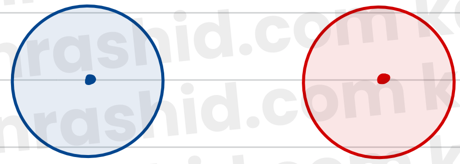
ELECTRIC FIELDS

Gravitation	Electric Field
$F = \frac{GMm}{r^2}$	$F = \frac{kq_1q_2}{r^2}$
$g = \frac{GM}{r^2}$	$E = \frac{kq}{r^2}$
$U_p = -\frac{GMm}{r}$	$EPE = \frac{kq_1q_2}{r}$
$\phi = -\frac{GM}{r}$	$V = \frac{kq}{r}$
$\Delta\phi \times m = \frac{1}{2}m(v^2-u^2)$	$\Delta V \times q = \frac{1}{2}m(v^2-u^2)$



point mass

All mass at the centre. No field or force inside.

point charge

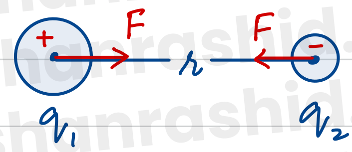
All charge at the centre. No field or force inside.

Charges

- Two types ; positive (protons) and negative (electrons)
- protons : +1 electron : −1
- $1p = +1.6\times10^{-19}C$ $1e = -1.6\times10^{-19}C$ (elementary charge)
- Like charges repel ; unlike attract.

Coulomb's Law

The force between two point charges is directly proportional to their product and inversely proportional to the square of their seperation.



$F \propto q_1q_2$, $F \propto \frac{1}{r^2}$ $\Big\}$ $F \propto \frac{q_1q_2}{r^2}$

$$\boxed{F = \frac{kq_1q_2}{r^2}}$$

$\boxed{k = 8.99\times10^9}$

$k = \frac{1}{4\pi\varepsilon_0}$

ε_0: Permittivity of free space, $\varepsilon_0 = 8.85\times10^{-12}$

q_1, q_2 : charges

r : distance between centres

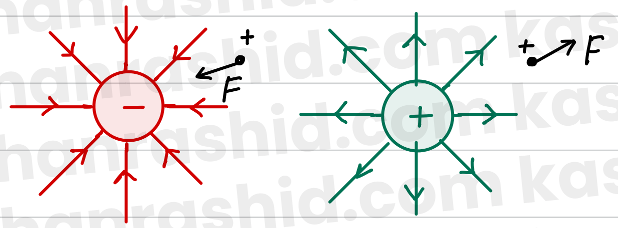
F : force

k : constant

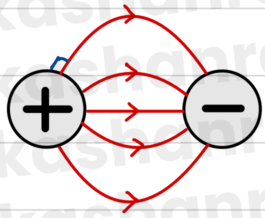
Comparing constants of gravitational and electric force, $G = 6.67 \times 10^{-11}$ } $\times 10^{20}$ larger!
$k = 8.99 \times 10^{9}$

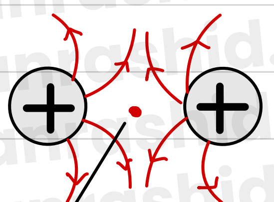
Electric Field

A region in space around a charge where another charge experiences electric force.

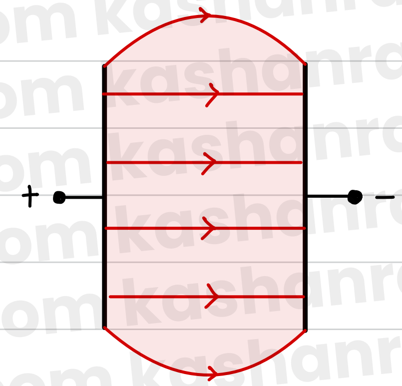


- ☑ Field lines are drawn radially inwards or outwards.
- ☑ The direction of field lines tell the direction of force on a small (test) positive charge.
  - (test) ↓ so small such that it doesnt effect the field lines of the bigger charge.
- ☑ Gap between field lines tell about the strength of field.
  - • less gap ⟶ strong field • more gap ⟶ weak field

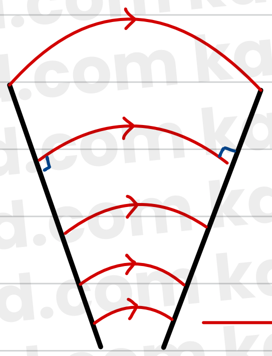




Null Point
Resultant Electric Field is zero!



Parallel field lines showing "Uniform" field

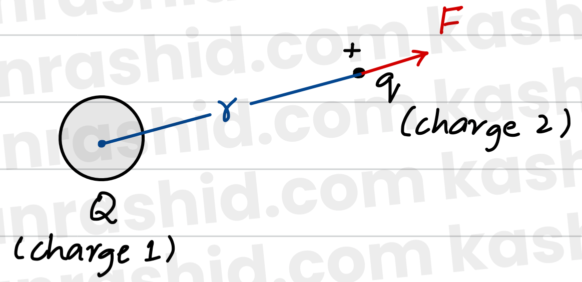


⟶ more gap; weak field

⟶ less gap; strong field

Electric Field Strength (E)

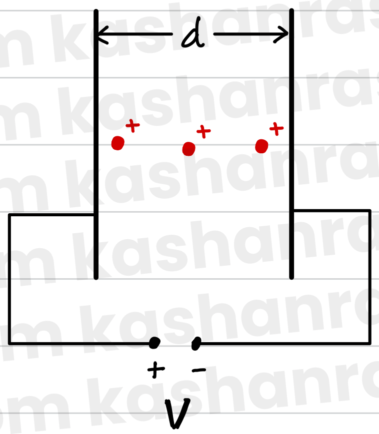
It is the force per unit positive charge at a point in an Electric field.



$$\boxed{E = \frac{F}{q}}$$

- SI Unit : NC^{-1} or Vm^{-1}
- Vector quantity.

For a uniform electric field (between parallel plates)



$$\boxed{E = \frac{V}{d}}$$

V → P.d. between plates

d → plate seperation

E → Electric Field Strength

$E = \frac{V}{d}$ also $E = \frac{F}{q}$

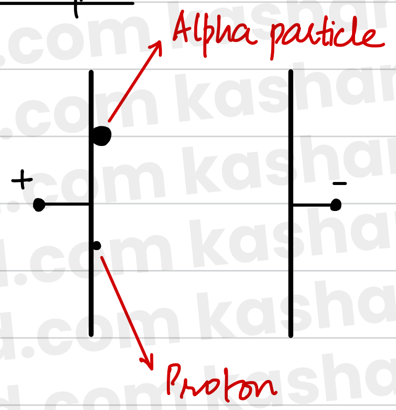
$\frac{F}{q} = \frac{V}{d} \rightarrow \boxed{F = \frac{Vq}{d}}$

as ↓ $F = ma$

$ma = \frac{Vq}{d} \rightarrow \boxed{a = \frac{Vq}{md}}$

As the field was uniform, a charged particle between plates would experience same magnitude of force & hence acceleration everywhere.

Example



An alpha particle & a proton are released from rest near positive plate. They accelerate towards the negative plate.

Alpha particle :

$m_\alpha = 4u \quad q_\alpha = +2e$

Proton : $m_p = 1u \quad q_p = +1e$

Determine

a) F_α / F_p b) a_α / a_p

{ $1u = 1.66 \times 10^{-27}$ kg }

{ $1e = 1.6 \times 10^{-19}$ C }

a) $E = \frac{F}{q}$ so $F = Eq$

$$\frac{F_\alpha}{F_p} = \frac{\cancel{E} q_\alpha}{\cancel{E} q_p}$$

$$\frac{F_\alpha}{F_p} = \frac{2\cancel{e}}{1\cancel{e}}$$

$$\boxed{\frac{F_\alpha}{F_p} = \frac{2}{1}}$$

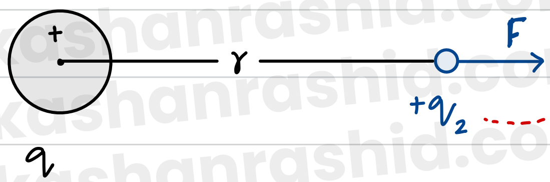
b) $\frac{F_\alpha}{F_p} = \frac{2}{1}$

$$\frac{m_\alpha a_\alpha}{m_p a_p} = \frac{2}{1}$$

$$\frac{4\cancel{u}}{\cancel{u}} \frac{a_\alpha}{a_p} = \frac{2}{1}$$

$$\boxed{\frac{a_\alpha}{a_p} = \frac{1}{2}}$$

For an Electric Field around point charges (sphere)



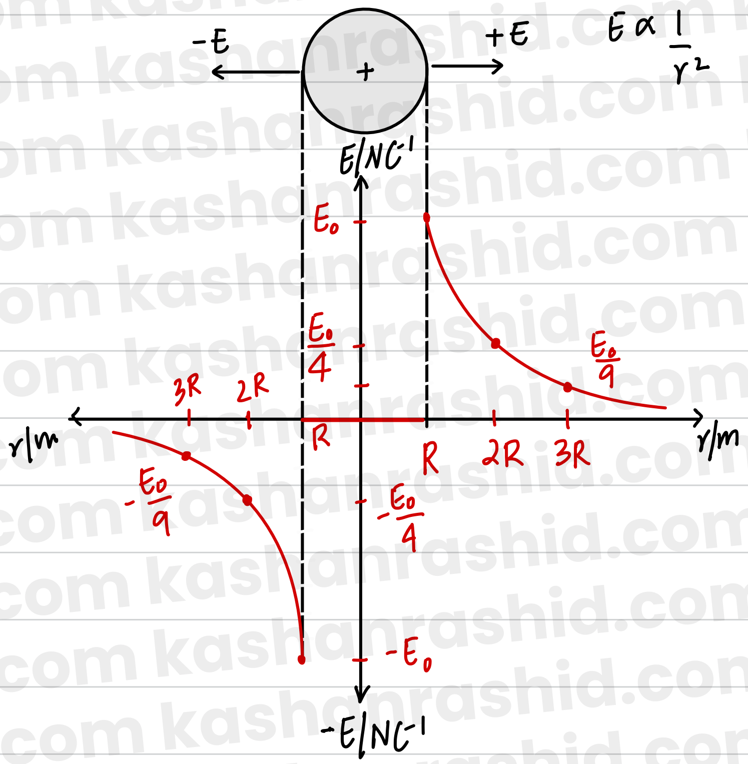
$E = \frac{F}{\underbrace{q}_{+q_2}}$ and $F = \frac{kq_1q_2}{r^2}$

$$E = \frac{kq_1\cancel{q_2}}{r^2 \times \underbrace{\cancel{q}}_{q_2}} \longrightarrow \boxed{E = \frac{kq}{r^2}}$$

q: charge whose field strength is to be determined

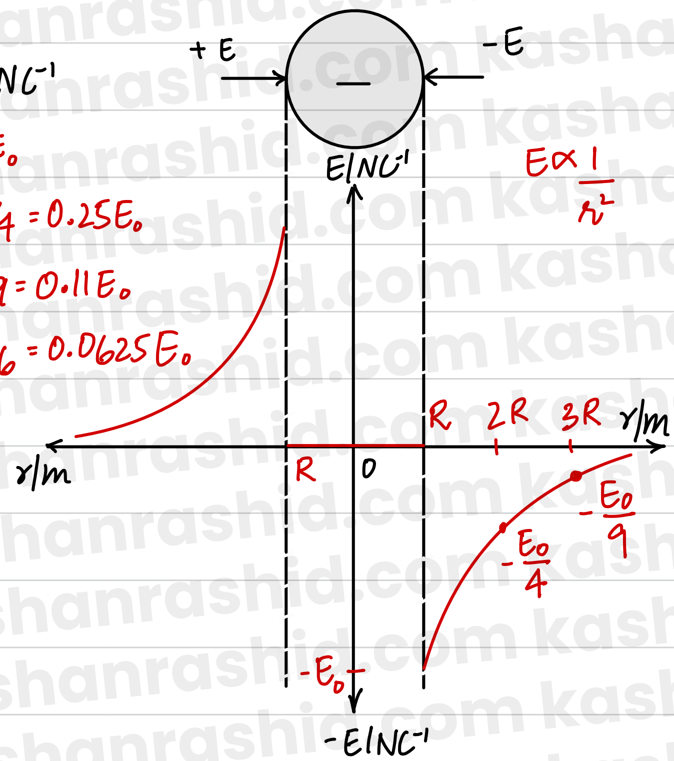
r: distance to the point from center.

E-r graphs for an isolated point charge

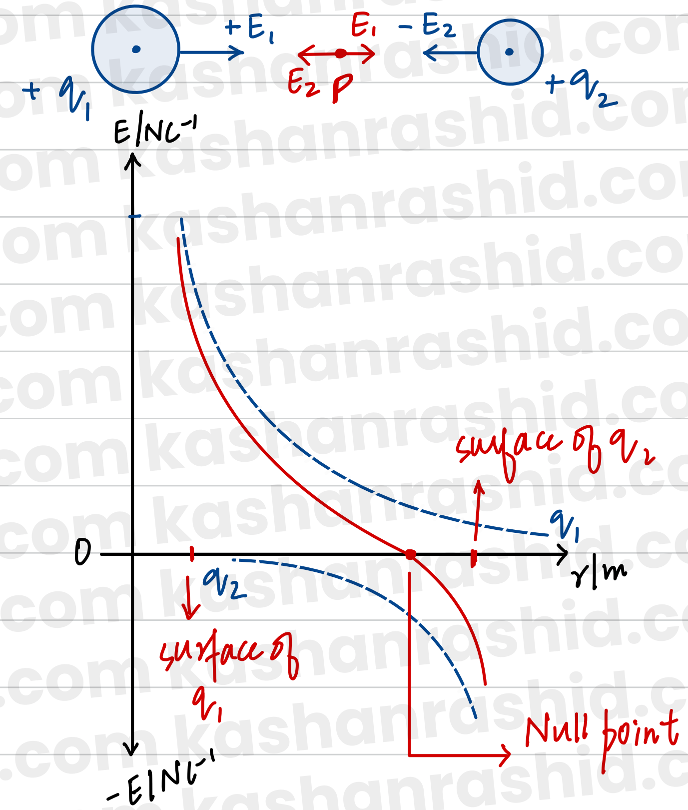


$E \propto \frac{1}{r^2}$

r/m	E/NC⁻¹
R	E_0
2R	$E_0/4 = 0.25E_0$
3R	$E_0/9 = 0.11E_0$
4R	$E_0/16 = 0.0625E_0$



E-r graph between like charges



$$\boxed{E_{net} = E_1 - E_2}$$

At Null point, $E_{net} = 0$

$$E_1 = E_2$$

$$\frac{\cancel{k}q_1}{r_1^2} = \frac{\cancel{k}q_2}{r_2^2}$$

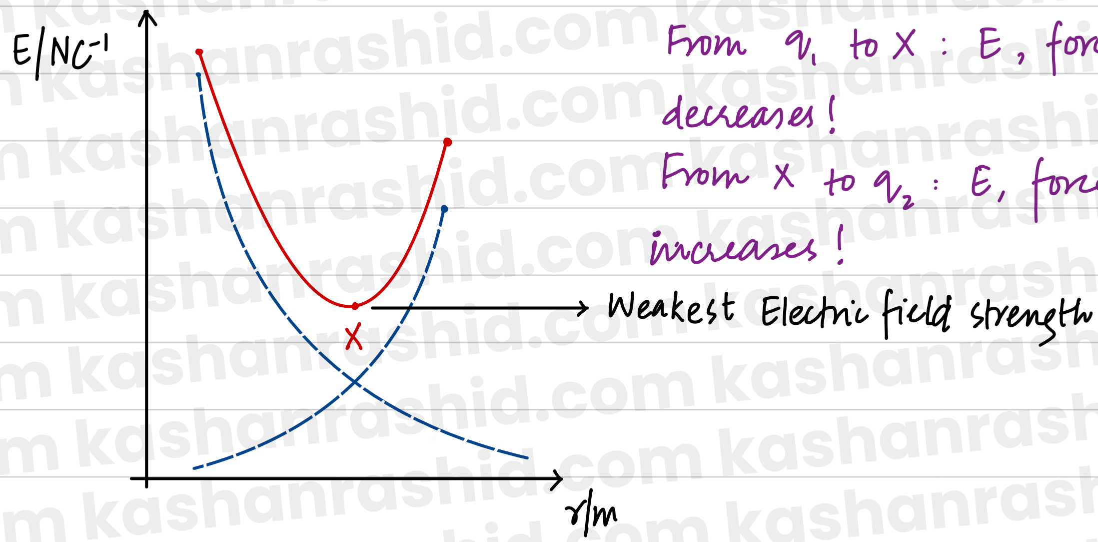
$$\boxed{\frac{q_1}{q_2} = \left(\frac{r_1}{r_2}\right)^2}$$

Null point is near the weaker charge.

E-r graph for unlike charges

$\xrightarrow{E_1}\bullet\xrightarrow{E_2}$

$E_{net} = E_1 + E_2$



From q_1 to X : E, force and acceleration decreases!

From X to q_2 : E, force and acceleration increases!

Electric Potential (V)

Workdone per unit positive charge to bring it from infinity to a point in an electric field.

$\phi = -\frac{GM}{r}$ similarly

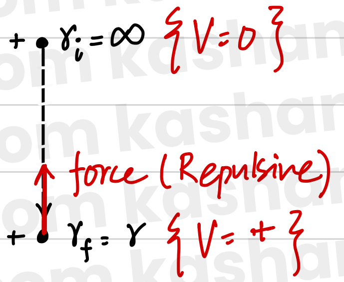
$$\boxed{V = \frac{kq}{r}}$$

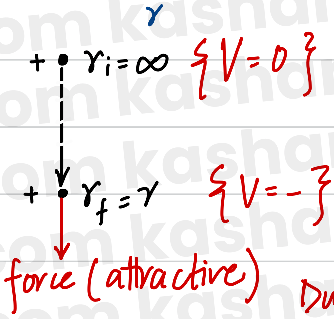
SI Unit: Volts (V)

Scalar quantity

positive charge: $V = +\frac{kq}{r}$

negative charge: $V = -\frac{kq}{r}$

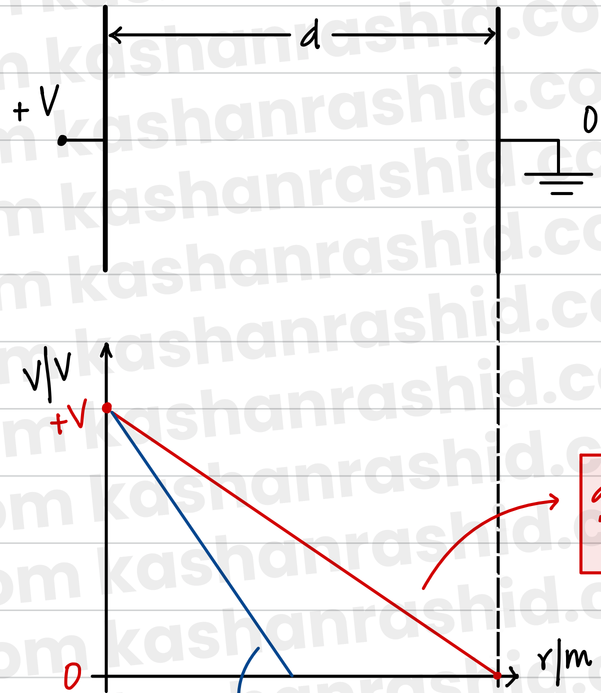




Due to repulsive force Work is done on the ref. positive charge to bring it closer to other charge.

Due to attractive force, work is done by the ref. positive charge as it gets closer to the negative charge.

V-r graph for parallel metal plates



"Electric field strength is equal to the negative of electric potential gradient."

$$\boxed{E = -\frac{\Delta V}{\Delta r}} \rightarrow E = \frac{V}{d}$$

V → p.d

$\boxed{\text{grad} = -\text{Electric field strength}}$

field strength uniform → gradient uniform

"stronger electric field"
more gradient

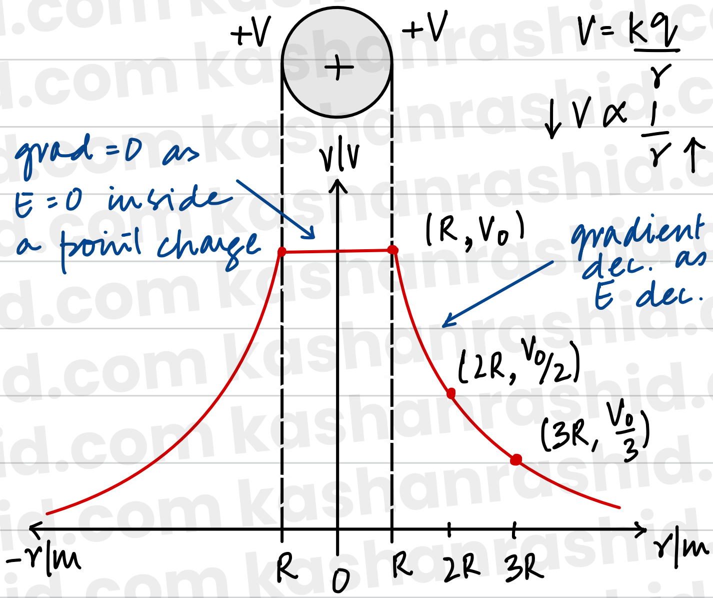
V-r graph for an isolated point charge

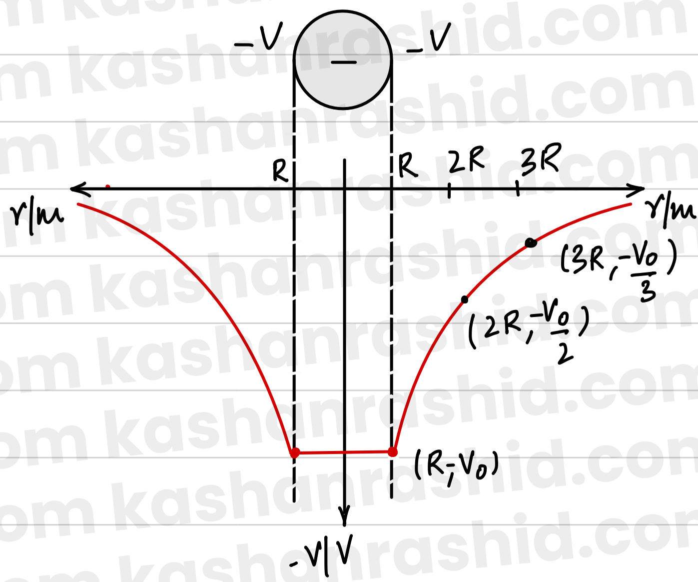
$V = \frac{kq}{r}$

$\downarrow V \propto \frac{1}{r} \uparrow$

grad = 0 as E = 0 inside a point charge

gradient dec. as E dec.





If a small positive charge (e.g. proton, alpha particle etc.) is brought close to another charge.

$$\boxed{\text{change in Electric Potential Energy} = \text{change in Kinetic Energy}}$$

loss in EPE = gain in K.E

$$-\Delta V \times q = \frac{1}{2} m (v^2 - u^2)$$

gain in EPE = loss in K.E

$$\Delta V \times q = -\frac{1}{2} m (v^2 - u^2)$$

where $\Delta V = V_f - V_i$

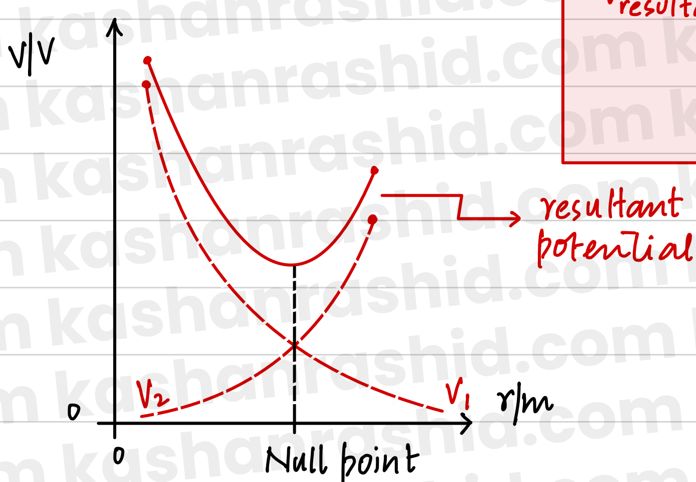
either $\Delta V = \frac{kq}{r_f} - \frac{kq}{r_i}$

or read from graph

$\frac{q}{m}$ ratio ⇒ charge-to-mass

V-r graph between two point like charges

$$\boxed{\begin{aligned} V_{resultant} &= V_1 + V_2 \\ V_r &= \frac{kq_1}{r_1} + \frac{kq_2}{r_2} \end{aligned}}$$



(grad = 0 | E = 0)

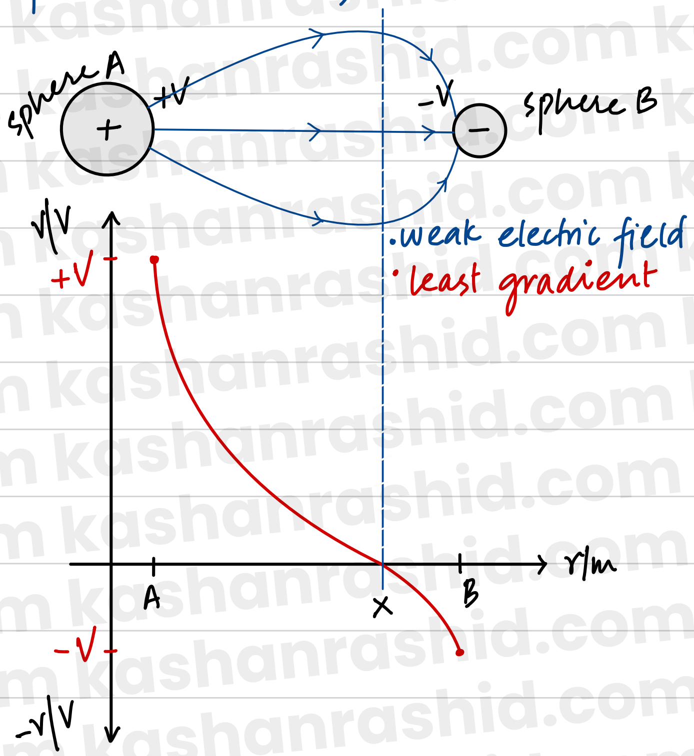
A proton released from rest from the surface of q_1 will have max speed at Null point.

$q_1 \longrightarrow$ Null point : speed inc.

Null point $\longrightarrow q_2$: speed dec.

V-r graph between two point unlike charges

$V = +\frac{kq}{r}$ (positive) $V = -\frac{kq}{r}$ (negative)



from A to X
→ E dec. → F dec. → a dec.

from X to B
→ E inc. → F inc. → a inc.

A to X to B : speed of the positive charge travelling between A to B keeps on increasing.

4 Two point charges A and B each have a charge of $+6.4 \times 10^{-19}$ C. They are separated in a vacuum by a distance of 12.0 μm, as shown in Fig. 4.1.

For Examiner's Use

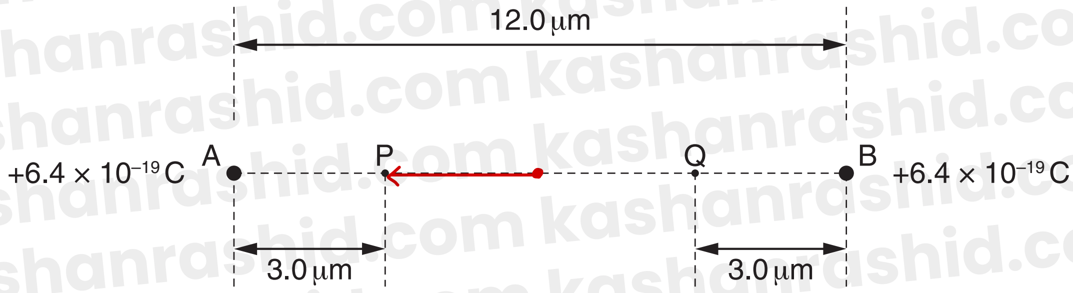


Fig. 4.1

Points P and Q are situated on the line AB. Point P is 3.0 μm from charge A and point Q is 3.0 μm from charge B.

(a) Calculate the force of repulsion between the charges A and B.

$F = \frac{kq_1q_2}{r^2}$

$F = \frac{8.99 \times 10^9 \times (6.4 \times 10^{-19})^2}{(12 \times 10^{-6})^2}$

$F = 2.6 \times 10^{-17}$

force = 2.6×10^{-17} N [3]

(b) Explain why, without any calculation, when a small test charge is moved from point P to point Q, the net work done is zero.

Both P and Q have the same electric potential, hence the change in potential and workdone is zero. [2]

(c) Calculate the work done by an electron in moving from the midpoint of line AB to point P.

Workdone = change in EPE
$= \Delta V \times q$
$= (V_f - V_i) \times q$

$V = \frac{kq}{r}$

final
$V_P = \frac{kq_A}{r_A} + \frac{kq_B}{r_B}$
$= 8.99 \times 10^9 \times 6.4 \times 10^{-19} \left(\frac{1}{3\mu m} + \frac{1}{9\mu m}\right)$
$V_P = 2.6 \times 10^{-3}$ V

initial
$V_m = \frac{kq_A}{r_A} + \frac{kq_B}{r_B}$
$= \left(\frac{8.99 \times 10^9 \times 6.4 \times 10^{-19}}{6 \times 10^{-6}}\right) \times 2$
$V_m = 1.92 \times 10^{-3}$ V

work done = ... J [4]

Work = $(2.6 \times 10^{-3} - 1.92 \times 10^{-3}) \times 1.6 \times 10^{-19}$ so Work = 1.1×10^{-22} J

4 **(a)** Define *electric potential* at a point.

Workdone per unit positive charge to move it from infinity to a point in an electric field.

[2]

$u=0$

(b) A charged particle is accelerated from rest in a vacuum through a potential difference V.
Show that the final speed v of the particle is given by the expression $\Delta V = V$

$$v = \sqrt{\left(\frac{2Vq}{m}\right)}$$

where $\frac{q}{m}$ is the ratio of the charge to the mass (the specific charge) of the particle.

change in EPE = change in KE

$$\Delta V \times q = \frac{1}{2} m (v^2 - u^2)$$

$$V \times q = \frac{1}{2} m (v^2 - 0)$$

$$v = \sqrt{\frac{2Vq}{m}}$$

[2]

q/m

(c) A particle with specific charge $+9.58 \times 10^7\,\mathrm{C\,kg^{-1}}$ is moving in a vacuum towards a fixed metal sphere, as illustrated in Fig. 4.1.

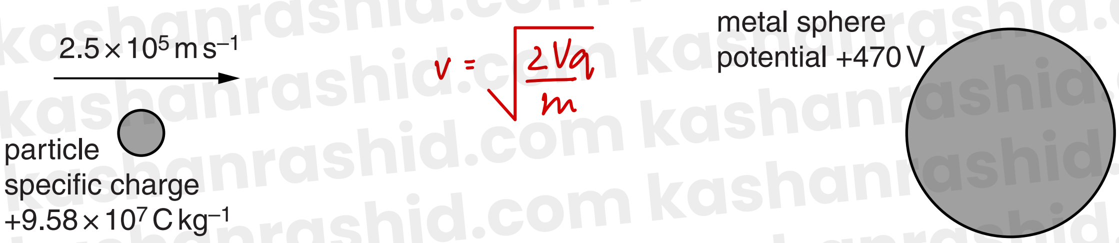


Fig. 4.1

infinity ($V_i = 0$)

The initial speed of the particle is $2.5 \times 10^5\,\mathrm{m\,s^{-1}}$ when it is a long distance from the sphere.
The sphere is positively charged and has a potential of +470 V.

Use the expression in **(b)** to determine whether the particle will reach the surface of the sphere.

$$v = \sqrt{2 \times 470 \times 9.58 \times 10^7}$$

$v = 3 \times 10^5$ m/s

will not reach.

$$(2.5 \times 10^5)^2 = 2V \times 9.58 \times 10^7$$

$V = 326$ V

will not reach

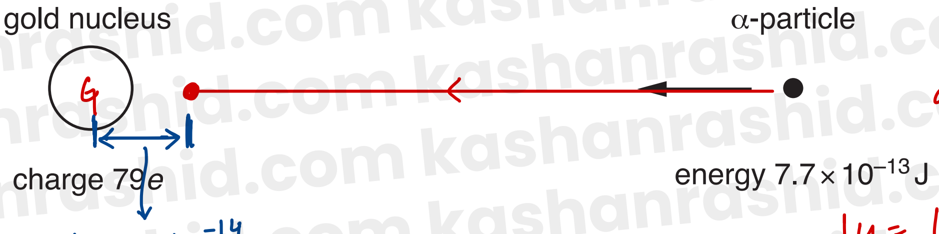
[3]

5 An α-particle is travelling in a vacuum towards the centre of a gold nucleus, as illustrated in Fig. 5.1.

$V = \frac{kq}{r}$

$k = 8.99 \times 10^9$

Alpha
$m = 4u$
$q = 2e$



4.7×10^{-14} m

$1u = 1.66 \times 10^{-27}$ kg
$1e = 1.6 \times 10^{-19}$ C

Fig. 5.1

The gold nucleus has charge 79*e*.
The gold nucleus and the α-particle may be assumed to behave as point charges.
At a large distance from the gold nucleus, the α-particle has energy 7.7×10^{-13} J.

∞ ; $V = 0$

(a) The α-particle does not collide with the gold nucleus. Show that the radius of the gold nucleus must be less than 4.7×10^{-14} m.

loss in K.E = gain EPE

$7.7 \times 10^{-13} = \Delta V \times q_\alpha$

$7.7 \times 10^{-13} = \left(\frac{kq_G}{r_G} - 0 \right) \times 2e$

$7.7 \times 10^{-13} = \frac{8.99 \times 10^9 \times 79e \times 2e}{r_G}$

$r_G = 4.7 \times 10^{-14}$ m

α-particle can reach as close as 4.7×10^{-14} m from the center of nucleus. If collision doesnot occur, this means radius is even smaller than this.

[3]

(b) Determine the acceleration of the α-particle for a separation of 4.7×10^{-14} m between the centres of the gold nucleus and of the α-particle.

$F = \frac{kq_1q_2}{r^2}$

$F = \frac{8.99 \times 10^9 \times 2e \times 79e}{(4.7 \times 10^{-14})^2}$

$F = 16.46$ N

$F = ma$

$16.46 = 4u \times a$

$a = 2.47 \times 10^{27}$ m/s²

acceleration = ...2.5×10^{27}...... m s⁻² [3]

(c) In an α-particle scattering experiment, the beam of α-particles is incident on a very thin gold foil.
Suggest why the gold foil must be very thin.

So that α-particles can cross through.

.... [1]

[Total: 7]

6 Two solid metal spheres A and B, each of radius 1.5 cm, are situated in a vacuum. Their centres are separated by a distance of 20.0 cm, as shown in Fig. 6.1.

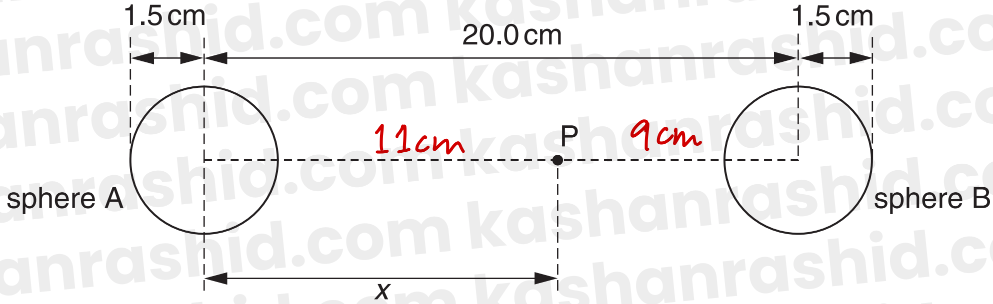


Fig. 6.1 (not to scale)

Both spheres are positively charged.

Point P lies on the line joining the centres of the two spheres, at a distance x from the centre of sphere A.

The variation with distance x of the electric field strength E at point P is shown in Fig. 6.2.

At the Null point

$E_A = E_B$

$\frac{\cancel{k} q_A}{r_A^2} = \frac{\cancel{k} q_B}{r_B^2}$

$\frac{q_A}{r_A^2} = \frac{q_B}{r_B^2}$

$\frac{q_A}{q_B} = \left(\frac{r_A}{r_B}\right)^2$

$\frac{q_A}{q_B} = \left(\frac{11}{9}\right)^2$

$\boxed{\frac{q_A}{q_B} = 1.5}$

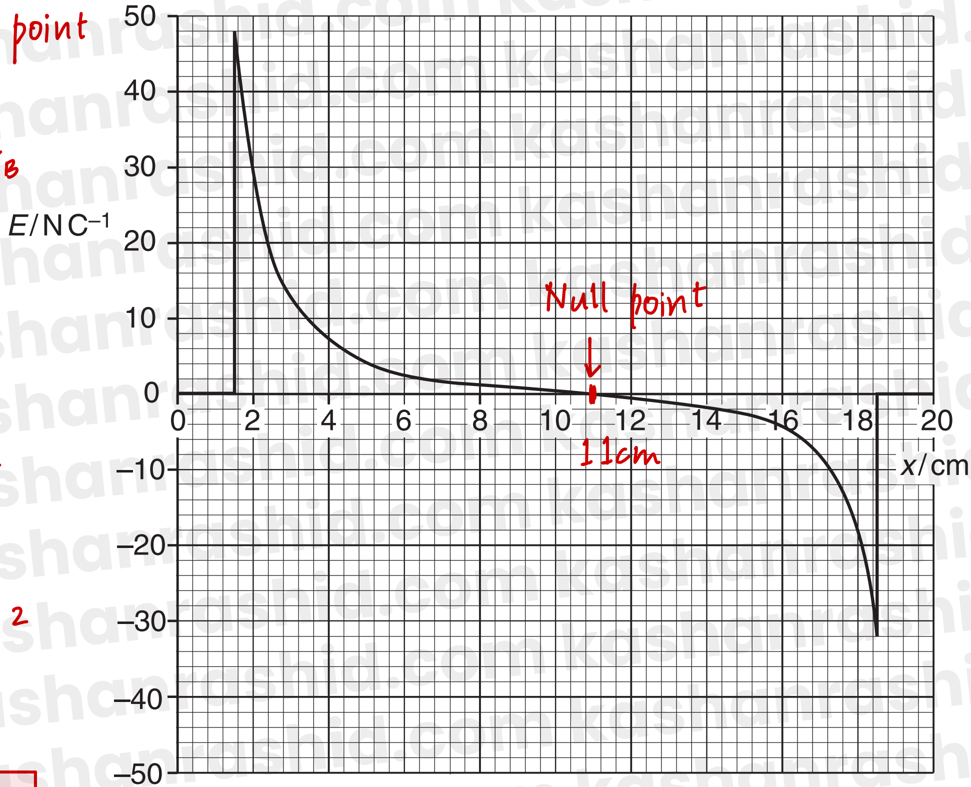


Fig. 6.2

(a) Use Fig. 6.2 to determine the ratio

$$\frac{\text{magnitude of charge on sphere A}}{\text{magnitude of charge on sphere B}}.$$

Explain your working.

ratio = ... [3]

(b) The variation with distance *x* of the electric potential *V* at point P is shown in Fig. 6.3.

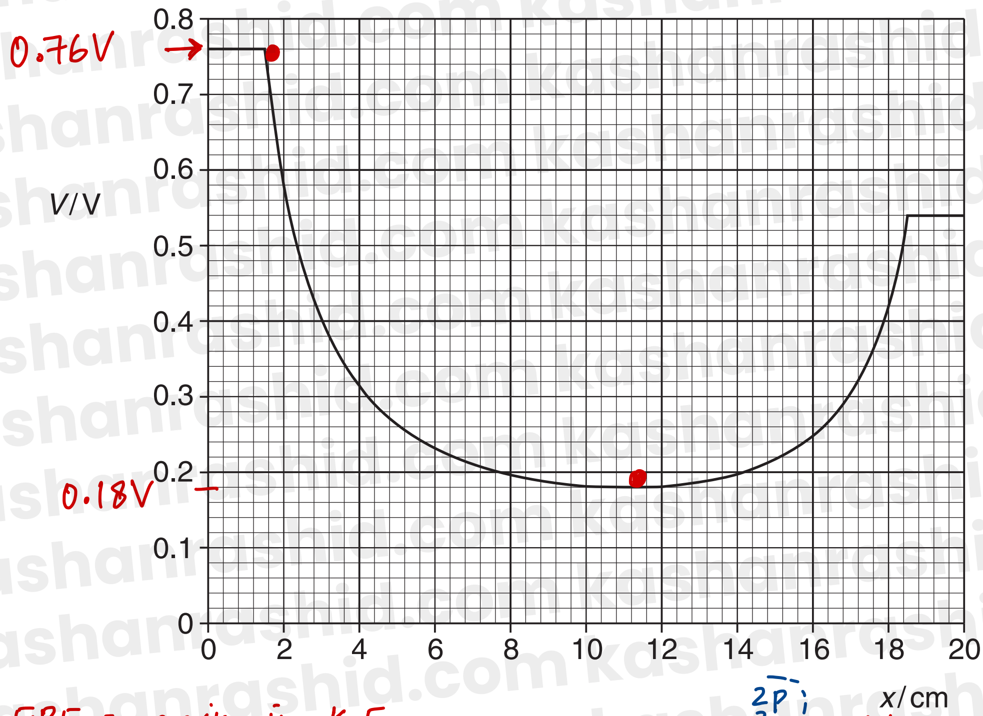


Fig. 6.3

(loss) in EPE = gain in K.E

$-\Delta V \times q = \frac{1}{2} m (v^2 - u^2)$

$-(V_f - V_i) \times q = \frac{1}{2} m (v^2 - u^2)$

$-(0.54 - 0.76) \times 2e = \frac{1}{2} \times 4u \times (v^2 - 0^2)$

$v = 4604 \approx 4.6 \times 10^3$ m/s

2p 2n

Alpha particle

$m = 4u$

$q = +2e$

$1e = 1.6 \times 10^{-19}$ C

$1u = 1.66 \times 10^{-27}$ kg

$u = 0$

An α-particle is initially at rest on the surface of sphere A.
The α-particle moves along the line joining the centres of the two spheres.

Determine, for the α-particle as it moves between the two spheres,

(i) its maximum speed,

loss in EPE = gain in K.E

$-\Delta V \times q = \frac{1}{2} m(v^2 - u^2)$

$-(V_f - V_i) \times q = \frac{1}{2} m(v^2 - u^2)$

$-(0.18 - 0.76) \times 2e = \frac{1}{2} \times 4u \times (v^2 - 0)$

$v = 7476.87 \simeq 7.5 \times 10^3 \text{ ms}^{-1}$

maximum speed = ... m s^{-1} [3]

(ii) its speed on reaching the surface of sphere B.

loss in EPE = gain in K.E

$-\Delta V \times q = \frac{1}{2} m(v^2 - u^2)$

$-(V_f - V_i) \times q = \frac{1}{2} m(v^2 - u^2)$

$-(0.54 - 0.76) \times 2e = \frac{1}{2} \times 4u \times (v^2 - 0^2)$

$v = 4604 \simeq 4.6 \times 10^3 \text{ m/s}$

speed = ... m s^{-1} [2]

[Total: 8]

