion

$$ma = mg - Ke - Kx$$
$$ma = \cancel{mg} - \cancel{mg} - Kx \quad (\text{from 1})$$

$$a = -\left(\frac{K}{m}\right)x$$

Step 4: But  $a = -\omega^2 x$ , so by comparison,

$$\omega^2 = \frac{K}{m}$$

$$\omega = \sqrt{\frac{K}{m}}$$

Step 5: Time period:  $T = \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{\sqrt{\frac{K}{m}}} \Rightarrow \boxed{T = 2\pi \sqrt{\frac{m}{K}}}$$

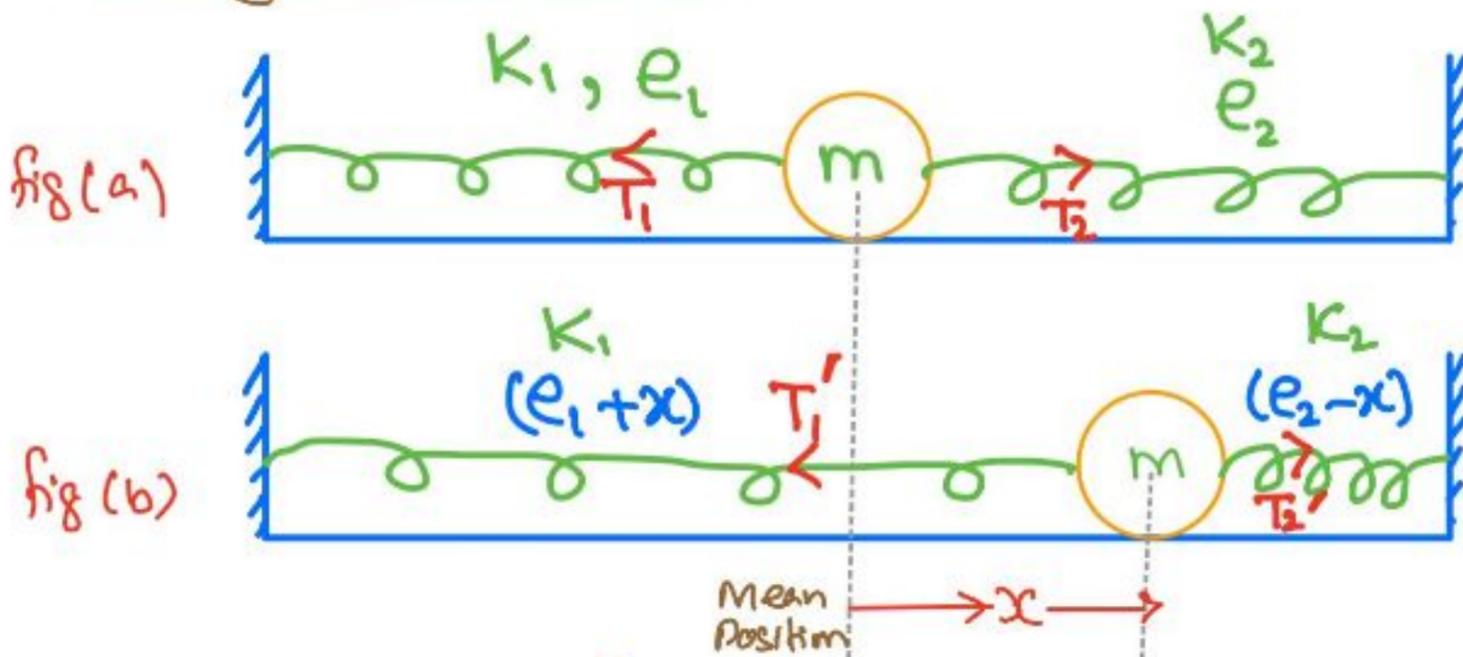
Frequency:  $f = \frac{\omega}{2\pi}$

$$\boxed{f = \frac{1}{2\pi} \sqrt{\frac{K}{m}}}$$

EX-2: Motion of a mass attached to a system of springs on a frictionless horizontal surface:-

Sp. Paper 2016

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Step 1: From fig (a), mass  $m$  is in equilibrium

$$T_1 = T_2$$

$$k_1 e_1 = k_2 e_2 \text{ ----- (1)}$$

Step 2: From fig (b), resultant force

(i) in the direction of increasing displacement =  $T_2' - T_1'$

OR

(ii) towards mean position =  $-(T_1' - T_2')$

$$F = T_2' - T_1'$$

$$F = k_2 (e_2 - x) - k_1 (e_1 + x)$$

$$F = k_2 e_2 - k_2 x - k_1 e_1 - k_1 x$$

Step 3: By Newton's second law of motion

$$ma = k_2 e_2 - k_2 x - k_1 e_1 - k_1 x$$

$$ma = \cancel{K_1 x_1} - K_2 x - \cancel{K_1 x_1} - K_1 x \quad \{ \text{from eq. 1} \}$$

$$ma = -(K_1 + K_2)x$$

$$a = -\left(\frac{K_1 + K_2}{m}\right)x$$

Step 4: In SHM,  $a = -\omega^2 x$

So by comparison,

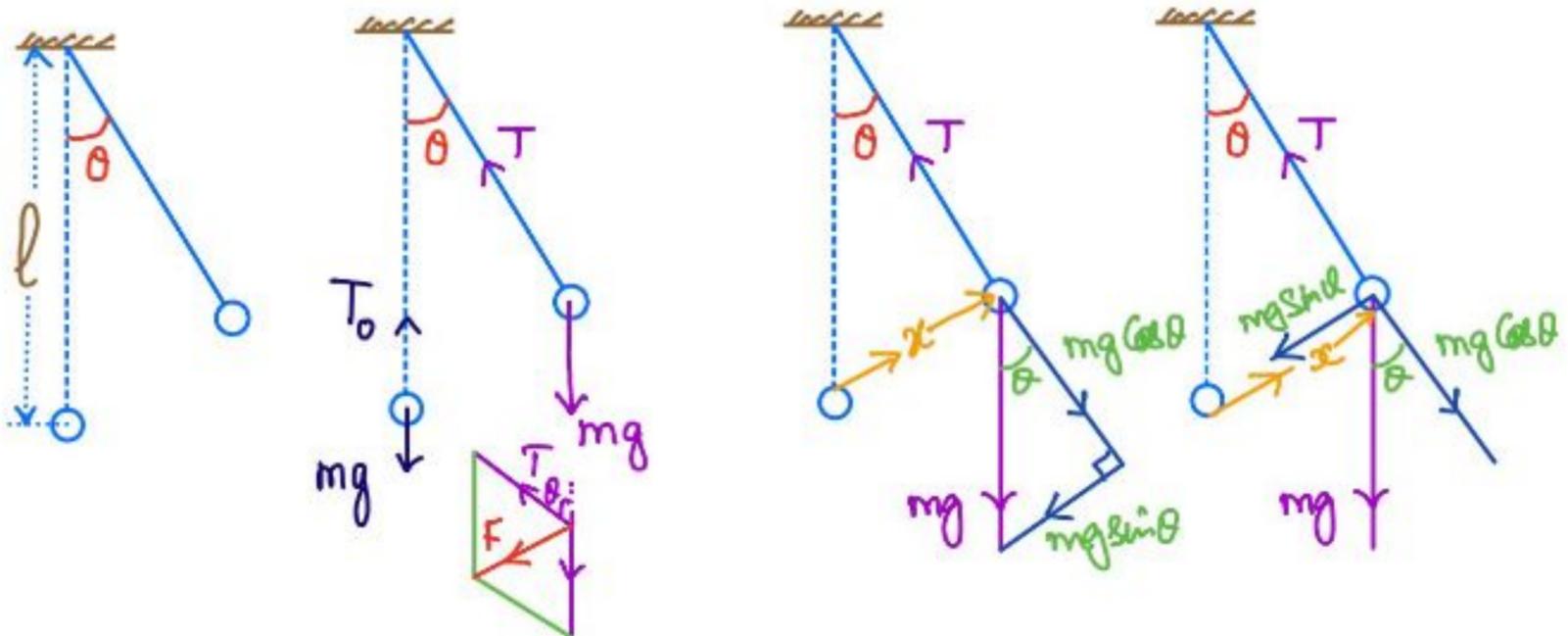
$$\omega^2 = \frac{K_1 + K_2}{m} \Rightarrow \omega = \sqrt{\frac{K_1 + K_2}{m}}$$

Step 5: Time period:  $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{K_1 + K_2}{m}}}$

$$T = 2\pi \sqrt{\frac{m}{K_1 + K_2}}$$

Frequency:  $f = \frac{\omega}{2\pi} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{K_1 + K_2}{m}}$

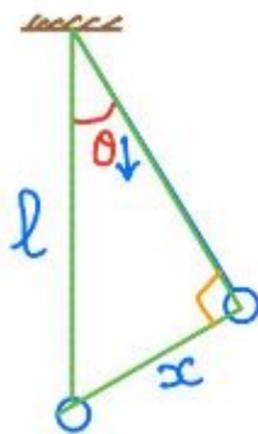
EX.3: Simple pendulum:-



Step 2: Resultant force towards mean position =  $-mg \sin \theta$   
 $F = -mg \sin \theta$

Step 3:  $\cancel{m}a = -\cancel{m}g \sin \theta$   
 $a = -g \sin \theta \text{ ----- (1)}$

To eliminate  $\sin \theta$ , we assume ' $\theta$ ' as a small angle so that approx. a right angled triangle is obtained.



$$\sin \theta = \frac{x}{l}$$

So eq (1) is modified as

$$a = -g \left( \frac{x}{l} \right)$$

$$a = - \left( \frac{g}{l} \right) x$$

Step 4: In S.H.M,  $a = -\omega^2 x$ , So

$$\omega^2 = \frac{g}{l} \Rightarrow \boxed{\omega = \sqrt{\frac{g}{l}}}$$

$$\text{Step 5: } T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{\sqrt{\frac{g}{l}}} \Rightarrow \boxed{T = 2\pi \sqrt{\frac{l}{g}}}$$

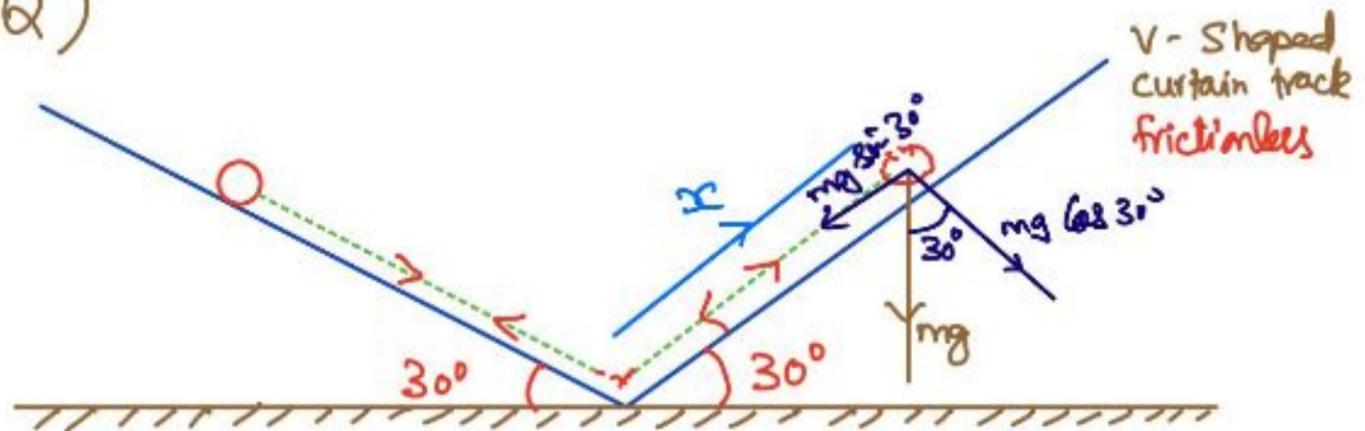
$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

Application:-  $T = 2\pi \sqrt{\frac{l}{g}} \Rightarrow T \propto \sqrt{l}$

$l \uparrow, T \uparrow$  ie Camel / Giraffe  
 $l \downarrow, T \downarrow$  ie mouse



EX. 4 (Q)



Justify with reason whether the vibration of sphere is simple harmonic or not.

Resultant force towards mean position =  $-mg \sin 30^\circ$

$$m \cdot a = -mg \sin 30^\circ$$

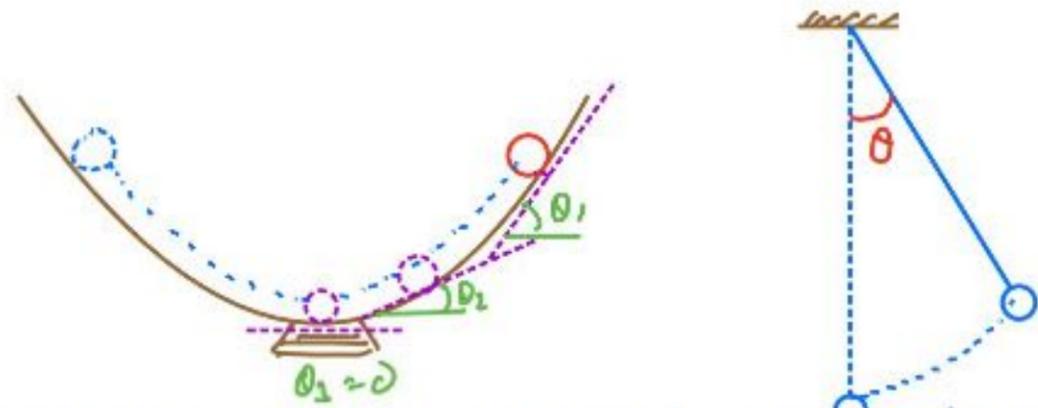
$$a = -g \sin 30^\circ$$

$$a = -(9.81)(0.5)$$

$$a = -4.9 \text{ m s}^{-2}$$

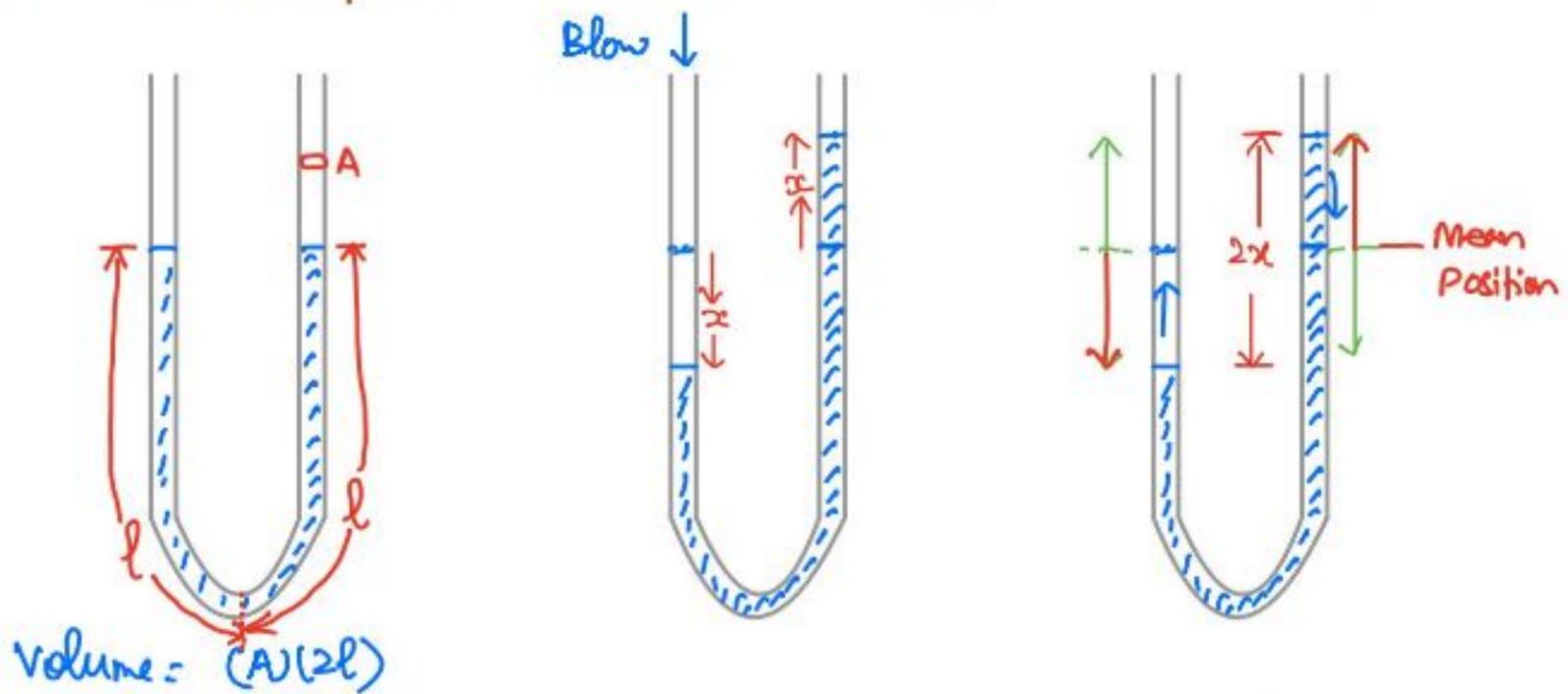
Since acceleration is constant and is independent of displacement, so motion of sphere on a straight V-shaped track is not S.H.M.

EX 5 / Q)



Motion of sphere on a curved track/bowl is S.H.M due to change of angle ( $\theta$ ).

EX.6) Motion of Mercury level in a limb of U-shaped tube (manometer) is S.H.M.



Step 2: Resultant force towards equilibrium position

$$F = -(\Delta P)(A)$$

$$F = -(2x \rho g)(A)$$

Step 3:

$$m a = -2x \rho g A$$

$$\rho V a = -2x \rho g A$$

$$a = -\left(\frac{2gA}{V}\right)x$$

But volume of liquid,  $V = (A)(2l)$   
So

$$a = - \left( \frac{\cancel{2gA}}{\cancel{2Al}} \right) x$$

$$a = - \left( \frac{g}{l} \right) x$$

Step 4: In S.H.M,  $a = -\omega^2 x$

So  $\omega^2 = \frac{g}{l} \Rightarrow \boxed{\omega = \sqrt{\frac{g}{l}}}$

Step 5:

Time period:  $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{l}}}$

$$\boxed{T = 2\pi \sqrt{\frac{l}{g}}}$$

Frequency:  $f = \frac{\omega}{2\pi}$

$$\boxed{f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}}$$

Free oscillation:- Periodic repeated motion of an object about its mean position and between two extreme positions without any variation of amplitude i.e. simple pendulum or vibrating mass suspended from a spring in vacuum.

Forced/Driven oscillation:- It is the vibration of external oscillator which provides energy to the oscillating system to vary its amplitude.

Natural frequency:- The periodic repeated motion of an object at its own (by nature). For example

(i) Heart beat 72 beats/min

(ii) Solar system

(iii) Simple pendulum  $f = \frac{1}{2\pi} \sqrt{\frac{g}{l}} \Rightarrow f_0 \propto \frac{1}{\sqrt{l}}$

(iv) Mass suspended from a spring,  $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$   
 $f_0 \propto \sqrt{\frac{k}{m}}$

Symbol:  $f_0$

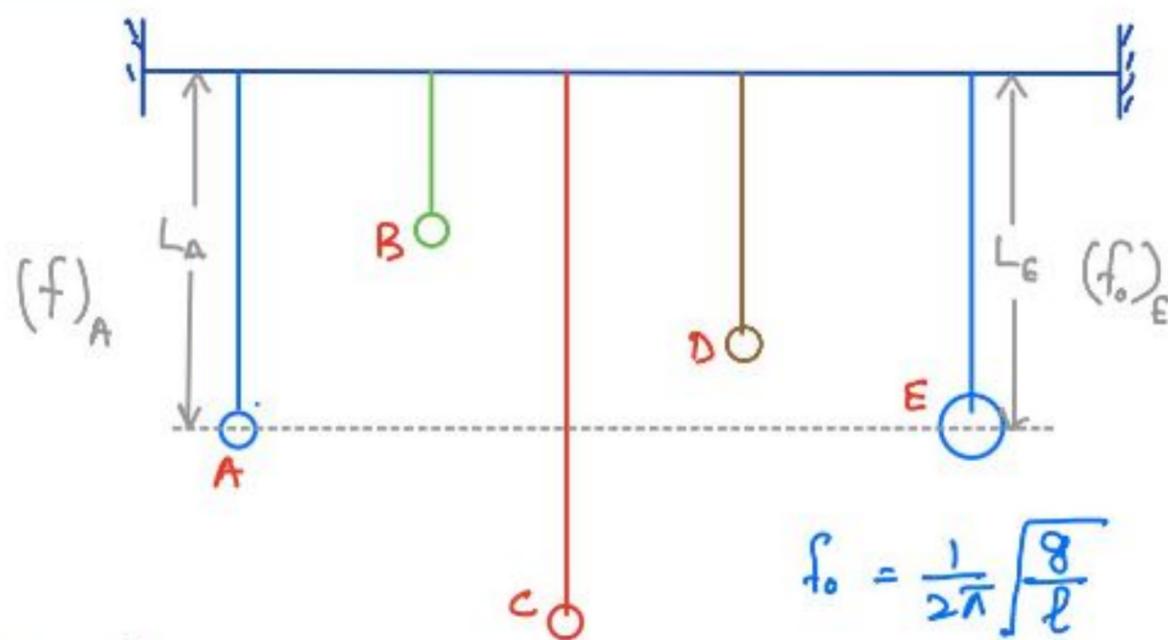
## Resonance:

Definition:- The phenomena in which there is remarkable increase in the amplitude of vibrating body when driving/forced frequency of an external oscillator is equal to its natural frequency.

### Condition:

Amplitude = maximum iff  
Driving frequency ( $f$ ) = natural frequency ( $f_0$ ) of  
oscillating system  
ie If  $f = f_0$  then  $x_0 = \text{maximum}$

### Experiment:

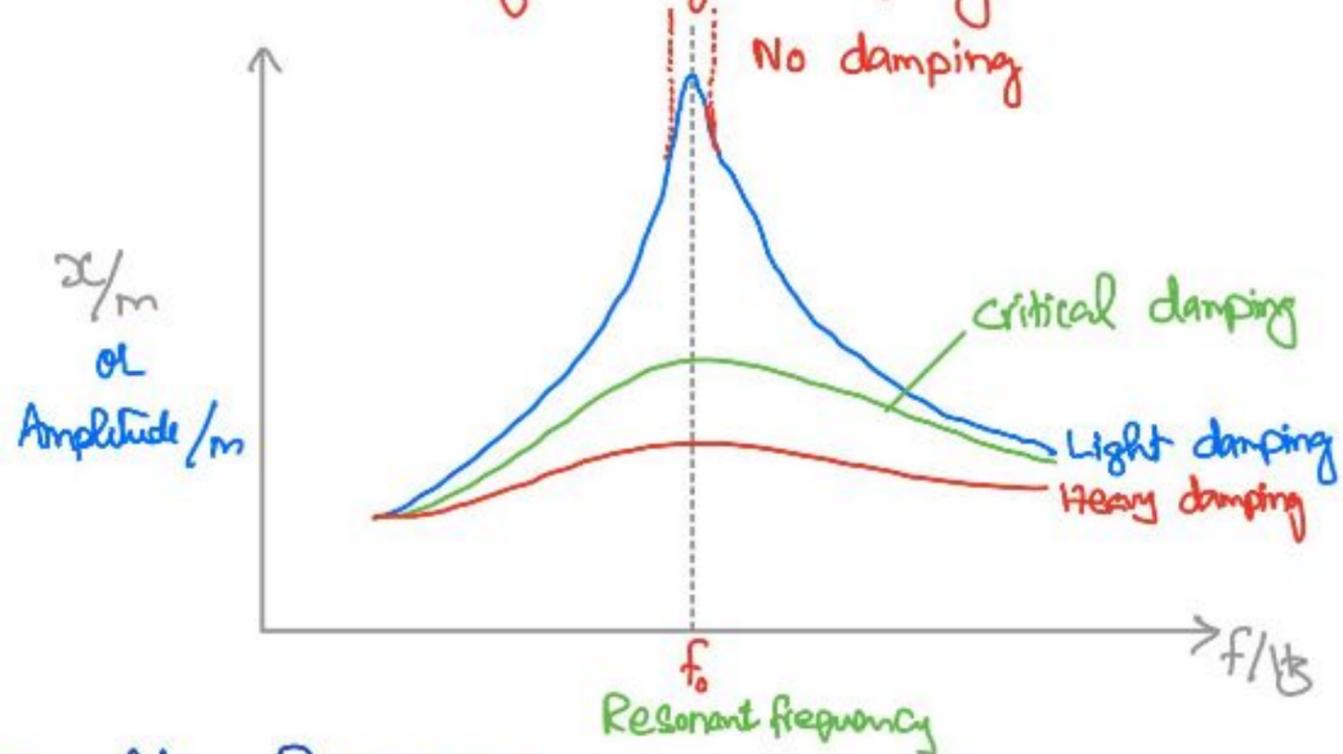


### Observation:-

Displace pendulum A towards yourself without disturbing other pendulums and release it so that it can perform back and forth oscillations in a vertical plane. It is observed after few seconds that amplitude of pendulum A decreases and all other start vibrating with different amplitudes but the amplitude of pendulum E is greatest among B, C and D.

Reason: Resonance occur in pendulum E only because driving frequency of pendulum A is equal to natural frequency of pendulum E due to same length i.e.  $f_0 \propto \frac{1}{\sqrt{l}}$

### Resonance and degree of damping:-



Note: At Resonance

- 1 - The sharpness of peak and its amplitude depends upon the degree of damping.
- 2 - The peak is very sharp with greatest amplitude for light damping.
- 3 - The peak is smooth at  $f_0$  and is well below the peak of light damping in case of critical damping.
- 4 - In heavy damping, the peak is smoothest with a least increase of amplitude a little before or at  $f_0$ .

## Simple Harmonic Motion

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Formula given in data sheet provided in CIE paper:

simple harmonic motion,

$$a = -\omega^2 x$$

velocity of particle in s.h.m.,

$$v = v_0 \cos \omega t$$

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

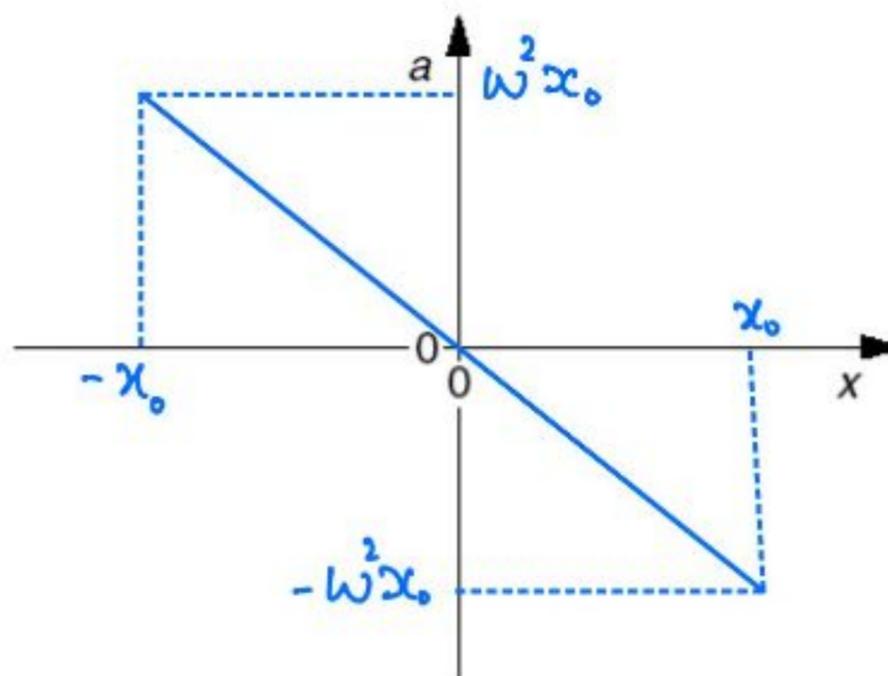
alternating current/voltage,

$$x = x_0 \sin \omega t$$

Q.1 (a) (i) Define simple harmonic motion.

The periodic motion in which acceleration is directly proportional to displacement and is opposite to the direction of displacement. [2]

(ii) On the axes of Fig. 1.1, sketch the variation with displacement  $x$  of the acceleration  $a$  of a particle undergoing simple harmonic motion.



$$a = -\omega^2 x$$

$$a \propto -x$$

fig. 1.1

(b) A strip of metal is clamped to the edge of a bench and a mass is hung from its free end as shown in Fig. 1.2.

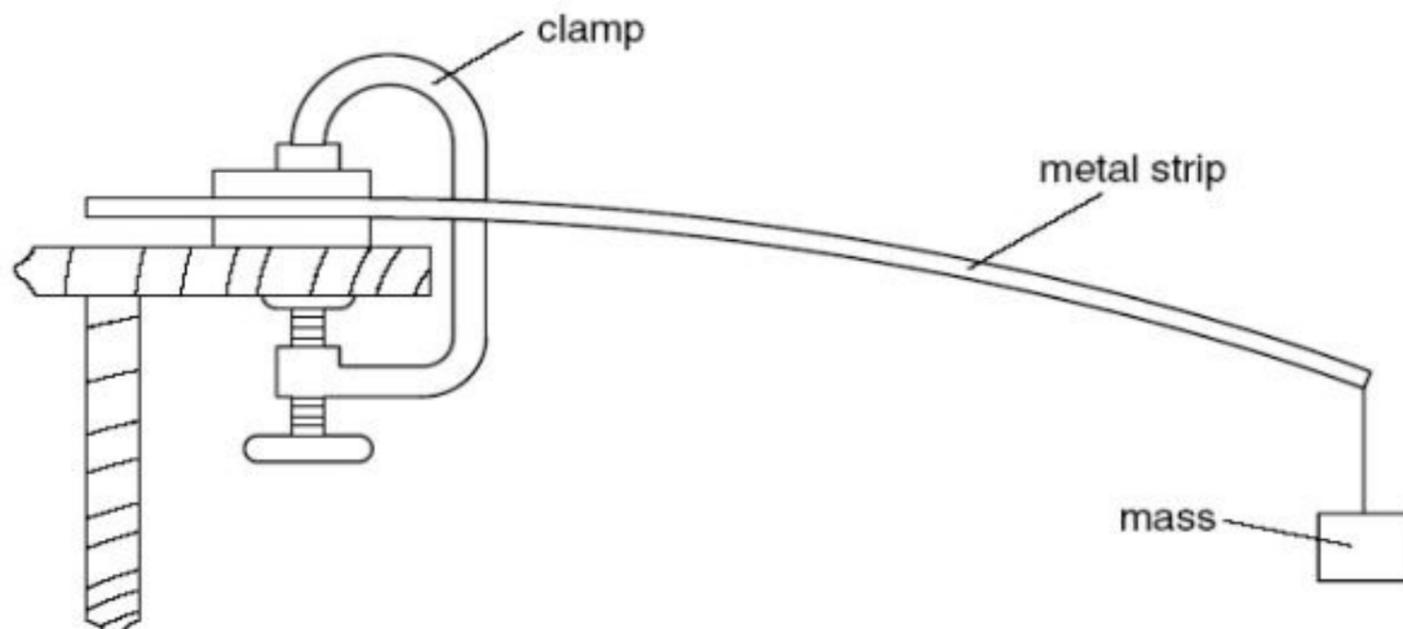


fig. 1.2

The end of the strip is pulled downwards and then released. Fig. 1.3 shows the variation with time  $t$  of the displacement  $y$  of the end of the strip.

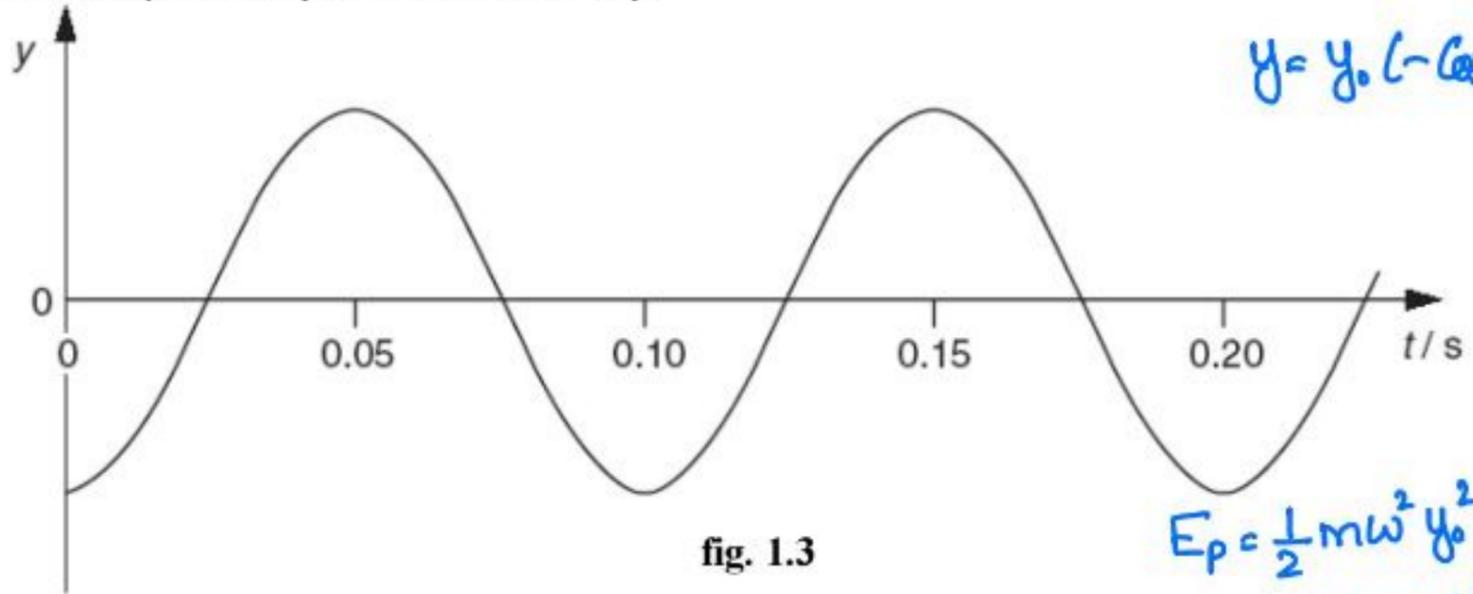


fig. 1.3

$$y = y_0 \cos(\omega t)$$

$$E_p = \frac{1}{2} m \omega^2 y_0^2$$

$$E_p \propto y_0^2$$

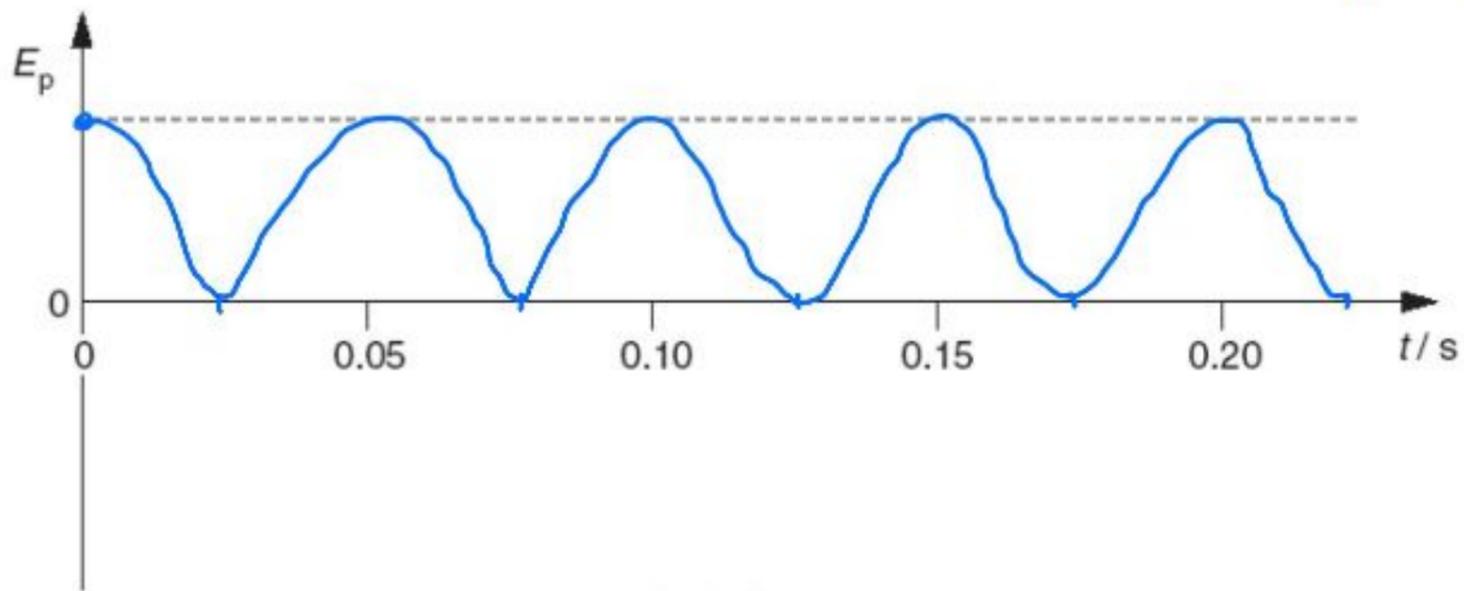


fig. 1.4

On Fig. 1.4, show the corresponding variation with time  $t$  of the potential energy  $E_p$  of the vibrating system. [3]

(c) The string supporting the mass breaks when the end of the strip is at its lowest point in an oscillation. Suggest what change, if any, will occur in the period and amplitude of the subsequent motion of the end of the strip.

period: Decreases  
 amplitude: Increases [2]

Q. 2 A student sets out to investigate the oscillation of a mass suspended from the free end of a spring, as illustrated in Fig. 2.1.

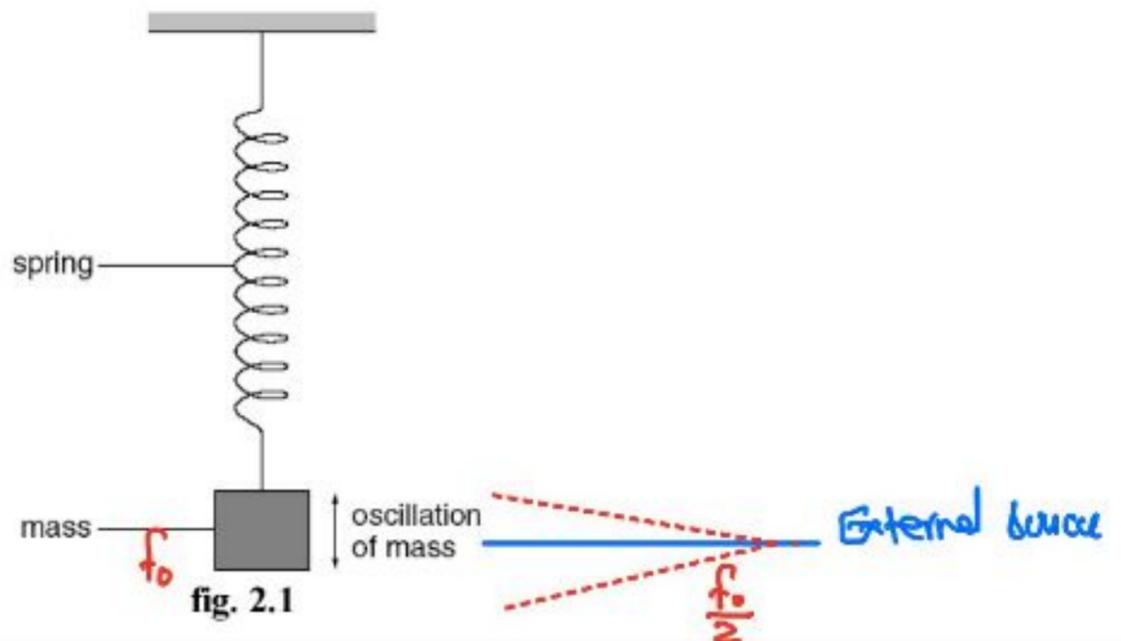


fig. 2.1

The mass is pulled downwards and then released. The variation with time  $t$  of the displacement  $y$  of the mass is shown in Fig. 2.2.

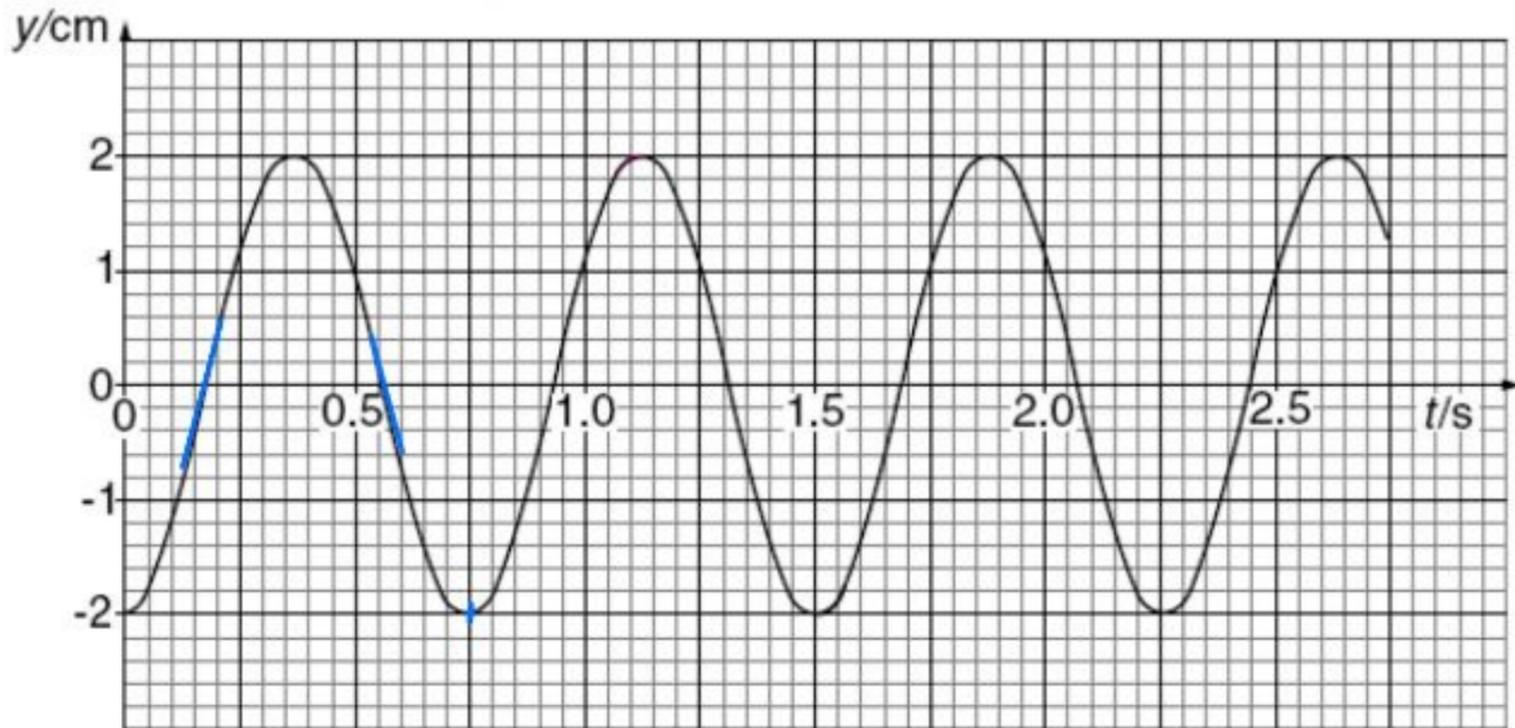


fig. 2.2

(a) Use information from Fig. 2.2

(i) to explain why the graph suggests that the oscillations are undamped,

Amplitude remain constant through out time.

(ii) to calculate the angular frequency of the oscillations,

$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{0.75}$$

angular frequency = 8.37 rad s<sup>-1</sup>

(iii) to determine the maximum speed of the oscillating mass.

$$V = \omega x_0 \\ = (8.37)(2.0 \times 10^{-2})$$

speed = 0.167 m s<sup>-1</sup> [6]

(b) (i) Determine the resonant frequency  $f_0$  of the mass-spring system. *is natural or driving frequency*

$$f_0 = \frac{1}{T} = \frac{1}{0.75}$$

$f_0 =$  1.33 Hz

(ii) The student finds that if short impulsive forces of frequency  $(1/2)f_0$  are impressed on the mass-spring system, a large amplitude of oscillation is obtained. Explain this observation.

Mass-spring system complete two oscillations when external source complete one. Impulsive forces change momentum and provides energy to increase amplitude. [3]

3. (a) The defining equation of simple harmonic motion is

$$a = -\omega^2 x$$

(i) Identify the symbols in the equation.

$a$  ... Acceleration towards mean position

$\omega$  ... Angular frequency

$x$  ... Displacement from mean position

(ii) State the significance of negative (-) sign in the equation.

Conventional significance: Acceleration is directed towards mean position.  
Vectorial significance: Acceleration and displacement are in opposite directions.

[3]

(b) A frictionless pulley of mass  $m$  is held on a horizontal surface by means of two similar springs, each of spring constant  $k$ . The springs are attached to fixed points as shown in fig. 3.1.

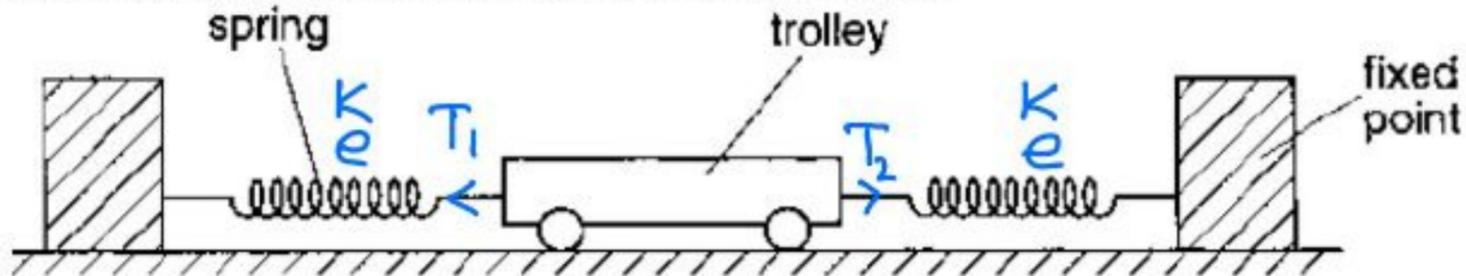


Fig. 3.1

When the trolley is in equilibrium, the extension of each spring is  $e$ .

The trolley is then displaced a small distance  $x$  to the right along the axis of the springs. Both springs remain extended.

(i) Show that the magnitude  $F$  of the restoring force acting on the trolley is given by

$$F = 2kx$$

Resultant force towards mean position:  $F = -(T_1 - T_2)$

$$F = -[K(e+x) - K(e-x)]$$

$$= -[Ke + Kx - Ke + Kx]$$

Magnitude of restoring force =  $F = 2Kx$

[2]

(ii) The trolley is then released. Show that the acceleration  $a$  of the trolley is given by,

$$a = -2kx/m$$

By Newton's second law  
 $ma = -2Kx$

$$a = -\left(\frac{2K}{m}\right)x$$

[2]

(iii) The mass  $m$  of the trolley is 900g and the spring constant  $k$  is 120 Nkg<sup>-1</sup>. By comparing your answer to a(i) and the equation in b(ii), determine the frequency of oscillation of the trolley.

Therefore,  $\omega^2 = \frac{2K}{m}$

$$2\pi f = \sqrt{\frac{2K}{m}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{2K}{m}}$$

$$f = \frac{1}{2(3.14)} \sqrt{\frac{2(120)}{900 \times 10^{-3}}}$$

frequency : ..... Hz [3]

(c) Suggest why the trolley in (b) provides a simple model for the motion of an atom in a crystal.

Atom in a crystal also vibrates at its mean position due to forces from the neighbouring atoms like that of trolley. [2]

4. A vertical spring supports a mass, as shown in Fig. 4.1.

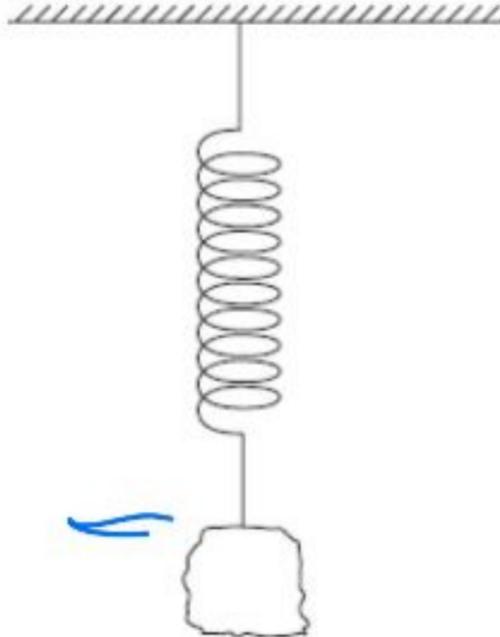


Fig. 4.1

The mass is displaced vertically then released. The variation with time  $t$  of the displacement  $y$  from its mean position is shown in Fig. 4.2.

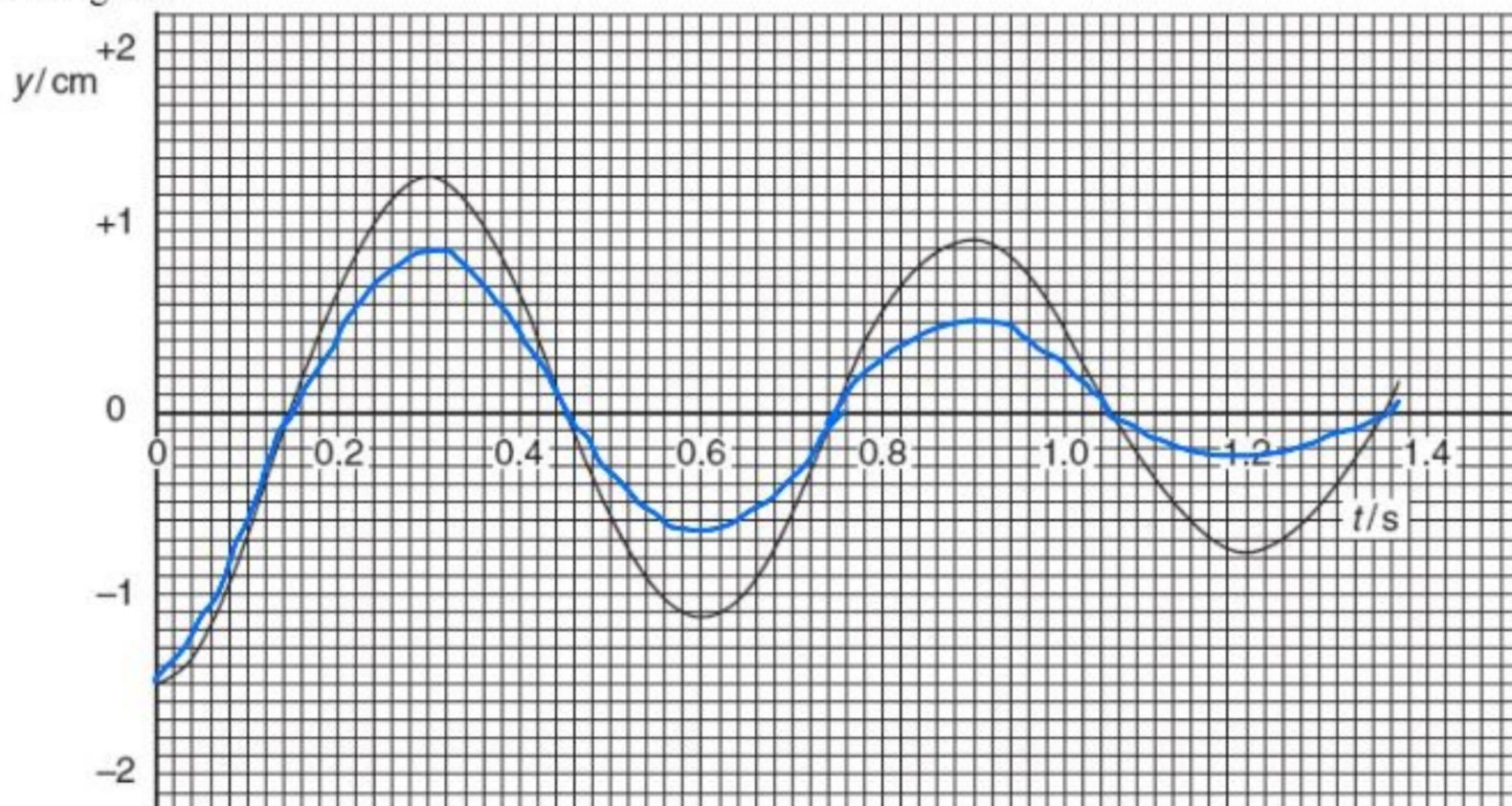


Fig. 4.2

A student claims that the motion of the mass may be represented by the equation

$$y = y_0 \sin \omega t.$$

(a) Give two reasons why the use of this equation is inappropriate.

1. Graph is of  $-y_0 e^{-\gamma t} \sin \omega t$  instead of  $\sin \omega t$ .

2. Amplitude  $y_0$  is not constant through out time.

(b) Determine the angular frequency  $\omega$  of the oscillations.

$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{0.60}$$

angular frequency = ..... 10.5 ..... rad s<sup>-1</sup> [2]

(c) The mass is a lump of plasticine. The plasticine is now flattened so that its surface area is increased. The mass of the lump remains constant and the large surface area is horizontal.

The plasticine is displaced downwards by 1.5 cm and then released.

On Fig. 4.2, sketch a graph to show the subsequent oscillations of the plasticine. [3]

5. The vibrations of a mass of 150 g are simple harmonic. Fig 5.1 shows the variation with displacement  $x$  of the kinetic energy  $E_k$  of the mass.

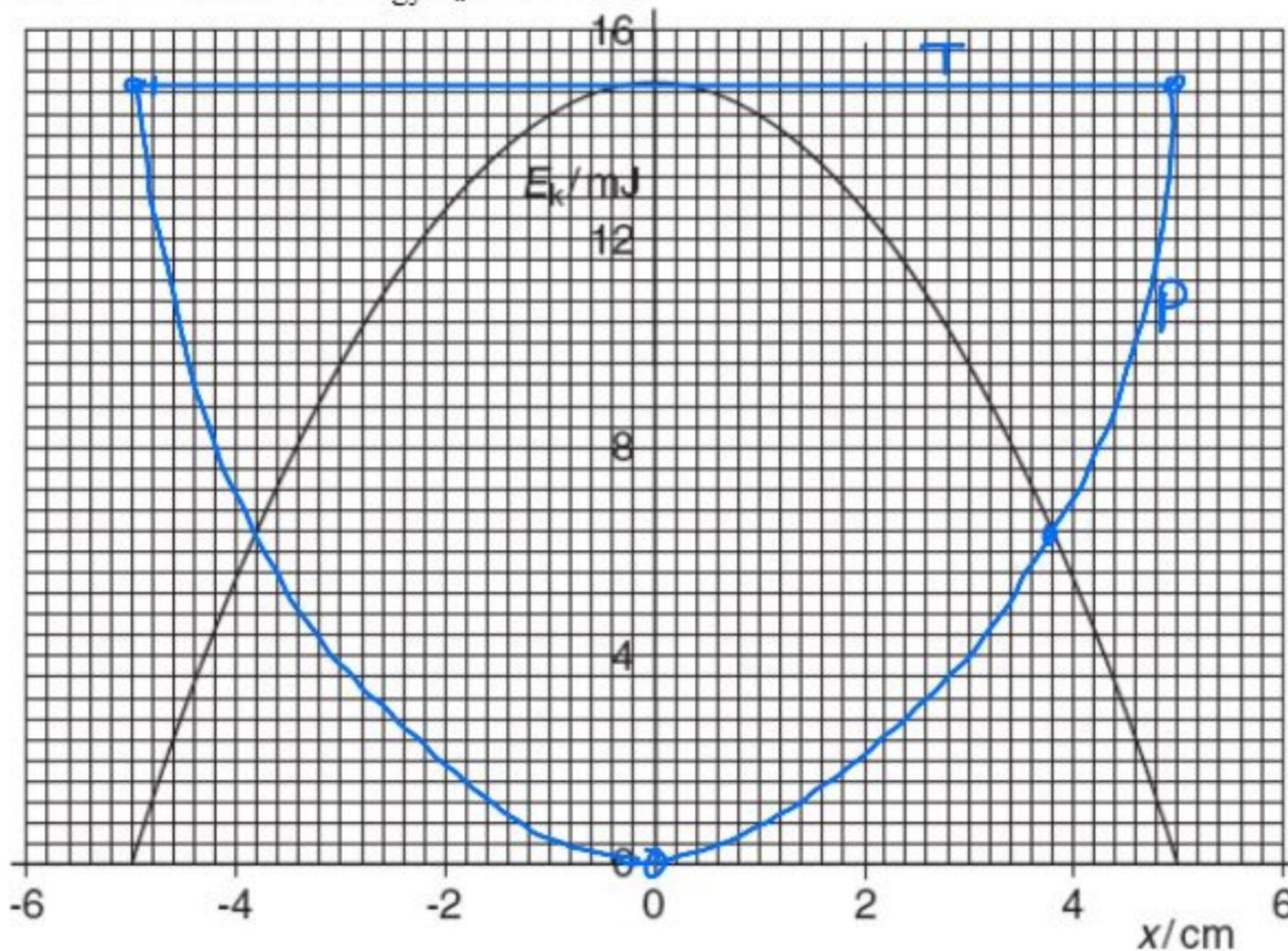


Fig. 5.1

(a) On Fig. 5.1, draw lines to represent the variation with displacement  $x$  of

(i) the potential energy of the vibrating mass (label this line P),

(ii) the total energy of the vibrations (label this line T).

[2]

(b) Calculate the angular frequency of the vibrations of the mass.

$$E_T = \frac{1}{2} m \omega^2 x_0^2$$

$$15 \times 10^{-3} = \frac{1}{2} (150 \times 10^{-3}) (\omega^2) (5.0 \times 10^{-2})^2$$

$$\omega =$$

angular frequency = ..... 8.94 ..... rad s<sup>-1</sup> [3]

- (c) The oscillations are now subject to damping.  
 (i) Explain what is meant by *damping*.

The decrease in the amplitude of vibrating body due to loss of energy in doing work against frictional/viscous forces. [2]

- (ii) The mass loses 20% of its vibrational energy. Use Fig. 5.1 to determine the new amplitude of oscillation. Explain your working.

$$80\% \text{ of } E_T = \frac{1}{2} m \omega^2 x_0^2$$

$$\left(\frac{80}{100}\right) (15 \times 10^{-3}) = \frac{1}{2} (150 \times 10^{-3}) (8.94)^2 x_0^2$$

$$x_0 =$$

amplitude = ..... cm [2]

6. A tube, closed at one end, has a constant area of cross-section  $A$ . Some lead shot is placed in the tube so that the tube floats vertically in a liquid of density  $\rho$ , as shown in Fig. 6.1.

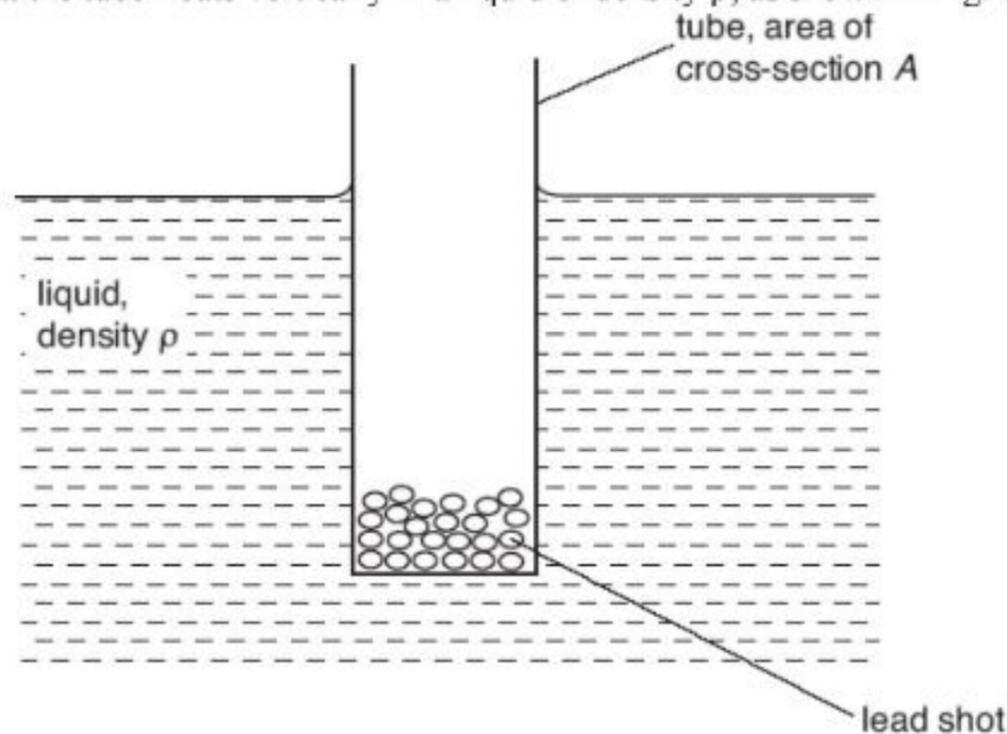


Fig. 6.1

The total mass of the tube and its contents is  $M$ .

When the tube is given a small vertical displacement and then released, the vertical acceleration  $a$  of the tube is related to its vertical displacement  $y$  by the expression

$$a = - \frac{A\rho g}{M} y,$$

where  $g$  is the acceleration of free fall.

- (a) Define *simple harmonic motion*.

The periodic motion in which acceleration is directly proportional to displacement and is opposite to the direction of displacement. [2]

(b) Show that the tube is performing simple harmonic motion with a frequency  $f$  given by

$$f = \frac{1}{2\pi} \sqrt{\frac{A\rho g}{M}}$$

By comparison with  $a = -\omega^2 y$

$$\omega^2 = \frac{A\rho g}{M}$$

$$2\pi f = \sqrt{\frac{A\rho g}{M}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{A\rho g}{M}}$$

[3]

(c) Fig. 6.2 shows the variation with time  $t$  of the vertical displacement  $y$  of the tube in another liquid.

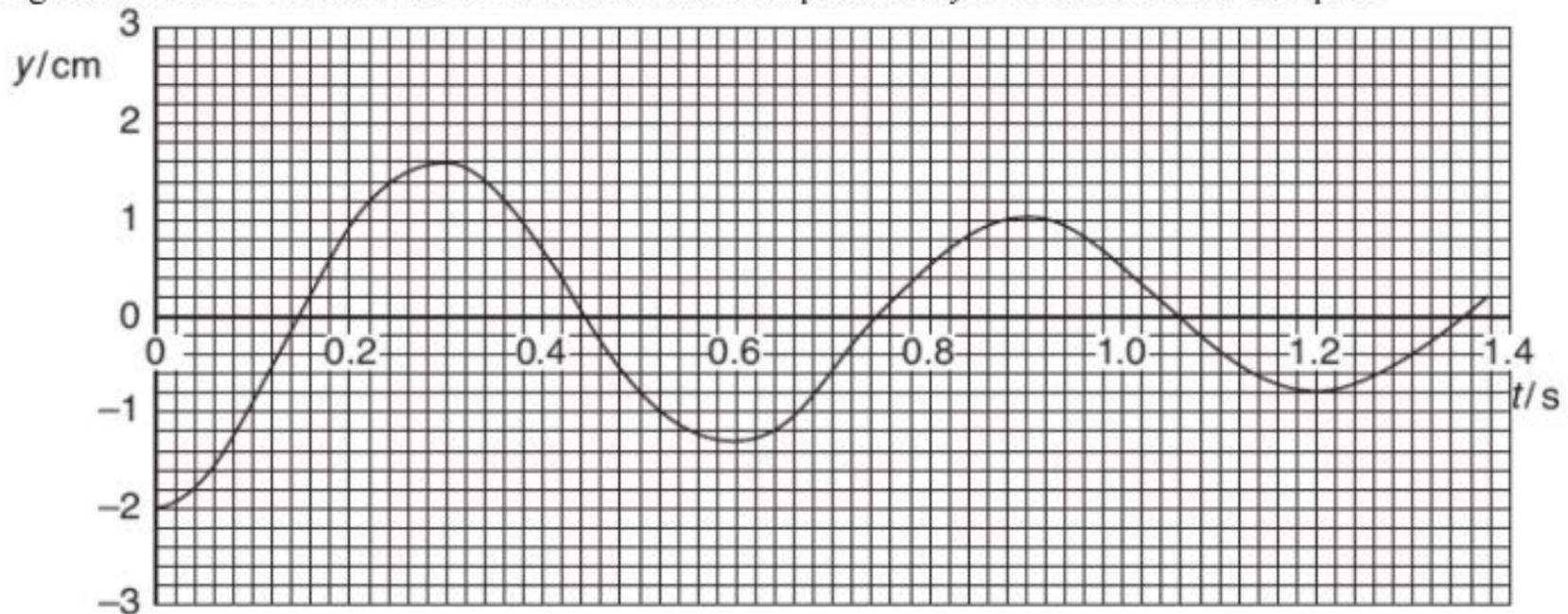


Fig. 6.2

(i) The tube has an external diameter of 2.4 cm and is floating in a liquid of density  $950 \text{ kgm}^{-3}$ . Assuming the equation in (b), calculate the mass of the tube and its contents.

$$\frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{(\frac{\pi d^2}{4})(\rho)(g)}{M}}$$

$$M = \frac{1}{4\pi^2} \left[ (\frac{\pi d^2}{4})(\rho)(g)T^2 \right]$$

$$M = \frac{1}{16(3.14)} \left[ (2.4 \times 10^{-2})^2 (950)(9.81)(0.60)^2 \right]$$

mass = 0.384 kg [3]

(ii) State what feature of Fig. 4.2 indicates that the oscillations are damped.

Amplitude is decreasing with time.

[1]

7. (a) Explain what is meant by the term *resonance*.

The phenomena in which amplitude of vibrating object becomes maximum if driving frequency is equal to natural frequency. [2]

(b) On Fig. 7.1, sketch a graph which shows how the amplitude  $a$  of a forced oscillation varies with frequency as the frequency varies from near zero to above the resonant frequency  $f_0$ . [2]

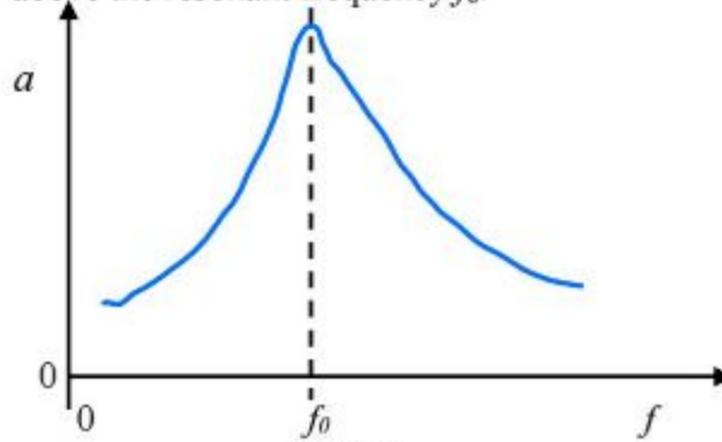


Fig. 7.1

(c) State one factor which determines the sharpness of the resonance.

Degree of damping [1]

(d) Name and outline one example where resonance is useful and one example where it is a nuisance or dangerous.

useful

.....  
.....

nuisance or dangerous

out of syllabus (2022)  
.....  
.....

[4]

Q.1 A metal ball is suspended from a fixed point by means of a string, as illustrated in Fig. 3.1.

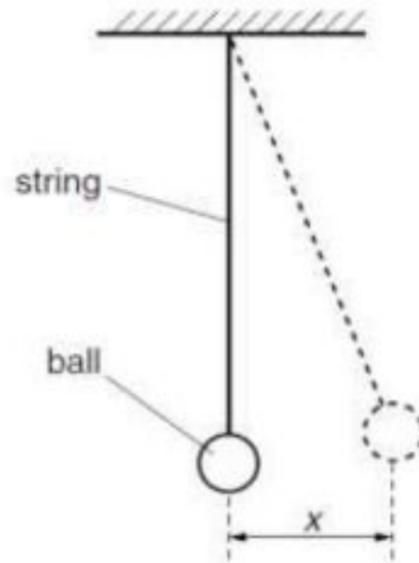


Fig. 3.1

The ball is given a small displacement and then released. The variation with time  $t$  of the displacement  $x$  of the ball is shown in Fig. 3.2.

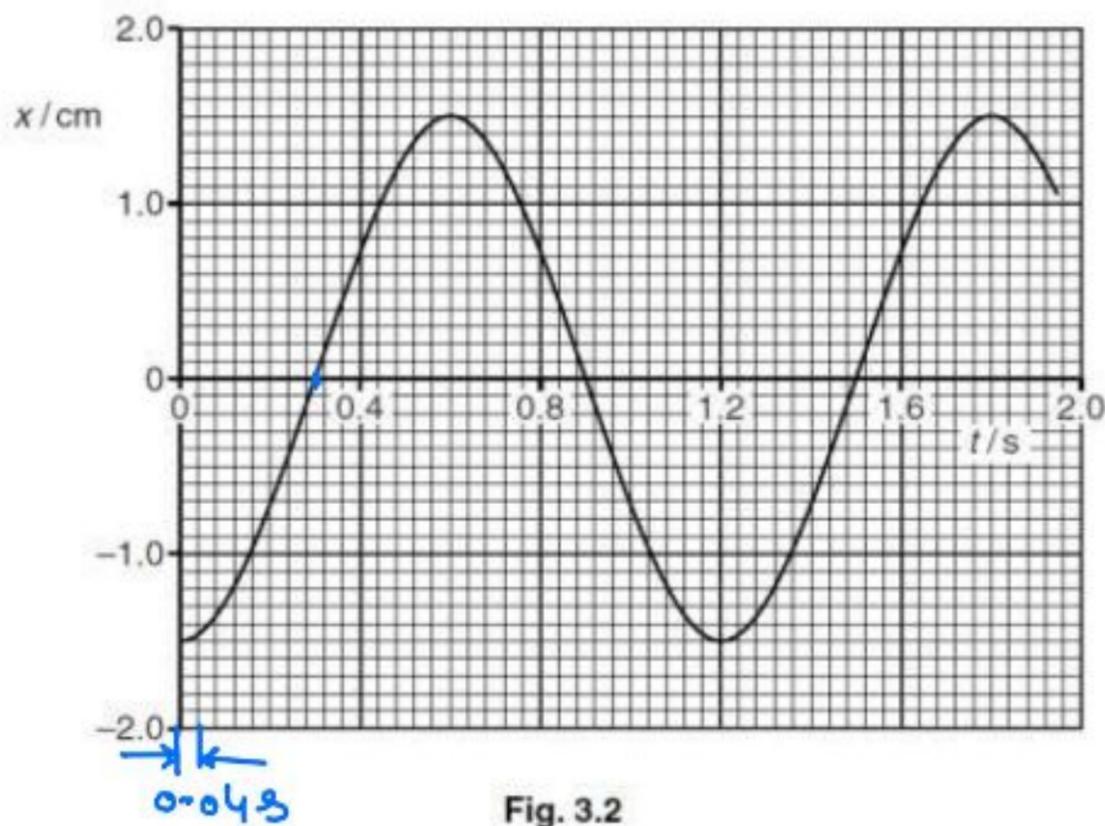


Fig. 3.2

(a) (i) State two times at which the speed of the ball is a maximum.

time = 0.30 s and time = 0.90 s [1] or 1.50 s

(ii) Show that the maximum speed of the ball is approximately  $0.08 \text{ m s}^{-1}$ .

$$\begin{aligned}
 v_{\text{max}} &= \omega x_0 \\
 &= \left(\frac{2\pi}{T}\right) (x_0) \\
 &= \frac{2(3.14)(1.5 \times 10^{-2})}{1.20} = 0.08 \text{ m s}^{-1}
 \end{aligned}$$

Q. 2

A ball is held between two fixed points A and B by means of two stretched springs, as shown in Fig. 4.1.

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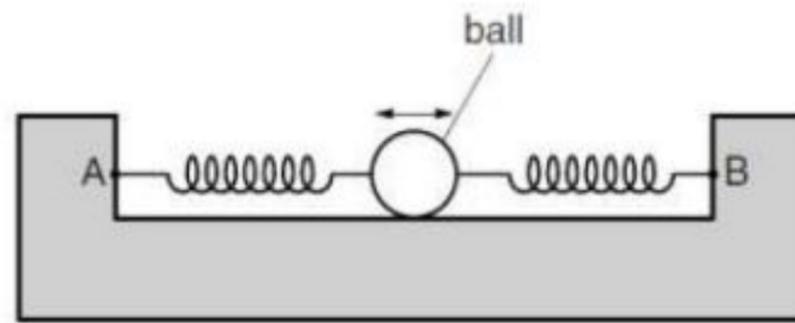


Fig. 4.1

The ball is free to oscillate horizontally along the line AB. During the oscillations, the springs remain stretched and do not exceed their limits of proportionality.

The variation of the acceleration  $a$  of the ball with its displacement  $x$  from its equilibrium position is shown in Fig. 4.2.

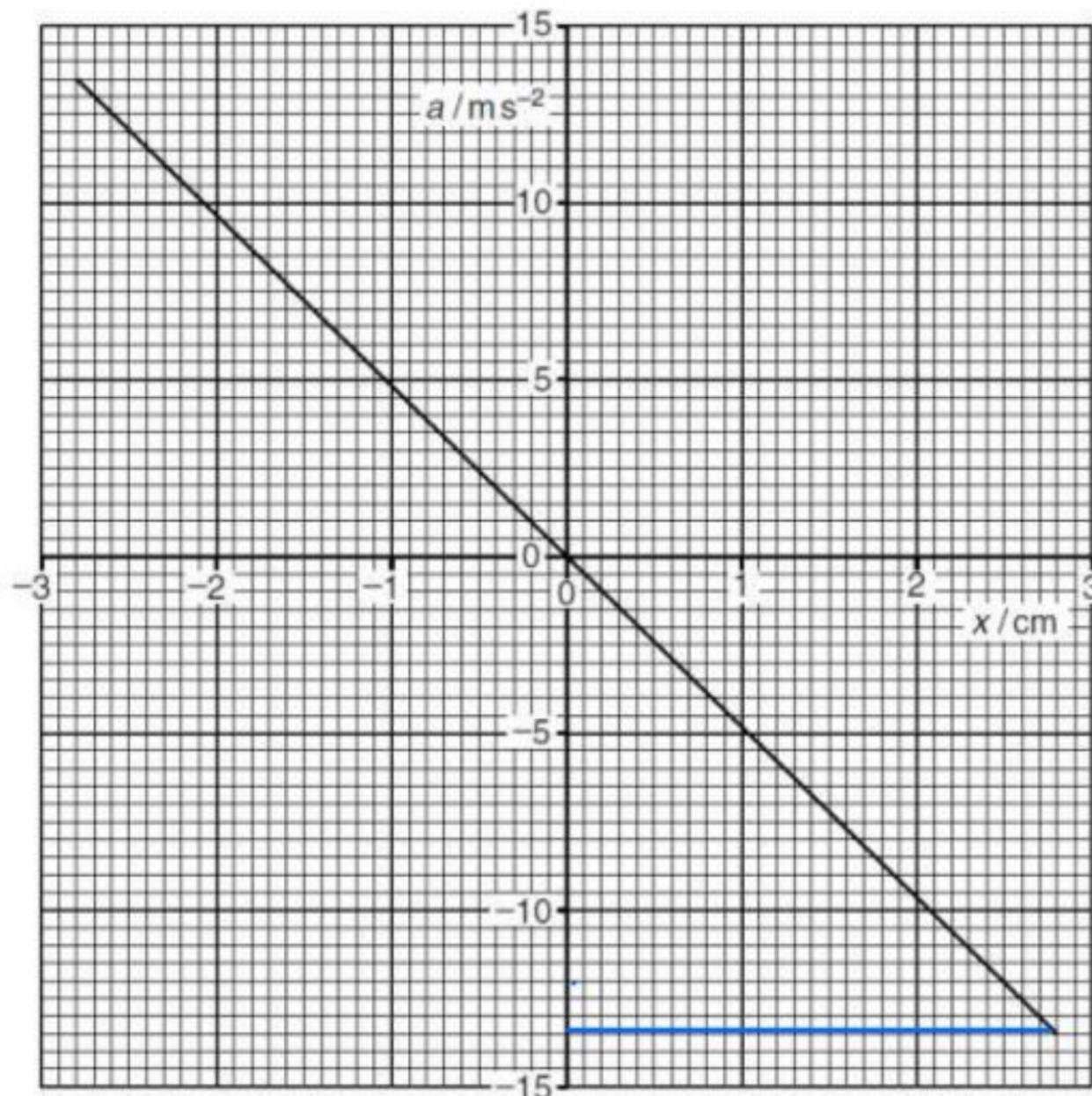


Fig. 4.2

(a) State and explain the features of Fig. 4.2 that indicate that the motion of the ball is simple harmonic.

\* Straight line from origin  $\rightarrow$  Acceleration  $\propto$  Displacement

- \* Gradient of straight line  $\rightarrow$  constant ( $\omega^2$ ) ie  $a = \omega^2 x$
- \* Signs of  $a/ms^2$  and  $x/m$   $\rightarrow$  if  $x = +ve, a = -ve$   
if  $x = -ve, a = +ve$
- \* Sign of gradient  $\rightarrow$   $-ve$  shows  $a = \omega^2(-x)$

.....[4]

(b) Use Fig. 4.2 to determine, for the oscillations of the ball,

(i) the amplitude,

amplitude = ..... 2.8 ..... cm [1]

(ii) the frequency.

$\omega^2 =$  Gradient of graph

$$2\pi f = \frac{-13.5 - (13.5)}{2.8 - (-2.8)} \Rightarrow f = \frac{-4.82}{2(3.14)}$$

$$f = -0.767$$

frequency = ..... 0.77 ..... Hz [3]

(c) The arrangement in Fig. 4.1 is now rotated through  $90^\circ$  so that the line AB is vertical. The ball now oscillates in a vertical plane.

Suggest one reason why the oscillations may no longer be simple harmonic.

The ball does not cross its equilibrium position every time. ....[1]

{9702/41 & 42/O/N/12}

Q.3

A small frictionless trolley is attached to a fixed point A by means of a spring. A second spring is used to attach the trolley to a variable frequency oscillator, as shown in Fig. 2.1.

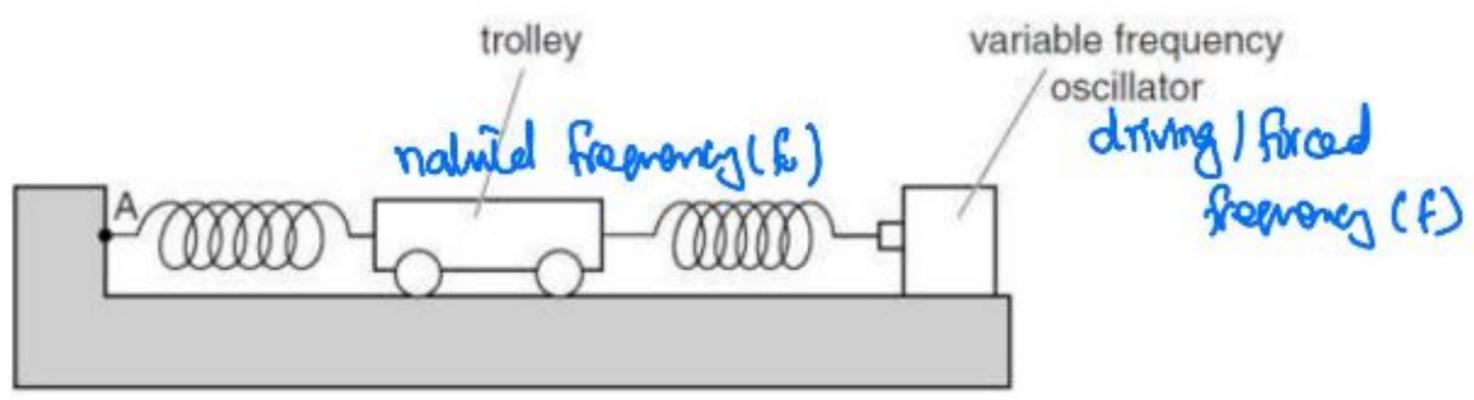


Fig. 2.1

Both springs remain extended within the limit of proportionality. Initially, the oscillator is switched off. The trolley is displaced horizontally along the line joining the two springs and is then released. The variation with time  $t$  of the velocity  $v$  of the trolley is shown in Fig. 2.2.

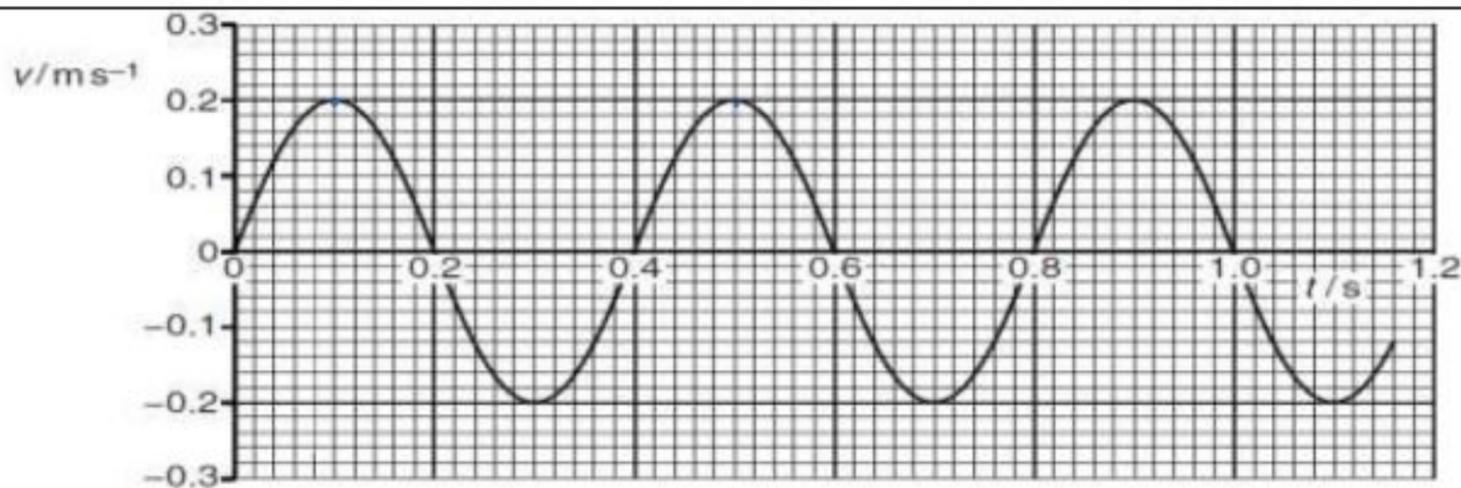


Fig 2.2

(a) (i) Using Fig. 2.2, state two different times at which

Akhtar Mahmood (0333-4281759)

1. the displacement of the trolley is zero,

$a \propto -x$

ie  $v=0$

time =  $0.10$  s and time =  $0.30$  s [1]

2. the acceleration in one direction is maximum.

time =  $0.40$  s and time =  $0.8$  s [1]

(b) The oscillator is now switched on. The amplitude of vibration of the oscillator is constant. The frequency  $f$  of vibration of the oscillator is varied. The trolley is forced to oscillate by means of vibrations of the oscillator. The variation with  $f$  of the amplitude  $a_0$  of the oscillations of the trolley is shown in Fig. 2.3.

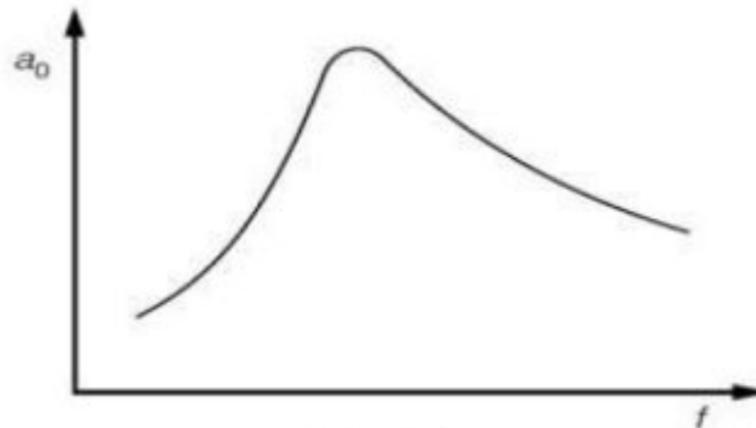


Fig. 2.3

By reference to your answer in (a), state the approximate frequency at which the amplitude is maximum.

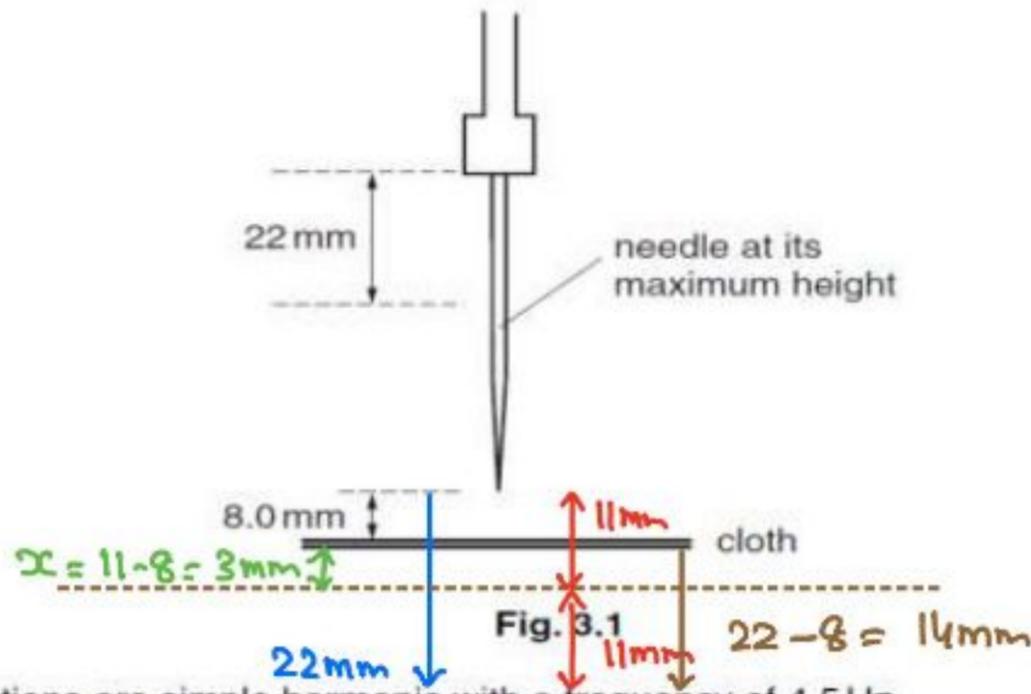
At Resonance,  $f_0 = f$  frequency =  $\frac{1}{T} = \frac{1}{0.40}$  =  $2.5$  Hz [1]

(c) The amplitude of the oscillations in (b) may be reduced without changing significantly the frequency at which the amplitude is a maximum. State how this may be done and give a reason for your answer. You may draw on Fig. 2.1 if you wish.

(i) Either increase the surface area of trolley perpendicular to the plane of motion or place the assembly in a viscous fluid to increase damping (loss of energy) [2]

(ii) [with out changing mass of trolley and spring] [2]

Q.4 The needle of a sewing machine is made to oscillate vertically through a total distance of 22 mm, as shown in Fig. 3.1.



The oscillations are simple harmonic with a frequency of 4.5 Hz. The cloth that is being sewn is positioned 8.0 mm below the point of the needle when the needle is at its maximum height.

(a) State what is meant by *simple harmonic motion*.

The periodic motion in which acceleration is directly proportional to displacement and is opposite to the direction of displacement. [2]

(b) The displacement  $y$  of the point of the needle may be represented by the equation

$$y = a \cos \omega t.$$

(i) Suggest the position of the point of the needle at time  $t = 0$ .

At highest point or 8.0 mm above cloth or 14 mm below cloth [1]

(ii) Determine the values of

1.  $a$ ,

$a = 11$  mm [1]

2.  $\omega$ .

$$\begin{aligned} \omega &= 2\pi f \\ &= 2(3.14)(4.5) \end{aligned}$$

$\omega = 28.3$  rads<sup>-1</sup> [2]

(c) Calculate, for the point of the needle,

(i) its maximum speed,

Max. speed at mean position where displacement = 0

$$v = \omega x_0$$

$$= (28.3)(11 \times 10^{-3})$$

speed = 0.031 ms<sup>-1</sup> [2]

(ii) its speed as it moves downwards through the cloth.

$$v = \omega \sqrt{x_0^2 - x^2}$$

$$v = (28.3) \sqrt{(11 \times 10^{-3})^2 - [(11-8) \times 10^{-3}]^2}$$

$$v = \underline{\hspace{2cm}} \text{ ms}^{-1}$$

speed = ..... ms<sup>-1</sup> [3]

{9702/04/O/N/08}

Q.5

A piston moves vertically up and down in a cylinder, as illustrated in Fig. 4.1.

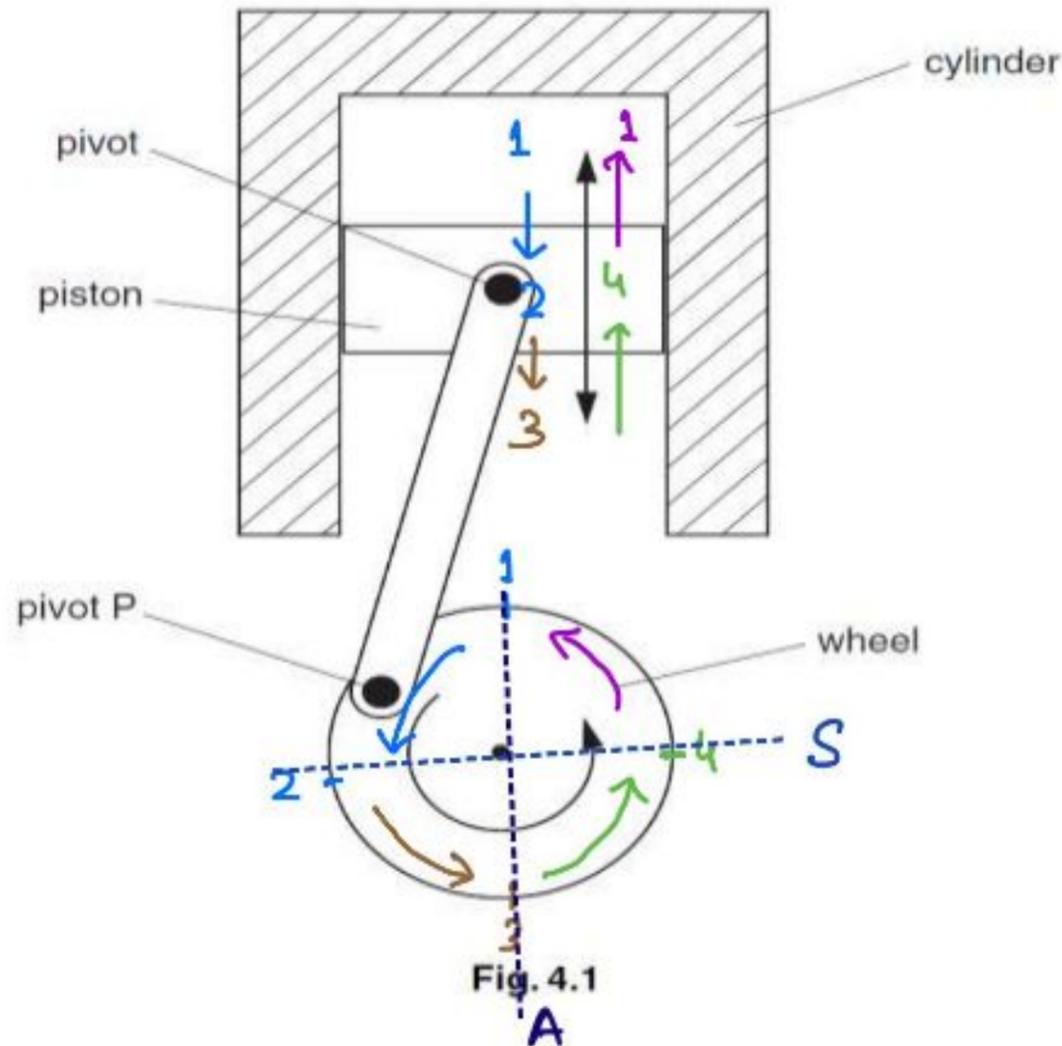


Fig. 4.1

A

The piston is connected to a wheel by means of a rod that is pivoted at the piston and at the wheel. As the piston moves up and down, the wheel is made to rotate.

- (a) (i) State the number of oscillations made by the piston during one complete rotation of the wheel.

number = ..... 1 ..... [1]

- (ii) The wheel makes 2400 revolutions per minute. Determine the frequency of oscillation of the piston.

$$f = \frac{n}{t}$$

$$= \frac{2400}{60}$$

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frequency = ..... 40 ..... Hz [1]

- (b) The amplitude of the oscillations of the piston is 42 mm.

Assuming that these oscillations are simple harmonic, calculate the maximum values for the piston of

- (i) the linear speed,

$$v_{\max} = \omega x_0 \Rightarrow v_{\max} = (2\pi f)(x_0)$$

$$= 2(3.14)(40)(42 \times 10^{-3})$$

speed = ..... 10.6 .....  $\text{ms}^{-1}$  [2]

- (ii) the acceleration.

$$a_{\max} = -\omega^2 x_0 \Rightarrow a = (2 \times 3.14 \times 40)^2 (42 \times 10^{-3})$$

acceleration = .....  $2.65 \times 10^3$  .....  $\text{ms}^{-2}$  [2]

- (c) On Fig. 4.1, mark a position of the pivot P for the piston to have

- (i) maximum speed (mark this position S), [1]  
 (ii) maximum acceleration (mark this position A). [1]

Q.6

- (a) State what is meant by *simple harmonic motion*.

{9702/04/M/J/06}

[2]

The periodic motion in which acceleration is directly proportional to displacement and is opposite to the direction of displacement.

(b) The variation with time  $t$  of the displacement  $x$  of two oscillators P and Q is shown in Fig. 4.1.

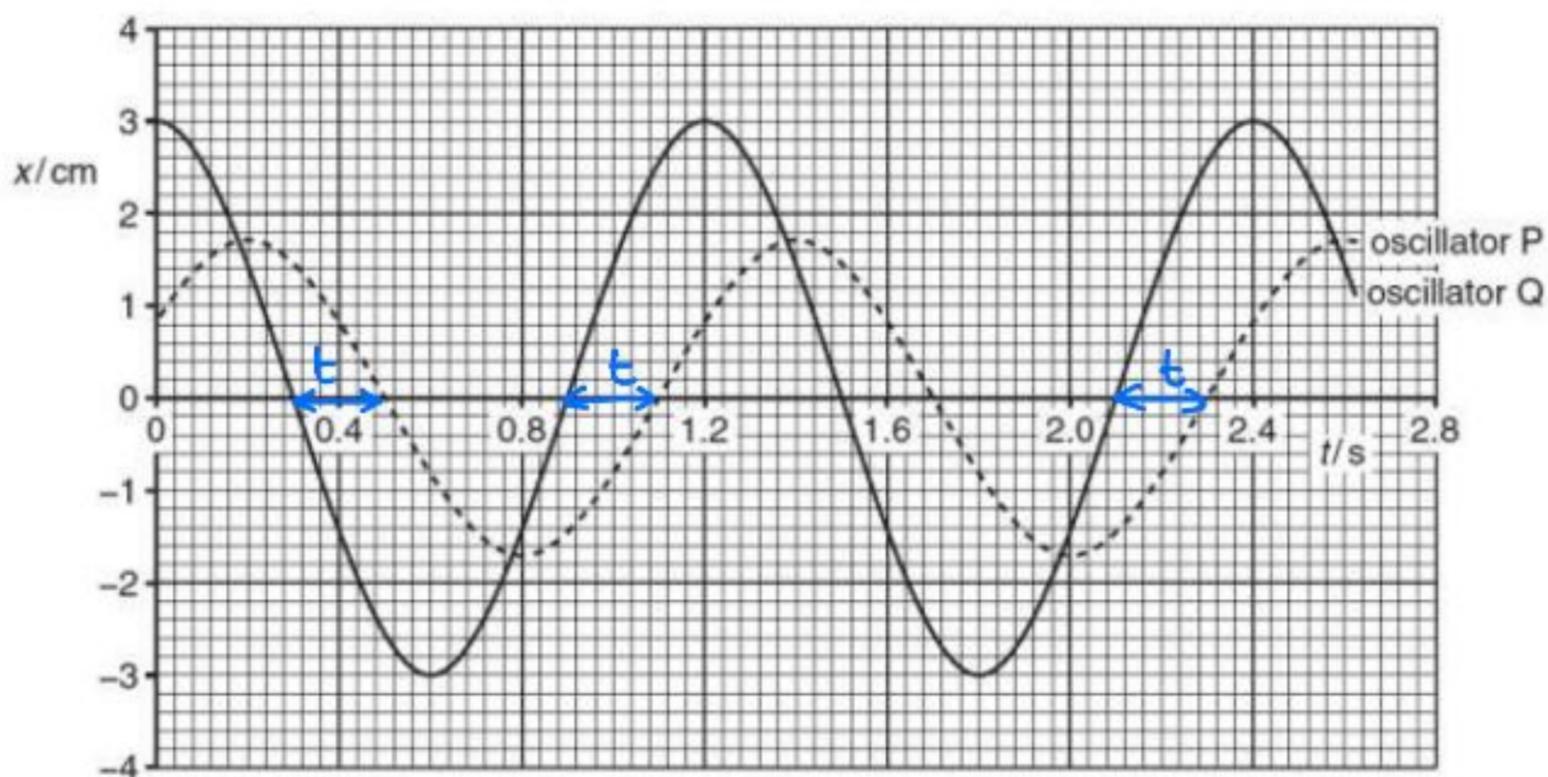


Fig. 4.1

The two oscillators each have the same mass.

Use Fig. 4.1 to determine

$$\phi = \left(\frac{t}{T}\right) 2\pi = \left(\frac{0.2}{1.2}\right) 2\pi$$

(i) the phase difference between the two oscillators,

phase difference = ..... 1.05 ..... rad [1]

(ii) the maximum acceleration of oscillator Q,

$$\begin{aligned} a_{\max} &= -\omega^2 x_0 \\ &= -\left(\frac{2\pi}{T}\right)^2 x_0 \\ &= \frac{4(3.14)^2 (3.0 \times 10^{-2})}{1.2} \end{aligned}$$

maximum acceleration = ..... 0.822 .....  $\text{ms}^{-2}$  [2]

(iii) the ratio

maximum kinetic energy of oscillations of Q  
maximum kinetic energy of oscillations of P

$$\frac{K_Q}{K_P} = \frac{\frac{1}{2} m \omega^2 (x_0^2)_Q}{\frac{1}{2} m \omega^2 (x_0^2)_P} = \frac{(3.0)^2}{(1.7)^2}$$

ratio = ..... 3.11 ..... [2]

(c) Use data from (b) to sketch, on the axes of Fig. 4.2, the variation with displacement  $x$  of the acceleration  $a$  of oscillator Q.

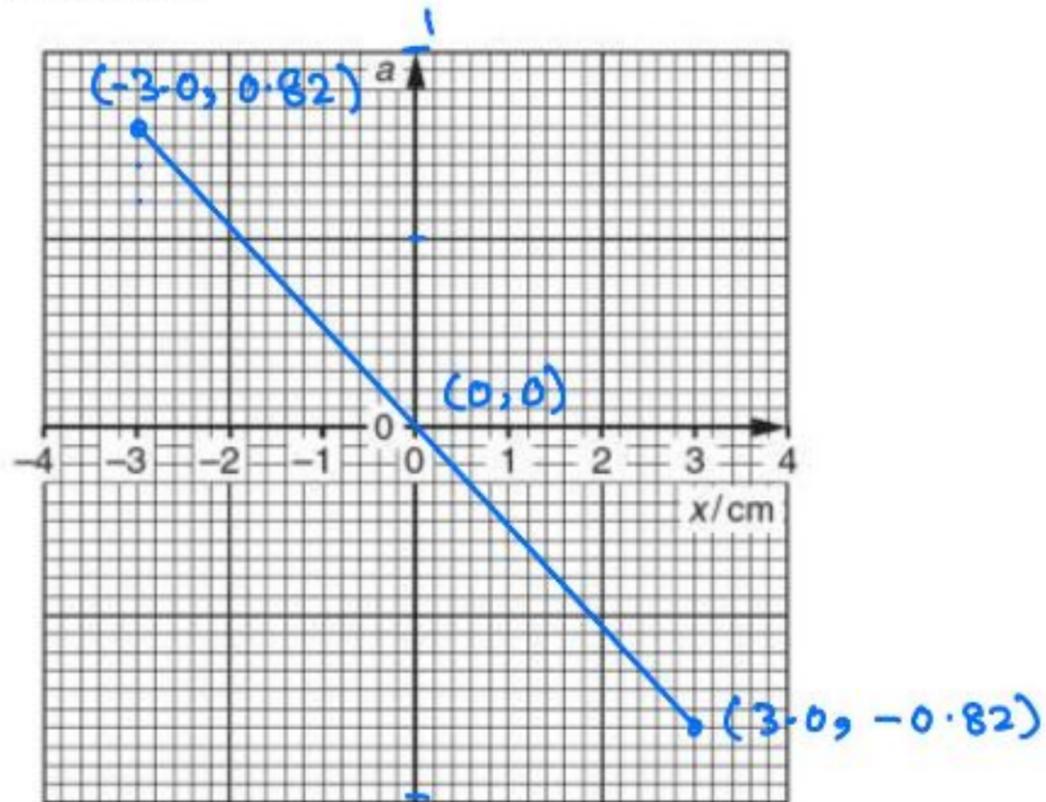


Fig. 4.2

[2]

{9702/42/M/J/2015}

J-20/41

- 3 The piston in the cylinder of a car engine moves in the cylinder with simple harmonic motion. The piston moves between a position of maximum height in the cylinder to a position of minimum height, as illustrated in Fig. 3.1.

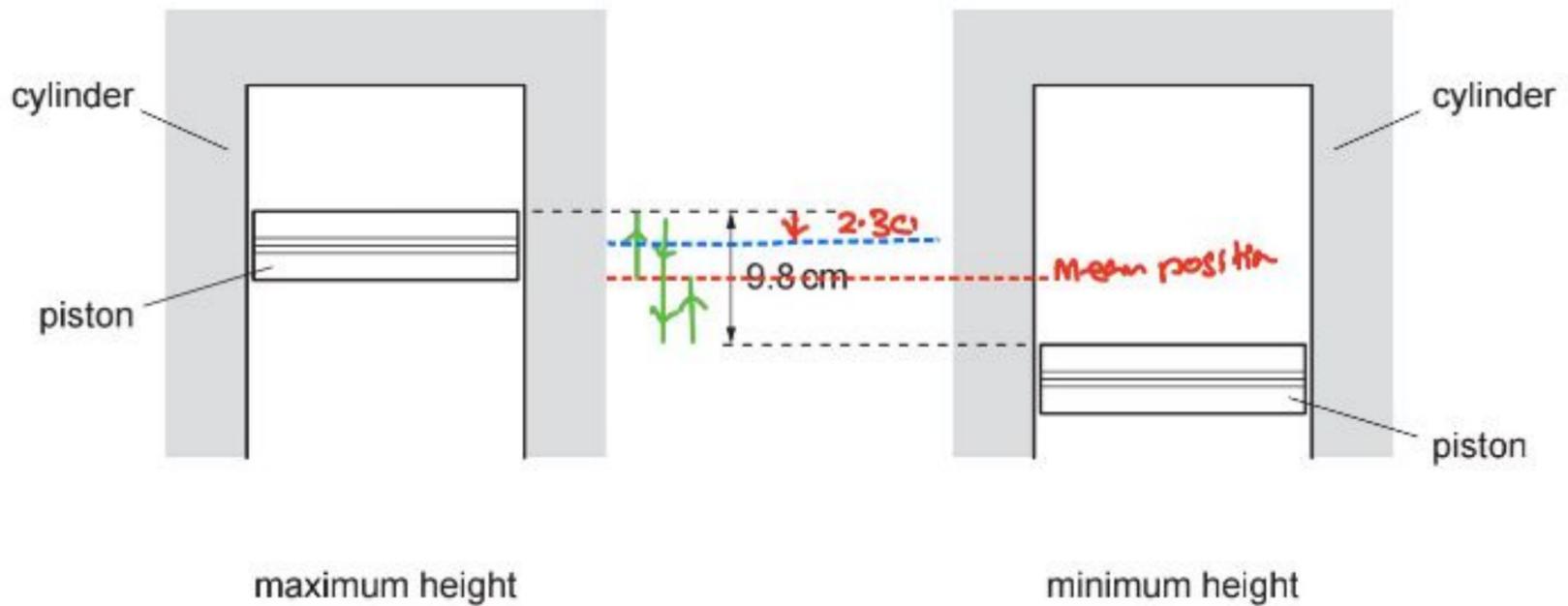


Fig. 3.1

The distance moved by the piston between the positions shown in Fig. 3.1 is 9.8 cm.

The mass of the piston is 640 g.

At one particular speed of the engine, the piston completes 2700 oscillations in 1.0 minute.

(a) For the oscillations of the piston in the cylinder, determine:

(i) the amplitude  $\frac{9.8}{2}$  amplitude = ..... 4.9 ..... cm [1]

(ii) the frequency  $f = \frac{n}{t} = \frac{2700}{60}$  frequency = ..... 45 ..... Hz [1]

(iii) the maximum speed  $V = \omega x_0 = (2\pi f)(x_0) = 2(3.14)(45)(4.9 \times 10^{-2}) = 13.85$  maximum speed = ..... 13.9 .....  $\text{ms}^{-1}$  [2]

- (iv) the speed when the top of the piston is 2.3 cm below its maximum height.

$$\begin{aligned}
 v &= \omega \sqrt{x_0^2 - x^2} \\
 &= (2\pi f) \sqrt{x_0^2 - x^2} \\
 &= 2(3.14)(45) \sqrt{(4.9 \times 10^{-2})^2 - [(4.9 - 2.3)10^{-2}]^2} \\
 v &= 11.74
 \end{aligned}$$

speed = ..... 11.7 .....  $\text{ms}^{-1}$  [2]

- (b) The acceleration of the piston varies.  $a \propto -x$

Determine the resultant force on the piston that gives rise to its maximum acceleration.

Max acceleration and hence force is at amplitude position

$$\begin{aligned}
 F &= ma_0 \\
 &= m(-\omega^2 x_0) \\
 F &= -m(2\pi f)^2(x_0) \\
 &= -(640 \times 10^{-3})(4)(3.14)^2(45)^2(4.9 \times 10^{-2}) \\
 F &= -2507
 \end{aligned}$$

force = .....  $-2.51 \times 10^3$  ..... N [3]

[Total: 9]

June 20/42

4 A dish is made from a section of a hollow glass sphere.

The dish, fixed to a horizontal table, contains a small solid ball of mass 45 g, as shown in Fig. 4.1.

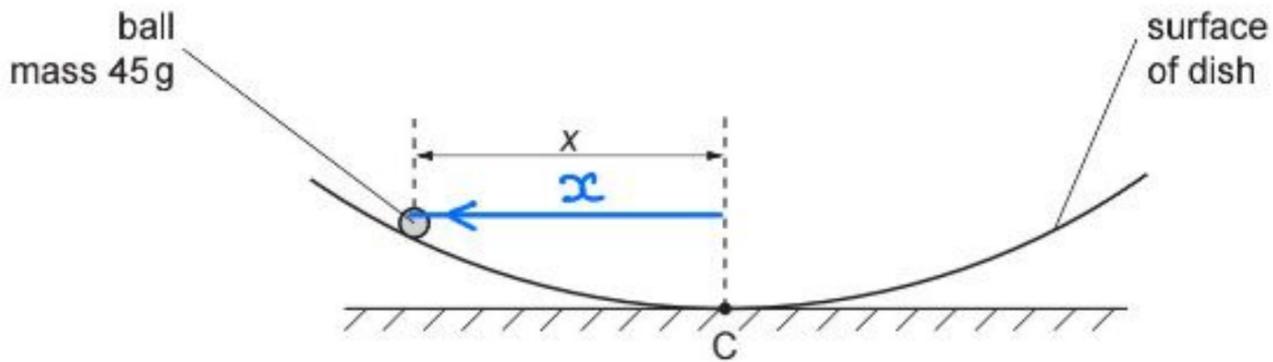


Fig. 4.1

The horizontal displacement of the ball from the centre C of the dish is x.

Initially, the ball is held at rest with distance  $x = 3.0$  cm.

The ball is then released. The variation with time  $t$  of the horizontal displacement  $x$  of the ball from point C is shown in Fig. 4.2.

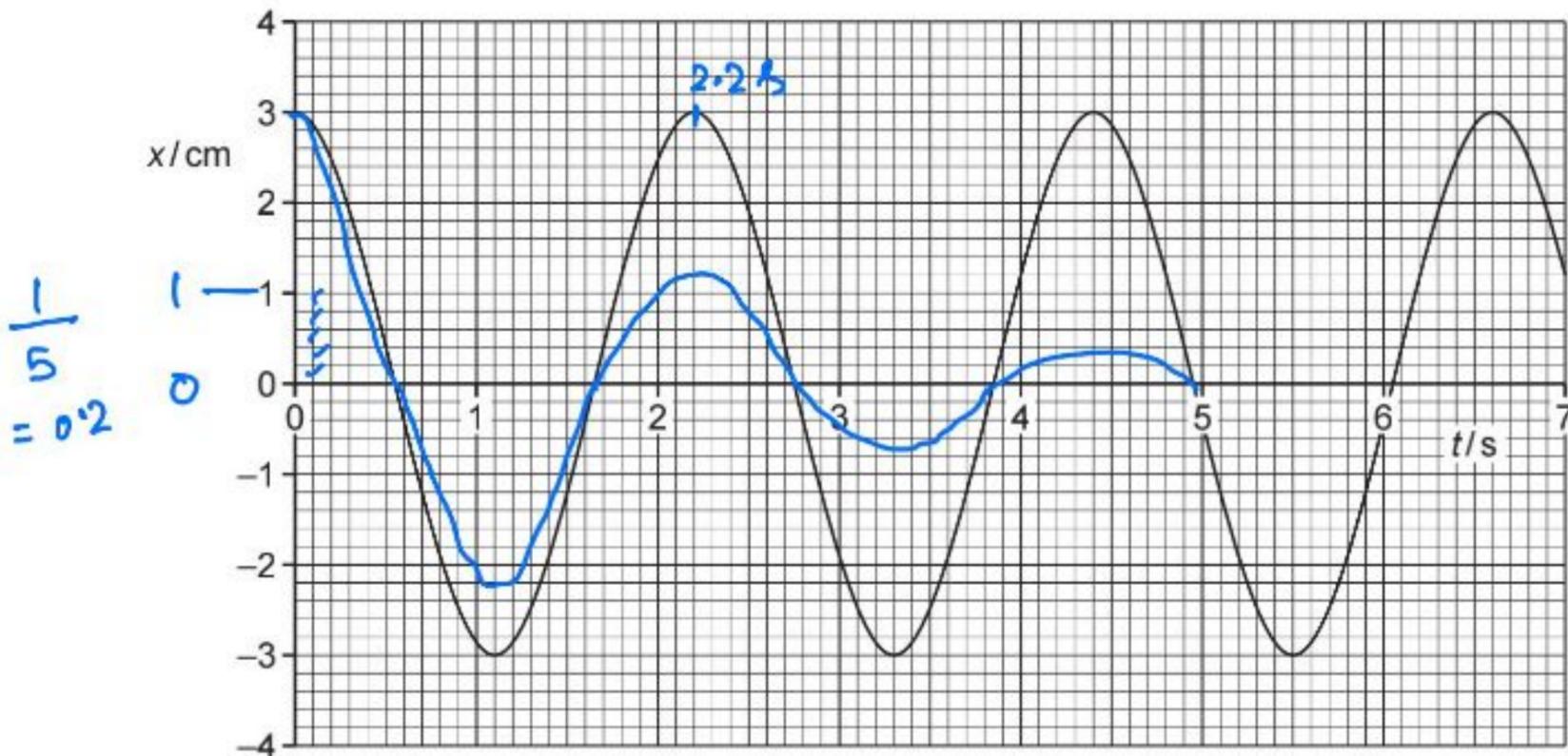


Fig. 4.2

The motion of the ball in the dish is simple harmonic with its acceleration  $a$  given by the expression

$$a = -\left(\frac{g}{R}\right)x$$

where  $g$  is the acceleration of free fall and  $R$  is a constant that depends on the dimensions of the dish and the ball.

- (a) Use Fig. 4.2 to show that the angular frequency  $\omega$  of oscillation of the ball in the dish is  $2.9 \text{ rad s}^{-1}$ .

$$\omega = \frac{2\pi}{T} = \frac{2(3.14)}{2.2}$$

$$\omega = 2.85$$

$$= 2.9 \text{ rad s}^{-1}$$

[1]

- (b) Use the information in (a) to:

- (i) determine  $R$       Since  $a = -\omega^2 x$

By comparison

$$\omega^2 = \frac{g}{R}$$

$$R = \frac{g}{\omega^2} = \frac{9.81}{(2.9)^2} = 1.17$$

$$R = \dots\dots\dots 1.17 \dots\dots\dots \text{ m [2]}$$

- (ii) calculate the speed of the ball as it passes over the centre C of the dish.

Speed is maximum at C

$$v_0 = \omega x_0$$

$$= (2.9)(3.0 \times 10^{-2})$$

$$= 0.087$$

$$\text{speed} = \dots\dots\dots 0.087 \dots\dots\dots \text{ ms}^{-1} [2]$$

- (c) Some moisture collects on the surface of the dish so that the motion of the ball becomes lightly damped.

On the axes of Fig. 4.2, draw a line to show the lightly damped motion of the ball for the first 5.0 s after the release of the ball. [3]

[Total: 8]

Nov 20/41

- 3 A pendulum consists of a metal sphere P suspended from a fixed point by means of a thread, as illustrated in Fig. 3.1.

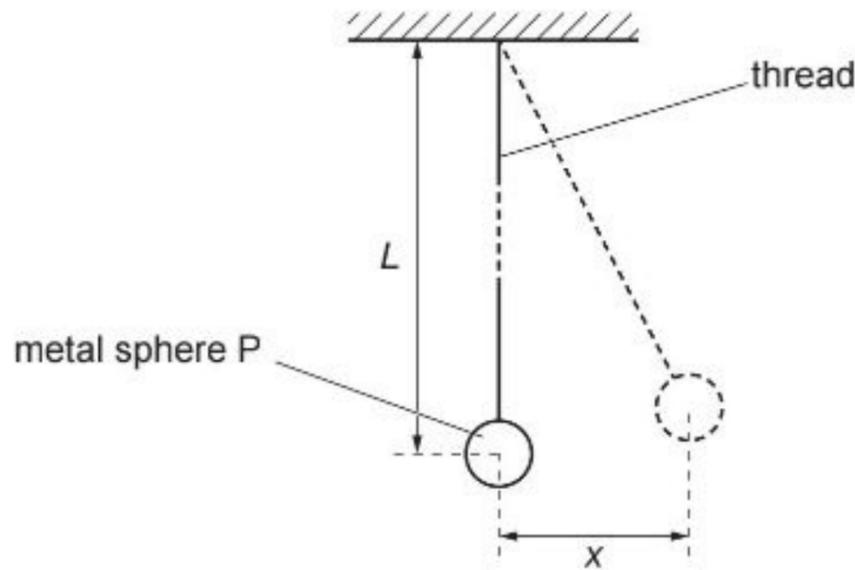


Fig. 3.1

The centre of gravity of sphere P is a distance  $L$  from the fixed point.

The sphere is pulled to one side and then released so that it oscillates. The sphere may be assumed to oscillate with simple harmonic motion.

- (a) State what is meant by *simple harmonic motion*.

The periodic motion in which acceleration is directly proportional to displacement and is directed towards mean position/or opposite to displacement. [2]

- (b) The variation of the velocity  $v$  of sphere P with the displacement  $x$  from its mean position is shown in Fig. 3.2.

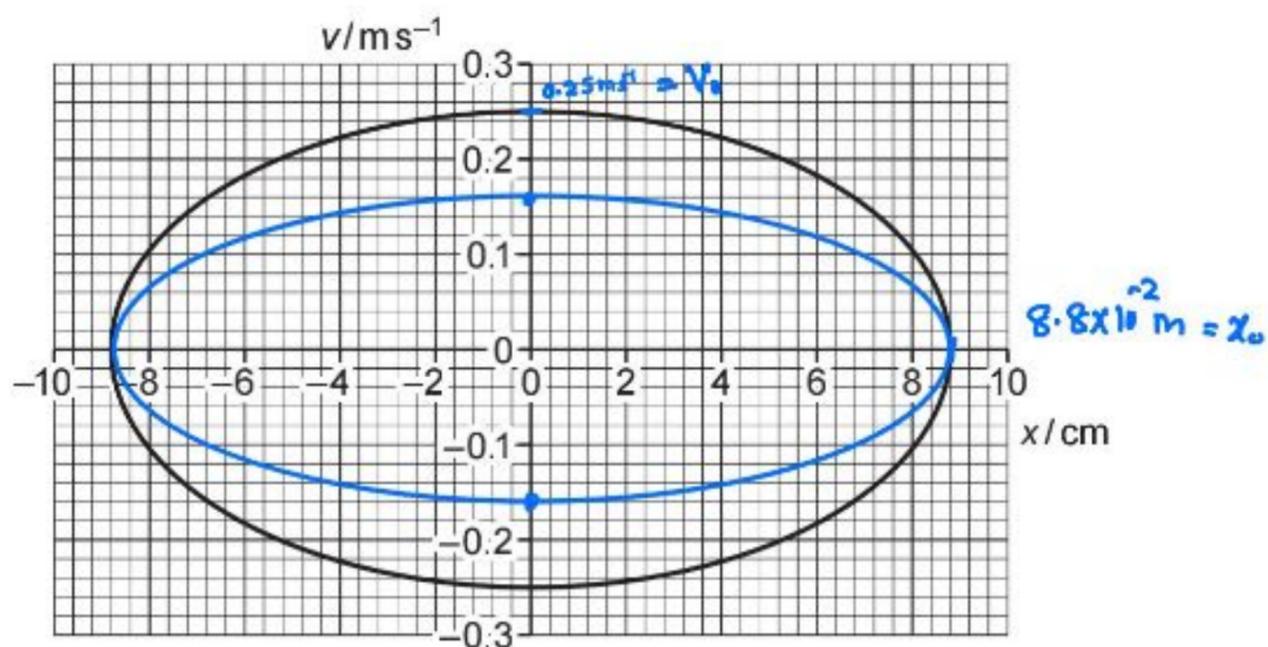


Fig. 3.2

Use Fig. 3.2 to determine the frequency  $f$  of the oscillations of sphere P.

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

At mean position,  $x = 0$ ,  $v = 0.25 \text{ m s}^{-1}$

$$v = (2\pi f) (\sqrt{x_0^2 - 0^2})$$

$$v = (2\pi f)(x_0)$$

$$0.25 = 2(3.14)(f)(8.8 \times 10^{-2})$$

$$f = 0.45 \text{ Hz} \quad f = \dots\dots\dots 0.45 \dots\dots\dots \text{ Hz [3]}$$

(c) The period  $T$  of the oscillations of sphere P is given by the expression

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where  $g$  is the acceleration of free fall.

Use your answer in (b) to determine the length  $L$ .

$$\frac{1}{f} = 2\pi \sqrt{\frac{L}{g}}$$

$$\left(\frac{1}{0.45}\right)^2 = 4(3.14)^2 \left(\frac{L}{9.81}\right)$$

$$L = 1.23$$

$$L = \dots\dots\dots 1.23 \dots\dots\dots \text{ m [2]}$$

(d) Another pendulum consists of a sphere Q suspended by a thread. Spheres P and Q are identical. The thread attached to sphere Q is longer than the thread attached to sphere P.

Sphere Q is displaced and then released. The oscillations of sphere Q have the same amplitude as the oscillations of sphere P.

On Fig. 3.2, sketch the variation of the velocity  $v$  with displacement  $x$  for sphere Q. [2]

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$L_Q > L_P$$

$$T_Q > T_P$$

$$f_Q < f_P$$

$$\omega_Q < \omega_P$$

$$v = \pm \omega \sqrt{x_0^2 - x^2}$$

So

$$v_Q < v_P$$

[Total: 9]

Nov. 20/42

- 3 A simple pendulum consists of a metal sphere suspended from a fixed point by means of a thread, as illustrated in Fig. 3.1.

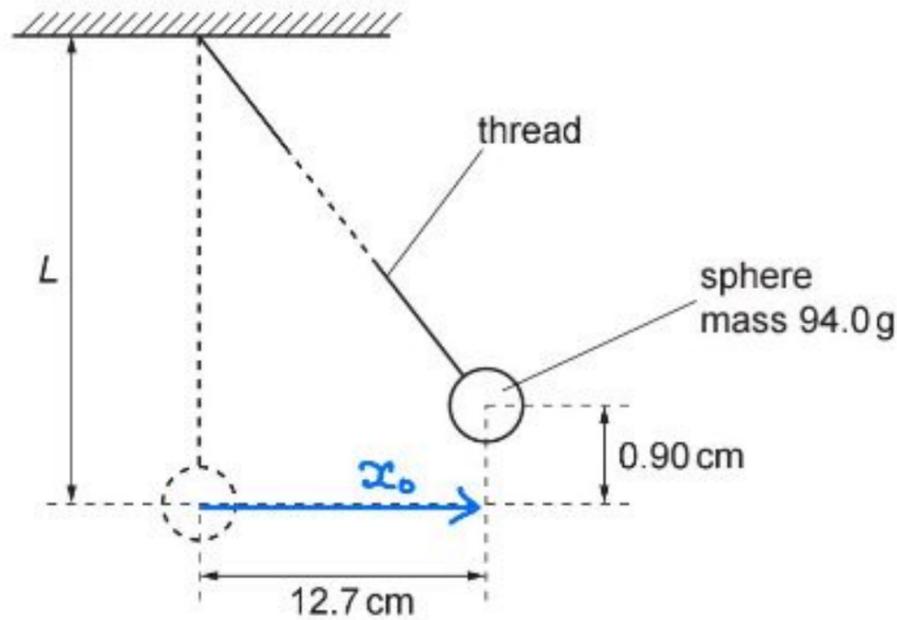


Fig. 3.1 (not to scale)

The sphere of mass 94.0g is displaced to one side through a horizontal distance of 12.7 cm. The centre of gravity of the sphere rises vertically by 0.90 cm.

The sphere is released so that it oscillates. The sphere may be assumed to oscillate with simple harmonic motion.

- (a) State what is meant by *simple harmonic motion*.

.....

.....

..... [2]

- (b) (i) State the kinetic energy of the sphere when the sphere returns to the displaced position shown in Fig. 3.1.

Speed is zero at amplitude position

kinetic energy = ..... 0 ..... J [1]

- (ii) Calculate the total energy  $E_T$  of the oscillations.

$$E_T = E_k + E_p$$

At amplitude position,  $E_k = 0$ ,  $E_p = \text{Max}$

$$E_T = E_p = mgh$$

$$E_T = (94.0 \times 10^{-3})(9.81)(0.90 \times 10^{-2})$$

$$= 8.29 \times 10^{-3} \text{ J}$$

$E_T = \dots\dots\dots$  J [2]

- (iii) Use your answer in (ii) to show that the angular frequency  $\omega$  of the oscillations of the pendulum is  $3.3 \text{ rad s}^{-1}$ .

$$E_T = \frac{1}{2} m \omega^2 x_0^2$$

$$8.29 \times 10^{-3} = \frac{1}{2} (94.0 \times 10^{-3}) (\omega^2) (12.7 \times 10^{-2})^2$$

$$\omega = 3.3 \text{ rad s}^{-1}$$

[2]

- (c) The period  $T$  of oscillation of the pendulum is given by the expression

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where  $g$  is the acceleration of free fall and  $L$  is the length of the pendulum.

Use data from (b) to determine  $L$ . Since  $T = \frac{2\pi}{\omega}$

So,

$$\frac{2\pi}{\omega} = 2\pi \sqrt{\frac{L}{g}}$$

$$\omega^2 = \frac{g}{L} \Rightarrow L = \frac{g}{\omega^2} = \frac{9.81}{(3.3)^2}$$

$$L = 0.895 \text{ m}$$

$L = \dots\dots\dots \text{ m}$  [3]

[Total: 10]

June 21/42

3 A U-shaped tube contains some liquid. The liquid column in each half of the tube has length  $L$ , as shown in Fig. 3.1.

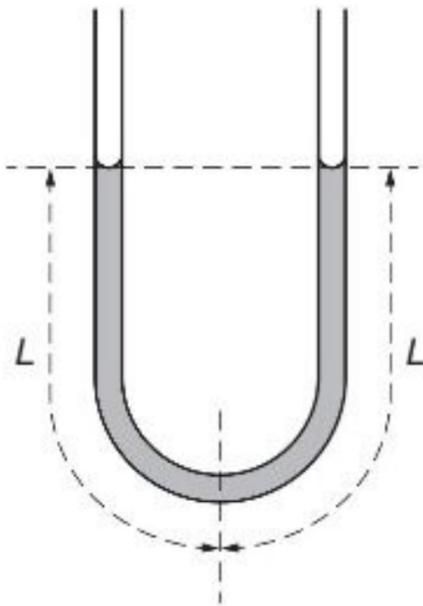


Fig. 3.1

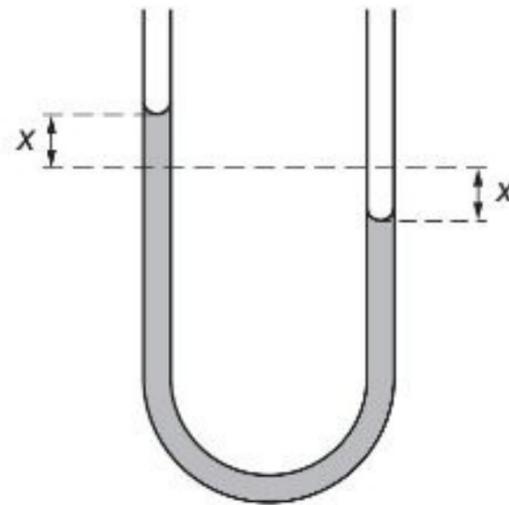


Fig. 3.2

The liquid columns are displaced vertically. The liquid then oscillates in the tube. The liquid levels are displaced from the equilibrium positions as shown in Fig. 3.2.

The acceleration  $a$  of the liquid in the tube is related to the displacement  $x$  by the expression

$$a = -\left(\frac{g}{L}\right)x$$

where  $g$  is the acceleration of free fall.

(a) Explain how the expression shows that the liquid in the tube is undergoing simple harmonic motion.

In S.H.M  $a = -\omega^2 x$

→ Here  $\omega^2 = \frac{g}{L} = \frac{9.81}{\text{Constant}} = \text{Constant}$

→ Acceleration is directly proportional to displacement.

→ Acceleration is in opposite direction to displacement due to -ve sign. [3]

(b) The length  $L$  of each liquid column is 18 cm.

Determine the period  $T$  of the oscillations.

$$\omega^2 = \frac{g}{L}$$
$$\frac{2\pi}{T} = \sqrt{\frac{g}{L}} \Rightarrow T = \frac{1}{2(3.14)} \sqrt{\frac{9.81}{18 \times 10^{-2}}}$$

T =

T = ..... s [3]

- (c) The oscillations of the liquid in the tube are damped.  
In any one complete cycle of the oscillations, the amplitude decreases by 6.0% of its value at the beginning of the oscillation.

Determine the ratio

$$\frac{\text{energy of oscillations after 3 cycles}}{\text{initial energy of oscillations}}$$

$$E_T = \frac{1}{2} m \omega^2 x_0^2$$

$$E_T = (\text{constant}) x_0^2$$

$$E_T \propto x_0^2$$

Let initial amplitude =  $x_0$

ratio = ..... [3]

After one oscillation, find amplitude =  $\left(\frac{100-6}{100}\right) x_0 = 0.94 x_0$  [Total: 9]

After 2<sup>nd</sup> oscillation =  $(0.94)(0.94 x_0)$

After 3<sup>rd</sup> oscillation =  $(0.94) [0.94(0.94 x_0)]$   
 $= (0.94)^3 x_0$

$$\frac{E_{3 \text{ cycl}}}{E_{\text{beginning}}} = \frac{[(0.94)^3 x_0]^2}{x_0^2} = \frac{[(0.94)^3]^2 x_0^2}{x_0^2}$$

$$= [(0.94)^3]^2$$

$$= (0.94)^6$$

$$= 0.69$$

June 21/43

3 (a) State what is meant by *simple harmonic motion*.

The periodic motion in which acceleration is directly proportional to displacement and is directed towards mean position/opposite to displacement. [2]

(b) A trolley of mass  $m$  is held on a horizontal surface by means of two springs. One spring is attached to a fixed point P. The other spring is connected to an oscillator, as shown in Fig. 3.1.

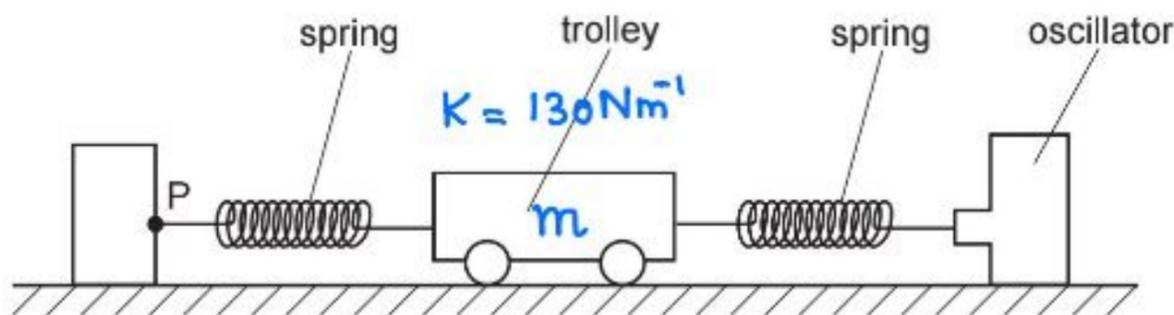


Fig. 3.1

The springs, each having spring constant  $k$  of  $130 \text{ N m}^{-1}$ , are always extended.

The oscillator is switched off. The trolley is displaced along the line of the springs and then released. The resulting oscillations of the trolley are simple harmonic.

The acceleration  $a$  of the trolley is given by the expression

$$a = -\left(\frac{2k}{m}\right)x$$

where  $x$  is the displacement of the trolley from its equilibrium position.

The mass of the trolley is 840 g.

Calculate the frequency  $f$  of oscillation of the trolley.

By comparison,  $a = -\omega^2 x$

$$\omega^2 = \frac{2k}{m}$$
$$2\pi f = \sqrt{\frac{2k}{m}} \Rightarrow f = \frac{1}{2\pi} \sqrt{\frac{2k}{m}}$$
$$f = \frac{1}{2(3.14)} \sqrt{\frac{2(130)}{840 \times 10^{-3}}} = \text{..... Hz}$$

$f = \text{..... Hz}$  [3]

- (c) The oscillator in (b) is switched on. The frequency of oscillation of the oscillator is varied, keeping its amplitude of oscillation constant.

The amplitude of oscillation of the trolley is seen to vary. The amplitude is a maximum at the frequency calculated in (b).

- (i) State the name of the effect giving rise to this maximum.

Resonance ..... [1]

- (ii) At any given frequency, the amplitude of oscillation of the trolley is constant.

Explain how this indicates that there are resistive forces opposing the motion of the trolley.

Amplitude would increase as oscillator supplies energy and if amplitudes remain constant then energy must be dissipated in doing work against frictional forces. [2]

[Total: 8]

Nov. 19/41

4 A mass is suspended vertically from a fixed point by means of a spring, as illustrated in Fig. 4.1.

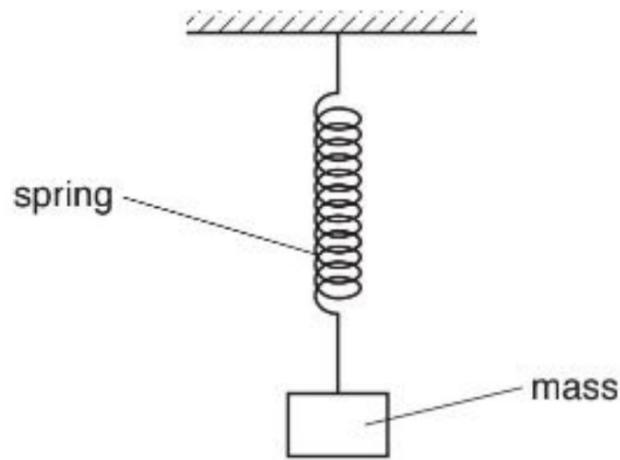


Fig. 4.1

The mass is oscillating vertically. The variation with displacement  $x$  of the acceleration  $a$  of the mass is shown in Fig. 4.2.

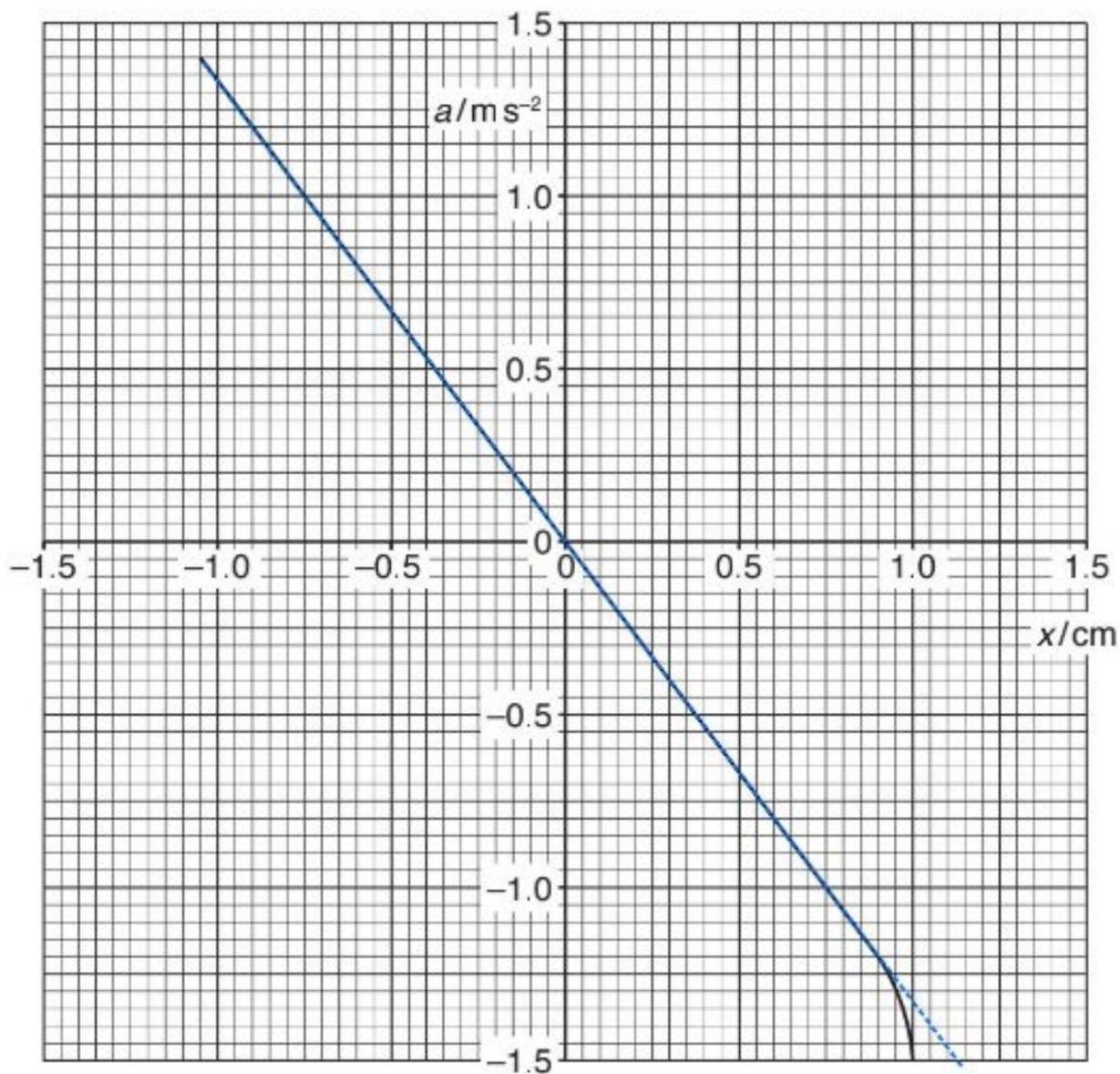


Fig. 4.2

(a) (i) State what is meant by the *displacement* of the mass on the spring.

Straight directed distance of vibrating mass from its equilibrium position. [1]

For SH.M  
 $a = -\omega^2 x$   
 $\frac{a}{x} = \text{constant}$   
 gradient = constant

(ii) Suggest how Fig. 4.2 shows that the mass is not performing simple harmonic motion.  
 Gradient of graph is not constant due to curve after  $x = 0.90 \text{ cm}$ . [1]

(b) (i) The amplitude of oscillation of the mass may be changed.

State the maximum amplitude  $x_0$  for which the oscillations are simple harmonic. *line must be straight*

$x_0 = 0.90 \text{ cm}$  [1]

(ii) For the simple harmonic oscillations of the mass, use Fig. 4.2 to determine the frequency of the oscillations.

$a = -\omega^2 x$   
 $a = -(2\pi f)^2 x \Rightarrow f^2 = -\frac{1}{4\pi^2} \left(\frac{a}{x}\right)$   
 $f = \frac{1}{2\pi} \sqrt{\text{Gradient of st. line}}$   
 $= \frac{1}{2(3.14)} \sqrt{\frac{-1.0 - 1.0}{(0.75 - (-0.75)) \times 10^{-2}}} \Rightarrow f = 1.838$

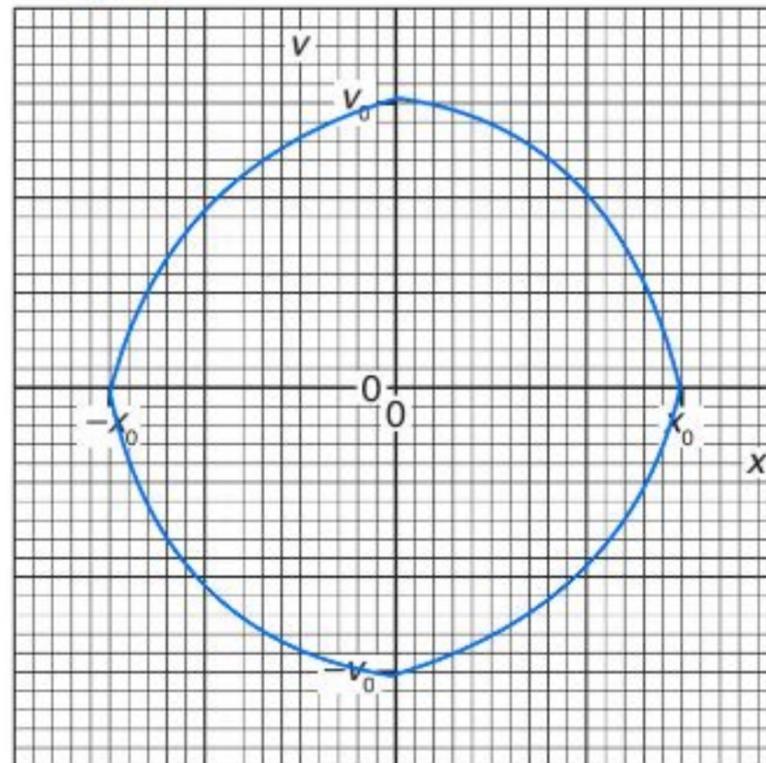
frequency = 1.84 Hz [3]

*-ve sign of gradient is neglected*

(c) The maximum speed of the mass when oscillating with simple harmonic motion of amplitude  $x_0$  is  $v_0$ .

On Fig. 4.3, show the variation with displacement  $x$  of the velocity  $v$  of the mass for displacements from  $+x_0$  to  $-x_0$ .

$v = 0, 0$



$v = \pm \omega \sqrt{x_0^2 - x^2}$

Fig. 4.3

[2]

[Total: 8]

Nov-19/42

- 4 A ball of mass  $M$  is held on a horizontal surface by two identical extended springs, as illustrated in Fig. 4.1.

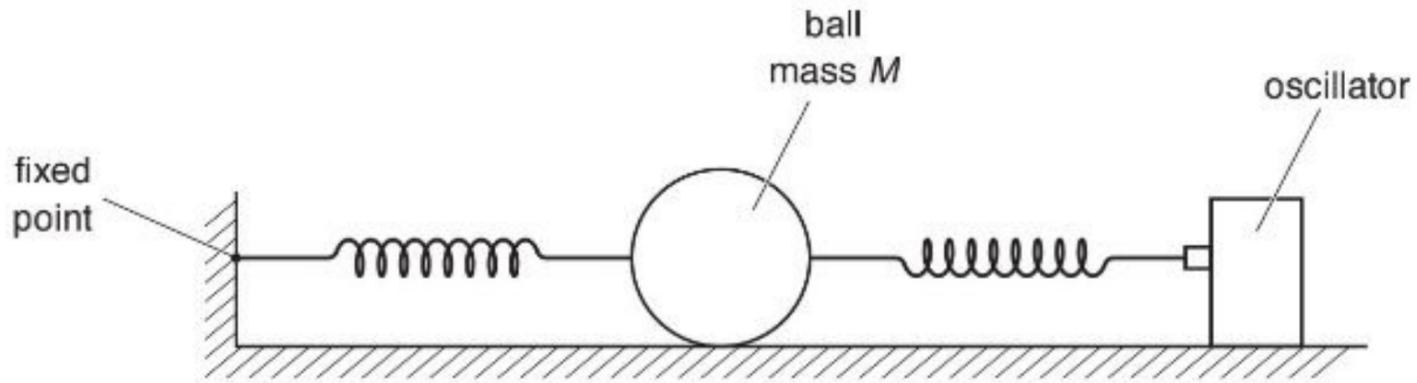


Fig. 4.1

One spring is attached to a fixed point. The other spring is attached to an oscillator.

The oscillator is switched off. The ball is displaced sideways along the axis of the springs and is then released. The variation with time  $t$  of the displacement  $x$  of the ball is shown in Fig. 4.2.

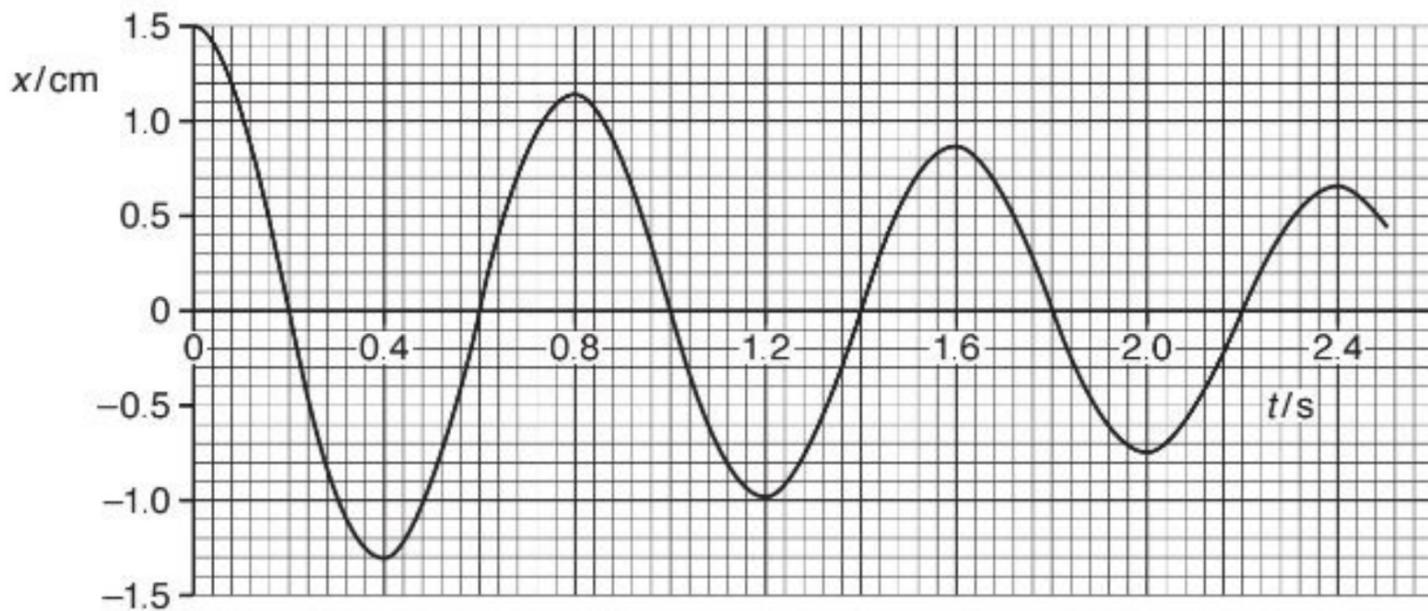


Fig. 4.2

(a) State:

- (i) what is meant by *damping*

Decrease in amplitude of an oscillating body due to loss of energy in doing work against frictional forces. [1]

- (ii) the evidence provided by Fig. 4.2 that the motion of the ball is damped.

Amplitude of oscillating body decreases with time. [1]

- (b) The acceleration  $a$  and the displacement  $x$  of the ball are related by the expression

$$a = -\left(\frac{2k}{M}\right)x$$

where  $k$  is the spring constant of one of the springs.

The mass  $M$  of the ball is 1.2 kg.

- (i) Use data from Fig. 4.2 to determine the angular frequency  $\omega$  of the oscillations of the ball.

$$\begin{aligned}\omega &= \frac{2\pi}{T} \\ &= \frac{2(3.14)}{0.80} \\ &= 7.85\end{aligned}$$

$$\omega = \dots\dots\dots \text{rads}^{-1} \quad [2]$$

- (ii) Use your answer in (i) to determine the value of  $k$ .

In S.H.M,  $a = -\omega^2 x$   
 So by comparison  
 $\omega^2 = \frac{2k}{M}$   
 $(7.85)^2 = \frac{2k}{1.2}$

$$k = 36.97 \quad k = \dots\dots\dots 37 \dots\dots\dots \text{Nm}^{-1} \quad [2]$$

- (c) The oscillator is switched on. The amplitude of oscillation of the oscillator is constant.

The angular frequency of the oscillations is gradually increased from  $0.7\omega$  to  $1.3\omega$ , where  $\omega$  is the angular frequency calculated in (b)(i).

- (i) On the axes of Fig. 4.3, show the variation with angular frequency of the amplitude  $A$  of oscillation of the ball.

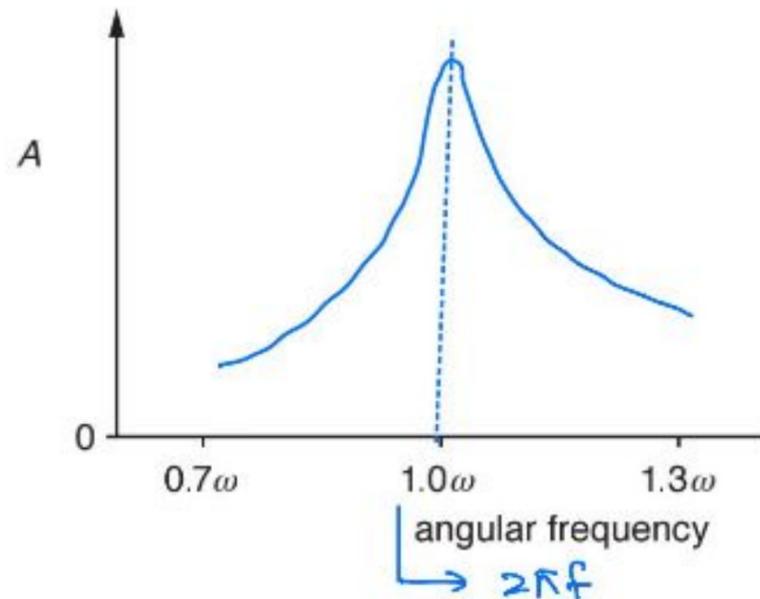


Fig. 4.3  $f = f_0$   
 $\omega = \omega_0$

[2]

- (ii) Some sand is now sprinkled on the horizontal surface.

The angular frequency of the oscillations is again gradually increased from  $0.7\omega$  to  $1.3\omega$ .

State **two** changes that occur to the line you have drawn on Fig. 4.3.

1. The new graph has lesser amplitude and is below the previous graph except at  $0.7\omega$ .
2. Peak is flatter due to increase in the degree of damping.

[2]

[Total: 10]

Nov. 18/42

4 A U-tube contains liquid, as shown in Fig. 4.1.

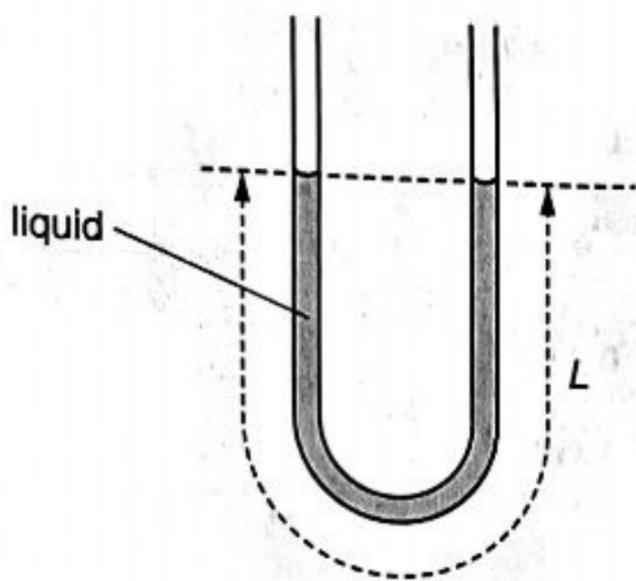


Fig. 4.1

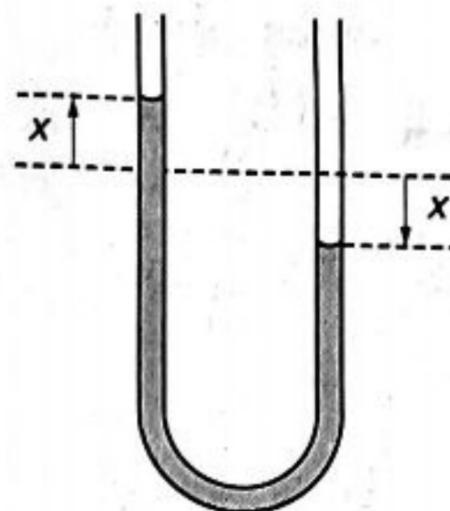


Fig. 4.2

The total length of the liquid column is  $L$ .

The column of liquid is displaced so that the change in height of the liquid level from the equilibrium position in each arm of the U-tube is  $x$ , as shown in Fig. 4.2.

The liquid in the U-tube then oscillates such that its acceleration  $a$  is given by the expression

$$a = -\left(\frac{2g}{L}\right)x$$

where  $g$  is the acceleration of free fall.

(a) Show that the liquid column undergoes simple harmonic motion.

The defining equation of s.h.m is:  $a = -\omega^2 x$   
 $a \propto -x$

So  $a = -\left(\frac{2g}{L}\right)x$

$g$  and  $L$  are constants, so  $a \propto -x$

Hence liquid in U-tube ~~oscillate~~ execute s.h.m.

[2]

(b) The variation with time  $t$  of the displacement  $x$  is shown in Fig. 4.3.

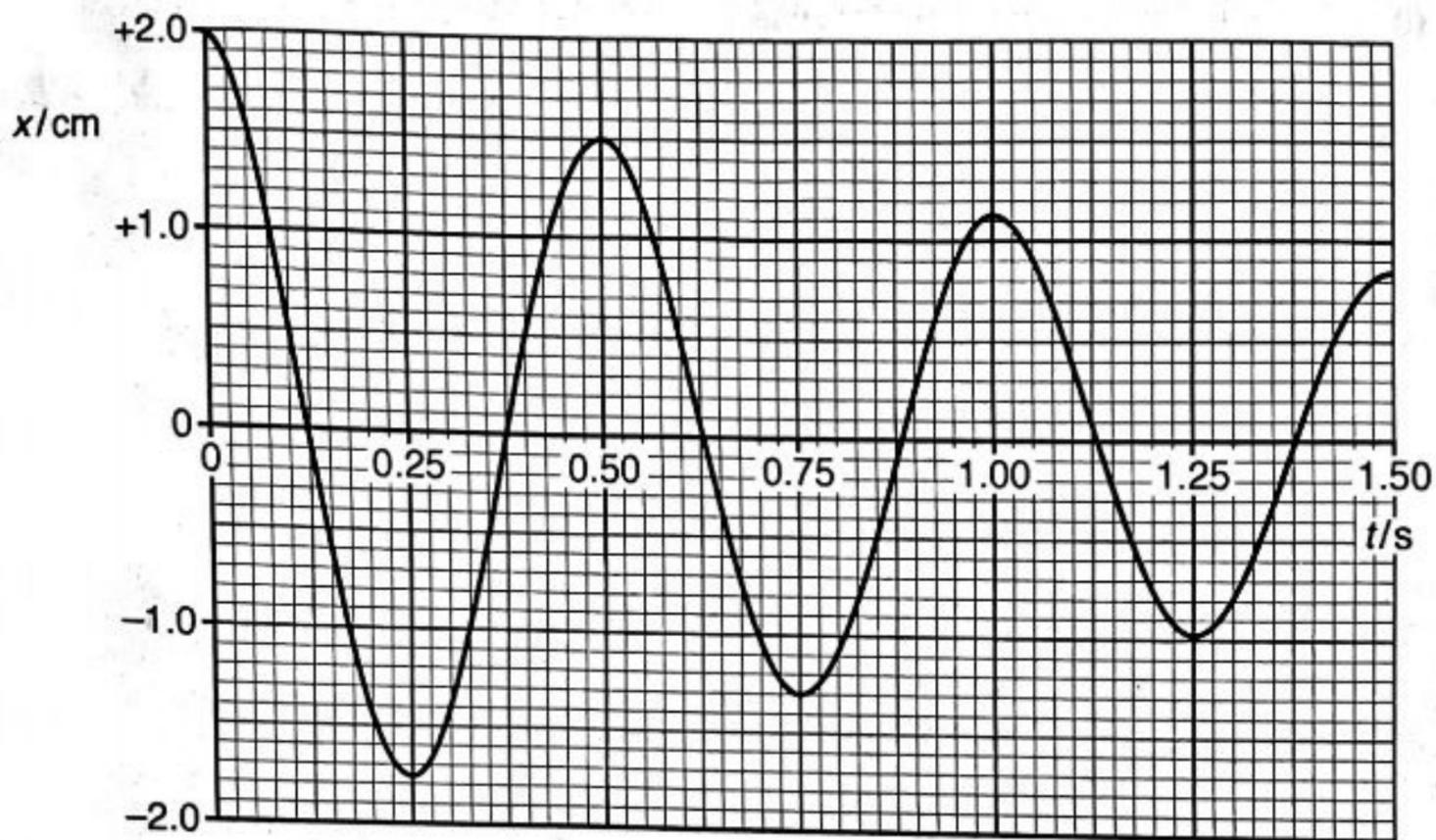


Fig. 4.3

Use data from Fig. 4.3 to determine the length  $L$  of the liquid column.

$$\omega = \frac{2\pi}{T} \Rightarrow \omega = \frac{2(3.14)}{0.50} = 12.6 \text{ rad s}^{-1}$$

$$\omega^2 = \frac{2g}{L} \Rightarrow L = \frac{2g}{\omega^2}$$

$$L = \frac{2(9.81)}{(12.6)^2} \Rightarrow L = 0.123$$

$$L = \dots\dots\dots \text{ m [3]}$$

(c) The oscillations shown in Fig. 4.3 are damped.

(i) Suggest one cause of this damping.

There is a friction between walls of the tube and liquid. [1]

(ii) Calculate the ratio

total energy of oscillations after 1.5 complete oscillations

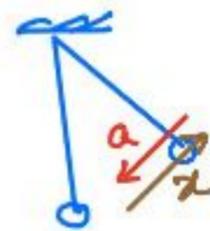
total initial energy of oscillations

Since  $E_T = \frac{1}{2} m \omega^2 x_0^2$

$$\text{Ratio} = \frac{\frac{1}{2} m \omega^2 (1.3)^2}{\frac{1}{2} m \omega^2 (2.0)^2} = 0.4225$$

ratio = 0.42 [2]

[Total: 8]



$a \propto -x$   
Acceleration ← Displacement →

3 (a) The defining equation for simple harmonic motion is

$$a = -\omega^2 x.$$

(i) State the relation between  $\omega$  and the frequency  $f$ .

$$\omega = 2\pi f$$

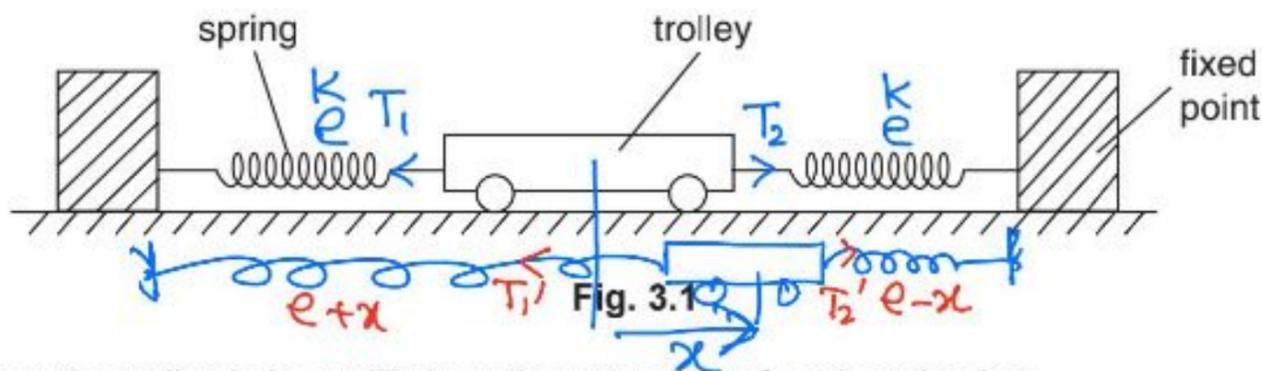
[1]

(ii) State the significance of the negative (-) sign in the equation.

Vectorial significance: Acceleration and displacement are in opposite directions. [1]

Conventional significance: Acceleration is directed towards mean position.

(b) A frictionless trolley of mass  $m$  is held on a horizontal surface by means of two similar springs, each of spring constant  $k$ . The springs are attached to fixed points, as illustrated in Fig. 3.1.



When the trolley is in equilibrium, the extension of each spring is  $e$ .

The trolley is then displaced a small distance  $x$  to the right along the axis of the springs. Both springs remain extended.

(i) Show that the magnitude  $F$  of the restoring force acting on the trolley is given by

Resultant force in the direction of increasing displacement

$$\begin{aligned} F &= T_2' - T_1' \\ &= k(e-x) - k(e+x) \\ &= ke - kx - ke - kx \\ F &= -2kx \end{aligned}$$

$$\begin{aligned} F &= 2kx. \\ \text{Resultant force towards mean} &= -(T_1' - T_2') \\ F &= T_2' - T_1' \end{aligned}$$

Magnitude of  $F = 2kx$

[2]

(ii) The trolley is then released. Show that the acceleration  $a$  of the trolley is given by

$$a = -\frac{2kx}{m}$$

Since

$$F = ma$$

$$ma = -2kx$$

$$a = -\frac{2kx}{m}$$

[2]

$$a = -\frac{2kx}{m}$$

$$a = -\omega^2 x$$

- (iii) The mass  $m$  of the trolley is 900g and the spring constant  $k$  is  $120\text{Nm}^{-1}$ . By comparing the equations in (a) and (b)(ii), determine the frequency of oscillation of the trolley.

$$\omega^2 = \frac{2k}{m} \Rightarrow 2\pi f = \sqrt{\frac{2k}{m}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{2k}{m}}$$

$$f = \frac{1}{2(3.14)} \sqrt{\frac{2(120)}{900 \times 10^{-3}}}$$

$$f = 2.60$$

frequency = ..... 2.60 ..... Hz [3]

[Total: 9]