3 A small wooden block (cuboid) of mass *m* floats in water, as shown in Fig. 3.1.

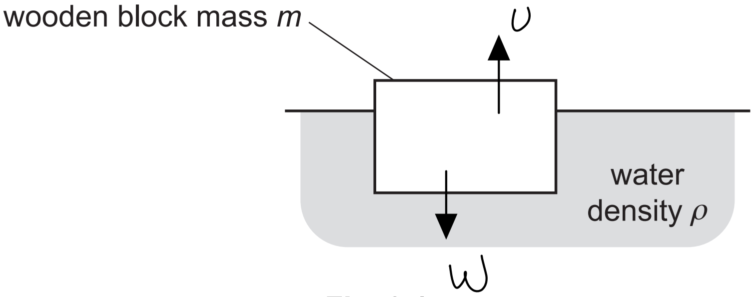


Fig. 3.1

The top face of the block is horizontal and has area *A*. The density of the water is ρ.

(a) State the names of the two forces acting on the block when it is stationary.

Upthrust and Weight .... [1]

(b) The block is now displaced downwards as shown in Fig. 3.2 so that the surface of the water is higher up the block.

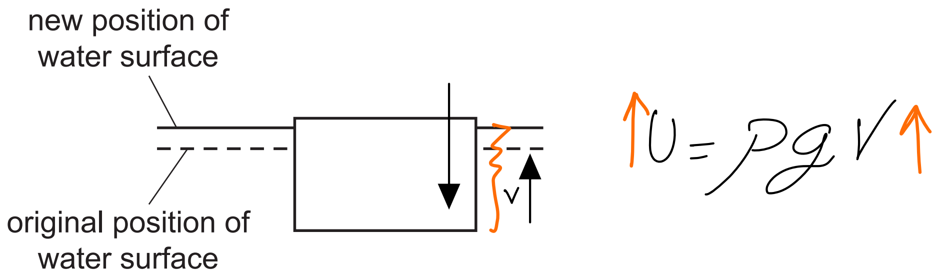


Fig. 3.2

State and explain the direction of the resultant force acting on the wooden block in this position.

Upthrust is now greater than weight so

resultant force is upwards. .... [1]

(c) The block in **(b)** is now released so that it oscillates vertically.

The resultant force F acting on the block is given by

$$F = -Ag\rho x \qquad F \propto -x$$

where g is the gravitational field strength and x is the vertical displacement of the block from the equilibrium position.

(i) Explain why the oscillations of the block are simple harmonic.

F is proportional to x since A, ρ, g are constant.
The minus sign shows that the direction of force is opposite to displacement.
.... [2]

(ii) Show that the angular frequency ω of the oscillations is given by

$$\omega = \sqrt{\frac{A\rho g}{m}}.$$

$$\therefore F = ma$$

$$a = -\omega^2 x$$

$$F = ma$$

$$\boxed{a = -\frac{Ag\rho x}{m}}$$

$$\omega^2 = \frac{Ag\rho}{m}$$

$$\omega = \sqrt{\frac{Ag\rho}{m}}$$

[2]

(d) The block is now placed in a liquid with a greater density. The block is displaced and released so that it oscillates vertically. The variation with displacement x of the acceleration a of the block is measured for the first half oscillation, as shown in Fig. 3.3.

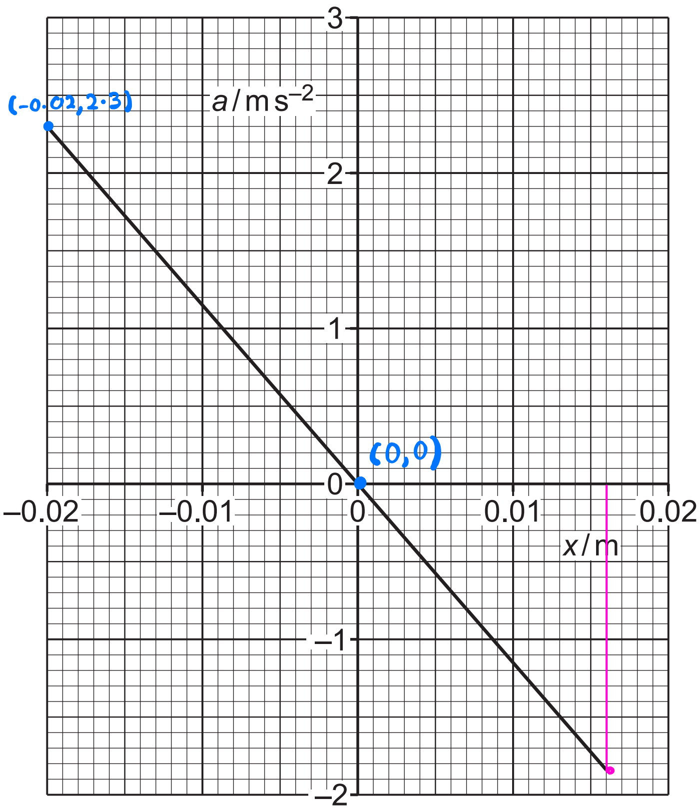


Fig. 3.3

m

$a = -w^2 x$

$m = \dfrac{2.3 - 0}{-0.02 - 0}$

$m = -115$

$w^2 = -115$

(i) Explain why the maximum negative displacement of the block is not equal to its maximum positive displacement.

The block is experiencing damping from viscous drag.

.... [1]

(ii) The mass of the block is 0.57 kg.

Use Fig. 3.3 to determine the decrease ΔE in energy of the oscillation for the first half oscillation.

$E = \frac{1}{2} m w^2 x_0^2$

$E_i = \frac{1}{2} \times 0.57 \times 115 \times 0.02^2$

$= 0.01311 \text{ J}$

$E_F = \frac{1}{2} \times 0.57 \times 115 \times 0.016^2$

$= 8.3904 \times 10^{-3} \text{ J}$

$\Delta E = E_F - E_i$

$= 8.3904 \times 10^{-3} - 0.01311$

E = 4.7×10^{-3} J [3]

[Total: 10]

7 2021 NOV P41 Q04

A trolley on a track is attached by springs to fixed blocks X and Y, as shown in Fig. 4.1. The track contains many small holes through which air is blown vertically upwards. This results in the trolley resting on a cushion of air rather than being in direct contact with the track.

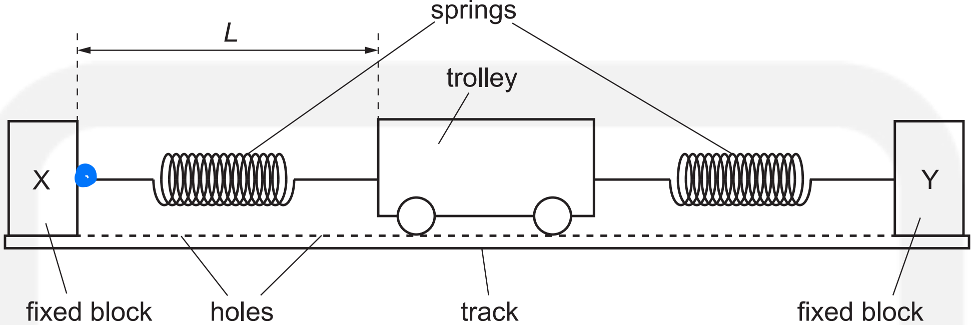


Fig. 4.1

The trolley is pulled to one side of its equilibrium position and then released so that it oscillates initially with simple harmonic motion. After a short time, the air blower is switched off. The variation with time t of the distance L of the trolley from block X is shown in Fig. 4.2.

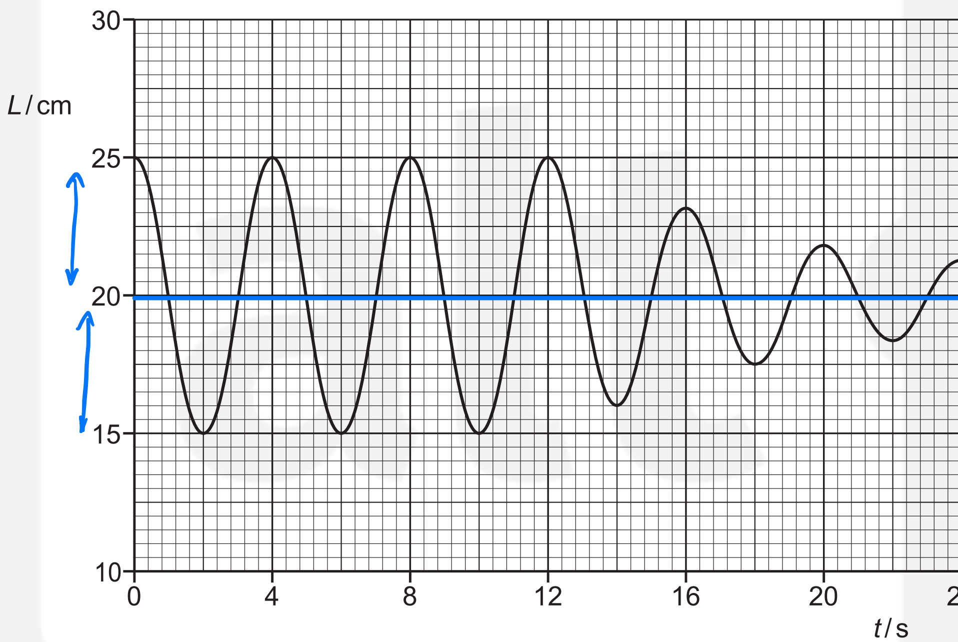


Fig. 4.2

(a) Use Fig. 4.2 to determine:

(i) the initial amplitude of the oscillations

amplitude = 5 cm [1]

(ii) the angular frequency ω of the oscillations

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4}$$

ω = 1.6 rad s^{-1} [2]

(iii) the maximum speed v_0, in cm s^{-1}, of the oscillating trolley.

$$V_0 = \pm \omega x_0$$
$$= 1.57 \times 5$$

v_0 = 7.9 cm s^{-1} [2]

(b) Apart from the quantities in **(a)**, describe what may be deduced from Fig. 4.2 about the motion of the trolley between time $t = 0$ and time $t = 24$ s. No calculations are required.

The trolley started out from near block Y. The time period of oscillation is 4 seconds. At time 12 seconds, light damping starts.

[3]

(c) On Fig. 4.3, sketch the variation with *L* of the velocity *v* of the trolley for its first complete oscillation.

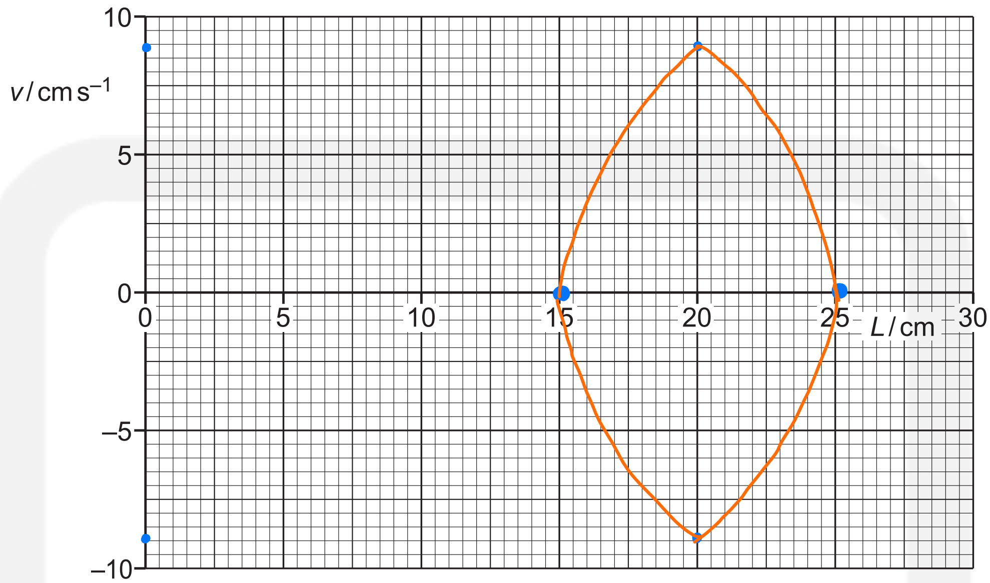


Fig. 4.3

[3]

1:42 PM – 1:50 PM

6 **2021 JUN P42 Q03**

A U-shaped tube contains some liquid. The liquid column in each half of the tube has length L, as shown in Fig. 3.1.

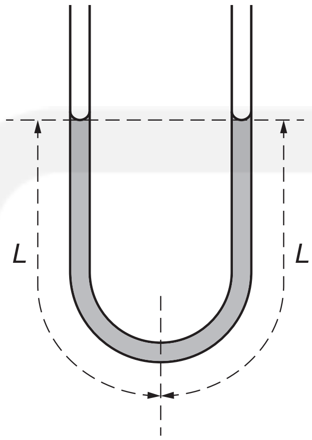


Fig. 3.1

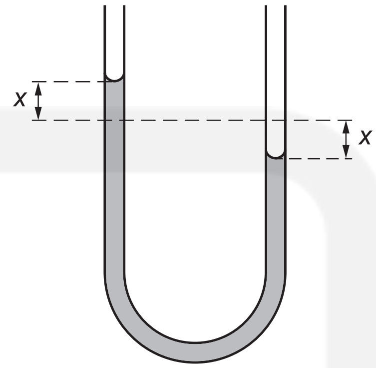


Fig. 3.2

The liquid columns are displaced vertically. The liquid then oscillates in the tube. The liquid levels are displaced from the equilibrium positions as shown in Fig. 3.2.

The acceleration a of the liquid in the tube is related to the displacement x by the expression

$$a = -\left(\frac{g}{L}\right)x$$

g/L constant

$a \propto -x$

where g is the acceleration of free fall.

(a) Explain how the expression shows that the liquid in the tube is undergoing simple harmonic motion.

acceleration is proportional to the displacement as g and L are constant. The minus sign shows that acceleration and displacement are in opposite directions.

.... [3]

$$\frac{4\pi^2}{T^2} = -\left(\frac{g}{L}\right)$$

$$\frac{4\pi^2 L}{g} = T^2$$

$$\frac{4\pi^2 \times 18\times10^{-2}}{9.81} = T^2$$

$T^2 = 0.724$

$T = \sqrt{0.724}$

(b) The length L of each liquid column is 18 cm.

Determine the period T of the oscillations.

$a = -\omega^2 x$

$\omega^2 = -\left(\frac{g}{L}\right)$

T = 0.85 s [3]

(c) The oscillations of the liquid in the tube are damped.
In any one complete cycle of the oscillations, the amplitude decreases by 6.0% of its value at the beginning of the oscillation.

94% of old value.

Determine the ratio

$$\frac{\text{energy of oscillations after 3 cycles}}{\text{initial energy of oscillations}}.$$

1cycle $\rightarrow$ 6%

$18 \xrightarrow[-6\%]{} 16.92 \xrightarrow[-6\%]{} 15.9 \xrightarrow[-6\%]{} 14.95$

$$\frac{\cancel{\tfrac{1}{2}}\cancel{m}\cancel{\omega^2}\times X_{0\,new}^2}{\cancel{\tfrac{1}{2}}\cancel{m}\cancel{\omega^2}\times X_{0\,old}^2} = \frac{X_{o\,new}^2}{X_{o\,old}^2} = \left(\frac{14.95}{18.0}\right)^2 = 0.69$$

ratio = 0.69 [3]

$E_{tot} = \frac{1}{2} m\omega^2 X_0^2$

$E \propto x^2$

Final $X_0 = (0.94)(0.94)(0.94)$

Amp $= 0.83 X_0$

Initial Amp $= X_0$

$$\frac{\text{Final } E}{\text{Initial } E} = \left(\frac{\text{Final Amp}}{\text{Initial Amp}}\right)^2 = \left(\frac{0.83\cancel{X_0}}{\cancel{X_0}}\right)^2 = 0.69$$

ALT.

6 **2021 JUN P42 Q03**

A U-shaped tube contains some liquid. The liquid column in each half of the tube has length L, as shown in Fig. 3.1.

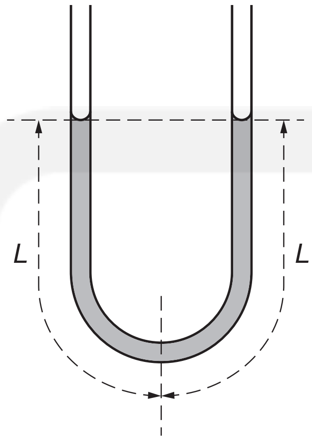


Fig. 3.1

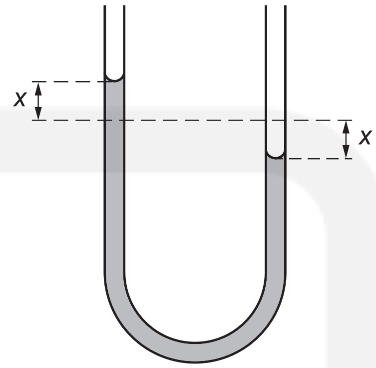


Fig. 3.2

The liquid columns are displaced vertically. The liquid then oscillates in the tube. The liquid levels are displaced from the equilibrium positions as shown in Fig. 3.2.

The acceleration a of the liquid in the tube is related to the displacement x by the expression

$$a = -\left(\frac{g}{L}\right)x$$

$a = -\omega^2 x$

$a \propto -x$

where g is the acceleration of free fall.

(a) Explain how the expression shows that the liquid in the tube is undergoing simple harmonic motion.

Acceleration is proportional to the displacement. The minus sign shows that the acceleration is opposite in direction to the displacement. g/L is constant.

[3]

(b) The length L of each liquid column is 18 cm.

Determine the period T of the oscillations.

$$\omega^2 = \frac{g}{L} = \frac{9.81}{18 \times 10^{-2}} = 54.5$$

$$\omega = 7.382 \text{ rad} = \frac{2\pi}{T}$$

$$T = 2\pi / 7.382$$

T = ...0.85........ s [3]

(c) The oscillations of the liquid in the tube are damped.
In any one complete cycle of the oscillations, the amplitude decreases by 6.0% of its value at the beginning of the oscillation.

Determine the ratio

$$\frac{\text{energy of oscillations after 3 cycles}}{\text{initial energy of oscillations}}.$$

$= x_0(0.94)(0.94)(0.94)$

Amp $= 0.83\, x_0$

Initial Amp $= x_0$

$$E_{TOT} = \frac{1}{2} m\omega^2 x_0^2$$

$$E \propto x_0^2$$

$100 \rightarrow 94 \rightarrow \boxed{} \rightarrow \boxed{}$

ratio = ...0.69........ [3]

$$\frac{\text{Final } E}{\text{Initial } E} = \left(\frac{\text{Final Amplitude}}{\text{Initial Amplitude}}\right)^2 = \left(\frac{0.83\cancel{x_0}}{\cancel{x_0}}\right)^2$$

4 A pendulum consists of a bob (small metal sphere) attached to the end of a piece of string. The other end of the string is attached to a fixed point. The bob oscillates with small oscillations about its equilibrium position, as shown in Fig. 4.1.

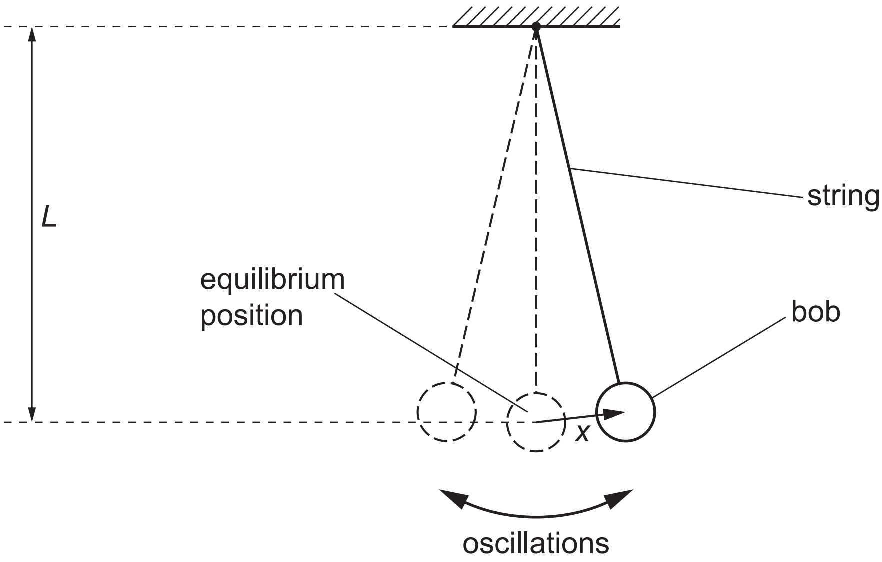


Fig. 4.1 (not to scale)

The length L of the pendulum, measured from the fixed point to the centre of the bob, is 1.24 m.

The acceleration a of the bob varies with its displacement x from the equilibrium position as shown in Fig. 4.2.

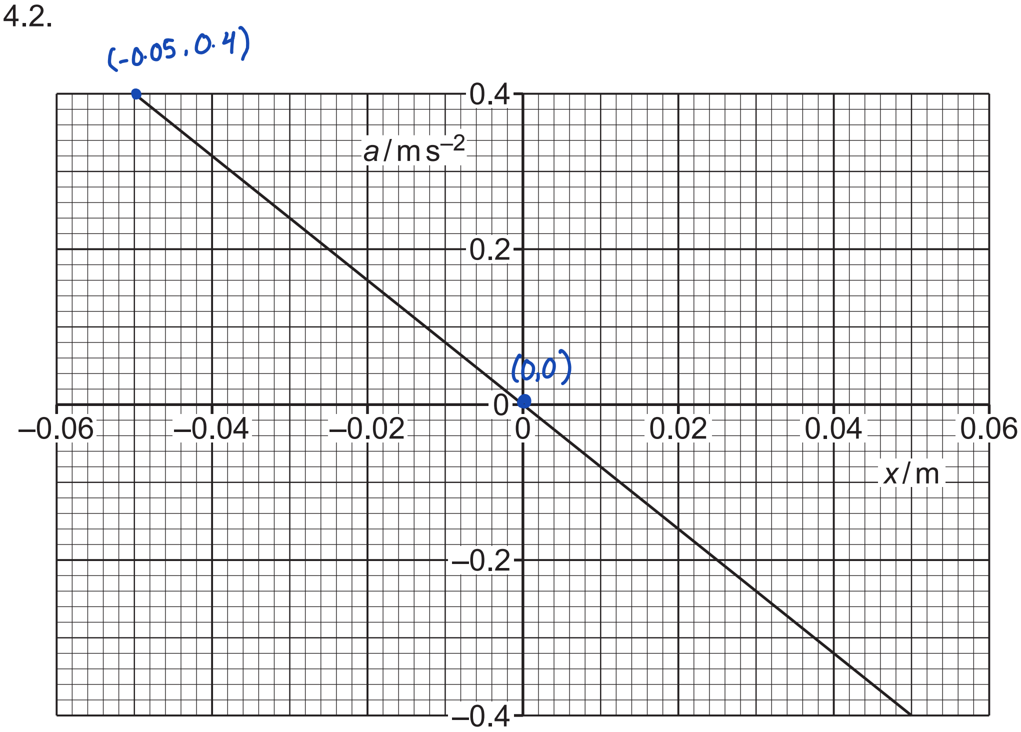


Fig. 4.2

$a = -\omega^2 x$

$y = m\ x$

$a \propto -x$

(a) State how Fig. 4.2 shows that the motion of the pendulum is simple harmonic.

acceleration is proportional to the displacement. The negative gradient shows that acceleration is opposite to displacement.

.... [2]

(b) **(i)** Use Fig. 4.2 to determine the angular frequency ω of the oscillations.

$$m = \frac{0.4 - 0}{-0.05 - 0} = 8 = w^2$$

$$\sqrt{8} = w$$

ω = 2.8 rad s^{-1} [2]

(ii) The angular frequency ω is related to the length L of the pendulum by

$$\omega = \sqrt{\frac{k}{L}}$$

$$(w)^2 = \left(\sqrt{\frac{K}{L}}\right)^2$$

where k is a constant.

Use your answer in **(b)(i)** to determine k. Give a unit with your answer.

$$w^2 = \frac{2\pi}{T} = \frac{1}{s} = s^{-1}$$

$$w^2 = \frac{K}{L}$$

$$K = w^2 L = 8 \times 1.24$$

$$(s^{-1})^2 (m)$$

k = 9.9 unit ms^{-2} [2]

(c) While the pendulum is oscillating, the length of the string is increased in such a way that the total energy of the oscillations remains constant.

Suggest and explain the qualitative effect of this change on the amplitude of the oscillations.

$E_{total} = \frac{1}{2} m w^2 X_0^2$. An increase in the length causes w to decrease. To keep the total energy constant, amplitude increases.

.... [2]

[Total: 8]

$$\downarrow w \propto \frac{1}{\sqrt{L}} \uparrow$$

$$w^2 \downarrow \quad X_0^2 \uparrow$$

$a = -\omega^2 x$

$a \propto -x$

2 **2016 JUN P41 Q03**

(a) State, by reference to displacement, what is meant by *simple harmonic motion*.

Acceleration is proportional but opposite in direction to the displacement of an object in simple harmonic motion.

[2]

(b) A mass is undergoing oscillations in a vertical plane.

The variation with displacement x of the acceleration a of the mass is shown in Fig. 3.1.

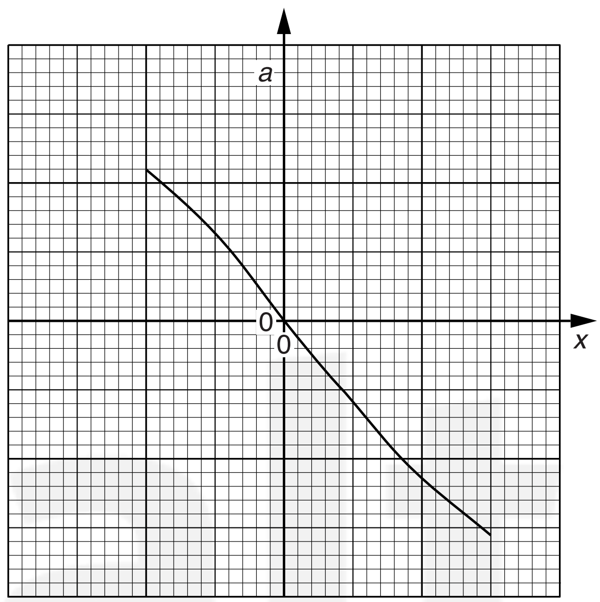


Fig. 3.1

State two reasons why the motion of the mass is not simple harmonic.

1. The graph is not a straight line.

2. Displacements on either side of the mean position are not equal

[2]

(c) A block of wood is floating in a liquid, as shown in Fig. 3.2.

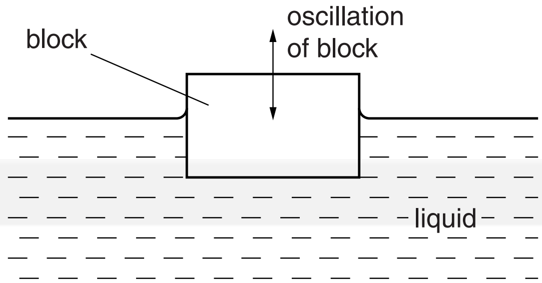


Fig. 3.2

The block is displaced vertically and then released.

The variation with time t of the displacement y of the block from its equilibrium position is shown in Fig. 3.3.

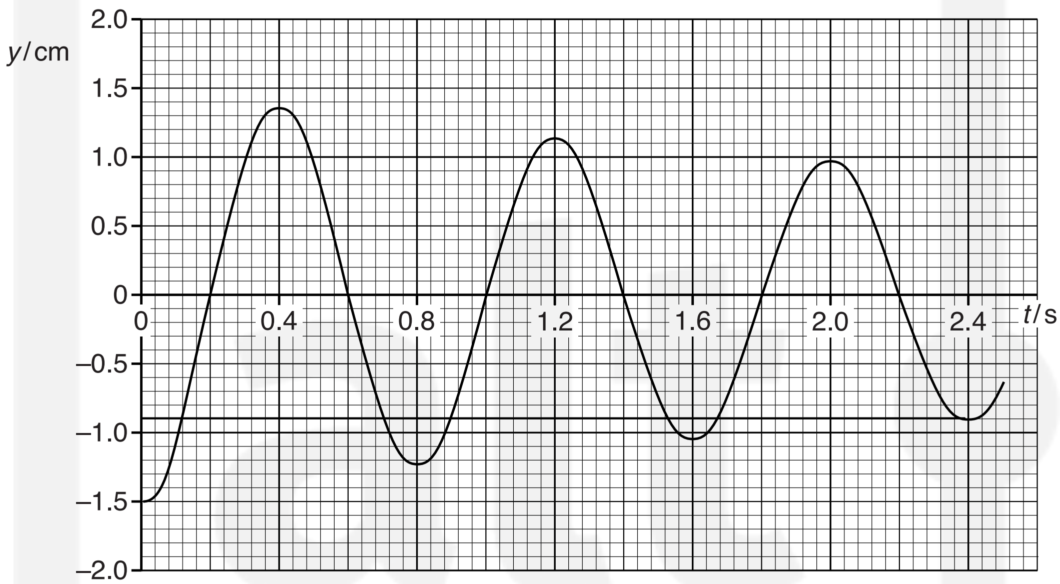


Fig. 3.3

Use data from Fig. 3.3 to determine

(i) the angular frequency ω of the oscillations,

$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8}$

ω = 7.9 rad s^{-1} [2]

(ii) the maximum vertical acceleration of the block.

$a = \pm\omega^2 X_0$

$= 7.85^2 \times 1.5 \times 10^{-2}$

maximum acceleration = 0.93 m s^{-2} [2]

(iii) The block has mass 120 g.

The oscillations of the block are damped. Calculate the loss in energy of the oscillations of the block during the first three complete periods of its oscillations.

$E = \frac{1}{2} m\omega^2 X_0^2$

$\Delta E = \frac{1}{2} m\omega^2 (X_{0\,ini}^2 - X_{0\,final}^2)$

$\frac{1}{2} \times 0.12 \times 7.85^2 \times \left(\left(\frac{1.5}{100}\right)^2 - \left(\frac{0.9}{100}\right)^2\right)$

energy loss = 5.3×10^{-4} J [3]

3 **2016 JUN P42 Q04**

A metal block hangs vertically from one end of a spring. The other end of the spring is tied to a thread that passes over a pulley and is attached to a vibrator, as shown in Fig. 4.1.

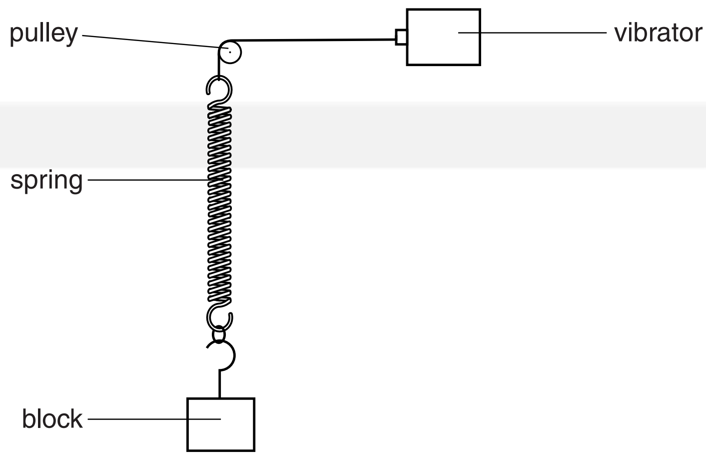


Fig. 4.1

(a) The vibrator is switched off.
The metal block of mass 120 g is displaced vertically and then released. The variation with time t of the displacement y of the block from its equilibrium position is shown in Fig. 4.2.

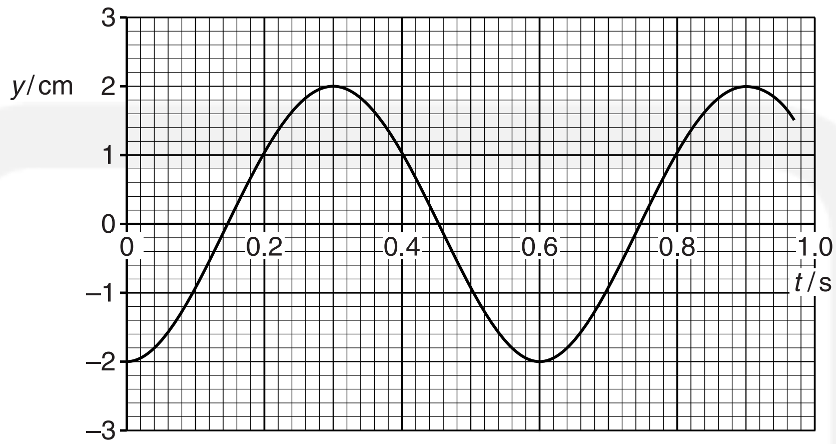


Fig. 4.2

For the vibrations of the block, calculate

(i) the angular frequency ω,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.6}$$

ω = 10.5 rad s^{-1} [2]

(ii) the energy of the vibrations.

$$E = \frac{1}{2} m \omega^2 x_0^2$$

$$= \frac{1}{2} \times 0.12 \times \left(\frac{2\pi}{0.6}\right)^2 (2 \times 10^{-2})^2$$

$$= 2.63 \times 10^{-3} \text{ J}$$

energy = 2.6×10^{-3} J [2]

(b) The vibrator is now switched on.

The frequency of vibration is varied from $0.7f$ to $1.3f$ where f is the frequency of vibration of the block in **(a)**.

For the block, complete Fig. 4.3 to show the variation with frequency of the amplitude of vibration. Label this line A. [3]

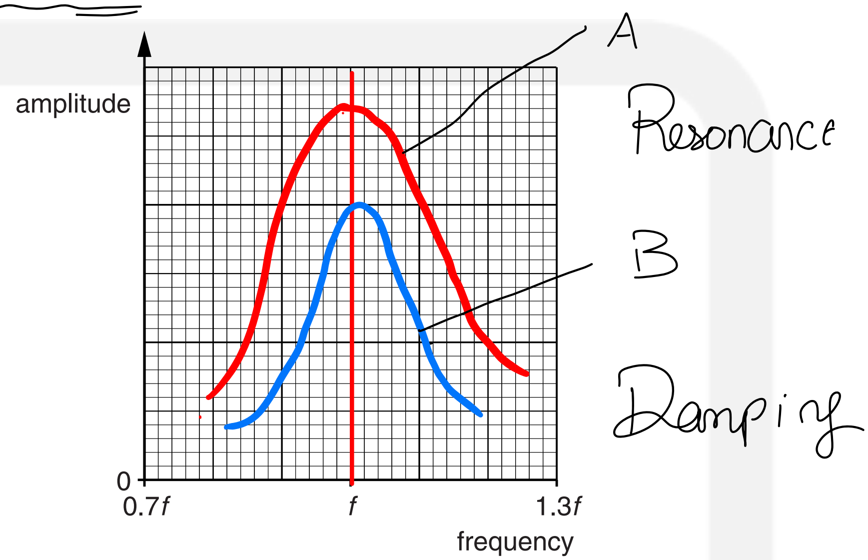


Fig. 4.3

(c) Some light feathers are now attached to the block in **(b)** to increase air resistance.

The frequency of vibration is once again varied from $0.7f$ to $1.3f$. The new amplitude of vibration is measured for each frequency.

On Fig. 4.3, draw a line to show the variation with frequency of the amplitude of vibration. Label this line B. [2]

(ii) The mass of the trolley is 250 g.

Calculate the maximum acceleration a of the trolley.

a = ... $m\,s^{-2}$ [1]

(iii) Use your answer in **(ii)** to determine the period T of the subsequent oscillation.

T = ... s [3]

(iv) The experiment is repeated with an initial displacement of the trolley of 2.4 cm.

State and explain the effect, if any, this change has on the period of the oscillation of the trolley.

....

....

.... [2]

5 2021 JUN P41 Q03

(a) State what is meant by *simple harmonic motion*.

it is a type of oscillatory motion in which acceleration is proportional to the displacement and both are in opposite directions to the other. [2]

(b) A trolley of mass m is held on a horizontal surface by means of two springs. One spring is attached to a fixed point P. The other spring is connected to an oscillator, as shown in Fig. 3.1.

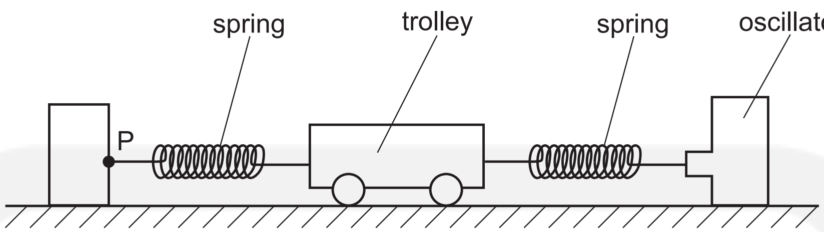


Fig. 3.1

The springs, each having spring constant k of 130 N m^{-1}, are always extended.

The oscillator is switched off. The trolley is displaced along the line of the springs and then released. The resulting oscillations of the trolley are simple harmonic.

The acceleration a of the trolley is given by the expression

$$a = -\left(\frac{2k}{m}\right)x$$

$a = -\omega^2 x$

where x is the displacement of the trolley from its equilibrium position.

The mass of the trolley is 840 g.

Calculate the frequency f of oscillation of the trolley.

$\Rightarrow \omega^2 = \frac{2k}{m}$

$(2\pi f)^2 = \frac{2k}{m}$

$4\pi^2 f^2 = \frac{2k}{m}$

$f^2 = \frac{2k}{4\pi^2 m}$

$\frac{2(130)}{4\pi^2\left(\frac{840}{1000}\right)} = 7.84$

$\sqrt{7.84}$

f = 2.8 Hz [3]

(c) The oscillator in **(b)** is switched on. The frequency of oscillation of the oscillator is varied, keeping its amplitude of oscillation constant.

The amplitude of oscillation of the trolley is seen to vary. The amplitude is a maximum at the frequency calculated in **(b)**.

(i) State the name of the effect giving rise to this maximum.

Resonance [1]

(ii) At any given frequency, the amplitude of oscillation of the trolley is constant.

Explain how this indicates that there are resistive forces opposing the motion of the trolley.

If there were no resistive forces, the amplitude should have increased since energy is provided by oscillator. However, it is not increasing so Resistive Forces are dissipating energy.
[2]

↑ Amp ∝ Energy ↑

6 2020 FEB P42 Q03

(a) A body undergoes simple harmonic motion.

The variation with displacement x of its velocity v is shown in Fig. 3.1.

$V = \omega\sqrt{X_0^2 - x^2}$

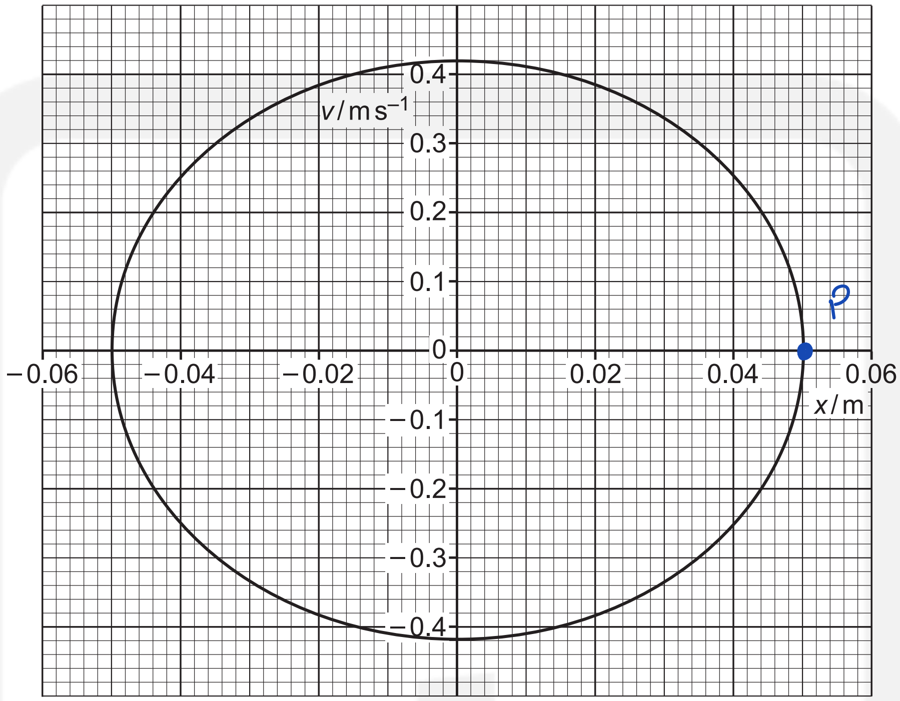


Fig. 3.1

(i) State the amplitude x_o of the oscillations.

x_o = 0.05 .... m [1]

(ii) Calculate the period T of the oscillations.

$\therefore \omega = \frac{2\pi}{T}$

$V_0 = \pm \omega X_0$

$0.42 = \omega \times 0.05$

$\omega = 8.4$

$\frac{2\pi}{T} = 8.4$

$T = 2\pi / 8.4$

T = 0.75 .... s [3]

(iii) On Fig. 3.1, label with a P a point where the body has **maximum** potential energy. [1]

$E_K = \frac{1}{2} m\omega^2 (X_0^2 - x^2)$

4 **2019 NOV P41 Q04**

A mass is suspended vertically from a fixed point by means of a spring, as illustrated in Fig. 4.1.

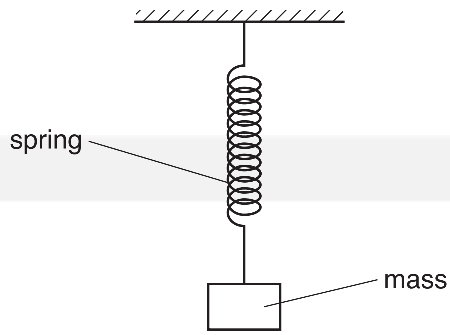


Fig. 4.1

The mass is oscillating vertically. The variation with displacement *x* of the acceleration *a* of the mass is shown in Fig. 4.2.

$a = -w^2 x$

$y = m X$

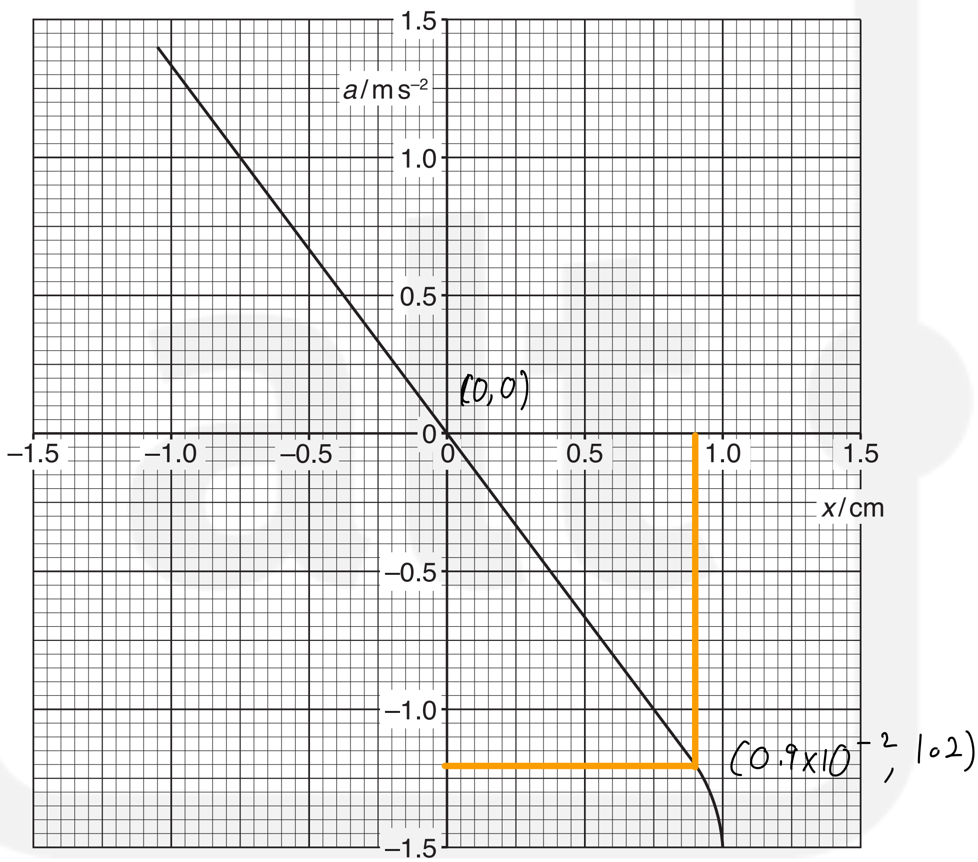


Fig. 4.2

(a) **(i)** State what is meant by the *displacement* of the mass on the spring.

Displacement is the distance of the mass from mean position in a specified direction .... [1]

(ii) Suggest how Fig. 4.2 shows that the mass is not performing simple harmonic motion.

Curve in the graph showing that gradient is not constant. .... [1]

(b) **(i)** The amplitude of oscillation of the mass may be changed.

State the maximum amplitude x_0 for which the oscillations are simple harmonic.

x_0 = 0.90 cm [1]

(ii) For the simple harmonic oscillations of the mass, use Fig. 4.2 to determine the frequency of the oscillations.

$\therefore \omega = 2\pi f$

$$m = \frac{1.2}{0.9 \times 10^{-2}} = 133.3 = \omega^2$$

$$\omega = \sqrt{133.3}$$

$$\omega = 11.547$$

$$2\pi f = 11.547$$

frequency = 1.84 Hz [3]

(c) The maximum speed of the mass when oscillating with simple harmonic motion of amplitude x_0 is v_0.

On Fig. 4.3, show the variation with displacement x of the velocity v of the mass for displacements from $+x_0$ to $-x_0$.

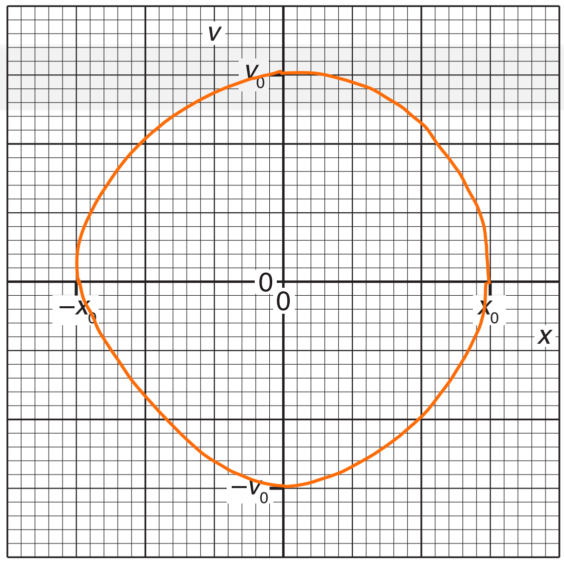


Fig. 4.3

[2]

Oscillations WS2

1 **2020 JUN P42 Q04**

A dish is made from a section of a hollow glass sphere.

The dish, fixed to a horizontal table, contains a small solid ball of mass 45 g, as shown in Fig. 4.1.

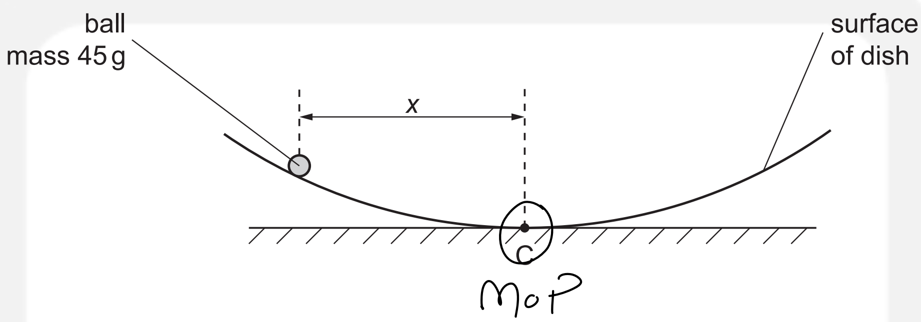


Fig. 4.1

The horizontal displacement of the ball from the centre C of the dish is x.

Initially, the ball is held at rest with distance x = 3.0 cm.

The ball is then released. The variation with time t of the horizontal displacement x of the ball from point C is shown in Fig. 4.2.

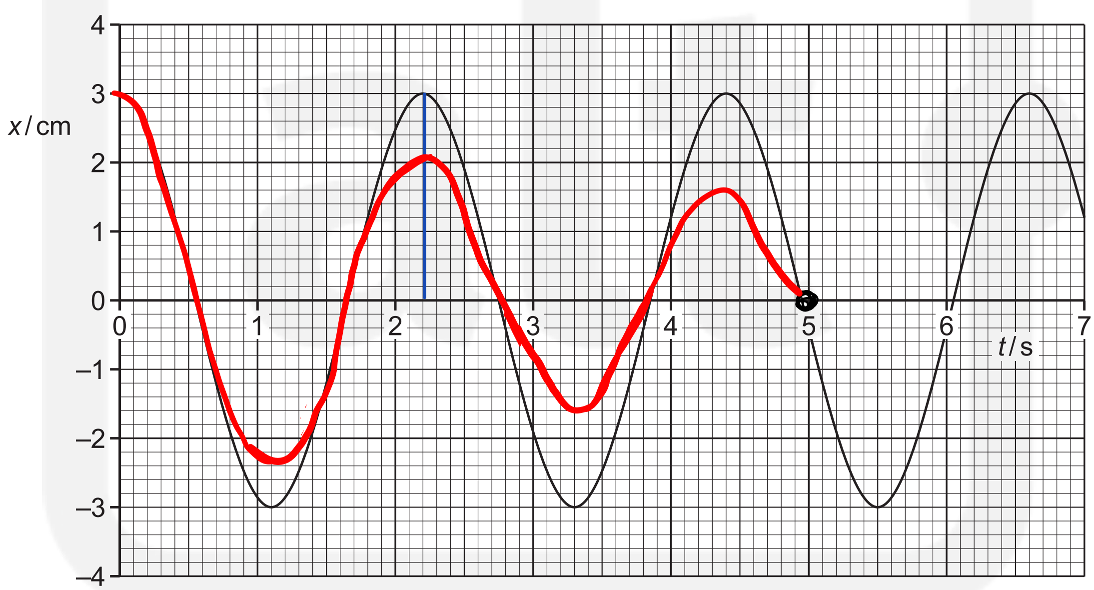


Fig. 4.2

(c) Some moisture collects on the surface of the dish so that the motion of the ball becomes lightly damped.

On the axes of Fig. 4.2, draw a line to show the lightly damped motion of the ball for the first 5.0 s after the release of the ball. [3]

The motion of the ball in the dish is simple harmonic with its acceleration a given by the expression

$$a = -\left(\frac{g}{R}\right)x$$

where g is the acceleration of free fall and R is a constant that depends on the dimensions of the dish and the ball.

(a) Use Fig. 4.2 to show that the angular frequency ω of oscillation of the ball in the dish is $2.9\,\text{rad}\,\text{s}^{-1}$.

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.2} = 2.855$$

$$\approx 2.9\ \text{rad}\,\text{s}^{-1}$$

[1]

(b) Use the information in **(a)** to:

(i) determine R

$$a = -\omega^2 x$$

$$\omega^2 = \frac{g}{R}$$

$$(2.855)^2 = \frac{9.81}{R}$$

R = 1.20 m [2]

(ii) calculate the speed of the ball as it passes over the centre C of the dish.

$$V_0 = \omega x_0$$

$$= 2.855 \times 3 \times 10^{-2}$$

$$= 0.08565$$

speed = 0.086 $\text{m}\,\text{s}^{-1}$ [2]

2 2017 JUN P42 Q03

A bar magnet of mass 250 g is suspended from the free end of a spring, as illustrated in Fig. 3.1.

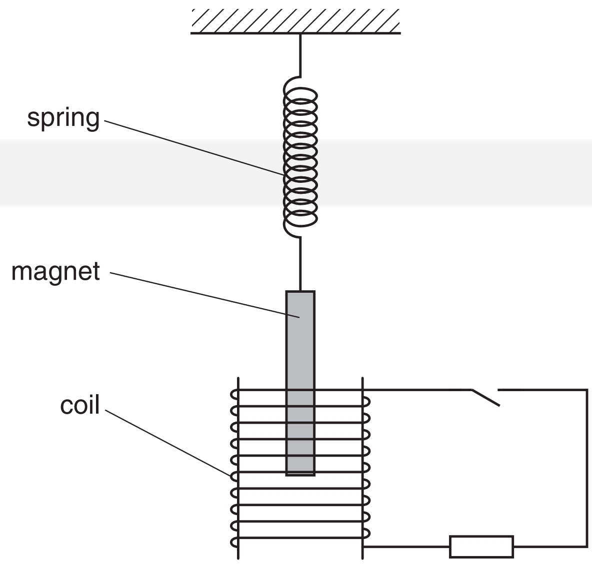


Fig. 3.1

The magnet hangs so that one pole is near the centre of a coil of wire.

The coil is connected in series with a resistor and a switch. The switch is open.

The magnet is displaced vertically and then allowed to oscillate with one pole remaining inside the coil. The other pole remains outside the coil.

At time $t = 0$, the magnet is oscillating freely as it passes through its equilibrium position. At time $t = 6.0\,\text{s}$, the switch in the circuit is closed.

The variation with time t of the vertical displacement y of the magnet is shown in Fig. 3.2.

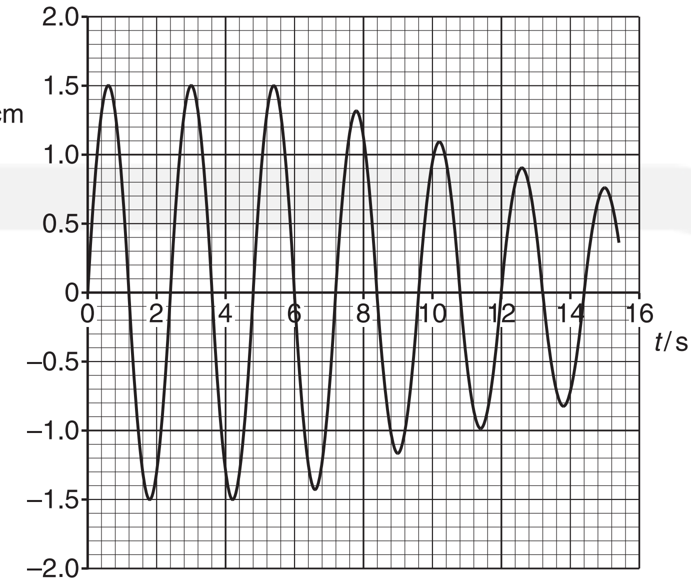


Fig. 3.2

(a) For the oscillating magnet, use data from Fig. 3.2 to calculate, to two significant figures,

(i) the frequency f,

$\omega = \frac{2\pi}{T}$

$\cancel{2\pi} f = \frac{\cancel{2\pi}}{T}$

$f = \frac{1}{T} = \frac{1}{2.2}$

$f =$ .... Hz [2]

(ii) the energy of the oscillations during the time $t = 0$ to time $t = 6.0$ s.

energy = ... J [3]

Oscillations WS2

1 2017 JUN P41 Q02

A bar magnet of mass 180 g is suspended from the free end of a spring, as illustrated in Fig. 2.1.

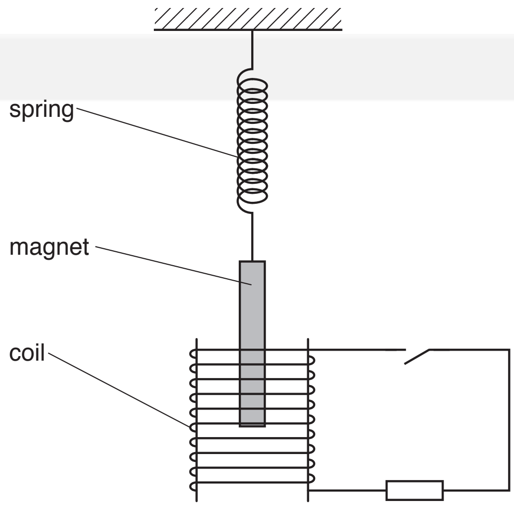


Fig. 2.1

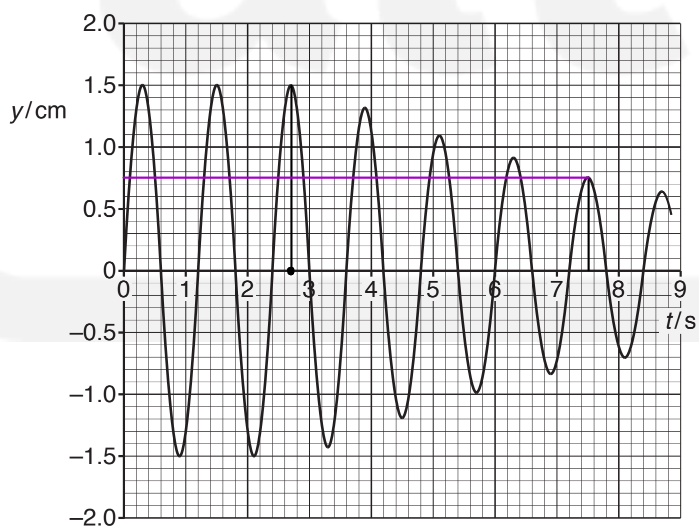
The magnet hangs so that one pole is near the centre of a coil of wire.

The coil is connected in series with a resistor and a switch. The switch is open.

The magnet is displaced vertically and then allowed to oscillate with one pole remaining inside the coil. The other pole remains outside the coil.

At time $t = 0$, the magnet is oscillating freely as it passes through its equilibrium position. At time $t = 3.0$ s, the switch in the circuit is closed.

The variation with time t of the vertical displacement y of the magnet is shown in Fig. 2.2.



(a) Determine, to two significant figures, the frequency of oscillation of the magnet.

$$f = 1/T = \frac{1}{1.2}$$

frequency = 0.83 Hz [2]

(b) State whether the closing of the switch gives rise to light, heavy or critical damping.

light [1]

(c) Calculate the change in the energy ΔE of oscillation of the magnet between time $t = 2.7$ s and time $t = 7.5$ s. Explain your working.

$t = 2.7$, $x_0 = 1.5 \times 10^{-2}$

$t = 7.5$, $x_0 = 0.75 \times 10^{-2}$

$\omega = 2\pi f = 2\pi \times 0.83 = \frac{5}{3}\pi$

$$\Delta E = \frac{1}{2} m \omega^2 \left(x_{0_{ini}}^2 - x_{0_{final}}^2\right)$$

$$\frac{1}{2} \times 0.180 \times \left(\frac{5}{3}\pi\right)^2 \left(0.015^2 - 0.0075^2\right)$$

$$= 4.1637 \times 10^{-4}$$

ΔE = 4.2×10^{-4} J [6]