THE EQUATIONS FOR

≈ OSCILLATIONS ≈

① $x = X_0 \sin \omega t$ — represents a function of displacement (x) against time (t) for any particle which performs S.H.M

x = Displacement X_0 = Amplitude
t = time ω = ang. velocity (C.M) OR ang. frequency (S.H.M)

② $V = X_0 \omega \cos \omega t$ → Velocity

③ $a = -\omega^2 \{X_0 \sin \omega t\}$

$a = -\omega^2 x$ → For an oscillator performing S.H.M $a \propto -x$

④ $X_{max} = \pm X_0$ ⑤ $V_{max} = \pm X_0 \omega$ ⑥ $a_{max} = \pm X_0 \omega^2$

⑦ $V = \pm \omega \sqrt{X_0^2 - x^2}$ → Formula to relate velocity with displacement for a particle performing S.H.M

ω = ang. freq x = disp at any instant
v = velocity at that instant X_0 = max disp / Amplitude of particle

⑧ $K.E\,(\text{gF}) = \frac{1}{2} m \omega^2 (X_0^2 - x^2)$

⑨ $K.E_{max} = \frac{1}{2} m \omega^2 X_0^2 = P.E_{(max)}$

⑩ $P.E\,(\text{gF}) = T.E - K.E\,(\text{gF})$

$\frac{1}{2} m \omega^2 X_0^2 - \frac{1}{2} m \omega^2 (X_0^2 - x^2)$

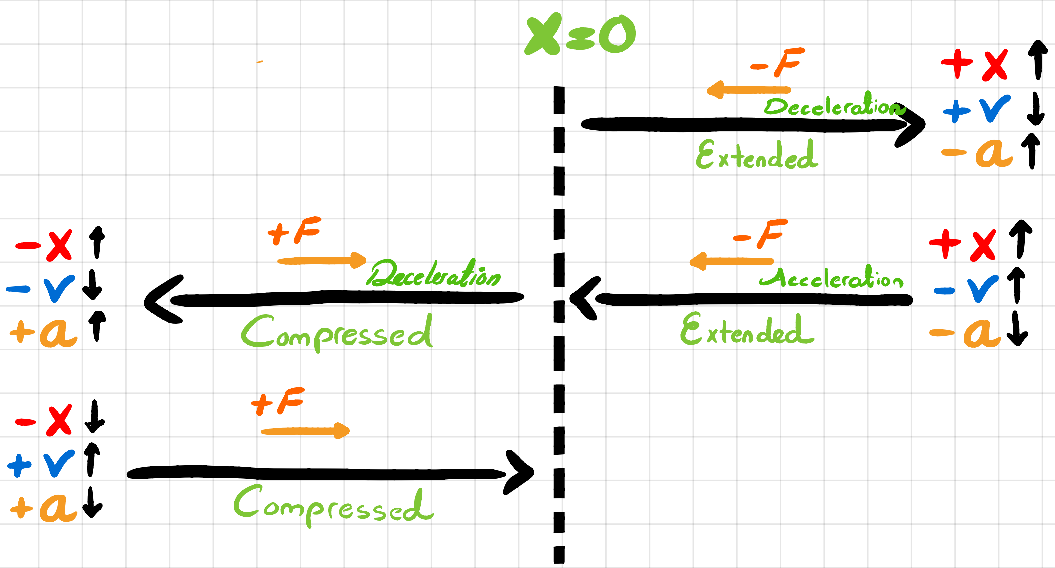
$\frac{1}{2} m \omega^2 x^2$

GRAPHICAL Analysis

Next up is something difficult to explain with lines... But thats the point. GRAPHS say everything... if only we can understand the language. So... if you need any help, i am an email away.

✓ The diagram shows one completo oscillation of a simple harmonic oscillator divided in FOUR QUARTERS.

CASE #1 => Oscillator is passing through its mean position at t=0



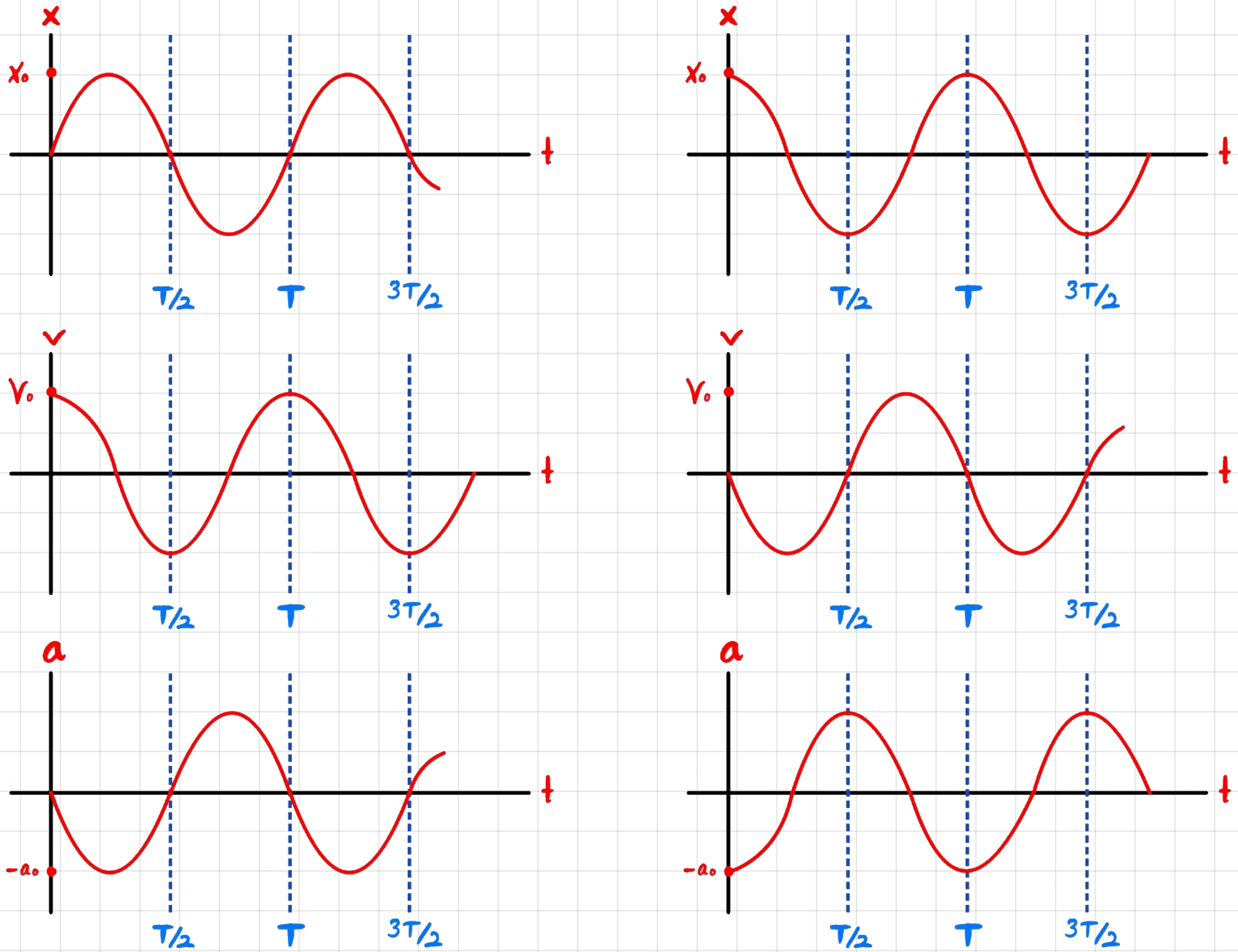
✓ Signs of acceleration and displacement (from mean point) Show that in 1st and 3rd quarters body decelerates at increasing rate AND in the 2nd and 4th quarter it accelerates at decreasing rate.

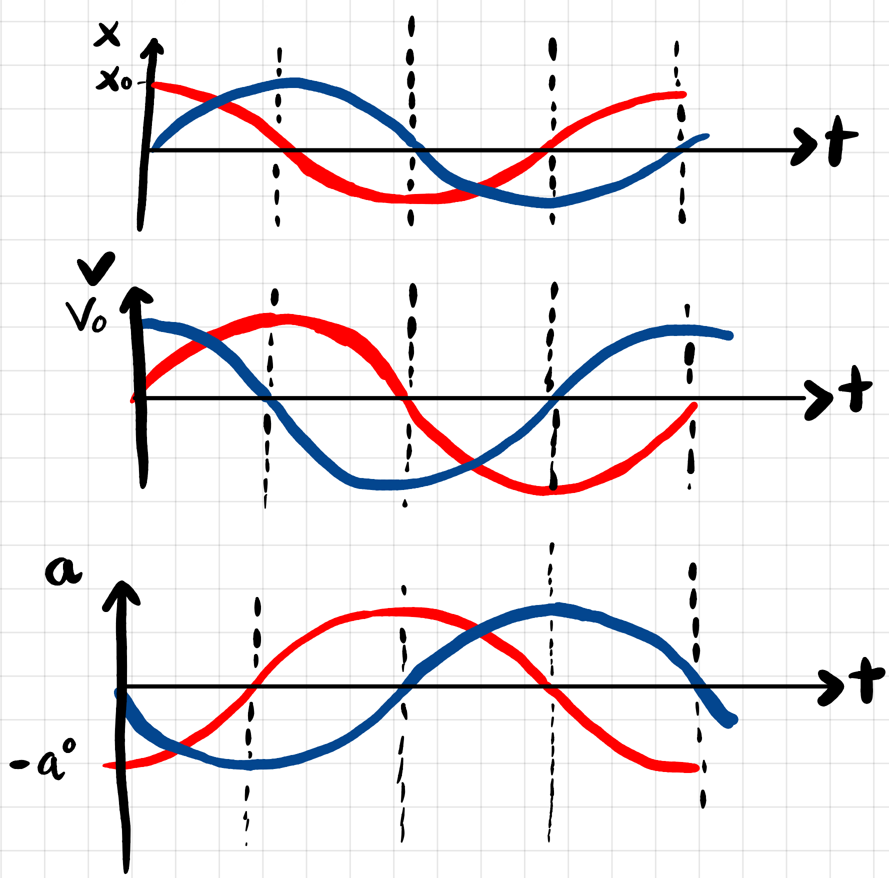
✓ 1st and 3rd quarters ⟶ Speed decreasing at increasing rate
✓ 2nd and 4th quarters ⟶ Speed increases at decreasing rate

✓ The acceleration and displacement (from mean point are always directly proportional and oppositely directed! ($a \propto -x$)

✓ The angular velocity "ω" is defined as Phase change per unit time

If you could not understand the reason for the specific wordings...
Just reach out... 10 minutes ---> Everything will well hopefully make sense.

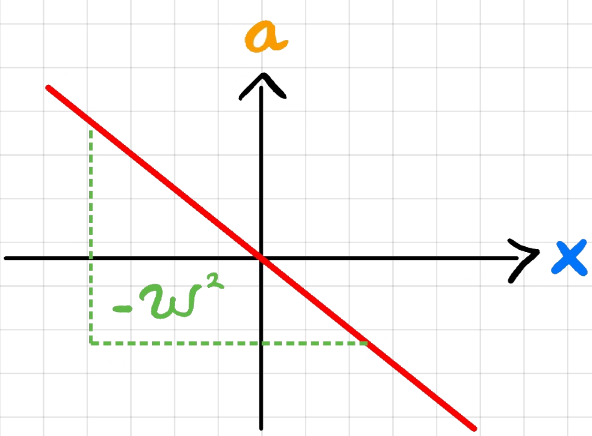




- ✓ The gradient of displacement-time graph shows variation of velocity.
- ✓ The gradient of velocity-time graph shows variation of acceleration.
- ✓ The variation of displacement, velocity and acceleration against time is well... mathematical → Sinusoidal.
- ✓ This sinusoidal variation of displacement, velocity and acceleration provides an evidence... that oscillator performs SIMPLE HARMONIC MOTION
- ✓ Phase difference b/w displacement-time and velocity-time graph OR b/w velocity-time and acceleration-time graph is $\pi/2$ radians.
- ✓ Phase difference b/w displacement-time and acceleration-time graph is π radians

MORE GRAPHS!

GRAPH of Acceleration against Displacement!



$$a \propto -x$$

$$a = -\omega^2 x + 0$$

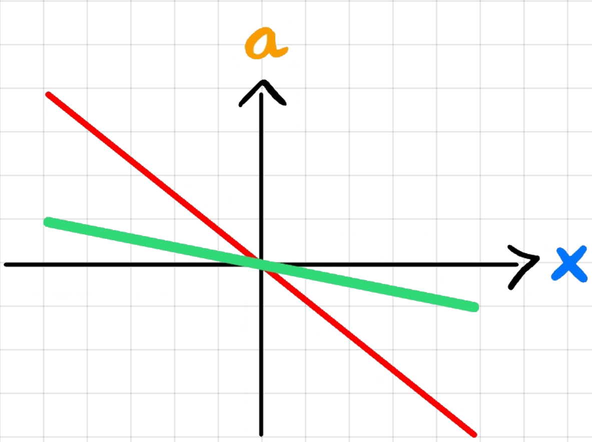
$a \to y$, $-\omega^2 \to m$, $x \to x$, $0 \to c$

$$\text{gradient} = -\omega^2$$

$$\text{gradient} = -(2\pi f)^2$$

$$\text{gradient} = -4\pi^2 f^2$$

gradient → m/4, f^2 → f/2

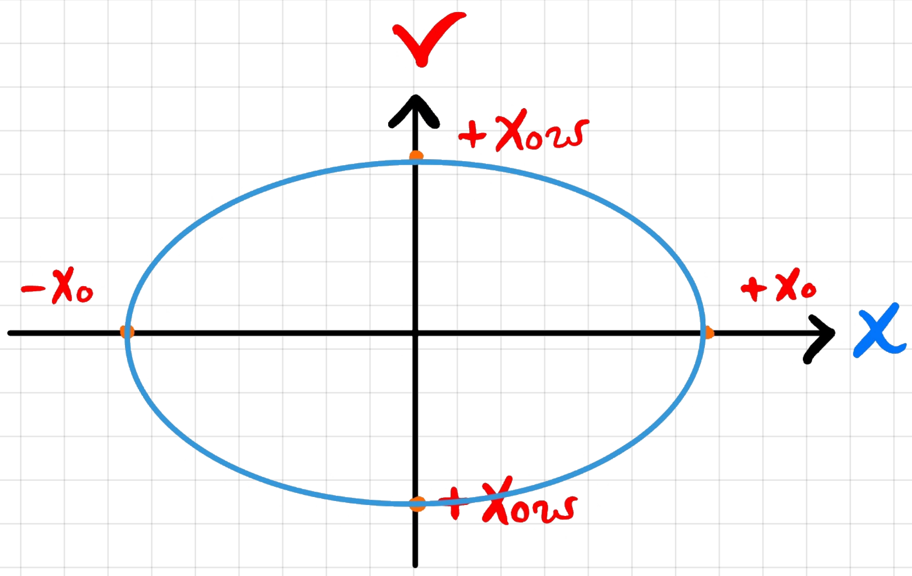


Sketch shows that a and x are in direct proportion but remain oppositely directed to each other.

The gradient represents square of angular frequency.

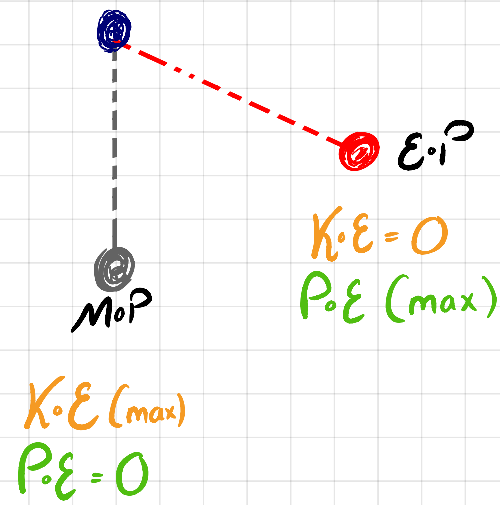
If frequency is reduced to half the gradient reduces to $1/4^{th}$

Graph of Velocity-Displacement



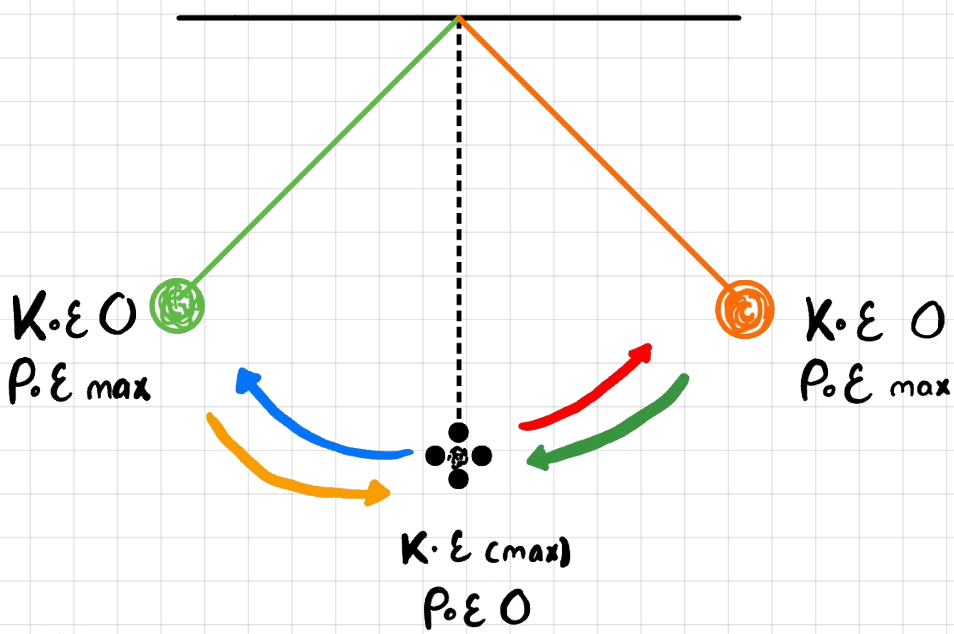
- Variation of velocity and displacement from mean point is a closed elliptical loop.
- Velocity and displacement do not change at the same rate.
- On a circular arc, quantities on x and y axis change uniformly at equal rates.

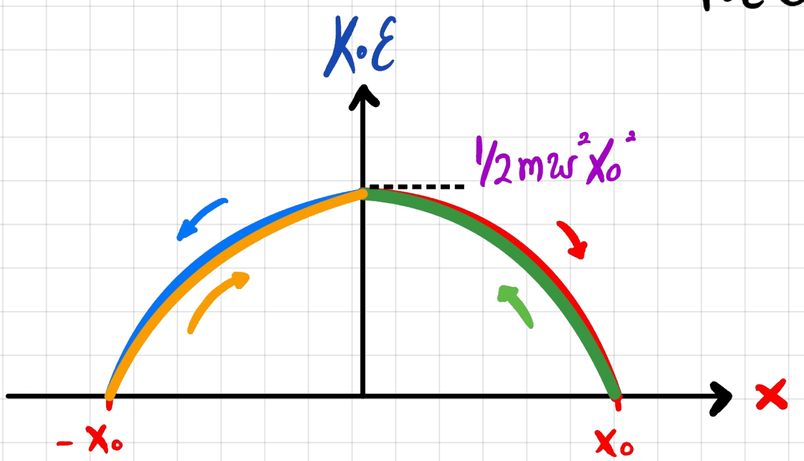
Energy of an Oscillator

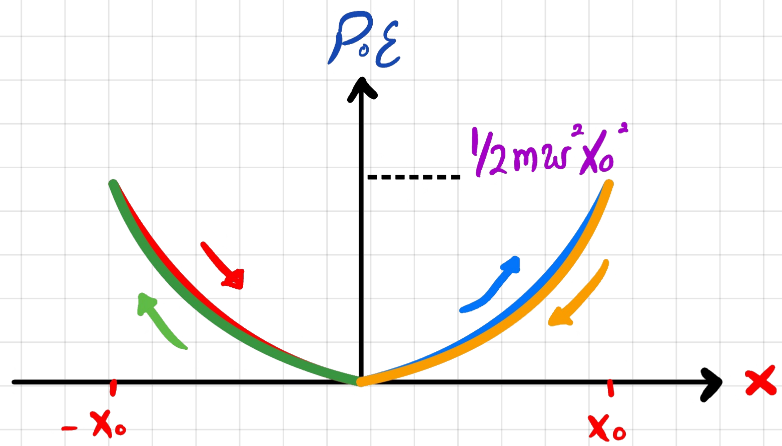


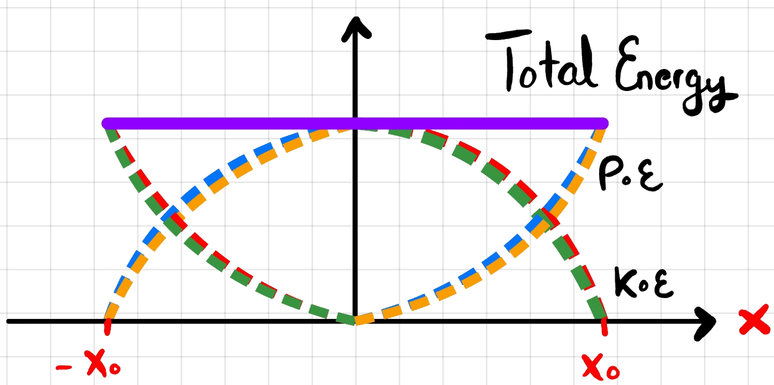
- Oscillators experience a continuous interchange between kinetic & potential energy.
- The type of potential energy depends upon the design of oscillator.
- In undamped oscillators i.e no resistive forces, total energy remains conserved, the amplitude does not change.
- The magnitude of maximum kinetic, maximum potential & total energy are same. At mean points, the total energy is all kinetic & at maximum displacement, total energy is all potential.

The Energy-Displacement & Energy-Time Graph

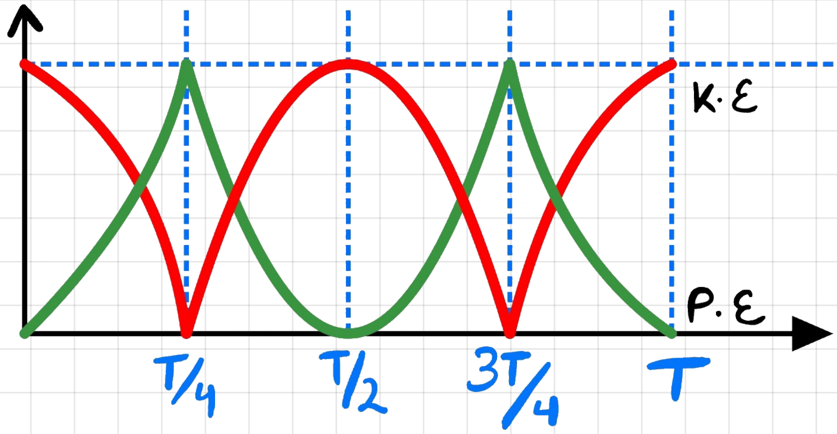


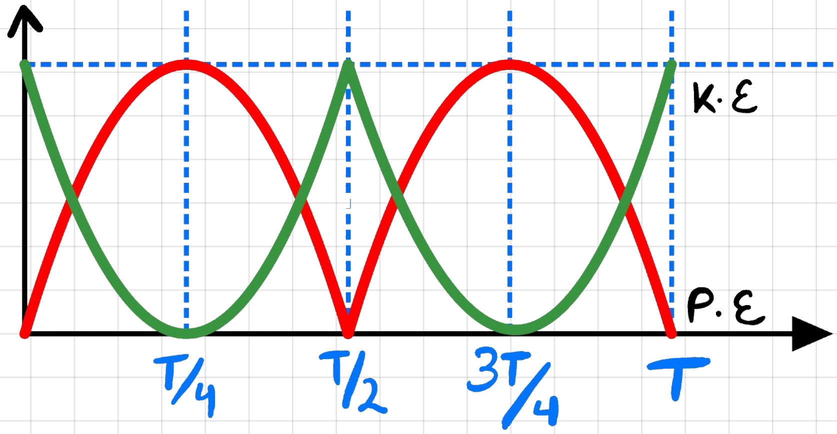


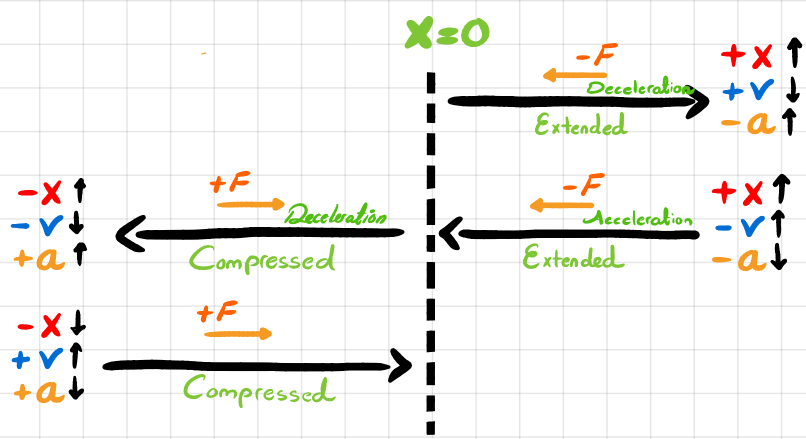


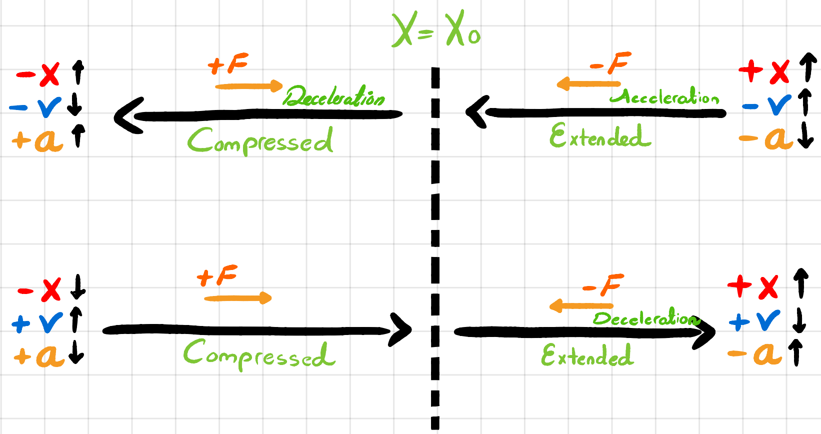


- ✓ The magnitude of Kinetic or Potential Energy changes more rapidly than the displacement of the oscillator from its mean point
- ✓ Independent of Time reference. K·E ↑ with ↓ rate P·E ↓ with ↓ rate
  - ✓ K·E ↓, P·E ↑; both occuring at a ↑ rate!


K.E
P.E
T/4
T/2
3T/4
T


K.E
P.E
T/4
T/2
3T/4
T


X=0
-F
Deceleration
Extended
+x
+v
-a
+F
Deceleration
Compressed
-x
-v
+a
-F
Acceleration
Extended
+x
-v
-a
+F
Compressed
-x
+v
+a


X= Xo
+F
Deceleration
Compressed
-x
-v
+a
-F
Acceleration
Extended
+x
-v
-a
+F
Compressed
-x
+v
+a
-F
Deceleration
Extended
+x
+v
-a