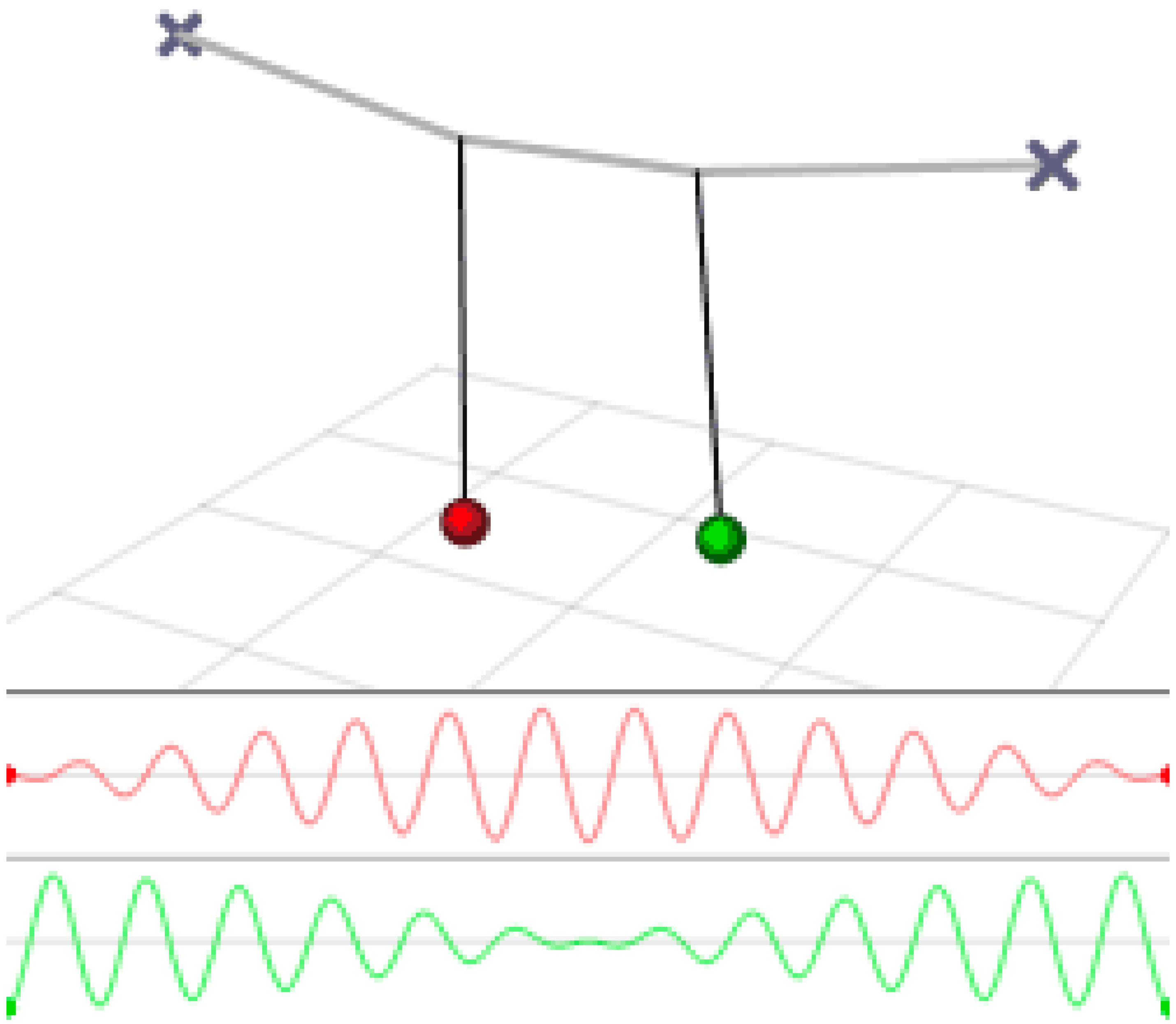
9702 C17 Oscillations



Describing Oscillations

Oscillation(aka vibration) = repeating back and forth movements on either side of any equilibrium position

Oscillating system can be represented by displacement-time grahs similar to transverse graph. The shape of the graph is **sine curve**. **The motion is described as sinusoidal**

Displacement(x) = distance of an oscillator from its equilibrium position

Amplitude(x_0) = **maximum displacement** of an oscillator from its equilibrium position

Angular frequency = **rate of change of angular displacement with respect to time** ($\omega=2\pi f$)

Phase difference = how much one oscillator is in front or behind another

Frequency = number of complete oscillations per unit time ($f=1/T = 2\pi/\omega$)

Time period = time taken for one complete oscillation, in seconds

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Phase difference = how much one oscillator is in front or behind another

-->in phase = position of two oscillators are equal (phase difference: $0\pi, 2\pi, 4\pi$, ...)

-->anti-phase = one oscillator is exactly half a cycle behing another (phase difference: $\pi, 3\pi, 5\pi$, ...)

Simple Harmonic Motion (SHM)

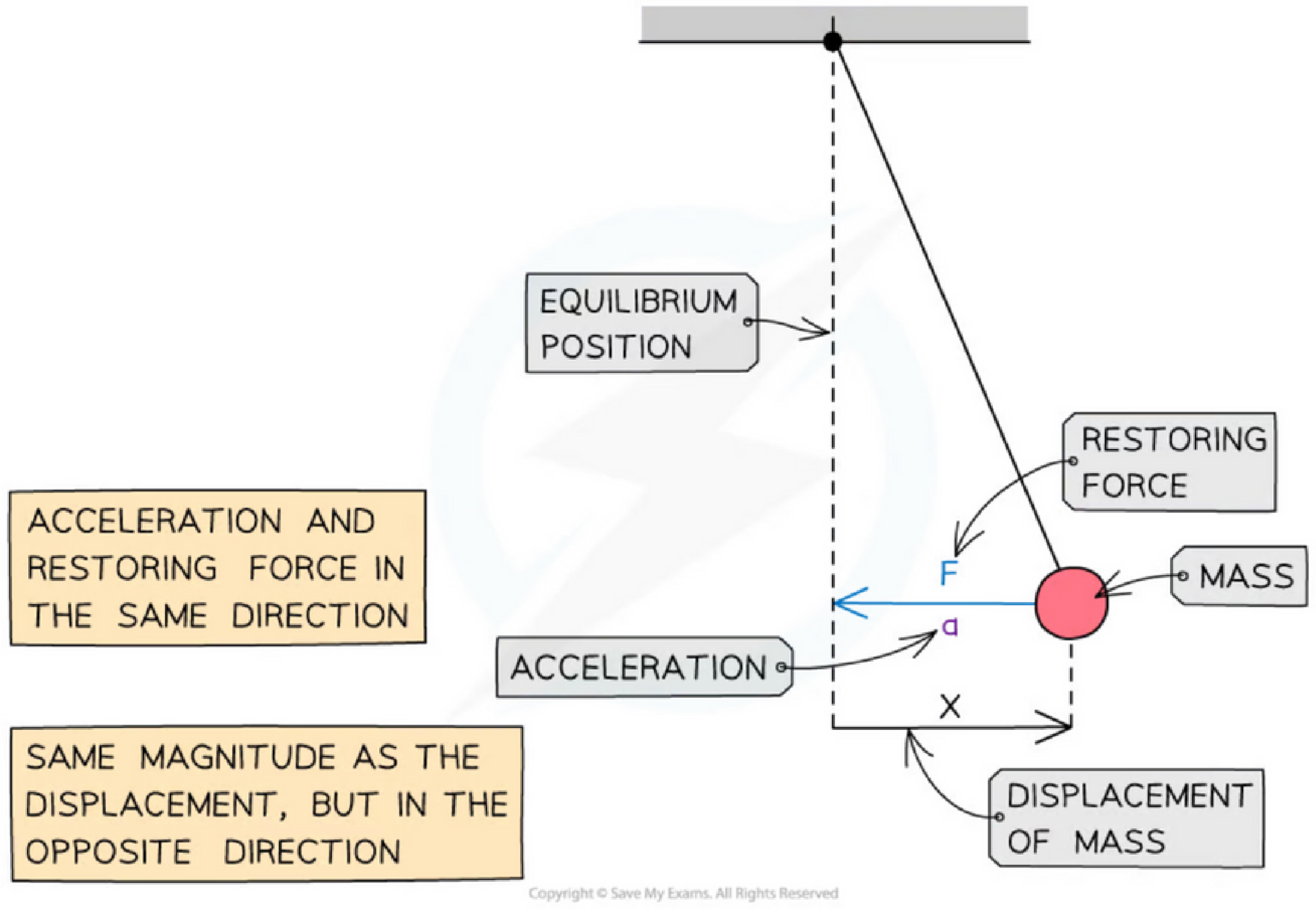
SHM = a type of oscillation in which the **acceleration** of a body **is proportional to its displacement from a fixed point and acts in the opposite direction**

$$a \propto -x$$

An object in SHM have a **restoring force** to return to its eqilibrium point

This restoring force will be directly proportional, but in opposite direction, to the displacement of the object from the equilibrium point. (Note: the direction of restoring force and acceleration is the same)

This is why a person jumping on a trampoline is not an example of SHM:
-the restoring force on the person is not proportional to their distance from the equilibrium position
-when the person is not in contact with the trampoline, the restoring force=weight which is constant -->This does not change, even if they jump higher



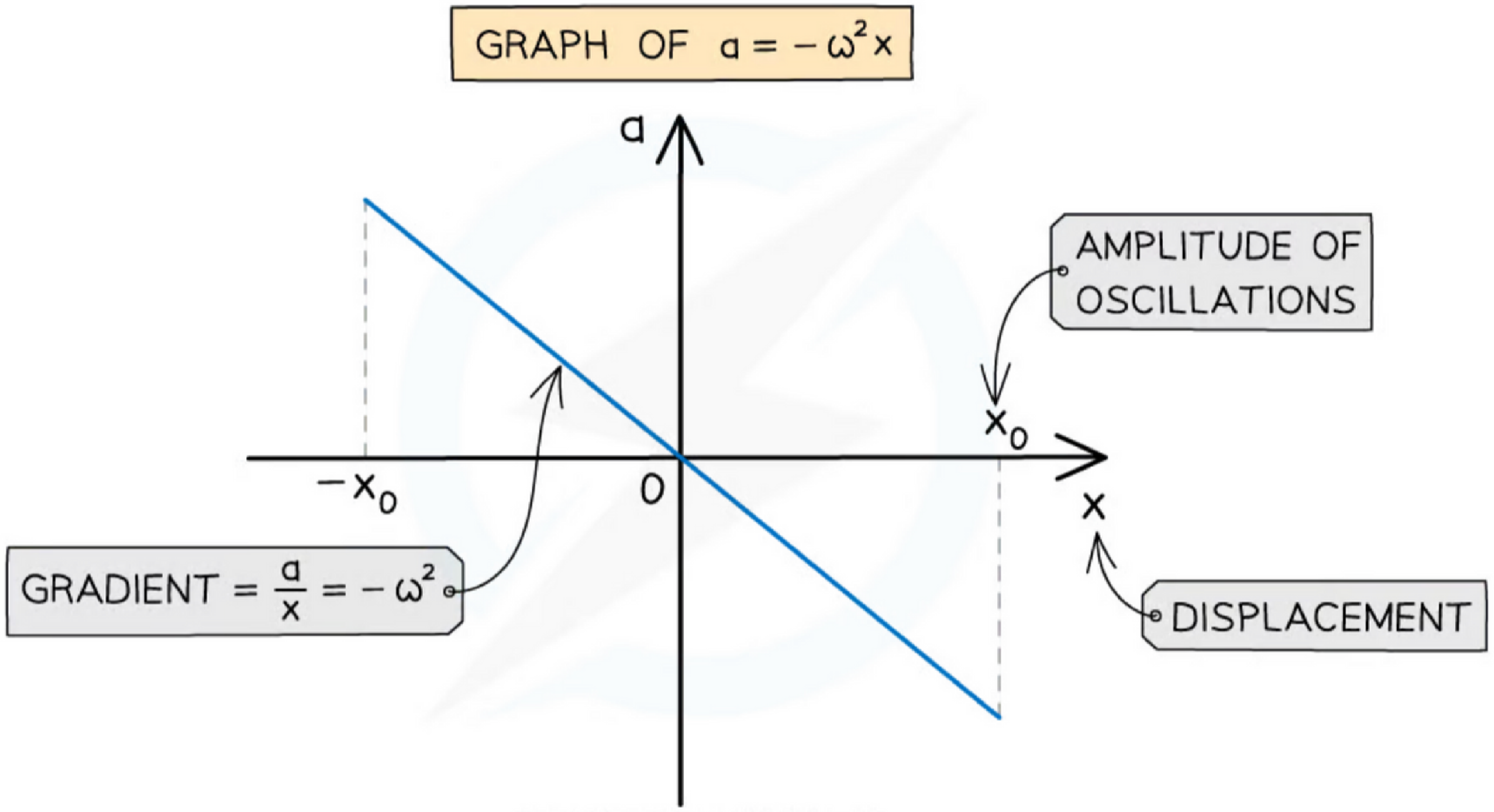
Force, acceleration and displacement of a pendulum in SHM

Acceleration and Displacement of an Oscillator

$$a = -\omega^2 x$$

Where:

- a = acceleration ($m\ s^{-2}$)
- ω = angular frequency ($rad\ s^{-1}$)
- x = displacement (m)



max = -x0

min = x0

A solution to the SHM acceleration equation is the displacement equation:

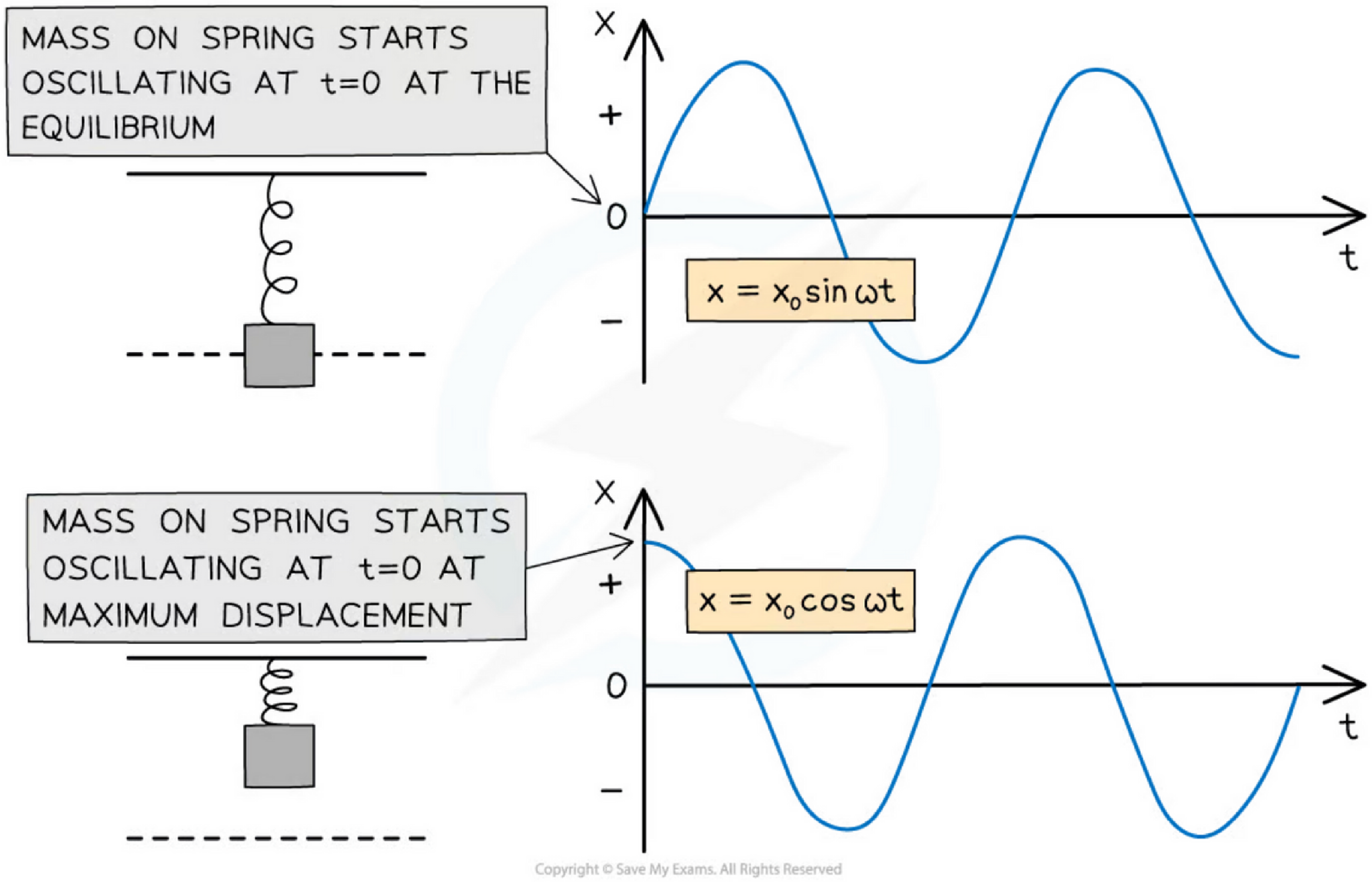
$x = x_0 \sin(\omega t)$

use sin when object begins oscillating from the equilibrium position(x=0 at t=0)

$x = x_0 \cos(\omega t)$

use cos when object begins oscillating from its amplitude position($x = x_0$ or $x = -x_0$ at $t = 0$)

- This is because the cosine graph starts at a maximum, whilst the sine graph starts at 0



Speed in SHM

The greatest speed of an oscillator is at the equilibrium position (when its displacement is 0, x=0)

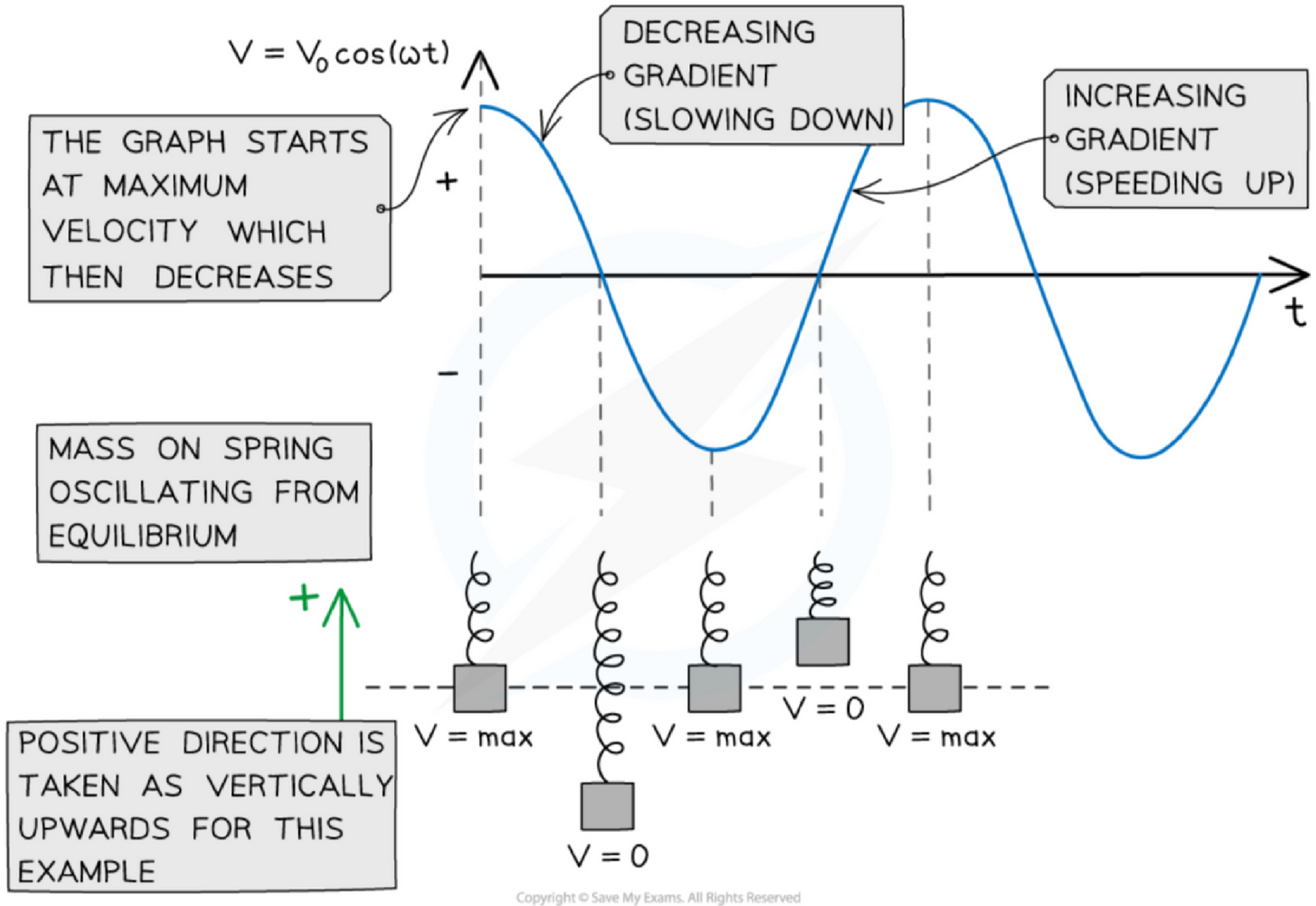
$$v = v_0 \cos(\omega t)$$

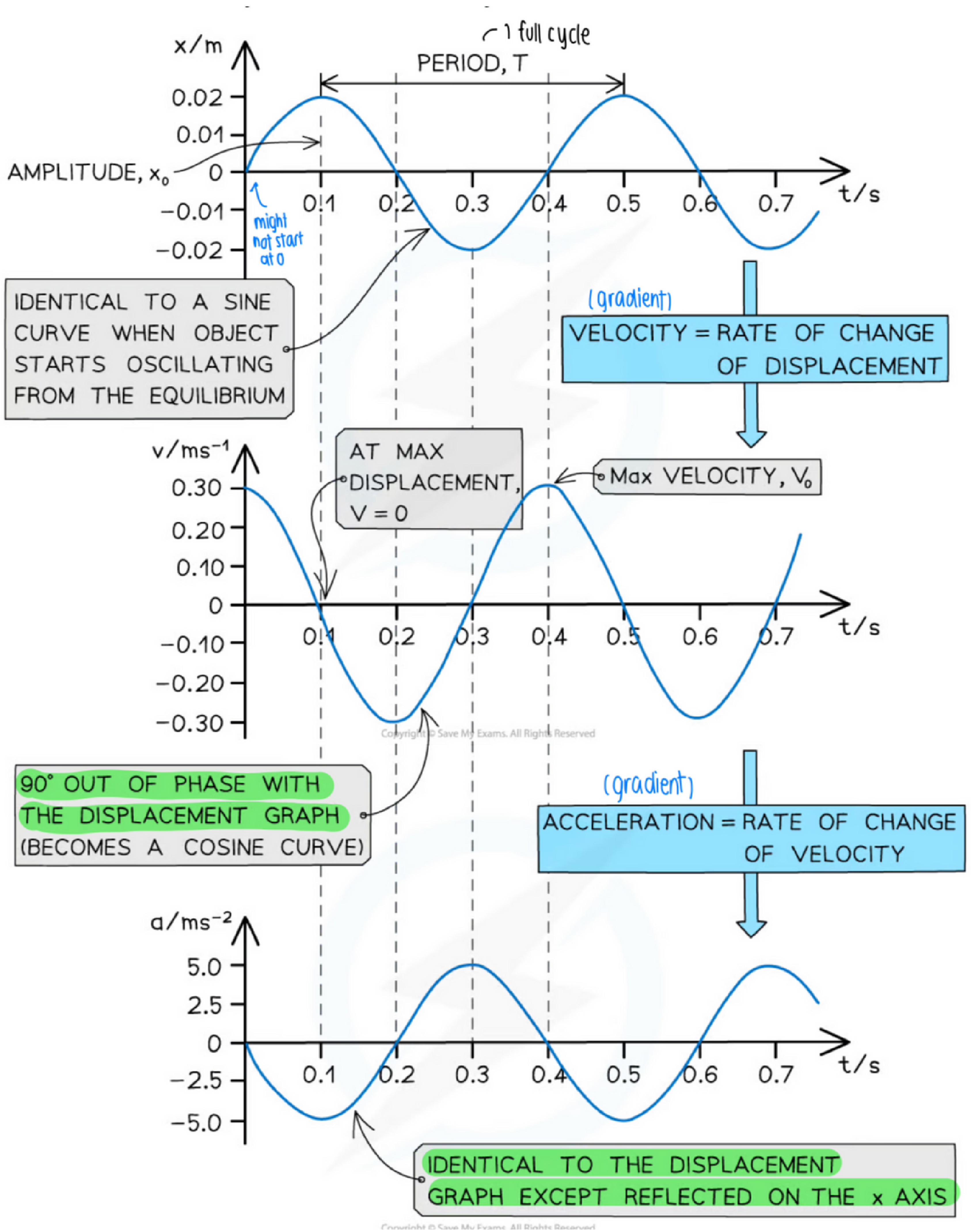
Where:

- v = speed (m s^{-1})
- v_0 = maximum speed (m s^{-1})
- ω = angular frequency (rad s^{-1})
- t = time (s)

$$v = \pm\, \omega \sqrt{(x_0^2 - x^2)}$$

→ max speed: $v_0 = \omega x_0$
(at $x = 0$)





The displacement, velocity and acceleration of an object in SHM can be represented by graphs against time

All **undamped** SMH graphs are represented by periodic functions (this means they can all be described by sine and cosine curves)

Ek and Ep in SHM

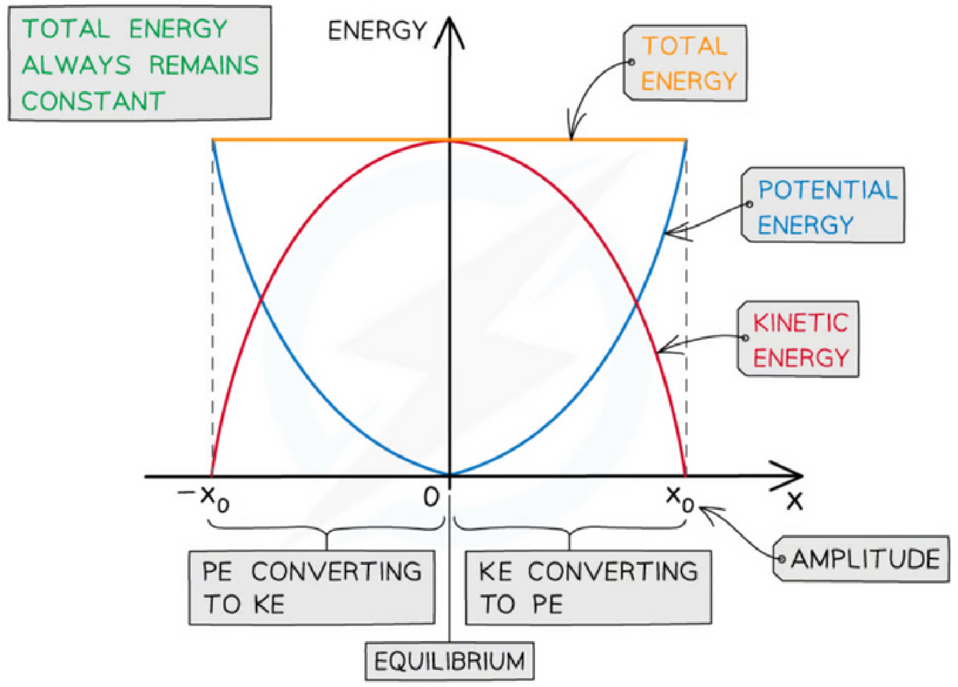
During SHM, energy is constantly exchanged between two forms Ek and Ep.
-->Ep could be in the form of gravitational potential energy(for a pendulum), elastic potential energy(for a horizontal mass on a spring) or both(for a vertical mass on a spring)

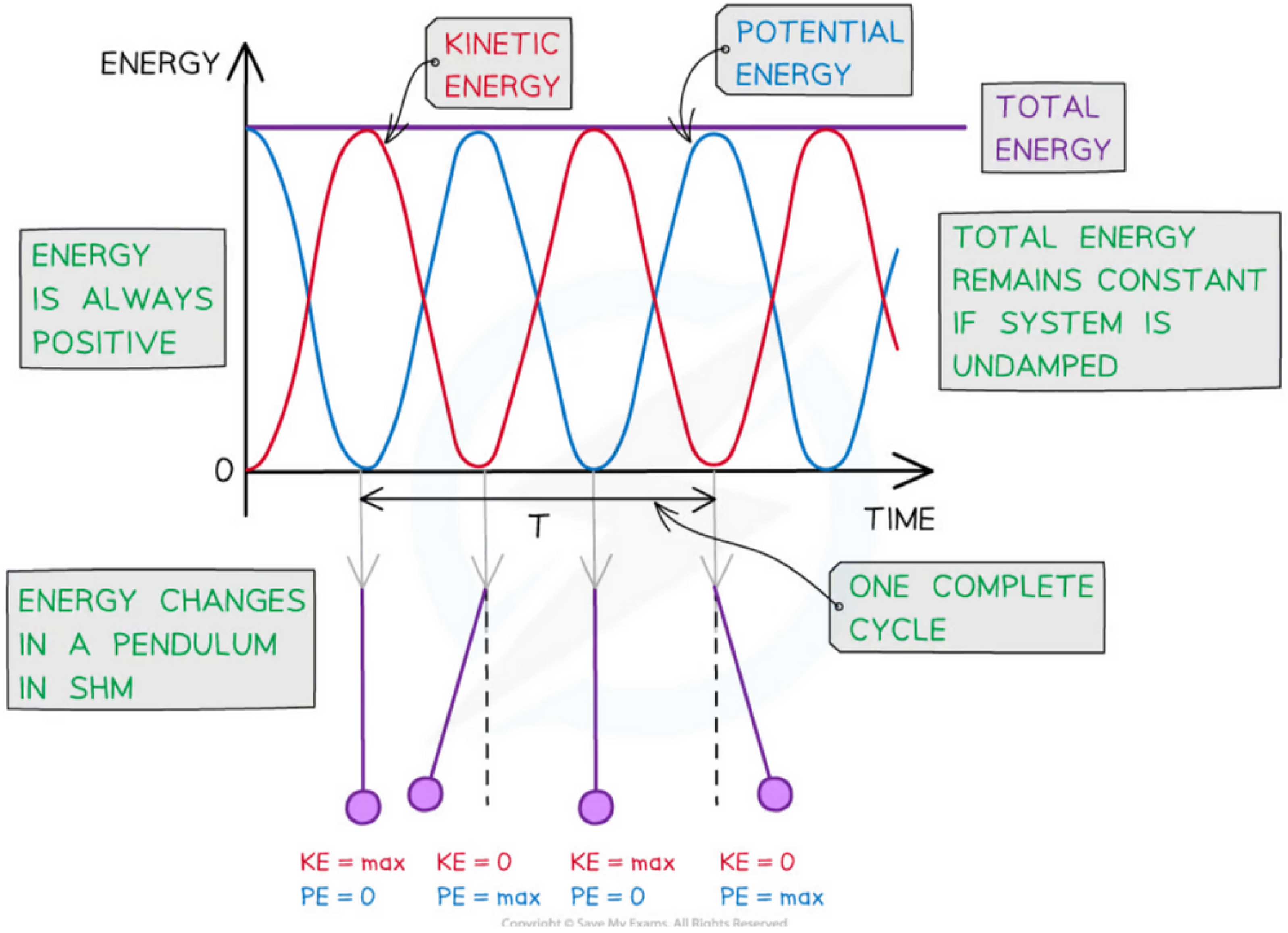
Speed(v) is maximum at displacement x=0 --> so maximum Ek because the oscillator is at its equilibrium poisiton and so moving at maximum velocity

Ep is maximum when the displacement is at a maximum

$x = \pm x_0$ ***(amplitude)***

The energy-displacement graph for **half** a cycle looks like:





Damping

In practice, all oscillators eventually stop oscillating, their amplitudes decrease rapidly/gradually

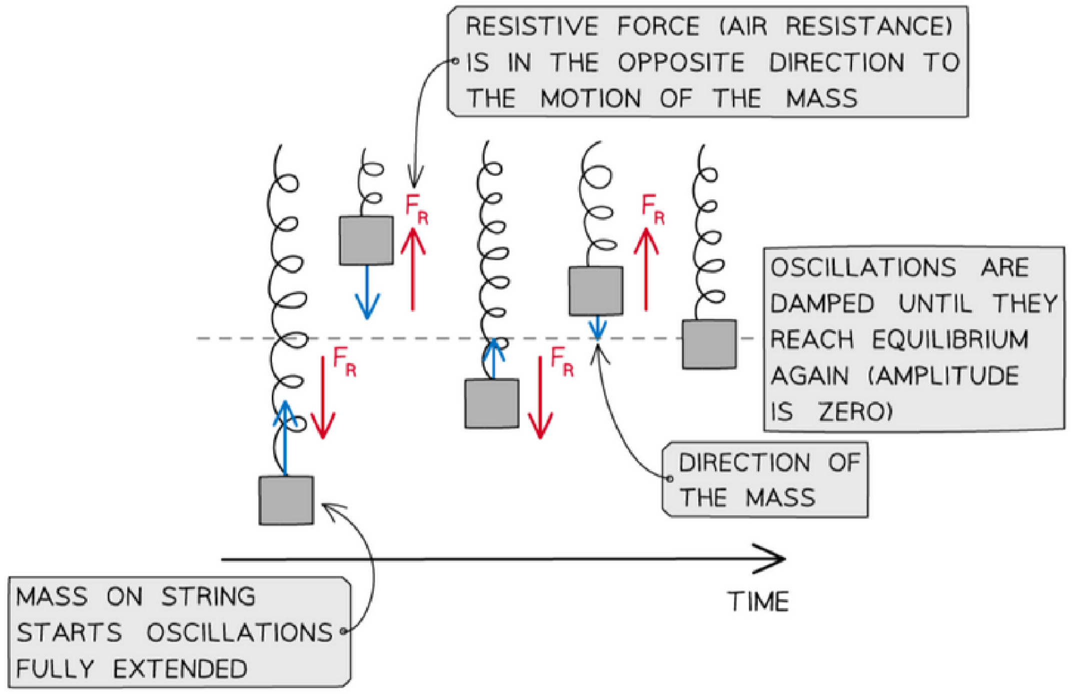
This is because resistive forces(e.g. air resistance or friction), which act in the opposite direction to the motion of an oscillator

Damping is due to resistive forces acting on an oscillating SHM system

Damping = reduction in energy and amplitude of oscillations due to resistive forces acting on the oscillating sytem

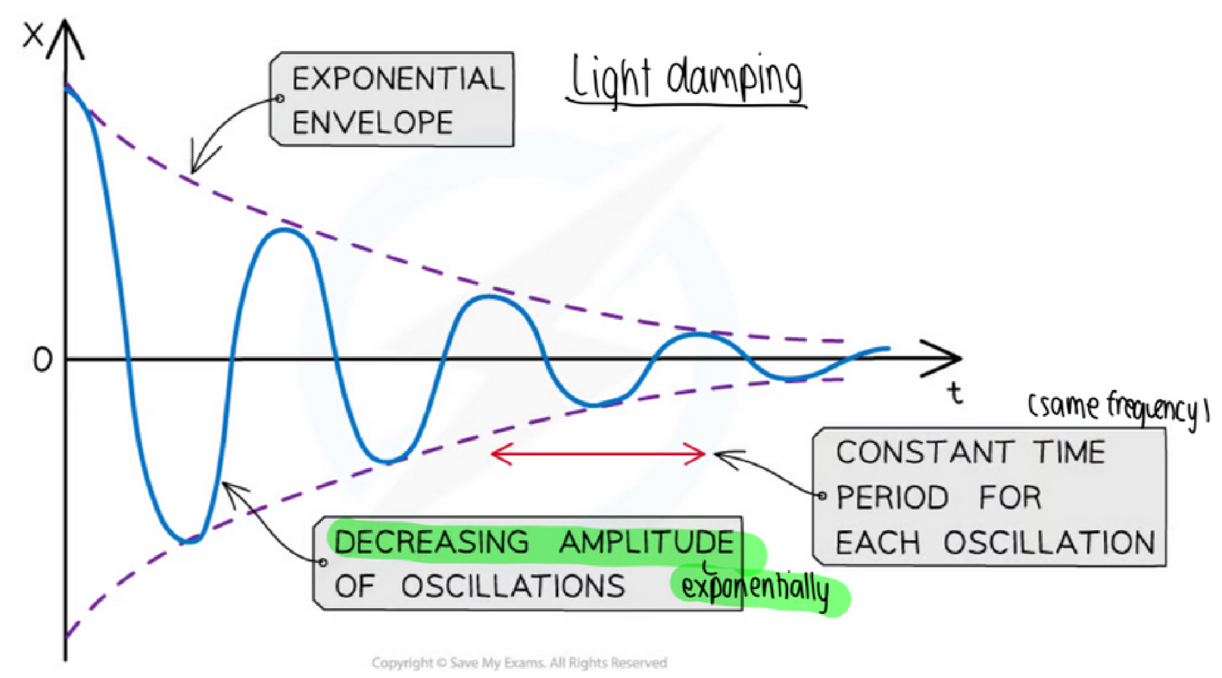
-->Damping continues until the oscillator cpme to rest at equlibrium poisition

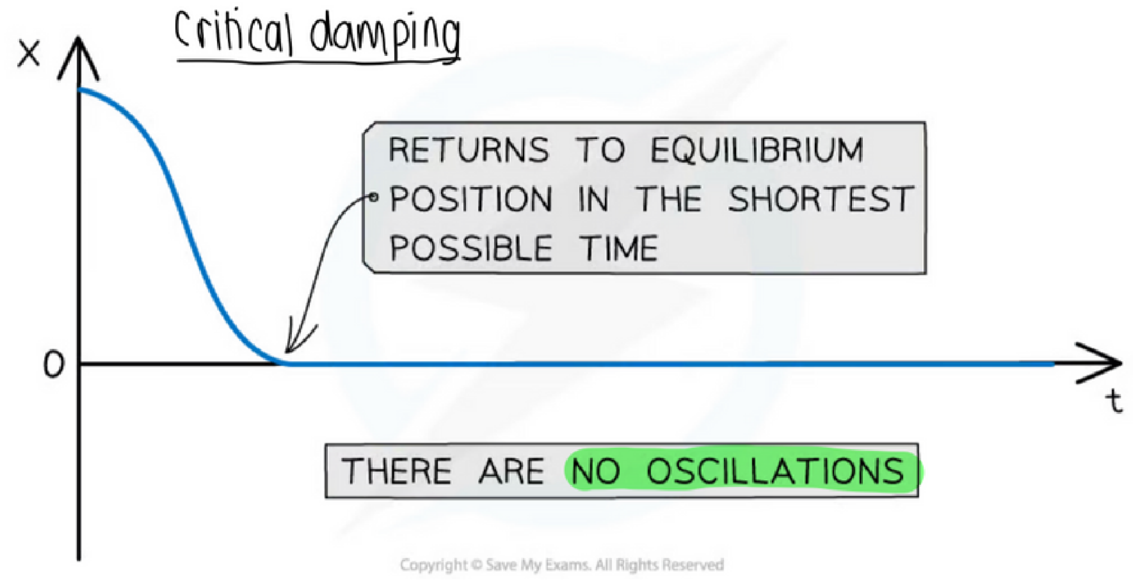
in SHM, **frequency(+time period)** of damped oscillations **does not change as the amplitude decrease**

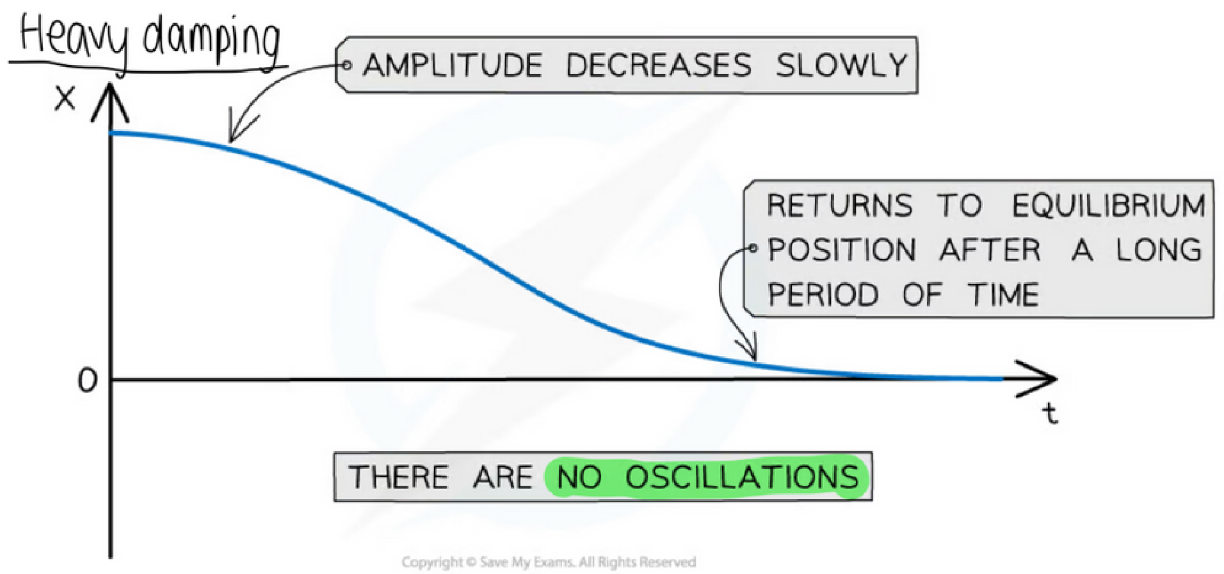


Types of Damping

1.Light damping
2.Critical damping
3.Heavy damping







note that all type of damping, the **frequency does not change as the amplitude decrease**

Resonance

Forced oscillations = periodic forces which are applied in order to sustain oscillations

Forced oscillations need to be applied to replace the energy lost in damping, to sustain oscillations in SHM

The frequency of forced oscillations is referred to as the driving frequency(f)

All oscillating systems have a natural frequency(f0).

Natural frequency = frequency of an oscillation when the oscillating system is allowed to oscillate freely

Resonance = when driving frequency(f) applied to an oscillating system is equal to its natural frequency(f0), the amplitude of resulting oscillations increases significantly

Higher amplitude = Higher energy
(so at resonance, the system will be transferring the max energy as possible)