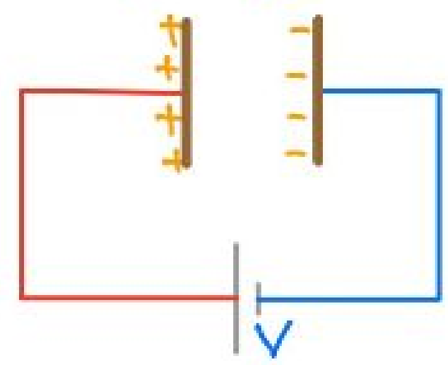
CAPACITANCE

0333-4281759
SALT Academy

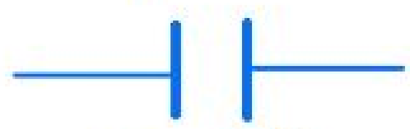
Capacitor:

Def. → Component
→ used to separate electric charge
→ for storage of electrical energy.

$E_p \propto -\left(\frac{Q_+ Q}{r}\right)$

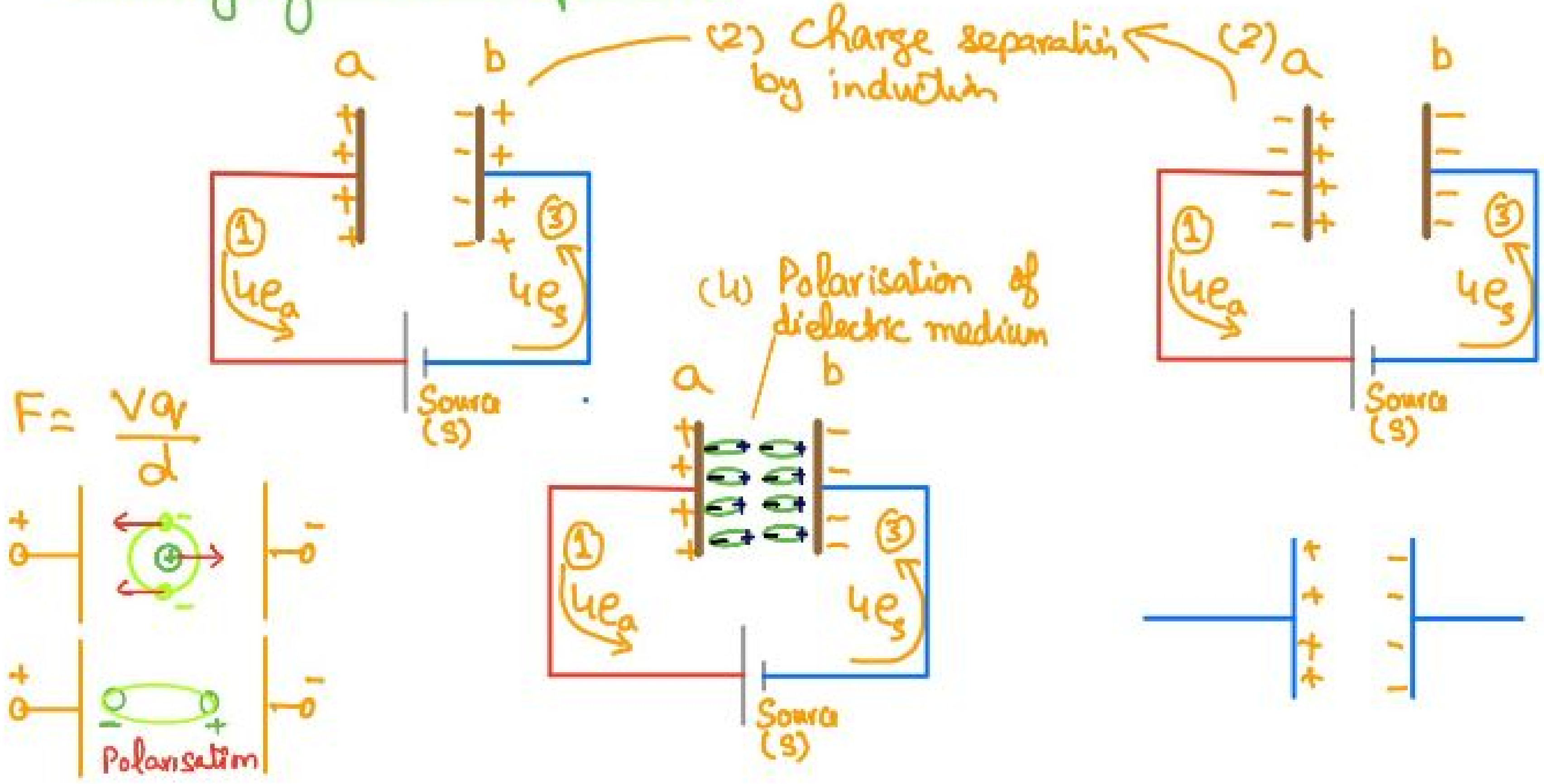


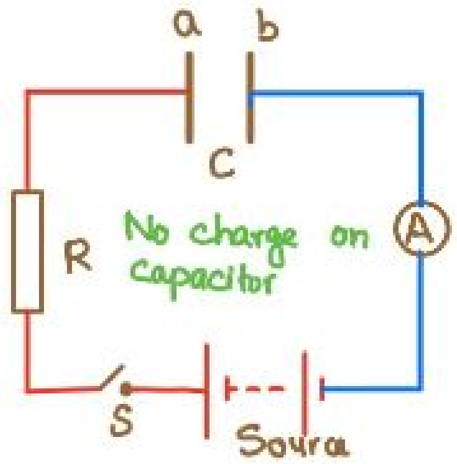
Symbol:

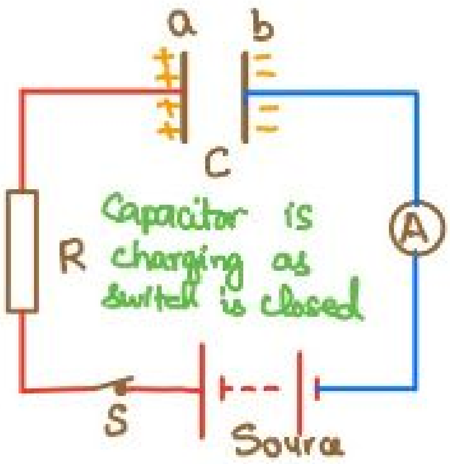


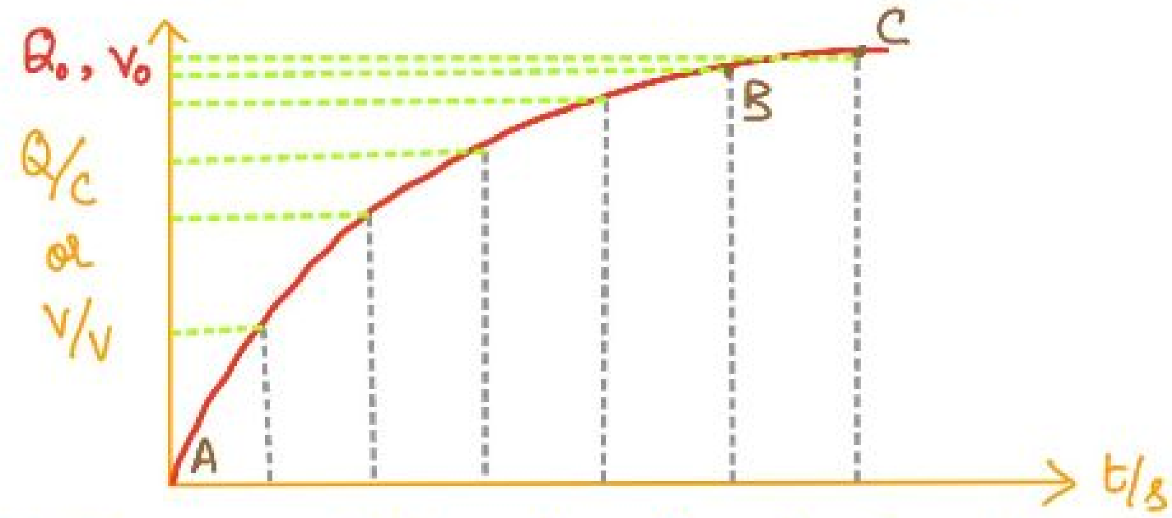
Construction: (1) Two parallel metallic plate either of same or different nature/material.
(2) Separated by a dielectric medium (an insulating medium which allow Electric and magnetic field lines to pass through it) i.e air, paper, mica etc

Charging a capacitor:









(i) At A:- When switch S is closed, there is no opposing potential, so there is a relatively large current, and charge rapidly builds up on the capacitor.

(ii) From A to B:- The charge on the capacitor causes its potential to increase; this p.d. opposes the cell p.d (V_0), and the current decreases as the charge increases.

(iii) At C:- Now, the p.d. across the capacitor becomes equal to the supply p.d (V_0). No further charge can flow, ($I = 0$) and the capacitor is now fully charged

Uses of Capacitor:-

1- To block Direct current (DC).

2- In tuning circuits to receive signal of maximum strength.

3- To smooth the output from a rectifier's circuit ie to reduce ripples.

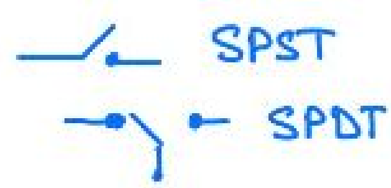
4- Temporary energy/Power source.

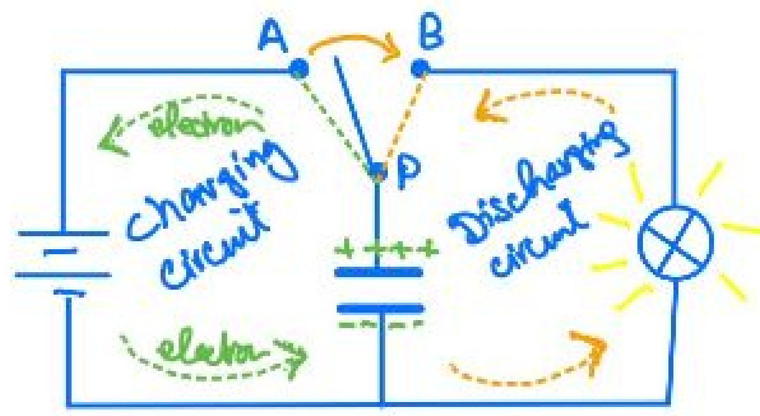
5- In time delay circuits.

6- For surge protection.

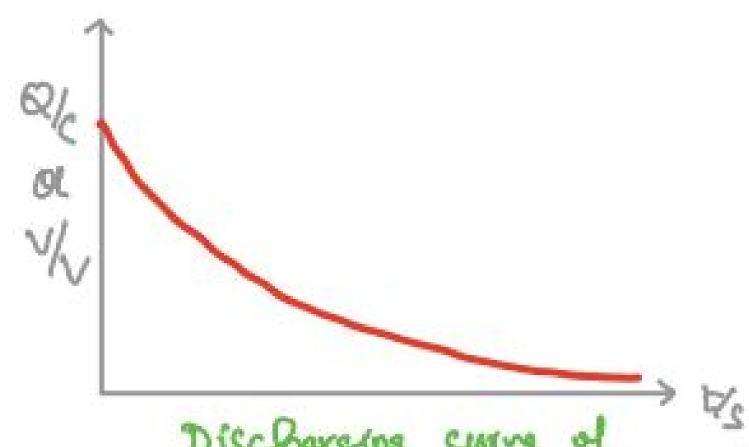
7- In oscillator circuit to produce electrical oscillations.

Discharging of a Capacitor: through a Camera flash:



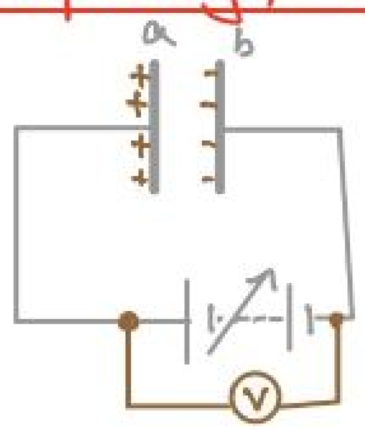


Charging curve of capacitor

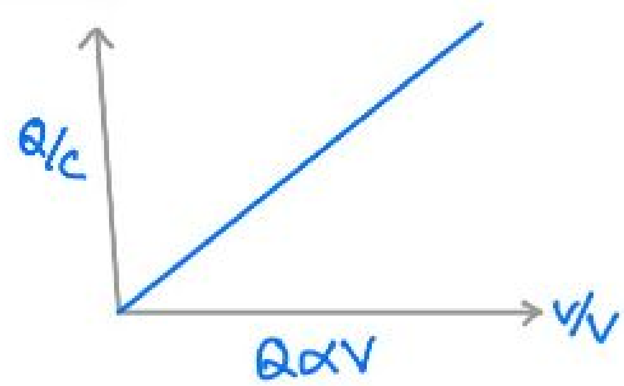


Discharging curve of capacitor

Capacity / Capacitance of a capacitor:-



V/V	Q/C
0	0
2	6
3	12
4	18



$Q \propto V$

(charge on a plate of capacitor) $\propto$ (p.d. across parallel plates)

$$Q \propto V$$

$$Q = CV$$

$C \rightarrow$ Capacitance / capacity.

Def: $C = \dfrac{Q}{V}$

Amount of charge stored on a plate of capacitor per unit p.d. across parallel plates.

P.S Scalar

Units: Farad (F) $\quad 1 \text{ Farad} = \dfrac{1 \text{ Coulomb}}{1 \text{ volt}}$

Dependance: $C = \dfrac{\varepsilon_0 A}{d}$, $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$

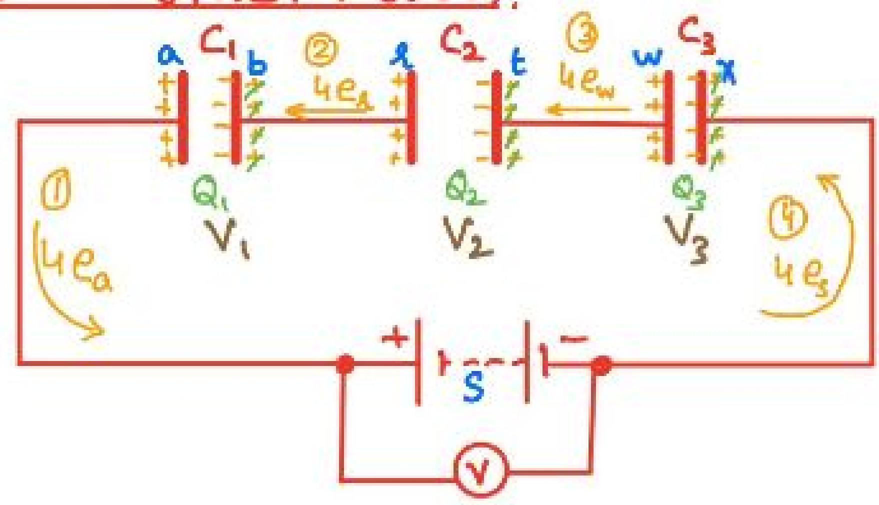
A – Surface area of a plate

d – Separation b/w parallel plate

Combination of Capacitors:-

$C_3 > C_2 > C_1$

(a) **Series Combination:**



Charge: Same charge on each capacitor

$$Q = Q_1 = Q_2 = Q_3$$

P.d / voltage: By Kirchhoff's second law $[C\uparrow, V\downarrow]$

$$V = V_1 + V_2 + V_3$$

Total Capacitance: decrease

$$V = V_1 + V_2 + V_3$$

But $Q = CV \Rightarrow V = \frac{Q}{C}$

$$\frac{Q}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$\frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \quad \left[\text{Series Combination}\right]$$

$$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

So total capacitance is series is even lesser than least capacity of a capacitor connected.

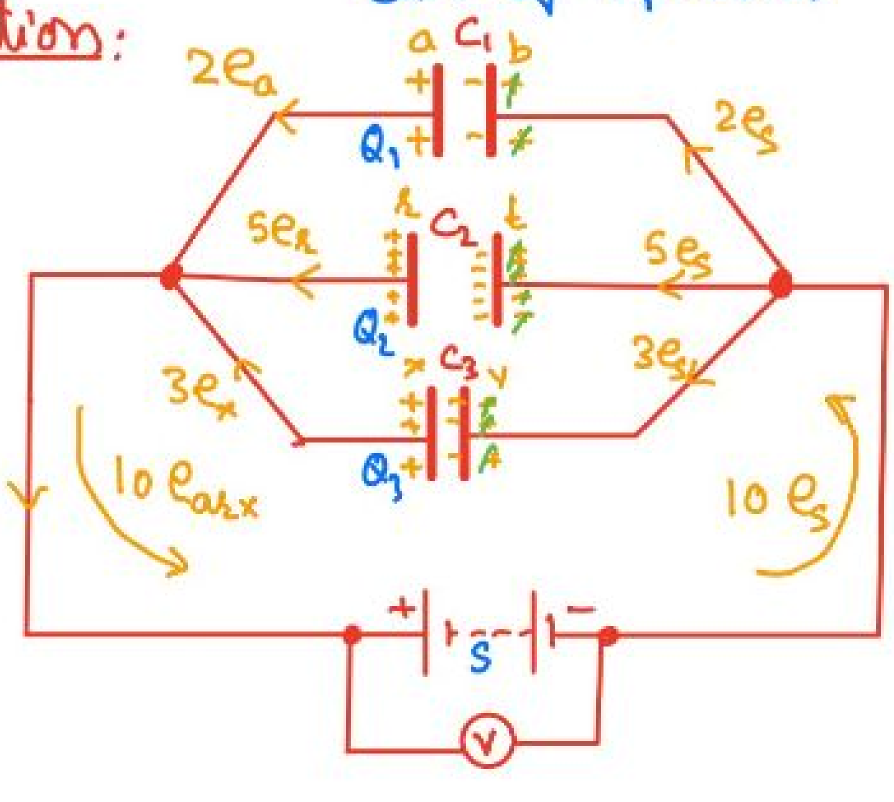
Total Capacity of two capacitor in series

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_T} = \frac{C_2 + C_1}{C_1 C_2} \Rightarrow \boxed{C_T = \frac{C_1 C_2}{C_1 + C_2}}$$

i.e. Total Capacitance $= \dfrac{\text{Product of capacities}}{\text{Sum of capacities}}$

(b) **Parallel Combination:**



Charge: Different as per capacity ($C\uparrow, Q\uparrow$)

$$Q = Q_1 + Q_2 + Q_3$$

P.d./voltage: Same by Kirchhoff's second law

$$V = V_1 = V_2 = V_3$$

Total Capacitance: Increase

$$Q = Q_1 + Q_2 + Q_3$$

$$C_T V = C_1 V_1 + C_2 V_2 + C_3 V_3$$

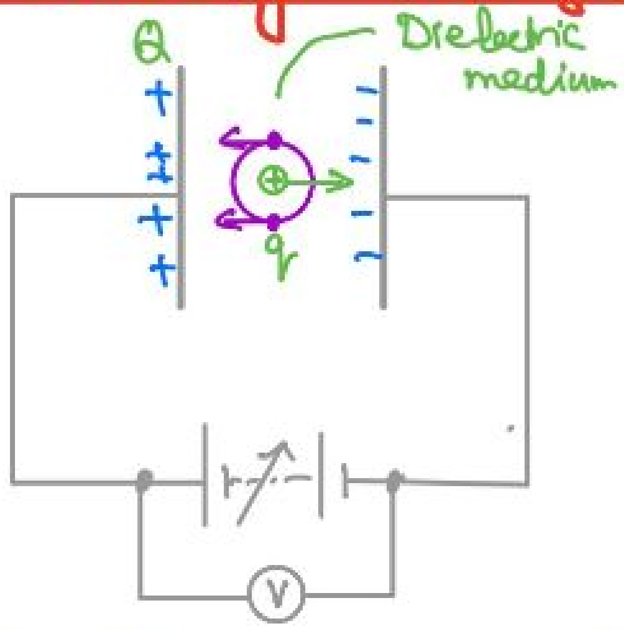
$$C_T V = C_1 V + C_2 V + C_3 V \qquad \left[\begin{matrix} V = V_1 = V_2 = V_3 \\ \text{parallel} \end{matrix}\right]$$

$$\boxed{C_T = C_1 + C_2 + C_3}$$

Note: For n-identical capacitors each of capacity C.

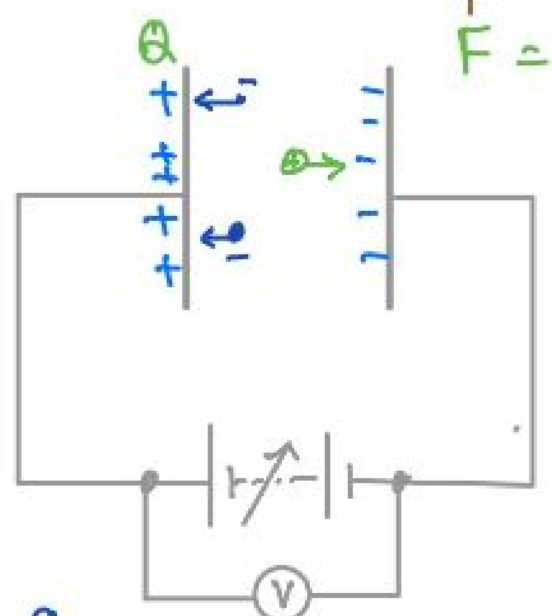
Series Combination	Parallel combination
$\boxed{C_T = \frac{C}{n}}$	$\boxed{C_T = nC}$

Safe working voltage across a Capacitor :-



$Q \propto V$

$$\vec{F} = \frac{\vec{V}\, q}{d} \quad \begin{matrix} q - \text{constant} \\ d - \text{constant} \end{matrix}$$



It is the maximum p.d that can be applied across a capacitor so that dielectric medium b/w two parallel plates is not conducting.

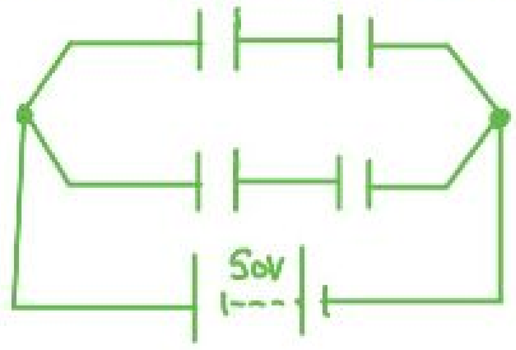
Note:

		Series Combination	Parallel combination
1.	Charge	All capacitors store equal charge	The charges stored on capacitors connected in parallel are in the same ratio as their capacitances i.e $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$
2.	P.d.	P.ds. across the capacitors are different depending upon capacities i.e $V\uparrow$ if $C\downarrow$ but add up up to the total p.d.	P.d. across each capacitor is same
3.	Total Capacitance	Expression for total capacitance is similar to resistors in parallel.	Expression for total capacitance is similar to resistors in series

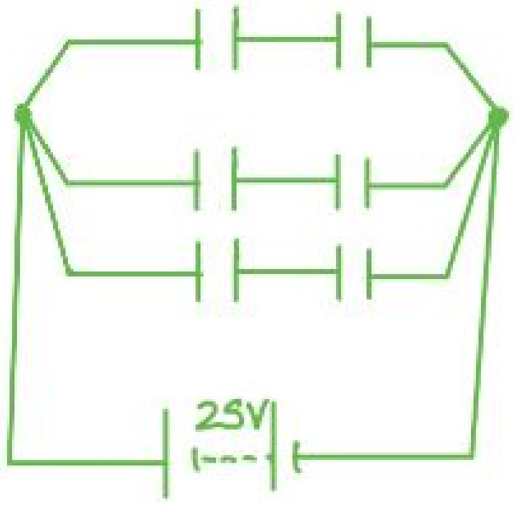
Q) A number of capacitor each having a rating of 48 μF, 25V are available. Show their connections in a combination so as to get a total capacity of

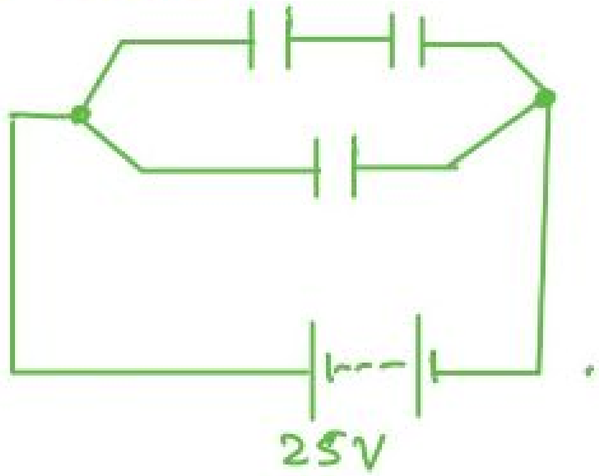
(a) 48 μF across a 50V source.

Series: $C_T = \frac{C}{n} = \frac{48}{2} = 24\mu F$



(b) 72 μF across a 25V source.





Nov. 20/41 variant

6 (a) (i) Define the *capacitance* of a parallel plate capacitor. $C = \frac{Q}{V}$

Amount of charge on a plate of capacitor per unit p.d. across parallel plates [2]

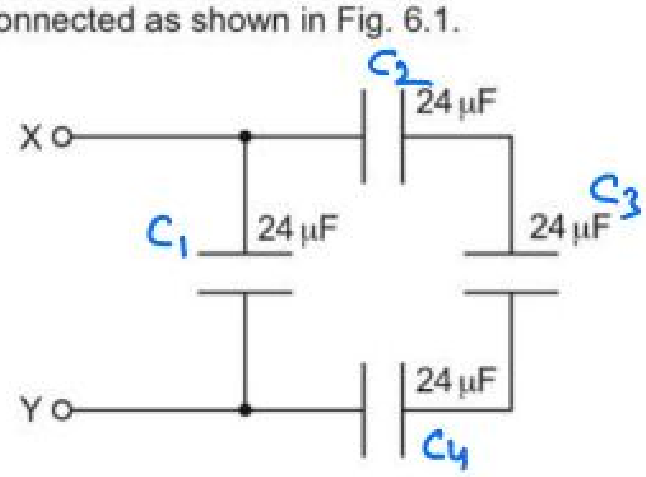
(ii) State **three** functions of capacitors in electrical circuits.

1. In tuning circuits to receive signal of max. strength.
2. To block DC
3. To smooth output from a rectifier's circuit.
4. Temporary energy / power source.
5. Time delay circuits

[3]

(b) A student has available four capacitors, each of capacitance 24 μF.

The capacitors are connected as shown in Fig. 6.1.



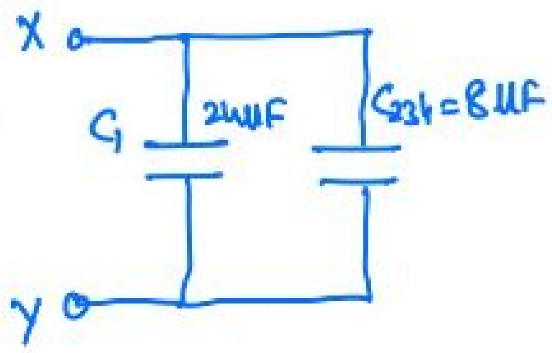


Fig. 6.1

Calculate the combined capacitance between the terminals X and Y.

Here C_2, C_3 and C_4 are in series

$$\frac{1}{C_{234}} = \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} \Rightarrow \frac{1}{C_{234}} = \frac{1}{24} + \frac{1}{24} + \frac{1}{24} \Rightarrow \frac{1}{C_{234}} = \frac{3}{24}$$

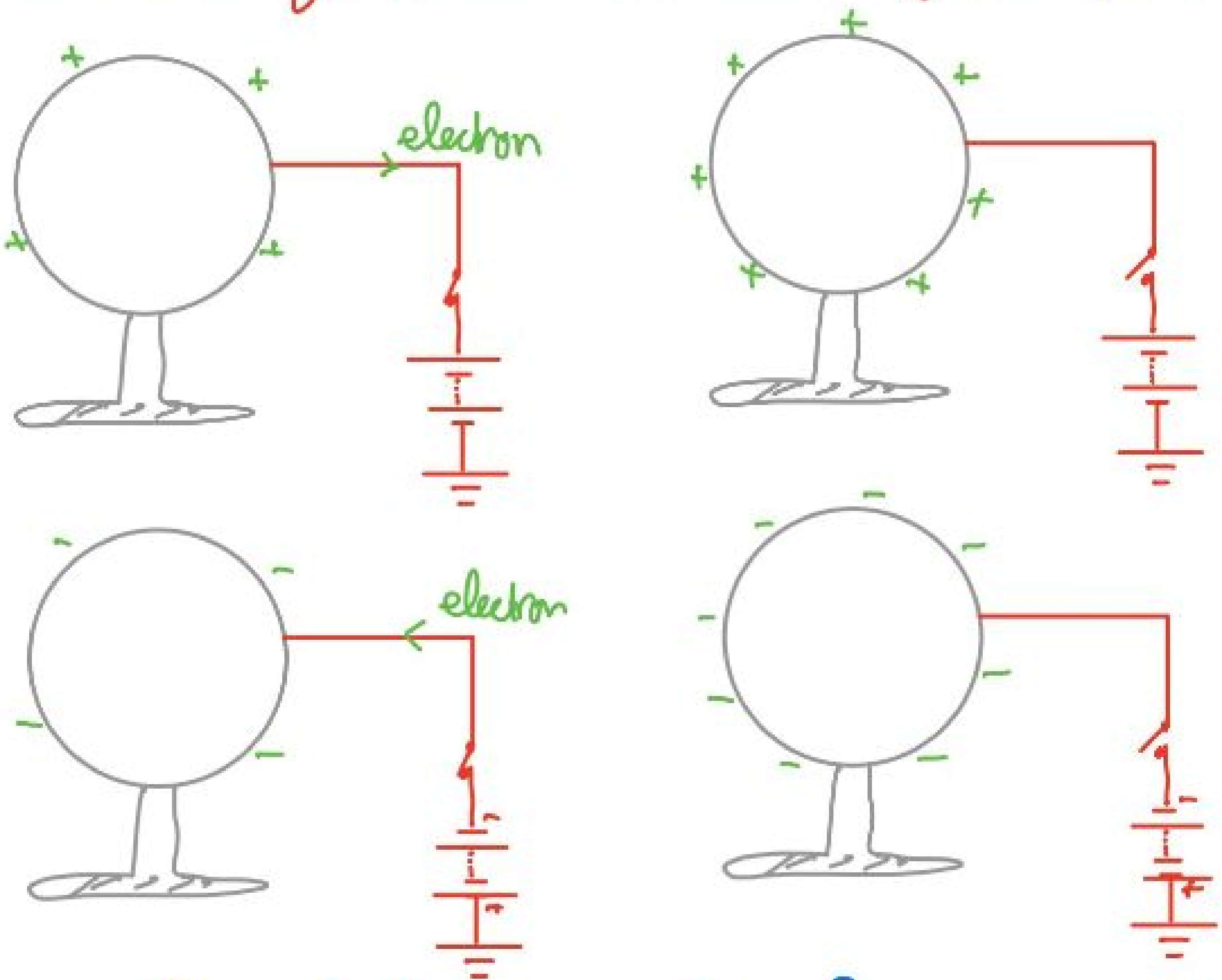
$$C_{234} = \frac{24}{3} = 8\ \mu F$$

$$C_{xy} = C_1 + C_{234} = 24 + 8$$

capacitance = 32 μF μF [2]

[Total: 7]

Capacitance of an isolated charged capacitor:-



$$Q = CV \Rightarrow V = \frac{Q}{C} \quad \text{------} (1)$$

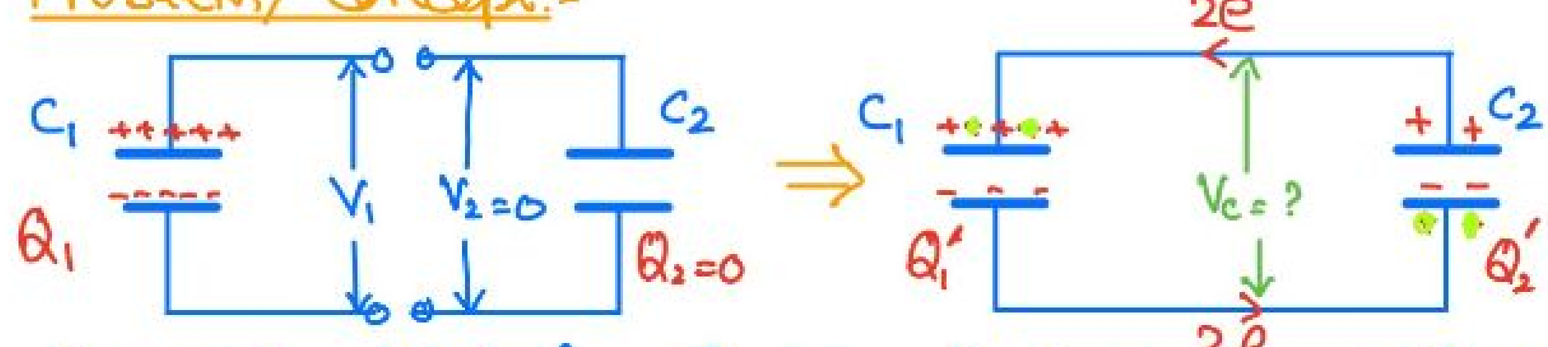
Also
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}\right) \quad \text{------} (2)$$

Comparing (1) and (2)

$$\boxed{C = 4\pi\varepsilon_0 r}$$

Common p.d. across a combination of capacitors:-

Problem/ Concept:-



Principle: Total charge before connection must be equal to total charge after connection.

$$Q_{\text{Before connection}} = Q_{\text{after connection}}$$

$$Q_1 + Q_2 = Q_1' + Q_2'$$

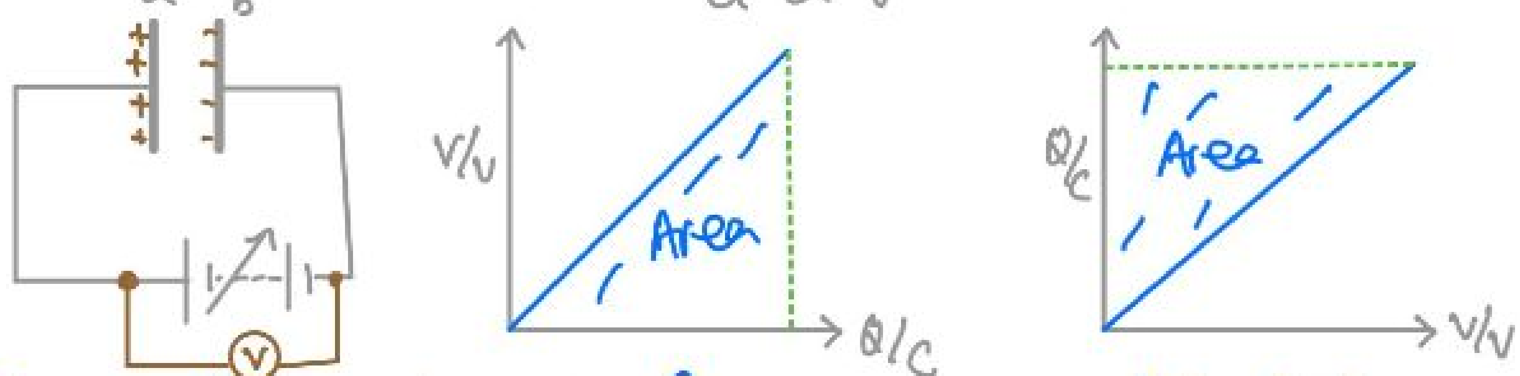
$$C_1V_1 + 0 = C_1V_c + C_2V_c$$

$$C_1V_1 = (C_1 + C_2)V_c$$

$$\boxed{V_c = \frac{C_1V_1}{C_1+C_2}}$$

Energy stored in a Capacitor:

$Q \propto V$



Since they is a charge separation between +ve and -ve plate of capacitor. So energy is stored in it. The variation of charge on a plate depends upon p.d. across parallel plates as $Q \propto V$.

E = Area of Q/C – V/v graph along with charge axis

$$\boxed{E = \frac{1}{2}(Q)(V)}$$

But $Q = CV$

$$E = \frac{1}{2}(CV)(V) \qquad\qquad E = \frac{1}{2}(Q)\left(\frac{Q}{C}\right)$$

$$\boxed{E = \frac{1}{2}(CV^2)} \qquad\qquad \boxed{E = \frac{Q^2}{2C}}$$

Energy delivered by an e.m.f source:

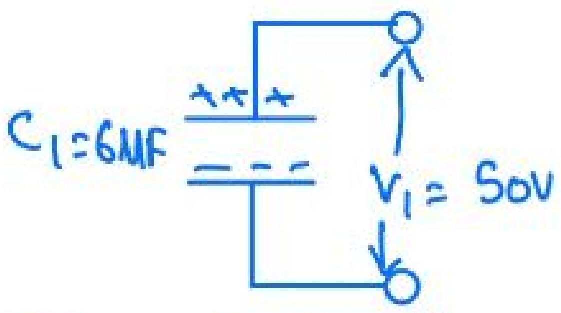
$$V = \frac{W}{Q} \Rightarrow V = \frac{E}{Q}$$

$$\boxed{E = VQ}$$

Note: Not all the energy delivered by an e.mf source becomes the electrical potential energy stored in a capacitor because half of this energy is dissipated as heat due to resistance of connecting wires during charging process.

Q) A capacitor C_1 of $6\mu F$ is charged to a p.d of 50V. Another capacitor C_2 of $8\mu F$ is charged to a p.d. of 100V.

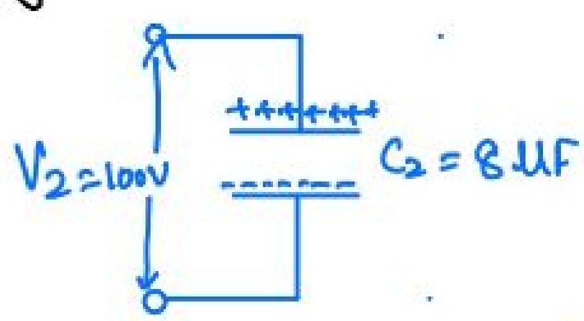
(a) Calculate the energy stored in each capacitor



$$E_1 = \frac{1}{2} C_1 V_1^2$$

$$= \frac{1}{2}(6\times10^{-6})(50)^2$$

$$= 7.5 \times 10^{-3}\,J$$



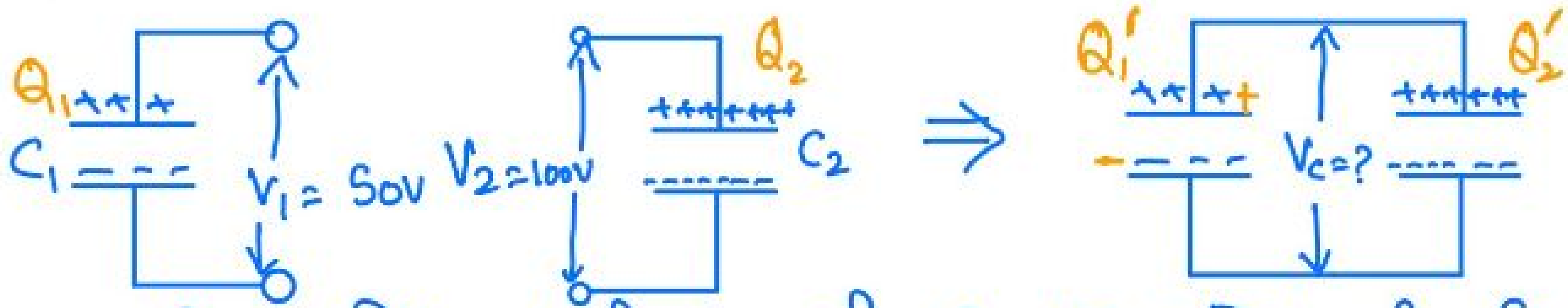
$$E_2 = \frac{1}{2} C_2 V_2^2$$

$$= \frac{1}{2}(8\times10^{-6})(100)^2$$

$$= 40 \times 10^{-3}\,J$$

(b) If the two capacitors are now joined with plates of like charges connected together, calculate

(i) the common p.d. across combination.



By Principle of Conservation of charge

Total charge before connection = Total charge after connection

$$Q_1 + Q_2 = Q_1' + Q_2'$$

$$C_1V_1 + C_2V_2 = C_1V_c + C_2V_2$$

$$V_c = \frac{C_1V_1 + C_2V_2}{C_1 + C_2} \Rightarrow V_c = \frac{(6\times10^{-6})(50) + (8\times10^{-6})(100)}{(6+8)10^{-6}}$$

$$V_c = 78.6\,V$$

(ii) Energy stored in the combination of capacitors.

$$E = \frac{1}{2}CV^2 = \frac{1}{2}(C_1 + C_2)(V_c)^2$$

$$E = \frac{1}{2}\left[(6+8)10^{-6}\right]\left[78.6\right]^2$$

$$= 0.0432\,J = 43.2\times10^{-3}\,J$$

(c) Calculate the difference of energy stored in part (a) and (b)-ii and explain the reason for difference of this energy.

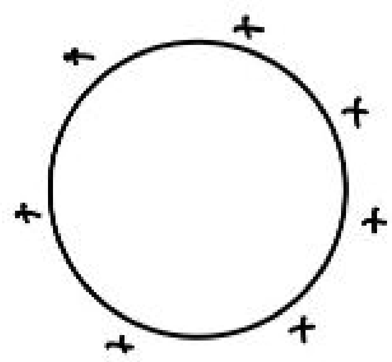
$$\Delta E = (E_1 + E_2) - E$$

$$= (7.5 + 40)10^{-3} - 43.2\times10^{-3}$$

$$\Delta E = 4.30 \times 10^{-3}\,J$$

This energy is dissipated as heat due to resistance of connecting wires during the charge re-distribution process.

Q)



$Q = 12.8 \times 10^{-19} C$

diameter = 8.6 cm

Calculate capacitance of this charge spher in Farad.

$C = 4\pi\varepsilon_0 r$

$C = 4(3.14)(8.85 \times 10^{-12})(4.3 \times 10^{-2})$

$C = 3.58 \times 10^{-12}$ F

$C = 3.58$ pF

March 19/42 Q/6 (b)

(b) A student has three capacitors. Two of the capacitors have a capacitance of 4.0 μF and one has a capacitance of 8.0 μF.

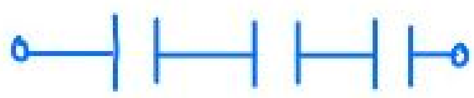
Draw labelled circuit diagrams, one in each case, to show how the three capacitors may be connected to give a total capacitance of:

(i) 1.6 μF

$\frac{1}{C_T} = \frac{1}{4} + \frac{1}{4} + \frac{1}{8}$

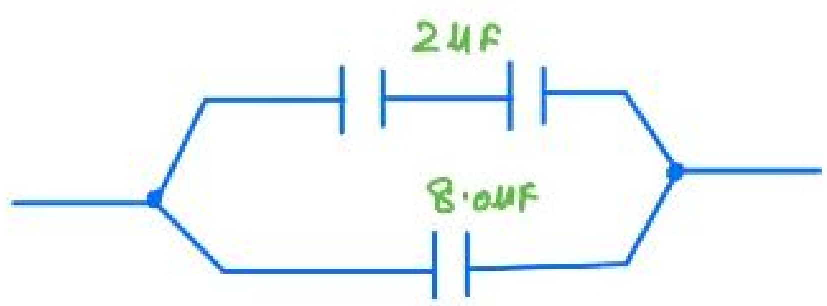
$= \frac{2+2+1}{8} = \frac{5}{8}$

$C_T = \frac{8}{5} = 1.6$



[1]

(ii) 10 μF.



$C_T = 2 + 8 = 10$

[1]

Nov.20/42/Q6)

6 (a) (i) Define the *capacitance* of a parallel plate capacitor. $C = \frac{Q}{V}$

Amount of charge on a plate of capacitor per unit p.d. across parallel plates [2]

(ii) State **three** functions of capacitors in electrical circuits.

1. In tuning circuits to receive signal of max. strength.
2. To block DC
3. To smooth output from a rectifier's circuit.
4. Temporary energy / Power source.
5. Time delay circuits

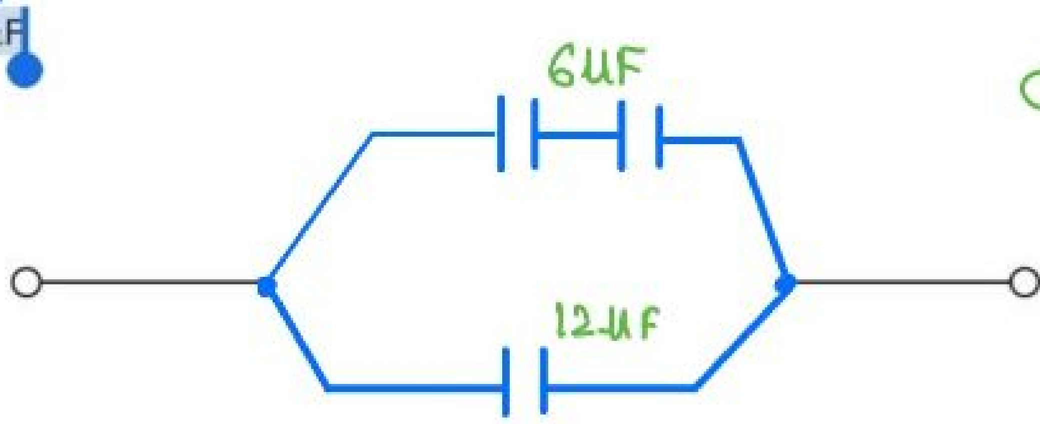
[3]

Series: $C_T = \frac{C}{n}$ | Parallel $C_T = nC$

(b) A student has available **three** capacitors, each of capacitance 12 μF.

Draw diagrams, one in each case, to show how the student connects the capacitors to give a combined capacitance between the terminals of:

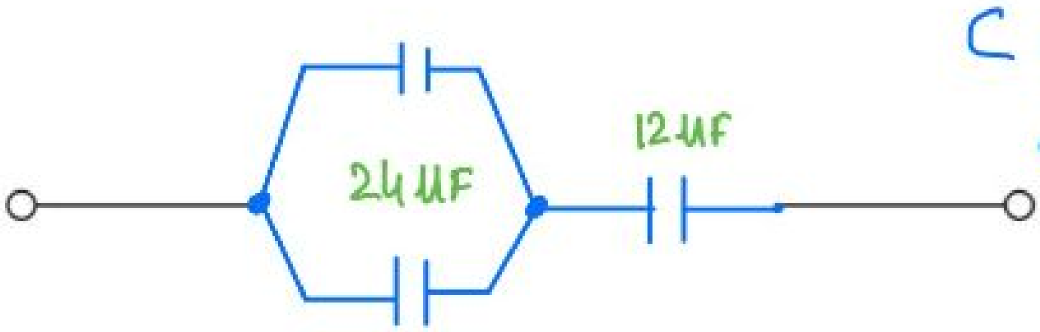
(i) 18 μF



$C_T = 6 + 12$
$= 18\,uF$

[1]

(ii) 8 μF.



$C = \frac{(24)(12)}{24+12} = \frac{288}{36}$

$C = 8\,uF$

[1]

[Total: 7]

June18/42 **6** **(a)** Explain what is meant by the *capacitance* of a parallel plate capacitor.

$C = \frac{Q}{V}$

Amount of charge stored on one plate of capacitor per unit p.d. across parallel plates. [3]

(b) Three parallel plate capacitors each have a capacitance of 6.0 μF.

Draw circuit diagrams, one in each case, to show how the capacitors may be connected together to give a combined capacitance of

(i) 9.0 μF, two in series, in parallel with the other

[1]

(ii) 4.0 μF. two in parallel connected to one in series

[1]

(c) Two capacitors of capacitances 3.0 μF and 2.0 μF are connected in series with a battery of electromotive force (e.m.f.) 8.0 V, as shown in Fig. 6.1.

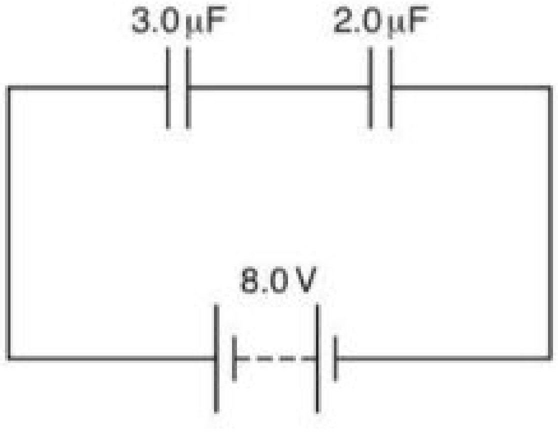


Fig. 6.1

(i) Calculate the combined capacitance of the capacitors.

$$\frac{1}{C_T} = \frac{1}{3\times10^{-6}} + \frac{1}{2.0\times10^{-6}}$$

$$= 1.2\times10^{-6}$$

capacitance = 1.2 μF [1]

(ii) Use your answer in **(i)** to determine, for the capacitor of capacitance 3.0 μF,

1. the charge on one plate of the capacitor,

$$Q = CV$$

$$Q = (1.2\times10^{-6})(8.0)$$

$$= 9.6\times10^{-6}\,C$$

charge = 9.6 μC

2. the energy stored in the capacitor.

$$E = \frac{1}{2}\left(\frac{Q^2}{C}\right)$$

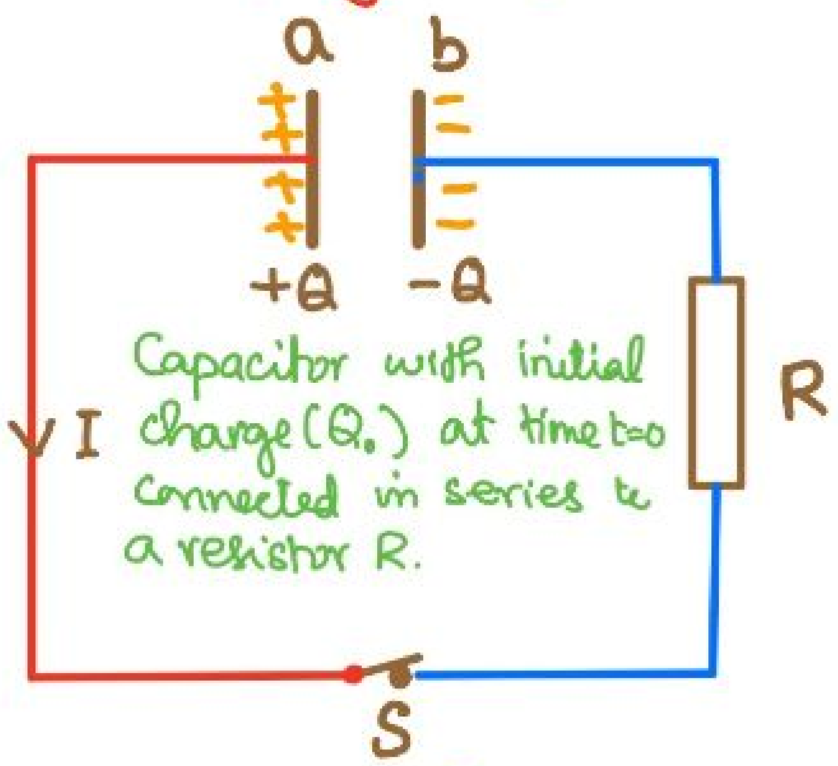
$$= \frac{1}{2}\left[\frac{(9.6\times10^{-6})^2}{3.0\times10^{-6}}\right]$$

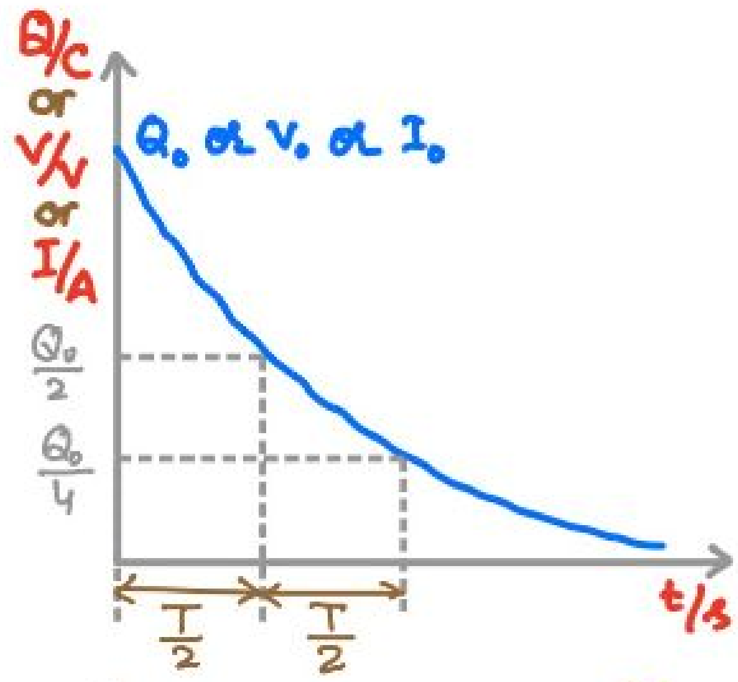
energy = 1.54×10^{-5} J

[4]

[Total: 10]

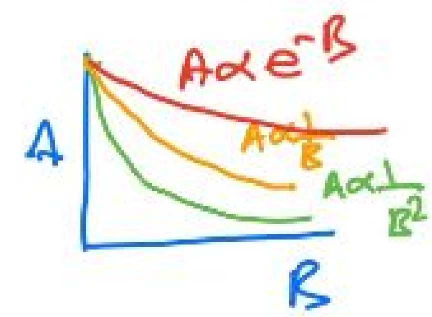
Discharging of a Capacitor – Decay of charge:-





When switch S is closed, a current flows in the circuit and discharging process occur. Energy is also dissipated to the surrounding in the form of heat due to current through the resistive part of the circuit.

(i) Variation of charge with time:-



Trend: Exponential curve

Equation: $Q = Q_0 e^{-\frac{t}{RC}}$

Here Q – Charge on capacitor at any time(t)

Q_0 – Initial charge on capacitor at t = 0.

(ii) Variation of voltage with time:- As $Q \propto V$

Trend: Exponential curve

Equation: $V = V_0 e^{-\frac{t}{RC}}$

Here, V – p.d. across the capacitor at any time(t)

(iii) **Variation of current with time:-** As $I = \frac{Q}{t}$

Trend: Exponential curve

Equation: $I = I_0 e^{-\frac{t}{RC}}$

Here, I - current flowing through R at any time(t)

(iv) **Half life ($T_{\frac{1}{2}}$) of a RC circuit:-**

The charge (and hence p.d. and discharge current) decreases by equal fractions in equal time intervals.

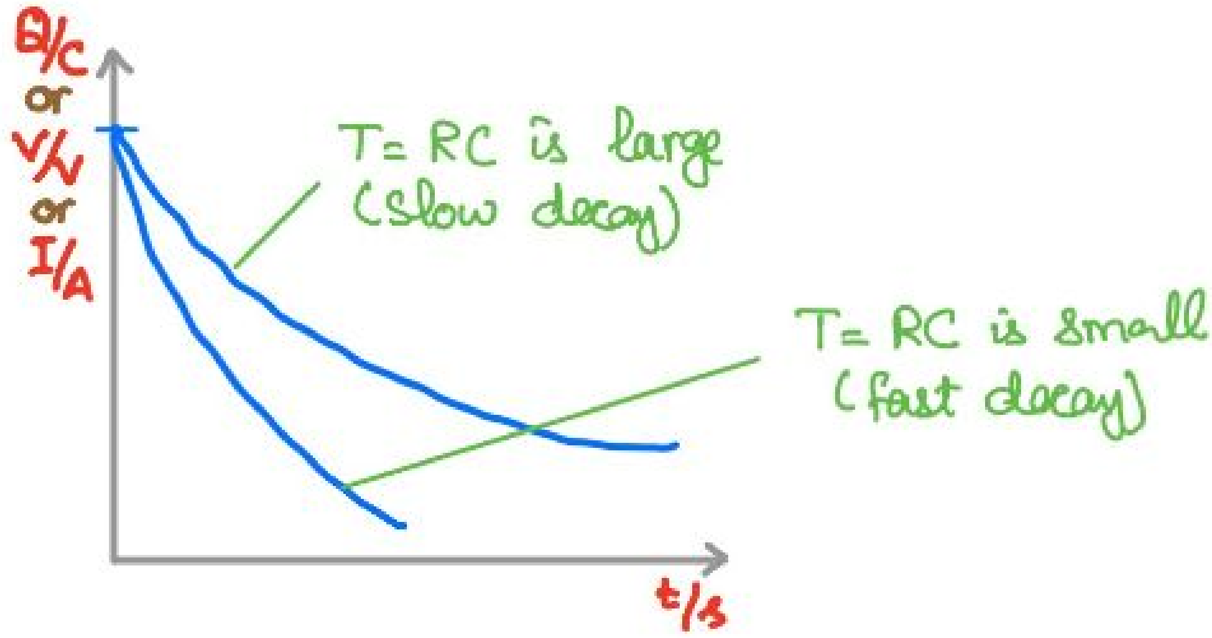
Def. The half life ($T_{\frac{1}{2}}$) of a RC circuit is defined as; the **time taken** for the charge on the capacitor (or the p.d. across it, or the discharge current through it to decrease to **half** of it's initial value.

Time constant (T) of RC circuit:-

Def.: It is the **time taken** for the charge on the capacitor (or the p.d. across it, or the discharge current through it to decrease to $(\frac{1}{e})$ of it's initial value.

Formula: $T = RC$

Note: If RC is large, the decay will be slow and vice versa



Q/C
or
V/V
or
I/A
T= RC is large
(slow decay)
T= RC is small
(fast decay)
t/s

5 A capacitor C is charged using a supply of e.m.f. 8.0V. It is then discharged through a resistor R.
The circuit is shown in Fig. 5.1.

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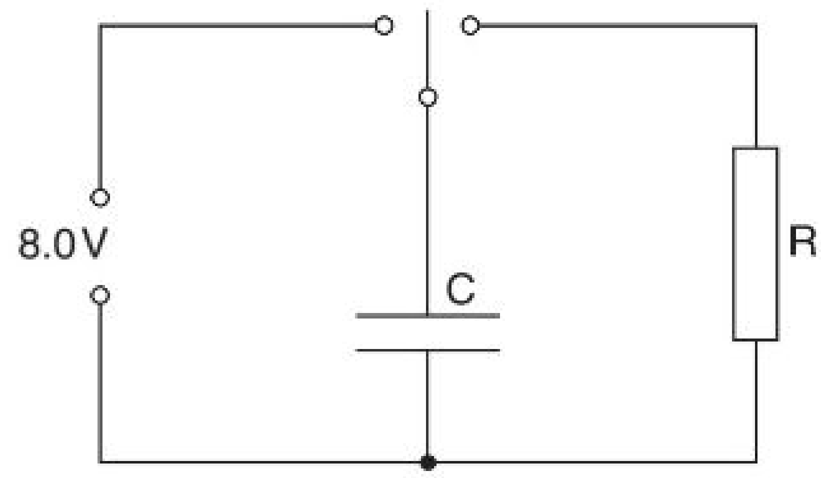

Fig. 5.1

The variation with time t of the potential difference V across the resistor R during the discharge of the capacitor is shown in Fig. 5.2.

$V = V_0 e^{-\frac{t}{RC}}$
$V = 8 e^{-\frac{t}{RC}}$

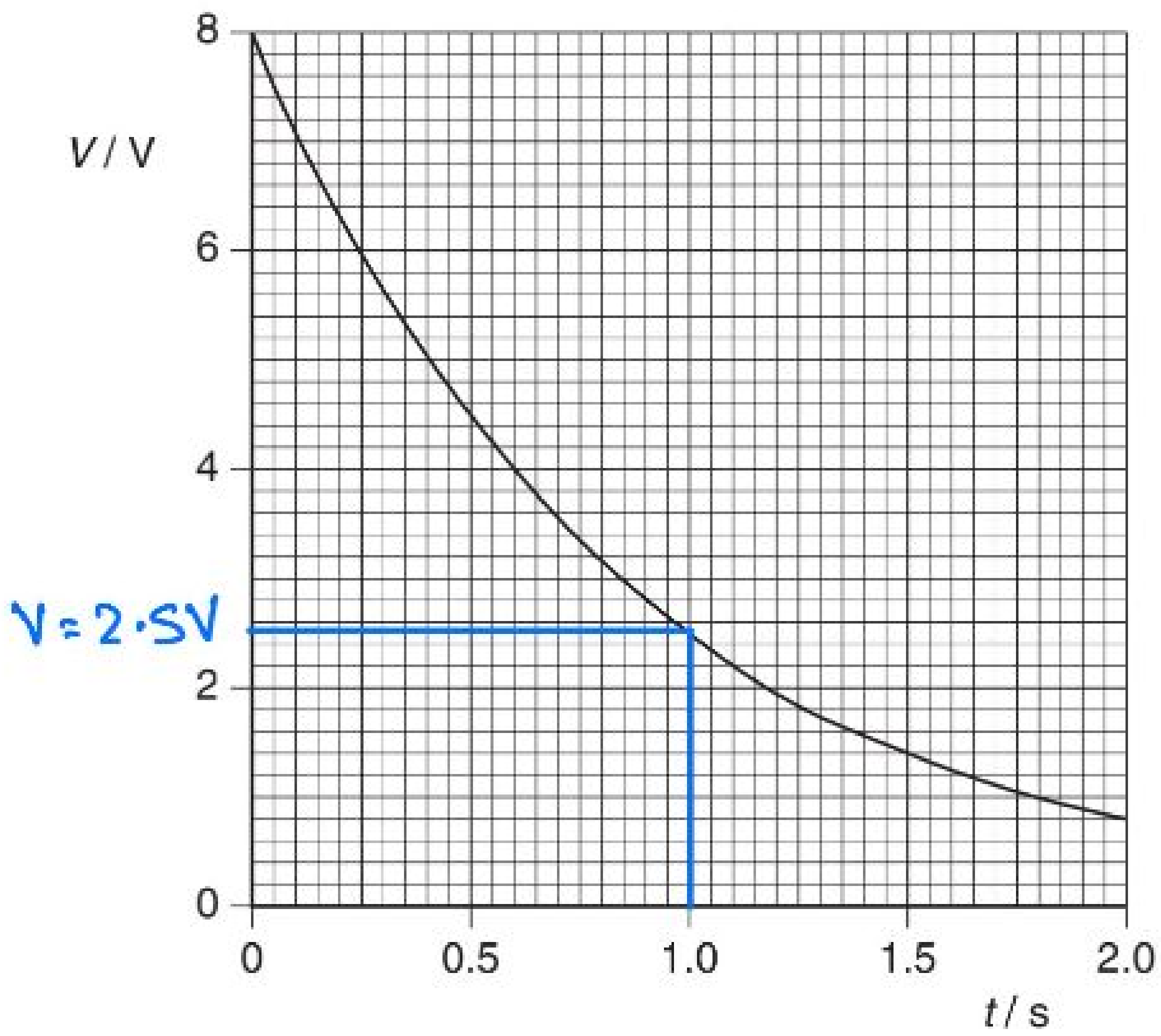

Fig. 5.2

(a) During the first 1.0 s of the discharge of the capacitor, 0.13 J of energy is transferred to the resistor R.
Show that the capacitance of the capacitor C is 4500 μF.

Loss of stored energy $= \frac{1}{2} C V_1^2 - \frac{1}{2} C V_2^2$

$0.13 = \frac{1}{2} C \left[(8.0)^2 - (2.5)^2 \right]$

$C = 4500 \mu F$

[3]

(b) Some capacitors, each of capacitance 4500 μF with a maximum working voltage of 6V, are available.

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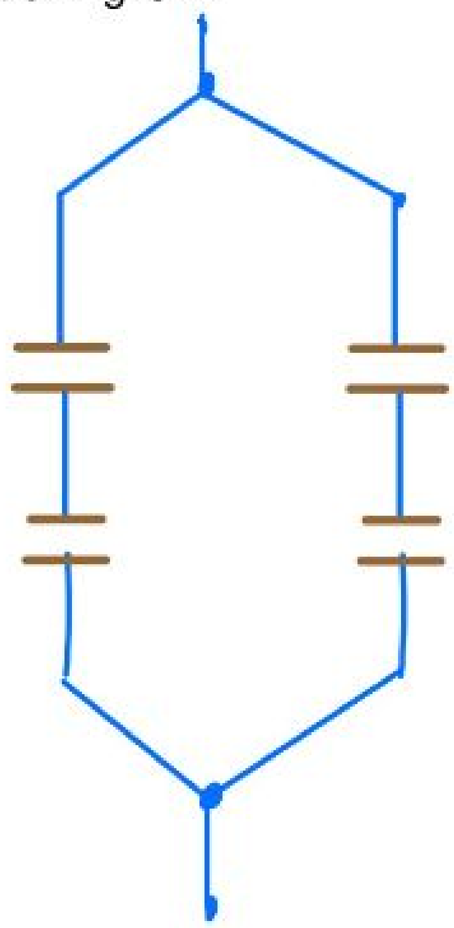
Draw an arrangement of these capacitors that could provide a total capacitance of 4500 μF for use in the circuit of Fig. 5.1.



Series

$C_T = \frac{C}{n} = \frac{4500}{2}$

$= 2250$

Parallel

$C_T = nC$

[2]

$C_T = 2(2250)$

$= 4500$

For Examiner's Use

5 **(a)** State one function of capacitors in simple circuits. Already done

....

....[1]

(b) A capacitor is charged to a potential difference of 15 V and then connected in series with a switch, a resistor of resistance 12 kΩ and a sensitive ammeter, as shown in Fig. 5.1.

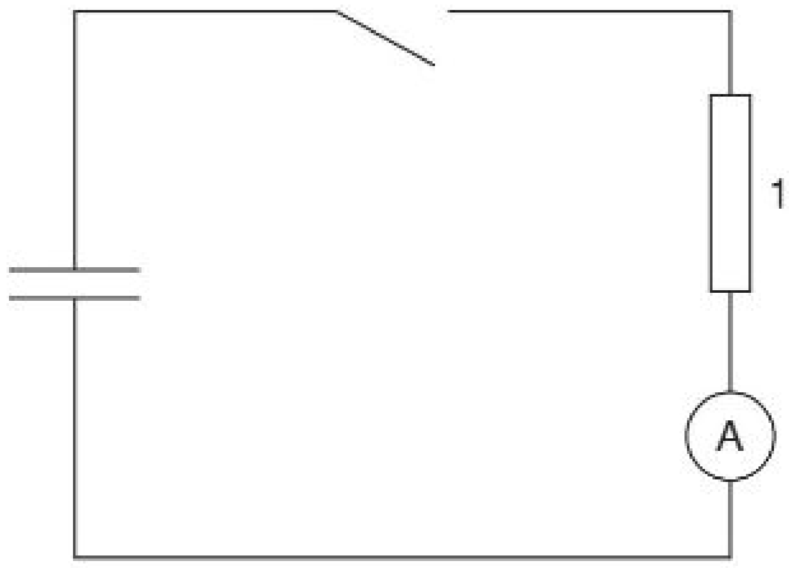


Fig. 5.1

The switch is closed and the variation with time t of the current I in the circuit is shown in Fig. 5.2.

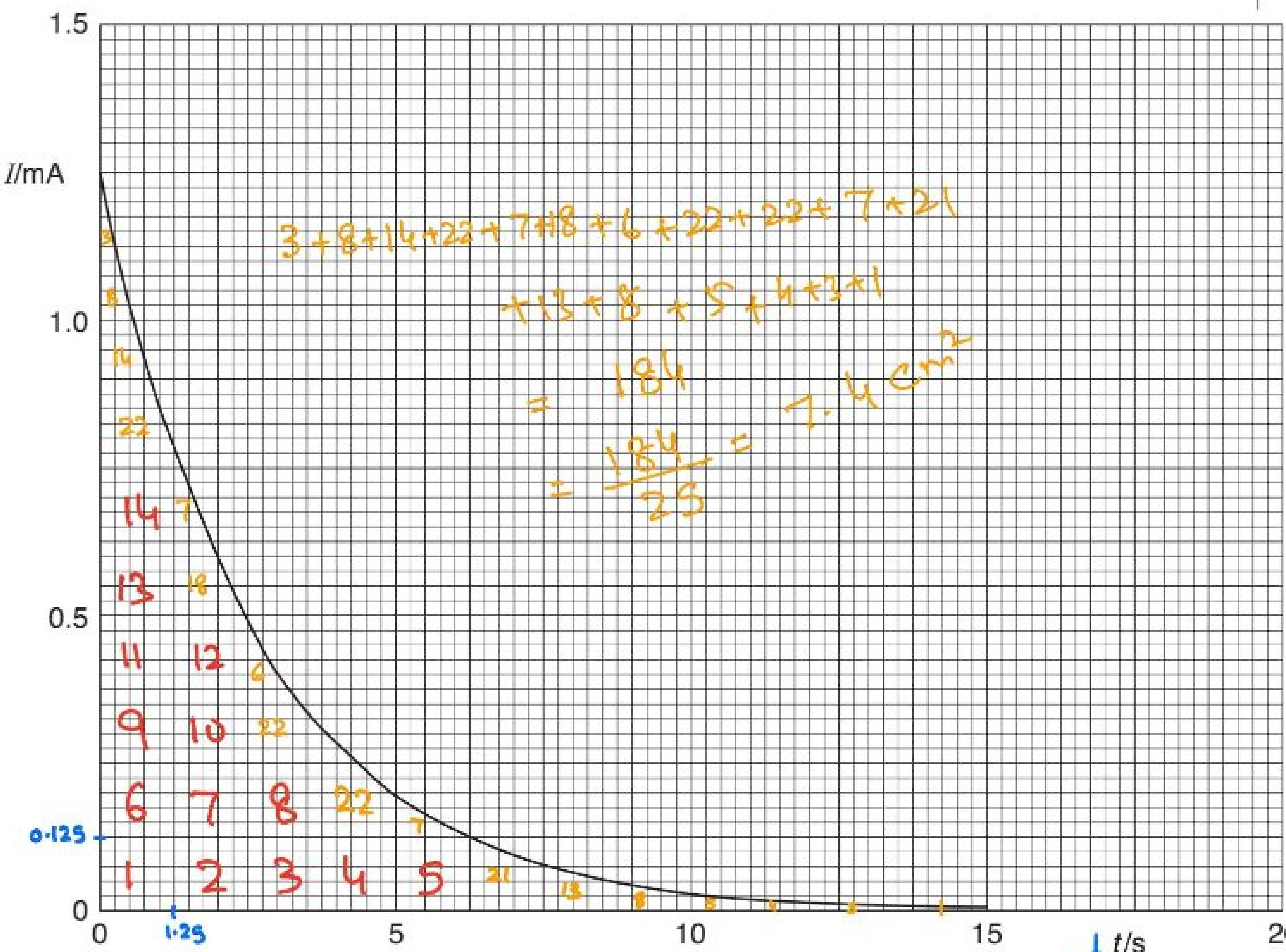


Fig. 5.2

$I = I_0 e^{-\frac{t}{RC}}$

For Examiner's Use

(i) State the relation between the current in a circuit and the charge that passes a point in the circuit.

$I = \frac{Q}{t}$

....[1]

(ii) The area below the graph line of Fig. 5.2 represents charge.
Use Fig. 5.2 to determine the initial charge stored in the capacitor.

1 cm^2 area = 25 small boxes = (0.125)(1.25) = 0.156×10^{-3} C

184 small boxes = $\frac{184}{25}$ = 7.4

Total Area in cm^2 = 14 + 7.4 = 21.4 cm^2

21.4 cm^2 area represents = (21.4)(0.156)
= 3.34×10^{-3}

charge = 3338 μC [4]

(iii) Initially, the potential difference across the capacitor was 15 V.
Calculate the capacitance of the capacitor.

$Q = CV$

$3338 \times 10^{-6} = (C)(15)$

$C = 222.6 \times 10^{-6}$

capacitance = 223 μF [2]

(c) The capacitor in **(b)** discharges one half of its initial energy. Calculate the new potential difference across the capacitor.

Remaing energy in capacitors = $\frac{1}{2}$(Total initial energy stored)

$\cancel{\frac{1}{2}} \cancel{C} V^2 = \frac{1}{2}\left[\cancel{\frac{1}{2}} \cancel{C} V_0^2\right]$

$V^2 = \frac{1}{2}(15)^2 \Rightarrow V = \sqrt{\frac{225}{2}}$

potential difference = 10.6 V [3]

For Examiner's Use

4 **(a)** Define *capacitance*.

$C = \frac{q}{V}$

[1]

(b) An isolated metal sphere has a radius *r*. When charged to a potential *V*, the charge on the sphere is *q*.
The charge may be considered to act as a point charge at the centre of the sphere.

(i) State an expression, in terms of *r* and *q*, for the potential *V* of the sphere.

$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$ [1]

(ii) This isolated sphere has capacitance. Use your answers in **(a)** and **(b)(i)** to show that the capacitance of the sphere is proportional to its radius.

Since $V = \frac{q}{C}$

By comparison

$C = 4\pi\varepsilon_0 r$

$C = (\text{Constant})\, r$

$C \propto r$

[1]

(c) The sphere in **(b)** has a capacitance of 6.8 pF and is charged to a potential of 220 V.

Calculate

(i) the radius of the sphere,

$C = 4\pi\varepsilon_0 r$

$r = \frac{C}{4\pi\varepsilon_0}$

$r = \frac{6.8 \times 10^{-12}}{4(3.14)(8.85 \times 10^{-12})}$

radius = 6.11×10^{-2} m [3]

(ii) the charge, in coulomb, on the sphere.

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$Q = CV$

$= (6.8 \times 10^{-12})(220)$

$= 1.496 \times 10^{-9}$

charge = 1.5×10^{-9} C [1]

(d) A second uncharged metal sphere is brought up to the sphere in **(c)** so that they touch. The combined capacitance of the two spheres is 18 pF.

Calculate

(i) the potential of the two spheres,

Total charge before connection = Total charge after connection

$1.5 \times 10^{-9} = Q$

$1.5 \times 10^{-9} = CV$

$1.5 \times 10^{-9} = (18 \times 10^{-12})V$

$V = 83.3$

potential = ... V [1]

(ii) the change in the total energy stored on the spheres when they touch.

$\Delta E = E_2 - E_1$

$= \frac{1}{2}C_2V_2^2 - \frac{1}{2}C_1V_1^2$

$= \frac{1}{2}(18 \times 10^{-12})(83.3)^2 - \frac{1}{2}(6.8 \times 10^{-12})(220)^2$

$= -1.02 \times 10^{-7}$ J

change = 1.02×10^{-7} J [3]

-ve sign shows the loss of energy.