# A2 PHYSCIS 9702
## Crash Course

PROSPERITY ACADEMY

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# CAPACITANCE

**COMPLETE NOTES**

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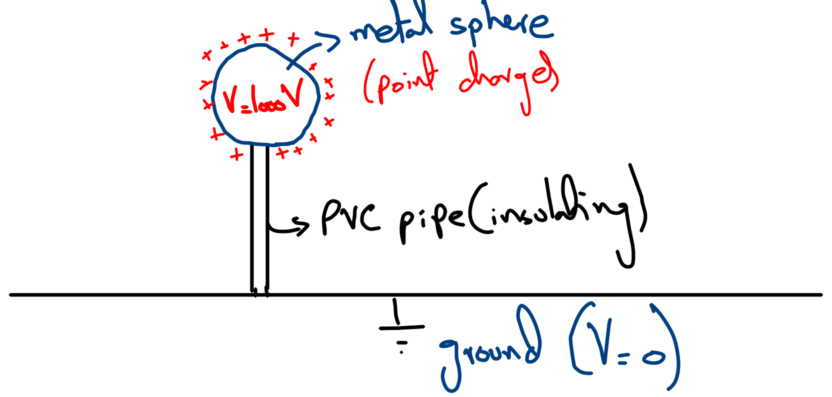
# Capacitance:-

Ratio of charge on a body to its electrical potential

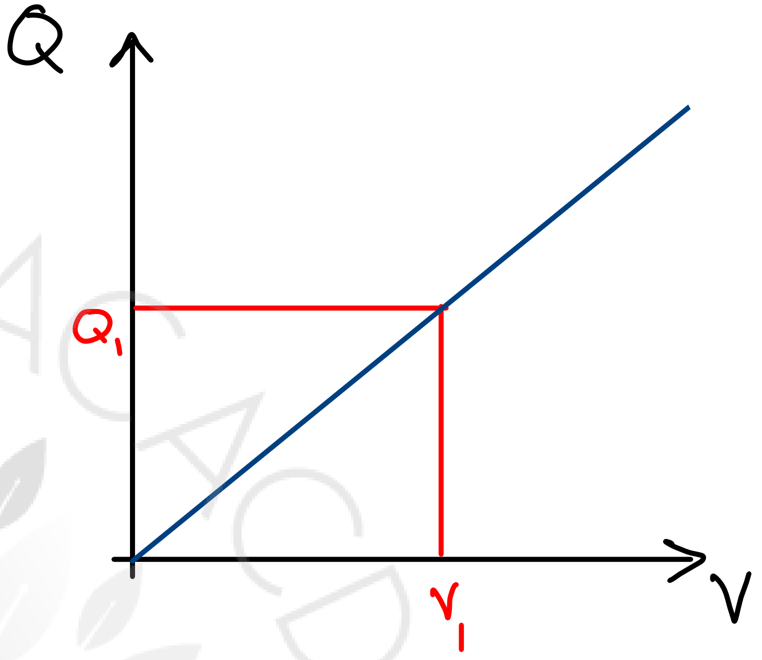
$$C = \frac{Q}{V}$$ (Scalar, measured in Farads)




Working out the capacitance of a metal sphere:-

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{K\cancel{Q}}{r}} \Rightarrow \frac{r}{K} = \boxed{4\pi\varepsilon_0 r = C}$$

$$K = \frac{1}{4\pi\varepsilon_0} = 9\times10^9$$



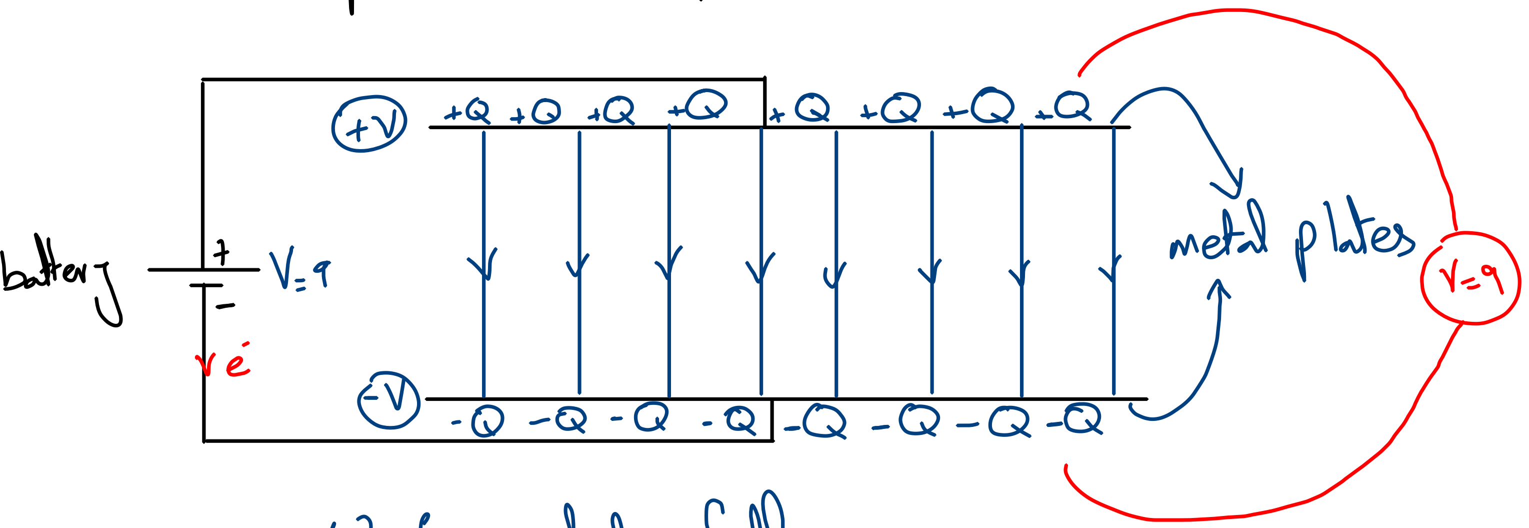

$$C = \frac{Q_1}{V_1}$$

Charge leaks naturally, you can minimise it by:-

1) Use a smooth surface to store charge
2) Use a dry environment

Capacitors store energy in the form of electrical potential energy due to seperation of charges

# Parallel plate Capacitor:-




Uniform electric field

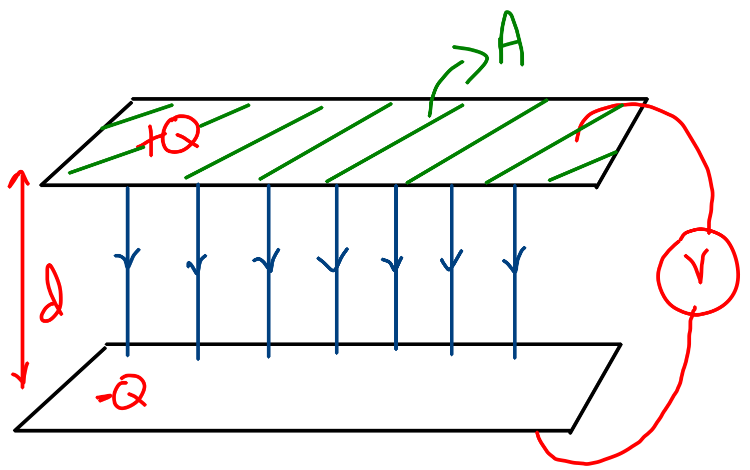
Capacitors donot store charge

$$Q_T = +Q - Q = 0$$

Capacitors store energy, not charge

Capacitance for a Parallel plate Capacitor:- Ratio of charge on one of the plates to the potential difference between the plates

Important relationships of a parallel plate capacitor:-



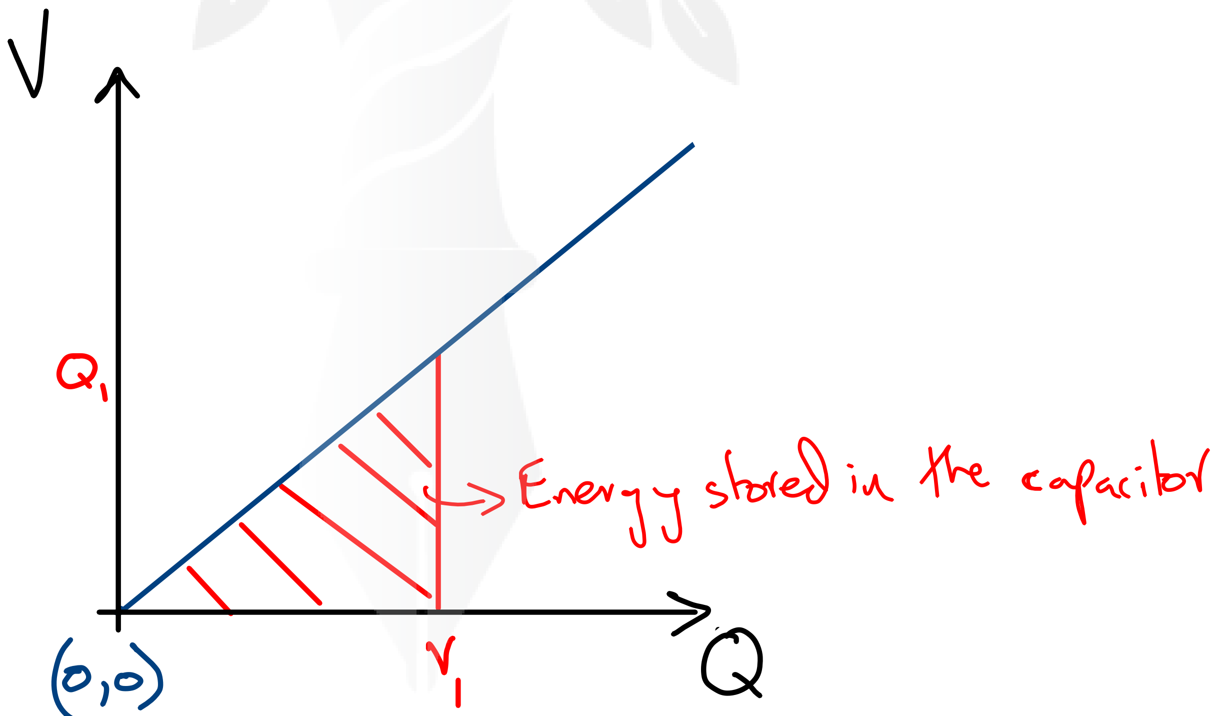

$* \; C \propto A$ $\quad * \; C \propto \frac{1}{d}$ $\quad * \; C \propto \frac{A}{d}$

We know:-

$$V = \frac{W}{Q}$$

$W = VQ$ (if the voltage is constant)

If the voltage is not constant, use Area under graph




Energy stored in a capacitor

$$\boxed{1)\; E = \frac{1}{2} VQ}$$

$$E = \frac{1}{2} \times \frac{Q}{C} \times Q$$

$$\boxed{2)\; E = \frac{1}{2} \frac{Q^2}{C}}$$

$$E = \frac{1}{2} V (CV)$$

$$\boxed{3)\; E = \frac{1}{2} CV^2}$$

$C = \frac{Q}{V}$

$V = \frac{Q}{C}$

$Q = CV$

5 A solid metal sphere, of radius $r$, is insulated from its surroundings. The sphere has charge $+Q$.
This charge is on the surface of the sphere but it may be considered to be a point charge at its centre, as illustrated in Fig. 5.1.

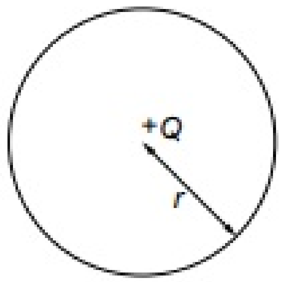



**Fig. 5.1**

**(a)** **(i)** Define *capacitance*.

ratio of charge on a body to its electrical potential

[1]

**(ii)** Show that the capacitance $C$ of the sphere is given by the expression

$C = 4\pi\varepsilon_0 r$.

$$C = \frac{Q}{V} = \frac{\cancel{Q}}{\frac{K\cancel{Q}}{r}} = \frac{r}{K} \quad \left(K = \frac{1}{4\pi\varepsilon_0}\right)$$

$$C = 4\pi\varepsilon_0 r$$

[1]

**(b)** The sphere has radius 36 cm.
Determine, for this sphere,

**(i)** the capacitance,

$$C = 4\pi\varepsilon_0 r$$
$$= 4\pi\,(8.85\times10^{-12})\times(36\times10^{-2})$$
$$= 4\times10^{-11}$$

capacitance = $4.0\times10^{-11}$ F [1]

**(ii)** the charge required to raise the potential of the sphere from zero to $7.0 \times 10^5$ V.

$$C = \frac{Q}{V} \Rightarrow 4\times10^{-11} = \frac{Q}{7\times10^{5}}$$

$$Q = 2.8\times10^{-5}\ C$$

charge = $2.8\times10^{-5}$ C [1]

**(c)** Suggest why your calculations in **(b)** for the metal sphere would not apply to a plastic sphere.

plastic is an insulator and so charge cannot flow on it. Therefore it cannot behave as a point charge and our calculations will not work.

[3]

**(d)** A spark suddenly connects the metal sphere in **(b)** to the Earth, causing the potential of the sphere to be reduced from $7.0 \times 10^5$ V to $2.5 \times 10^5$ V.

Calculate the energy dissipated in the spark.

$$\Delta E = \frac{1}{2}C(\Delta V^2)$$
$$= \frac{1}{2}C(V_f^2 - V_i^2)$$
$$= \frac{1}{2}(4\times10^{-11})\left[(2.5\times10^5)^2 - (7.0\times10^5)^2\right]$$
$$= -8.55\ J$$

energy = $-8.6$ J [3]

$$E = \frac{1}{2}VQ$$
$$= \frac{1}{2}\frac{Q^2}{C}$$
$$3) = \frac{1}{2}CV^2$$

**4** **(a)** **(i)** State what is meant by *electric potential* at a point.

It is the work done per unit charge in bringing that charge from infinity to a point within an electric field. [2]

**(ii)** Define *capacitance*.

Ratio of charge on a body to its potential [1]

**(b)** The variation of the potential $V$ of an isolated metal sphere with charge $Q$ on its surface is shown in Fig. 4.1.

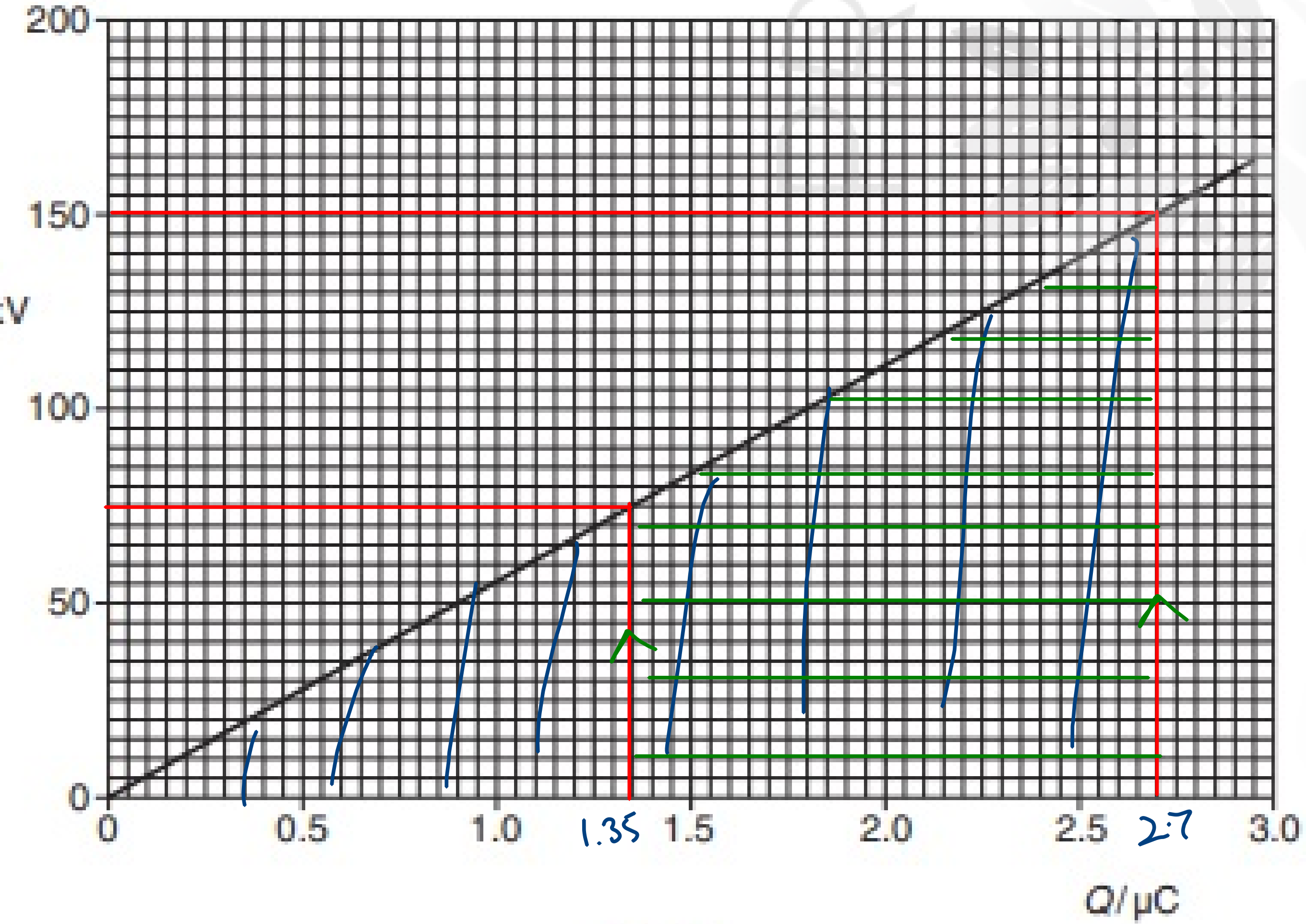



**Fig. 4.1**

An isolated metal sphere has capacitance.

Use Fig. 4.1 to determine

**(i)** the capacitance of the sphere,

$$C = \frac{Q}{V} = \frac{2.5\times10^{-6}}{140\times10^{3}} = 1.79\times10^{-11}$$

capacitance = $1.8\times10^{-11}$ F [2]

**(ii)** the electric potential energy stored on the sphere when charged to a potential of 150 kV.

Area under graph:-

$E = \frac{1}{2}CV^2$

$$E = \frac{1}{2}\times(2.7\times10^{-6})\times 150\times10^{3}$$

$= 0.2025$

energy = 0.20 J [2]

**(c)** A spark reduces the potential of the sphere from 150 kV to 75 kV. Calculate the energy lost from the sphere.

Area under graph:-

$$A = \frac{1}{2}\times(a+b)\times h = \frac{1}{2}\times\left[(75+150)\times10^{3}\right]\times\left[(2.7-1.35)\times10^{-6}\right]$$

$= 0.151875$

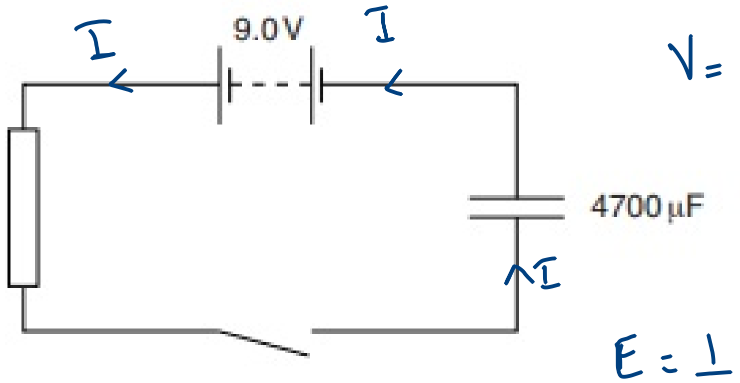
alt

$$\Delta E = \frac{1}{2}C(V_f^2 - V_i^2)$$

energy = 0.15 J [2]

6 An uncharged capacitor is connected in series with a battery, a switch and a resistor, as shown in Fig. 6.1.

$Q = It$




$V = \frac{W}{Q}$

$VQ = W$

$E = \frac{1}{2}CV^2$

**Fig. 6.1**

The battery has e.m.f. 9.0 V and negligible internal resistance. The capacitance of the capacitor is 4700 μF.

battery
capacitor

The switch is closed at time $t = 0$.

During the time interval $t = 0$ to $t = 4.0$ s, the charge passing through the resistor is 22 mC.

**(a)** **(i)** Calculate the energy transfer in the battery during the time interval $t = 0$ to $t = 4.0$ s.

$W = VQ$

$W = 9 \times (22 \times 10^{-3})$

$= 0.198$

energy transfer = 0.20 J [2]

**(ii)** Determine, for the capacitor at time $t = 4.0$ s,

**1.** the potential difference $V$ across the capacitor,

$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} = \frac{22 \times 10^{-3}}{4700 \times 10^{-6}} = 4.68$

$V$ = 4.7 V [2]

**2.** the energy stored in the capacitor.

$E = \frac{1}{2}\frac{Q^2}{C} \Rightarrow \frac{1}{2} \times \frac{(22 \times 10^{-3})^2}{(4700 \times 10^{-6})} = 5.1 \times 10^{-2}$

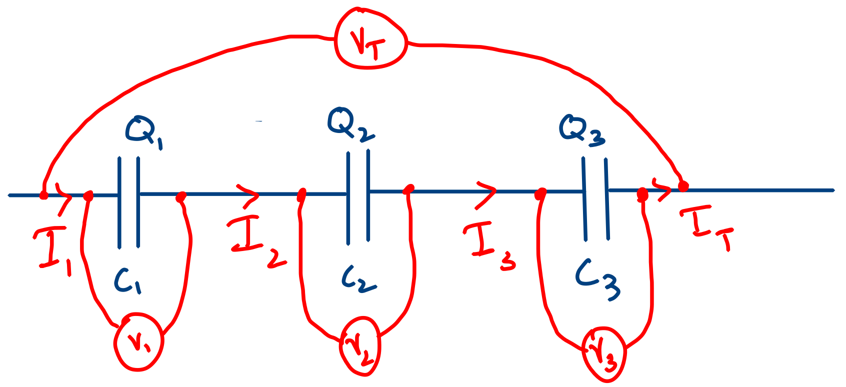
energy = $5.1 \times 10^{-2}$ J [2]

**(b)** Suggest why your answers in **(a)(i)** and **(a)(ii) part 2** are different.

Energy lost as thermal heat in resistor

[1]

# Capacitors in series:-




* $I$ is constant / same throughout

$$I_1 = I_2 = I_3 = I_T$$

* $Q = I \times \Delta t \Rightarrow I = \dfrac{\Delta Q}{\Delta t}$

$$\frac{Q_1}{\cancel{\Delta t}} = \frac{Q_2}{\cancel{\Delta t}} = \frac{Q_3}{\cancel{\Delta t}} = \frac{Q_T}{\cancel{\Delta t}}$$

* $Q_1 = Q_2 = Q_3 = Q_T$ (charge is constant)

* $V_T = V_1 + V_2 + V_3$

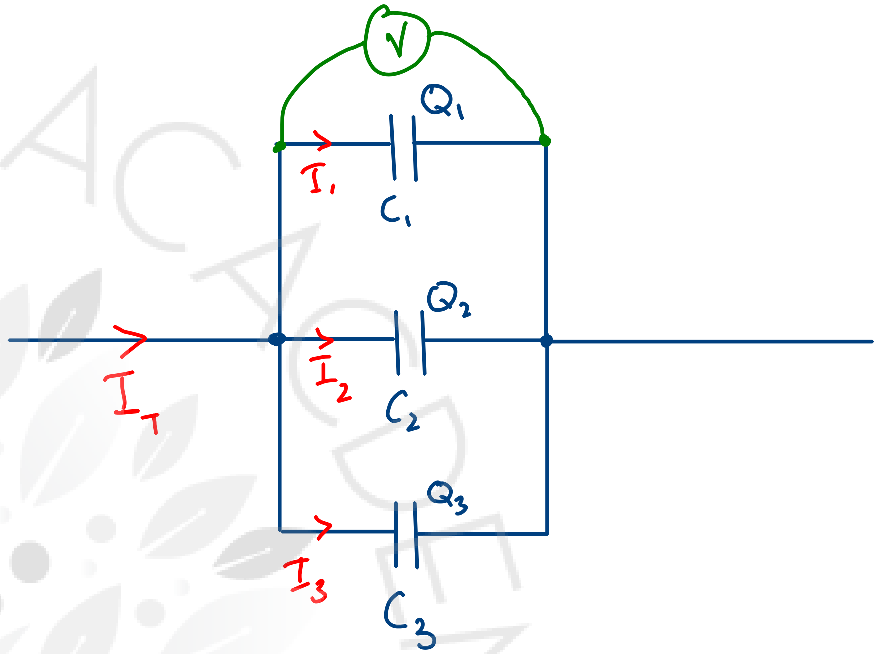
$$C = \frac{Q}{V}$$

$$\frac{\cancel{Q_T}}{C_T} = \frac{\cancel{Q_1}}{C_1} + \frac{\cancel{Q_2}}{C_2} + \frac{\cancel{Q_3}}{C_3}$$

$$V = \frac{Q}{C}$$

* $$\boxed{\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

# Capacitors in parallel:-




* $V$ remains constant

* $I_T = I_1 + I_2 + I_3$

$$I = \frac{\Delta Q}{\Delta t}$$

$$\frac{Q_T}{\cancel{\Delta t}} = \frac{Q_1}{\cancel{\Delta t}} + \frac{Q_2}{\cancel{\Delta t}} + \frac{Q_3}{\cancel{\Delta t}}$$

* $\boxed{Q_T = Q_1 + Q_2 + Q_3}$ total charge = sum of individual charges

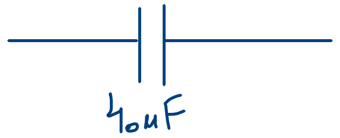
$$C_T\cancel{V} = C_1\cancel{V} + C_2\cancel{V} + C_3\cancel{V}$$

$$C = \frac{Q}{V} \Rightarrow Q = CV$$

$$\boxed{C_T = C_1 + C_2 + C_3}$$

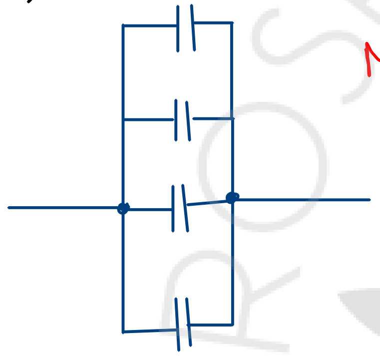
Q. You have been given several capacitors rated 40 μF and 6 V max. You are required to specifically arrange the capacitors in a circuit to achieve the desired capacitance and also calculate the maximum voltage that can be applied to the configuration.

1) 40 μF




Max Voltage = 6V

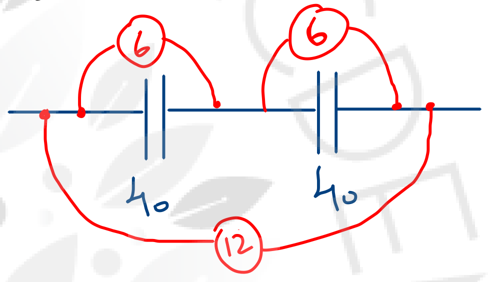
2) 160 μF



Max Voltage = 6V

$C_T = 40+40+40+40 = 160\mu F$

3) 20 μF




Max Voltage = 12V

$C_T = \left(\frac{1}{40} + \frac{1}{40}\right)^{-1}$

$C_T = 20\mu F$

In case of series, work with charge

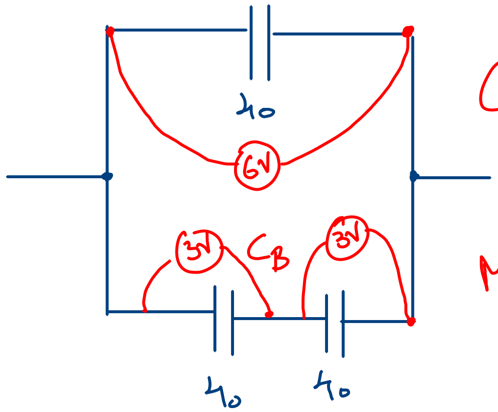
$Q_T = Q_1 = Q_2 = Q_3$

$C_1V_1 = C_2V_2 \Rightarrow \cancel{40}V_1 = \cancel{40}V_2$

$V_1 = V_2$

1 : 1

4) 60 μF




$C_T = 40+20$

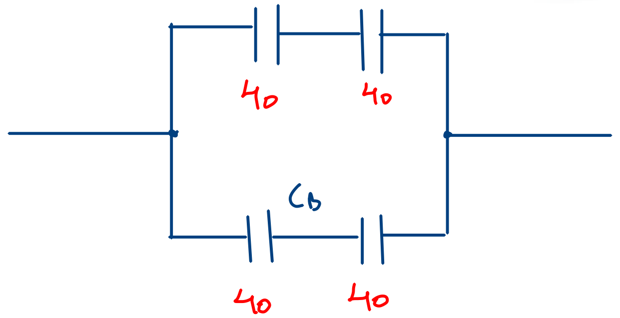
$= 60\mu F$

Max Voltage = 6V

$C_1V_1 = C_2V_2$

$C_B = \left(\frac{1}{40} + \frac{1}{40}\right)^{-1} = 20$

5) 40 μF using 4 capacitors

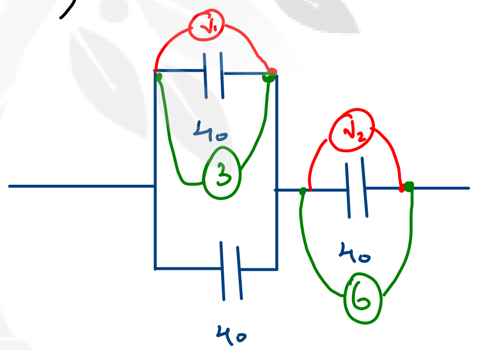



$C_B = \left(\frac{1}{40} + \frac{1}{40}\right)^{-1} = 20$

$C_T = 20+20 = 40\mu F$

Max Voltage = 12V

6) 26.67 μF




$C_T = \left(\frac{1}{80} + \frac{1}{40}\right)^{-1} = 26.67\mu F$

Max Voltage = 9V

$V_T = 3+6 = 9$

$C_1V_1 = C_2V_2$

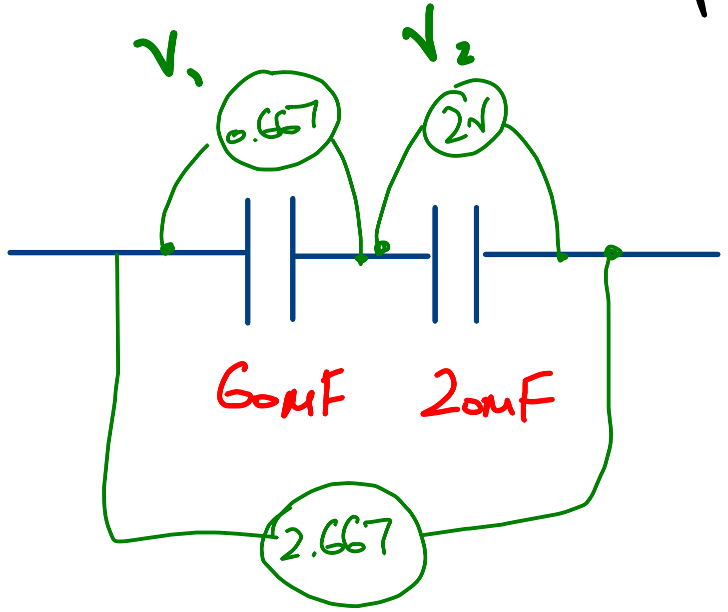
$2\,\cancel{80}V_1 = {}^1\cancel{40}V_2$

$V_1 = V_2$

1 : 2

3 : 6

A 60μF capacitor (6V max) is connected in series to a 20μF capacitor (2V max). Calculate the total capacitance and total voltage that can be applied




$$C_T = \left(\frac{1}{60} + \frac{1}{20}\right)^{-1}$$

$$C_T = 15\,\mu F$$

Max voltage = 2.667 V

Q const

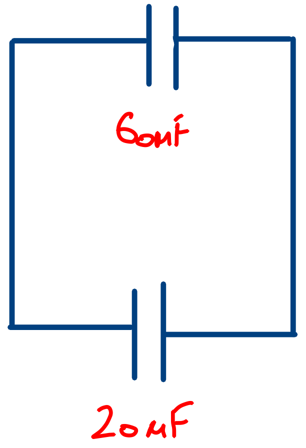
$$C_1V_1 = C_2V_2$$

$$\overset{3}{\cancel{60}}\,V_1 = \overset{1}{\cancel{20}}\,V_2$$

$$3V_1 = V_2$$

| | | |
|---|---|---|
| $V_1$ | : | $V_2$ |
| 1 | : | 3 |
| 6 | : | 18 |
| 0.667 | : | 2 |

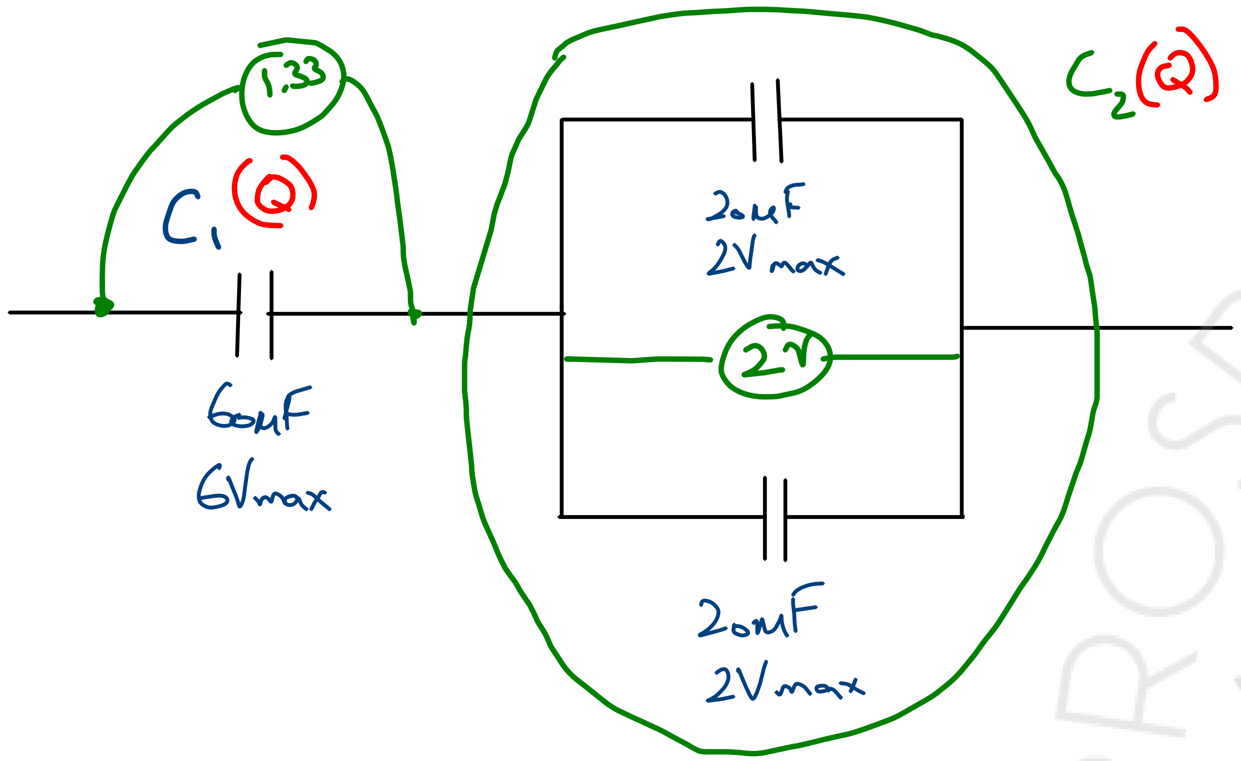
A 60μF capacitor (6V max) is connected in parallel to a 20μF capacitor (2V max). Calculate the total capacitance and total max voltage that can be applied




$$C_T = 60 + 20 = 80\,\mu F$$

Max Voltage = 2V

Calculate the total capacitance and maximum voltage that can be applied to the capacitors below:-




$$C_2 = 40\,\mu F$$

$$C_T = \left(\frac{1}{60} + \frac{1}{40}\right)^{-1} = 24\,\mu F$$

$$V_{max} = 2 + 1.33 = 3.33V$$

$$C_1 V_1 = C_2 V_2$$

$$3\,\cancel{60}\,V_1 = 2\,\cancel{40}\,V_2$$

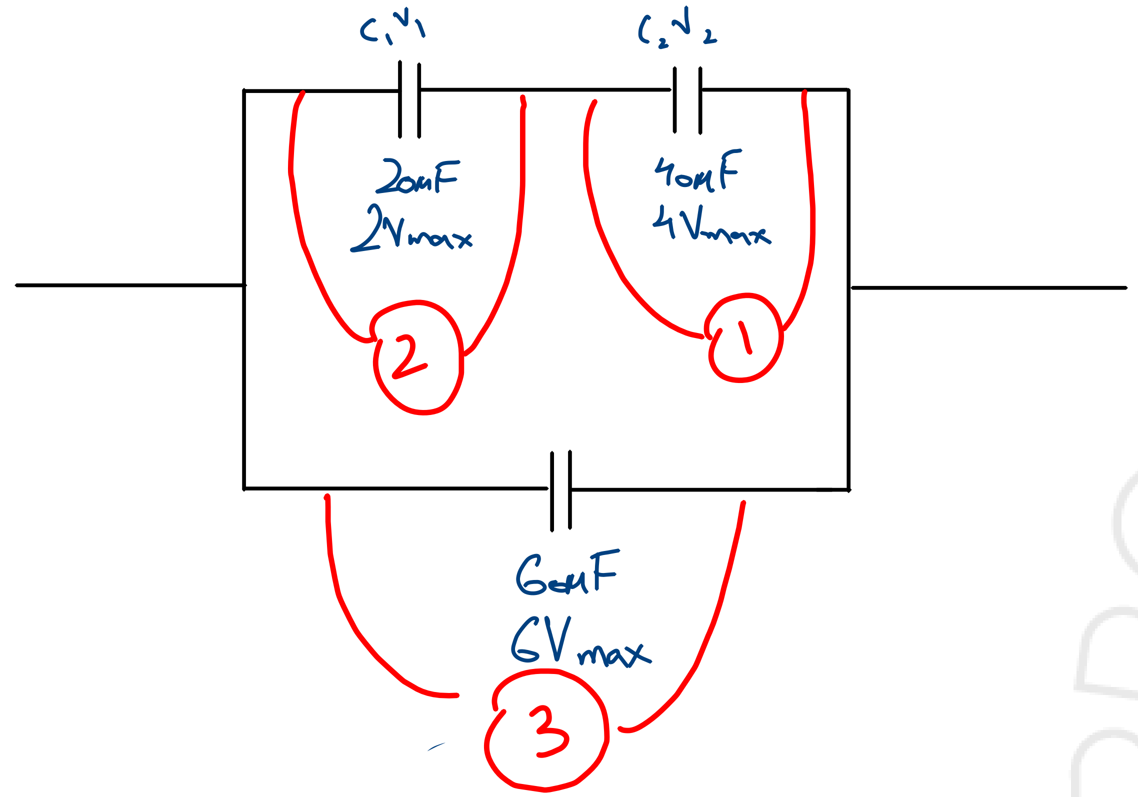
$$V_1 : V_2$$

$$1 : 1.5$$

$$6 : 9$$

$$1.33 : 2$$

Calculate the total capacitance and maximum voltage that can be applied to the capacitors below:-




$$C = \left(\frac{1}{20} + \frac{1}{40}\right)^{-1} = 13.33 \ \mu F$$

$$C_T = 13.33 + 60 = 73.33 \ \mu F$$

$$C_1 V_1 = C_2 V_2$$

$$\cancel{20}^{\,1} V_1 = \cancel{40}^{\,2} V_2$$

$V_1 : V_2$

$2 : 1$

$8 : 4$ ✗

$2 : 1$ ✓

Max Voltage = 3V

5 (a) (i) Define *capacitance*.

.............................................................................................................

.............................................................................................................[1]

(ii) A capacitor is made of two metal plates, insulated from one another, as shown in Fig. 5.1.

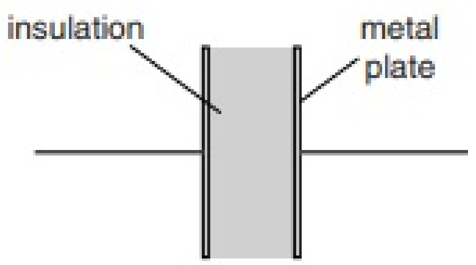



**Fig. 5.1**

Explain why the capacitor is said to store energy but not charge.

The plates get oppositely charged to equal magnitude which makes the net charge = 0. Due to the separation of the charges, the capacitor is able to store electrical potential energy.

.............................................................................................................[4]

(b) Three uncharged capacitors X, Y and Z, each of capacitance 12 μF, are connected as shown in Fig. 5.2.

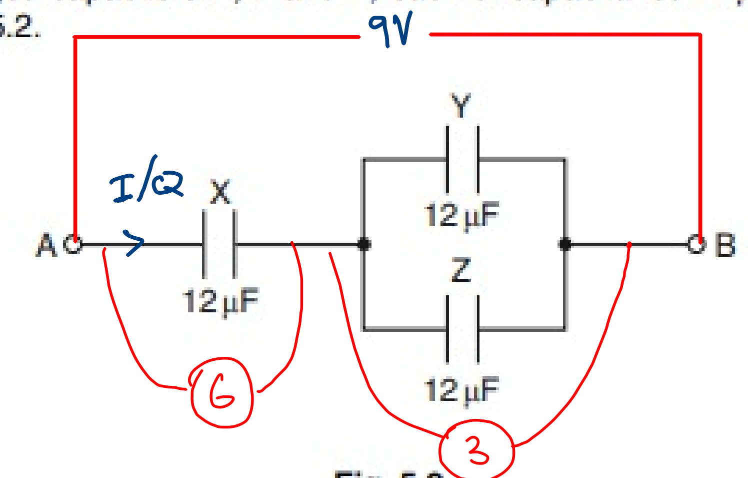



**Fig. 5.2**

$C = \frac{Q}{V}$

$8 = \frac{Q}{9 \times 10^{-6}}$

$Q = 72 \times 10^{-6}$ C

$Q_T = 72\ \mu C$

A potential difference of 9.0 V is applied between points A and B.

(i) Calculate the combined capacitance of the capacitors X, Y and Z.

$C_T = \left(\frac{1}{12} + \frac{1}{24}\right)^{-1} = 8\ \mu F$

capacitance = ............8............ μF [2]

(ii) Explain why, when the potential difference of 9.0 V is applied, the charge on one plate of capacitor X is 72 μC.

In series, charge is constant as current is constant so the charge on one plate of capacitor X is equal to the total charge.

.............................................................................................................[2]

(iii) Determine

1. the potential difference across capacitor X,

$C = \frac{Q}{V} \Rightarrow 12 \times \cancel{10^{-6}} = \frac{72 \times \cancel{10^{-6}}}{V}$

$V = \frac{72}{12} = 6$

potential difference = ............6............ V [1]

2. the charge on one plate of capacitor Y.

$C = \frac{Q}{V}$

$12 \times 10^{-6} = \frac{Q}{3} \Rightarrow Q = 36 \times 10^{-6}$

charge = ............$36 \times 10^{-6}$............ μC [2]

**7** **(a)** State two uses of capacitors in electrical circuits, other than for the smoothing of direct current.

1. ..............................................................................................................

2. .............................................................................................................. [2]

**(b)** The combined capacitance between terminals A and B of the arrangement shown in Fig. 7.1 is 4.0 μF.

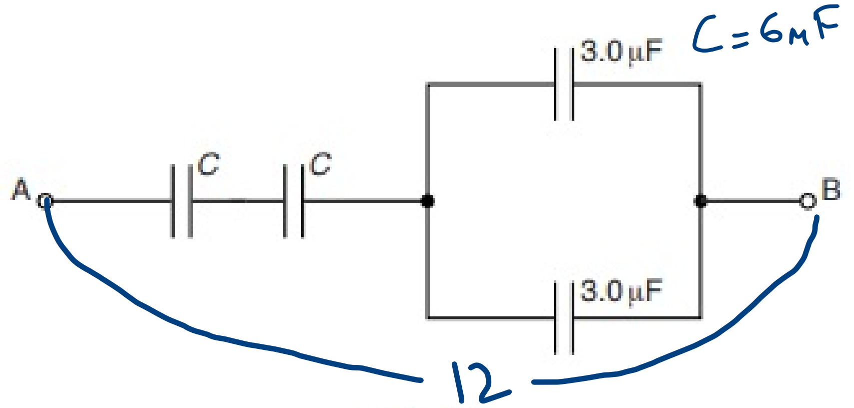



**Fig. 7.1**

Two capacitors each have capacitance $C$ and the remaining capacitors each have capacitance 3.0 μF.

The potential difference (p.d.) between terminals A and B is 12 V.

**(i)** Determine the capacitance $C$.

$$C_T = 4\mu F = \left(\frac{1}{C} + \frac{1}{C} + \frac{1}{6}\right)^{-1}$$

$$\frac{1}{4} = \frac{2}{C} + \frac{1}{6} \Rightarrow C = \frac{2}{\left(\frac{1}{4} - \frac{1}{6}\right)} = 24$$

$C$ = .............24............. μF [2]

**(ii)** Calculate the magnitude of the total positive charge transferred to the arrangement.

$$C = \frac{Q}{V} \Rightarrow 4 \times 10^{-6} = \frac{Q}{12}$$

$$Q = 48 \times 10^{-6}\ C$$

charge = .............48............. μC [2]



**(iii)** Use your answer in **(ii)** to state the magnitude of the charge on one plate of

**1.** a capacitor of capacitance $C$,

charge = .............48............. μC

**2.** a capacitor of capacitance 3.0 μF.

charge = .............24............. μC [2]

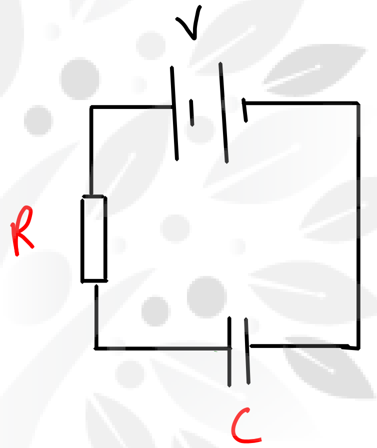
[Total: 8]

# Charging and discharging capacitors:-

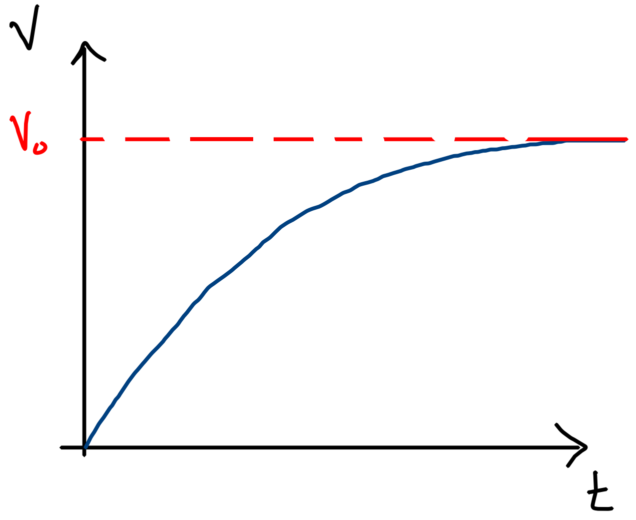
**Time constant:-** It is the time required for the capacitor to charge to 2/3 of the supply voltage.

It is the time required for the capacitor to discharge by 2/3 of its initial voltage

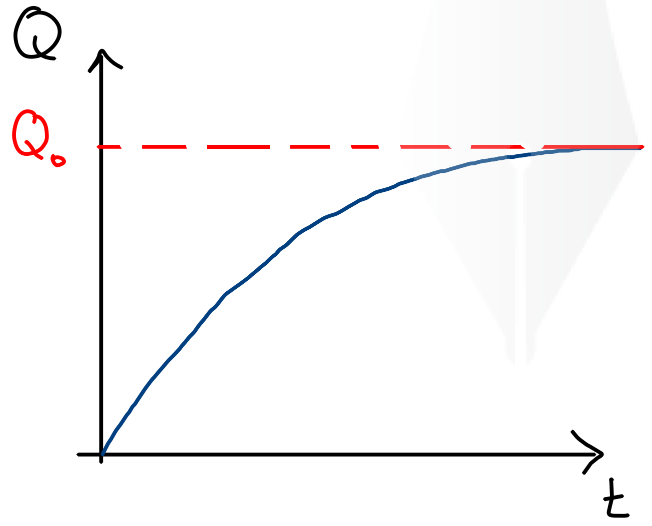
$$T = R \times C$$



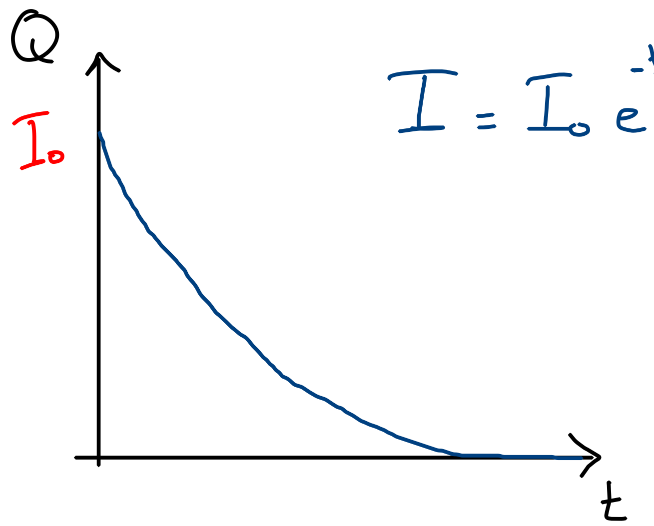

## Charging:-




$$V = V_o\left(1 - e^{-t/RC}\right)$$

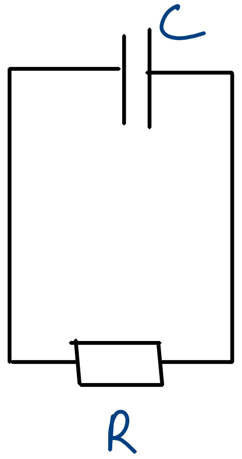



$$Q = Q_o\left(1 - e^{-t/RC}\right)$$

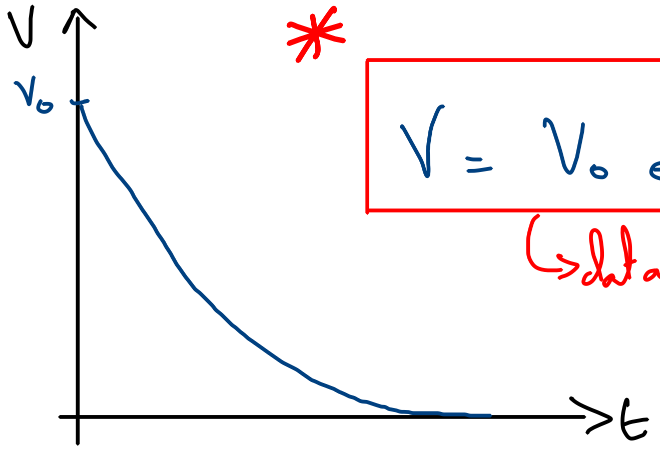



$$I = I_o e^{-t/RC}$$

# Discharging:-



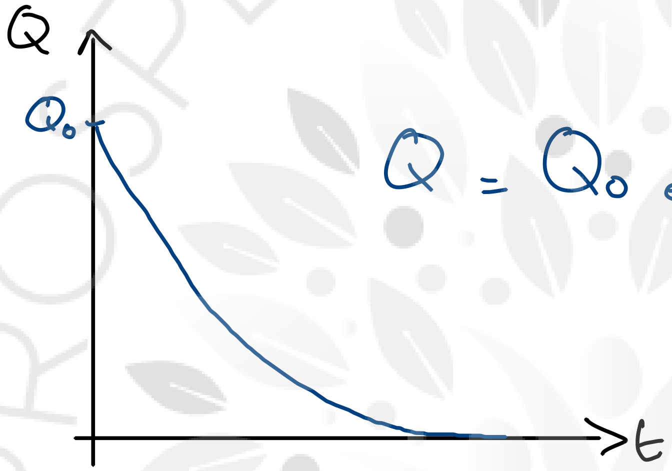

$T = R \times C$



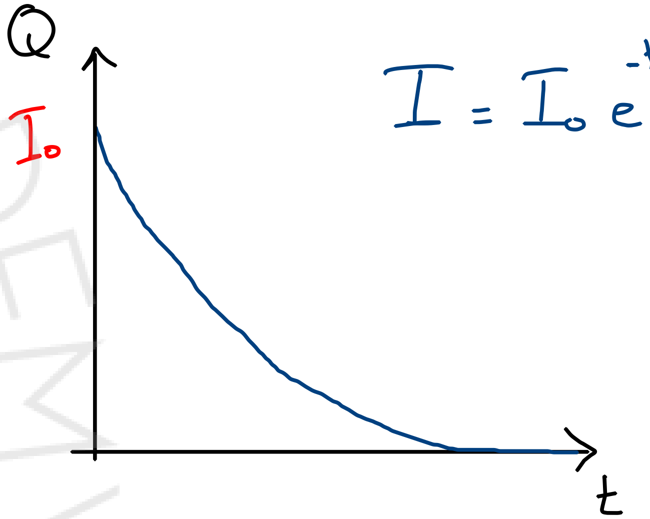

*

$$\boxed{V = V_0 e^{-t/RC}}$$

↳ data booklet




$$Q = Q_0 e^{-t/RC}$$




$$I = I_0 e^{-t/RC}$$



## Understanding ln and e

$\ln(e) = 1$ and $\ln(e^a) = a(1) = a$

a) $e^x = 2$

$\ln(e^x) = \ln(2)$

$x \ln(e) = \ln 2$

$x = \ln 2 = 0.693$

b) $e^{9/2} = 1 - x$

$x = 1 - e^{9/2}$

$x = -89.02$

c) $e^x \cdot e^{3x} = 2$

$\ln(e^{4x}) = \ln(2)$

$4x \cancel{\ln e} = \ln 2$

$x = \dfrac{\ln 2}{4}$

$x = 0.17$

d) $e^5 = 20 e^x$

$\dfrac{e^5}{e^x} = 20$

$\ln(e^{5-x}) = \ln(20)$

$5 - x \cancel{\ln e} = \ln 20$

$x = 5 - \ln 20$

$x = 2.00$

# Half life of a capacitor:-

The time taken by a capacitor to charge to half of the supply voltage.

The time taken by a capacitor to discharge to half of its initial value.

$$V = V_o e^{-t/RC}$$

$t_{\frac{1}{2}}$ = half life

$$\frac{\cancel{V_o}}{2} = \cancel{V_o} e^{-t_{\frac{1}{2}}/RC}$$

$$\frac{1}{2} = e^{-t_{\frac{1}{2}}/RC} \Rightarrow \frac{1}{2} = \frac{1}{e^{t_{\frac{1}{2}}/RC}}$$

$$\ln\left(e^{t_{\frac{1}{2}}/RC}\right) = \ln(2)$$

$$\frac{t_{\frac{1}{2}}}{RC} = \ln 2$$

$$\boxed{t_{\frac{1}{2}} = RC \ln 2}$$

5 A sinusoidal alternating potential difference (p.d.) from a supply is rectified using a single diode. The variation with time $t$ of the rectified potential difference $V$ is shown in Fig. 5.1.

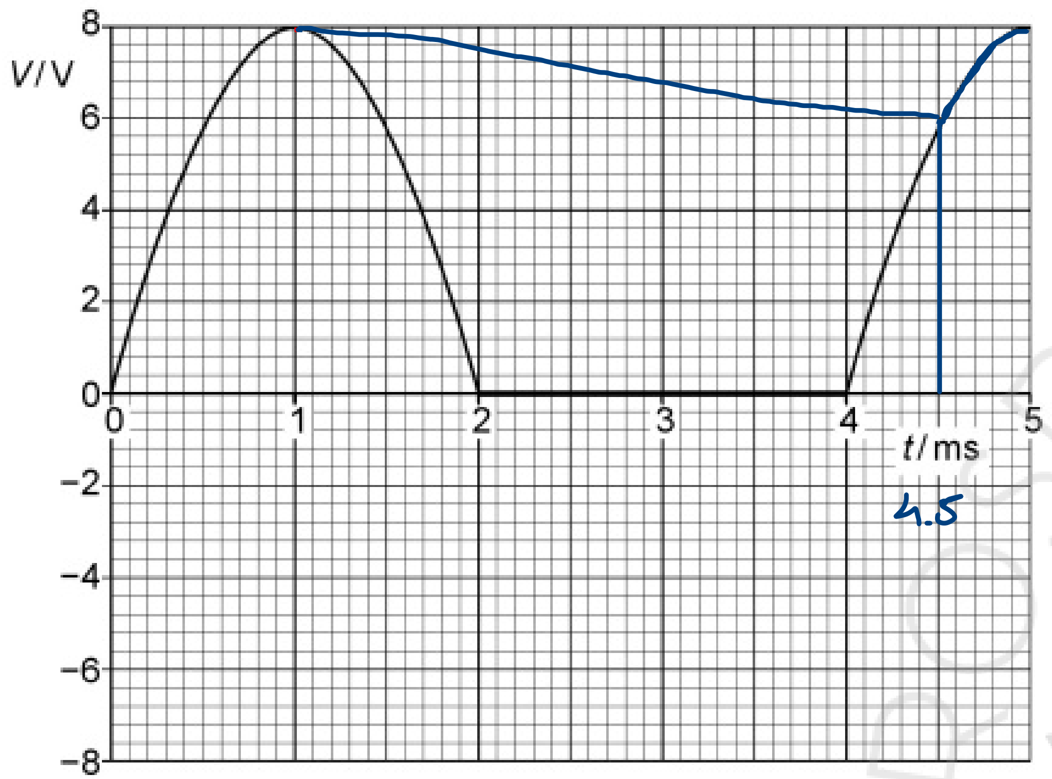



**Fig. 5.1**

**(a) (i)** Determine the root-mean-square (r.m.s.) value of the supply potential difference before rectification.

$$V_{r.m.s} = \frac{V_0}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 5.65$$

r.m.s. potential difference = 5.7 V [2]

**(ii)** State the type of rectification shown in Fig. 5.1.

Half wave rectification [1]

**(b)** The alternating potential difference is rectified and smoothed using the circuit in Fig. 5.2.

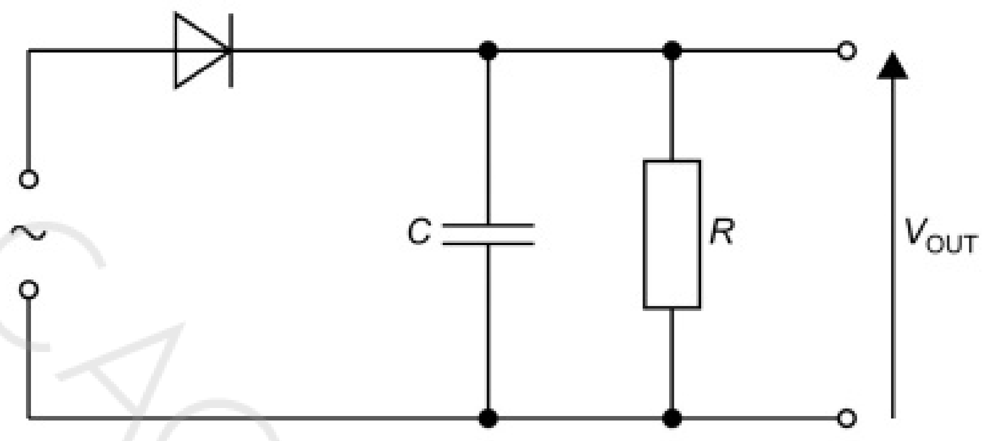



**Fig. 5.2**

The capacitor has capacitance $C$ of 85 μF and the resistor has resistance $R$.

The effect of the capacitor and the resistor is to produce a smoothed output potential difference $V_{OUT}$. The difference between maximum and minimum values of $V_{OUT}$ is 2.0 V.

**(i)** On Fig. 5.1, draw a line to show $V_{OUT}$ between times $t$ = 1.0 ms and $t$ = 5.0 ms. [3]

**(ii)** Determine the time, in s, for which the capacitor is discharging between times $t$ = 1.0 ms and $t$ = 5.0 ms.

time = $3.5 \times 10^{-3}$ s [1]

**(iii)** Use your answers in **(b)(i)** and **(b)(ii)** to calculate the resistance $R$.

$$V = V_0 e^{-t/RC}$$

$$6 = 8 e^{-t/RC}$$

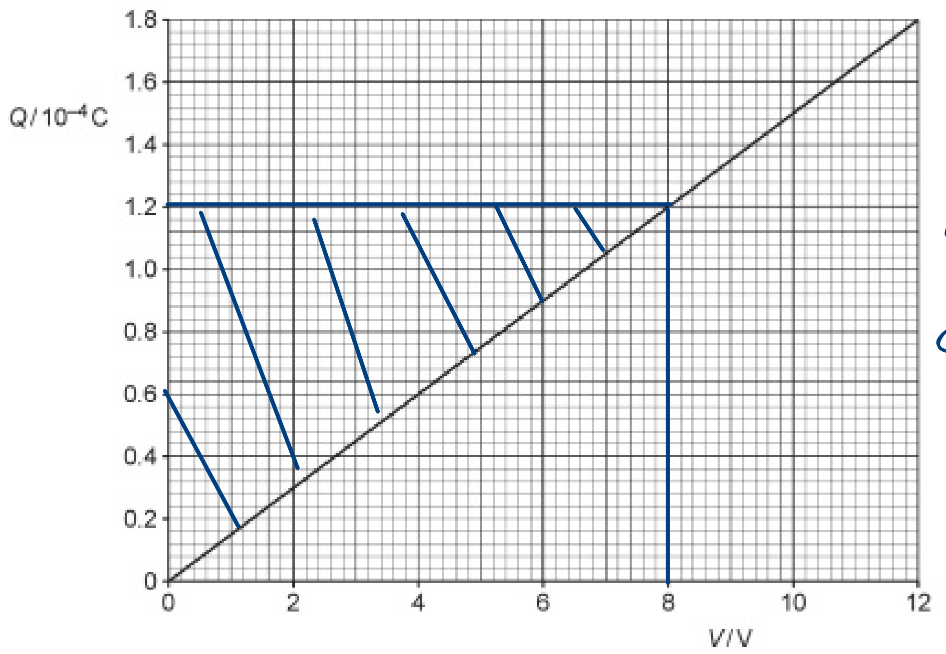
$$\ln\left(\frac{6}{8}\right) = \ln\left(e^{-t/RC}\right)$$

$$\frac{-t}{RC} = \ln\left(\frac{6}{8}\right)$$

$$R = \frac{-(3.5 \times 10^{-3})}{(85 \times 10^{-6})\left(\ln\left(\frac{6}{8}\right)\right)} = 143.13\,\Omega$$

$$\boxed{R = 140\,\Omega}$$

5 The variation with potential difference $V$ of the charge $Q$ on one of the plates of a capacitor is shown in Fig. 5.1.




$C = \frac{Q}{V}$

$C = \frac{1.2\times10^{-4}}{8}$

**Fig. 5.1**

The capacitor is connected to an 8.0 V power supply and two resistors R and S as shown in Fig. 5.2.

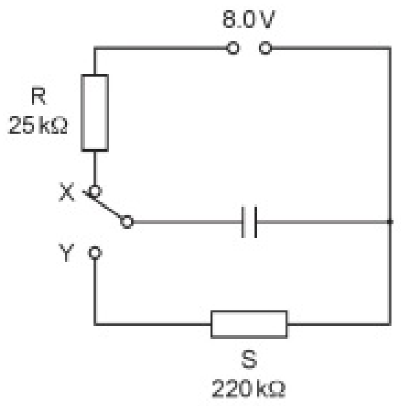



**Fig. 5.2**

The resistance of R is 25 kΩ and the resistance of S is 220 kΩ.

The switch can be in either position X or position Y.

**(a)** The switch is in position X so that the capacitor is fully charged.

Calculate the energy $E$ stored in the capacitor.

$E = \frac{1}{2} VQ$

$= \frac{1}{2} \times 8 \times (1.2 \times 10^{-4})$

$= 4.8 \times 10^{-4}$ J

$E$ = ....$4.8 \times 10^{-4}$.... J [2]

**(b)** The switch is now moved to position Y.

**(i)** Show that the time constant of the discharge circuit is 3.3 s.

$T = R \times C$

$T = 220\times10^{3} \times \left(\frac{1.2\times10^{-4}}{8}\right)$

$T \simeq 3.3$s

[2]

**(ii)** The fully charged capacitor in **(a)** stores energy $E$. $= E_0$

Determine the time $t$ taken for the stored energy to decrease from $E$ to $E/9$. $= E$

$E = \underbrace{\frac{1}{2} C}_{\text{const}} V^2$

$E \propto V^2$

$\frac{E_1}{V_1^2} = \frac{E_2}{V_2^2}$

$\frac{\cancel{E}}{V^2} = \frac{\cancel{E}/9}{V_2^2}$

$\sqrt{9V_2^2} = \sqrt{V^2}$

$3V_2 = V$

$V_2 = \frac{V}{3}$

$\frac{\cancel{V}}{3} = \cancel{V} e^{-t/3.3}$

$\ln\left(\frac{1}{3}\right) = \ln\left(e^{-\frac{t}{3.3}}\right)$

$\ln\left(\frac{1}{3}\right) = \frac{-t}{3.3}$

$3.3 \times \ln\left(\frac{1}{3}\right) = -t \Rightarrow t = 3.62$

$V = V_0 e^{-t/RC}$

$Q = Q_0 e^{-t/RC}$

$I = I_0 e^{-t/RC}$

$t$ = ....3.6.... s [4]

**(c)** A second identical capacitor is connected in parallel with the first capacitor.

State and explain the change, if any, to the time constant of the discharge circuit.

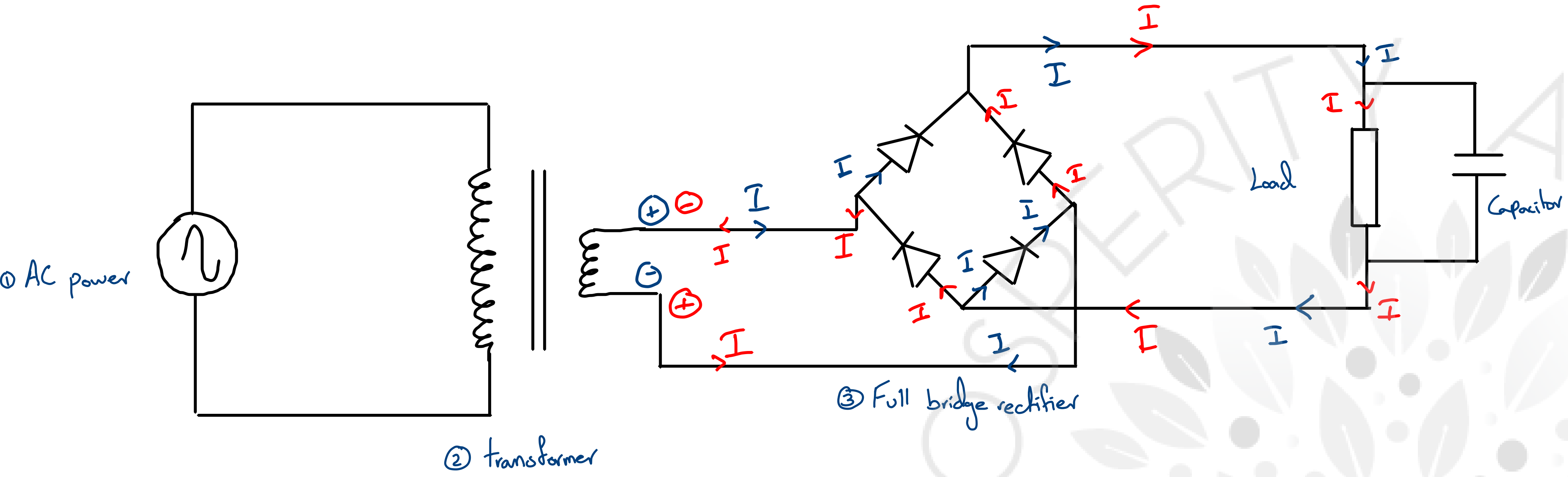
Total capacitance = 2C

$2T = R \times 2C \Rightarrow$ Time constant doubles

.......................................................................................................................... [2]

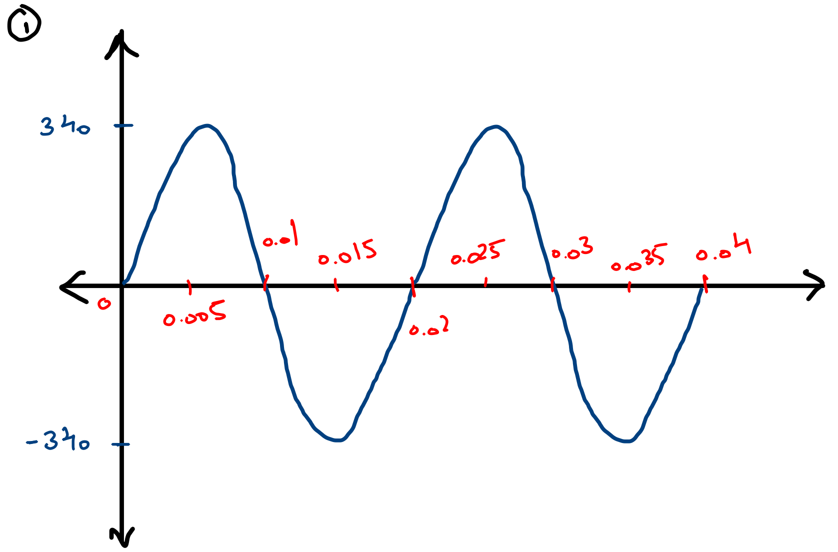
[Total: 10]

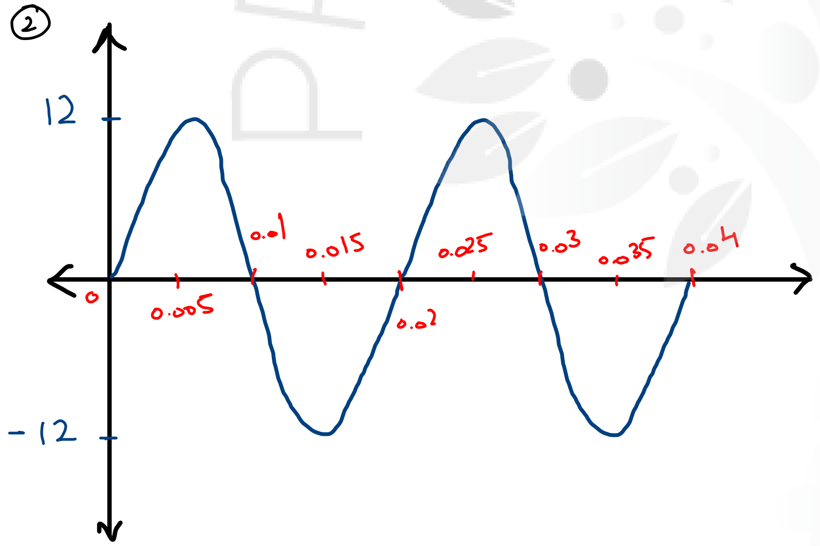
# Rectification :-

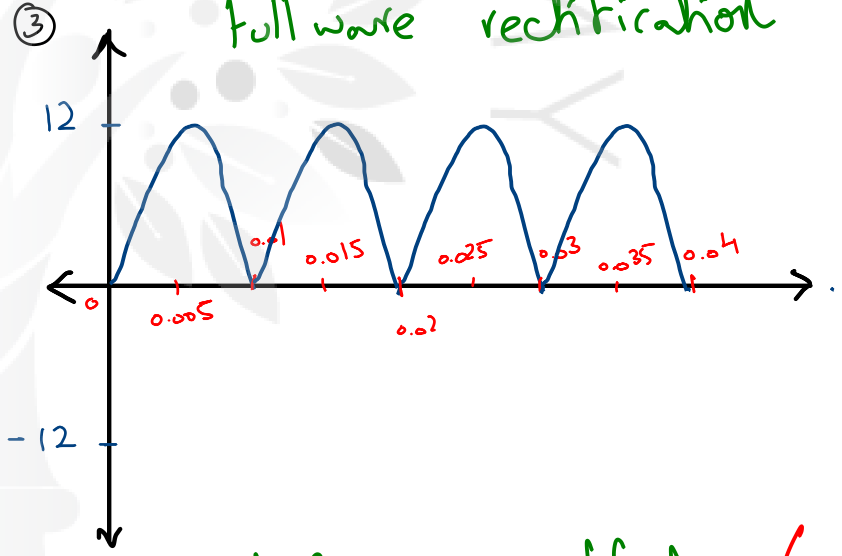


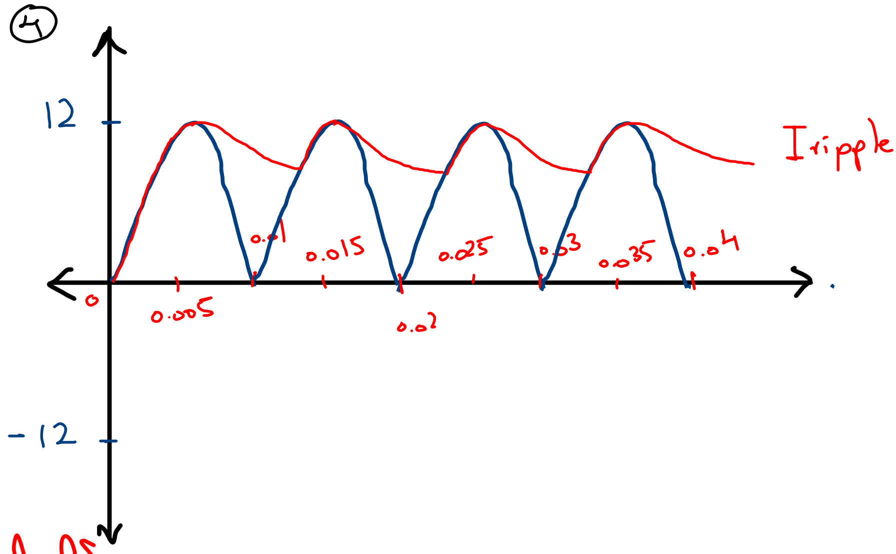

→ full bridge rectifier

Full wave rectification




$V_{r.m.s} = V_0/\sqrt{2} = 340/\sqrt{2} = 240V$

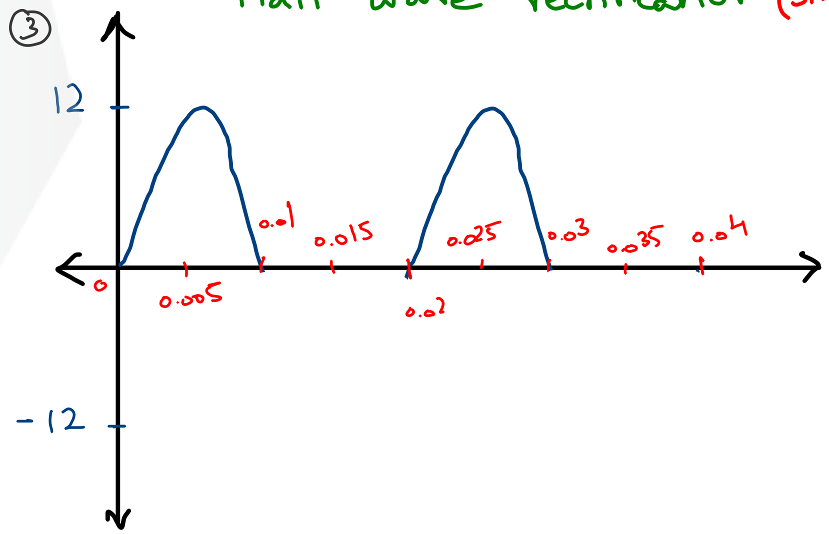
$T = 0.02$

$f = 1/T = 1/0.02 = 50Hz$

$\omega = 2\pi/T = 2\pi/0.02 = 100\pi$

$V = V_0 \sin(\omega t) \Rightarrow V = 340 \sin(100\pi t)$

Half wave rectification (single diode)



Uses of capacitors:-

- smoothing
- store energy (flash capacitors)
- blocking DC
- Oscillatory circuits

6 **(a)** Define the *capacitance* of a parallel-plate capacitor.

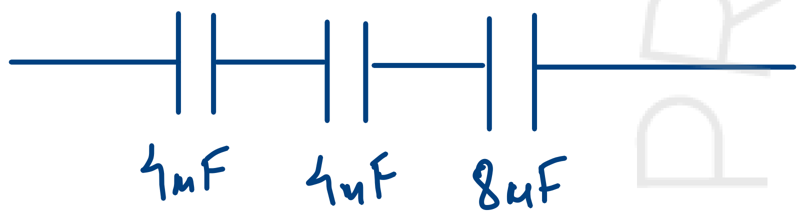
Ratio of charge on one of the plates to the potential difference between the plates

[2]

**(b)** A student has three capacitors. Two of the capacitors have a capacitance of 4.0 μF and one has a capacitance of 8.0 μF.

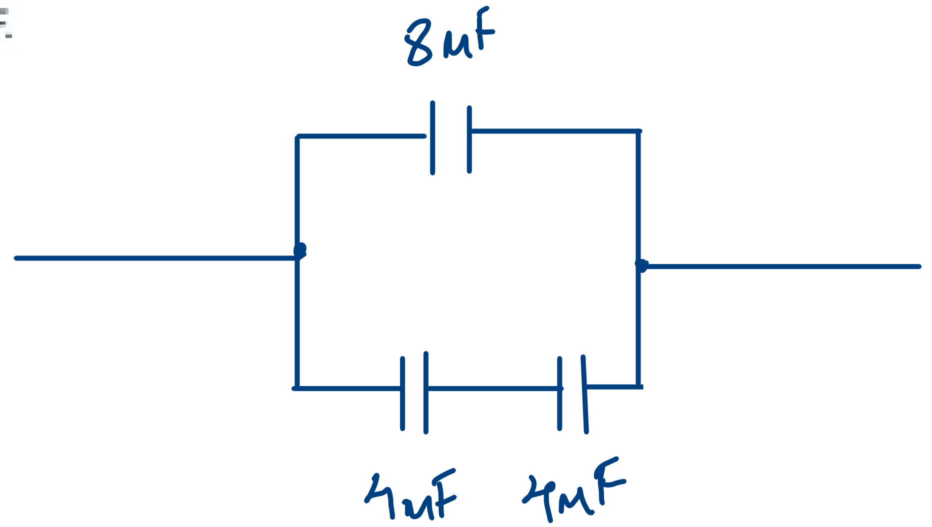
Draw labelled circuit diagrams, one in each case, to show how the three capacitors may be connected to give a total capacitance of:

(i) 1.6 μF




[1]

(ii) 10 μF.




[1]

**(c)** A capacitor C of capacitance 47 μF is connected across the output terminals of a bridge rectifier, as shown in Fig. 6.1.

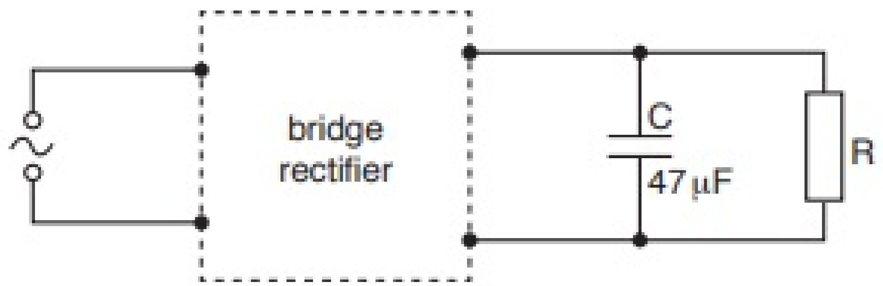



**Fig. 6.1**

The variation with time $t$ of the potential difference $V$ across the resistor R is shown in Fig. 6.2.

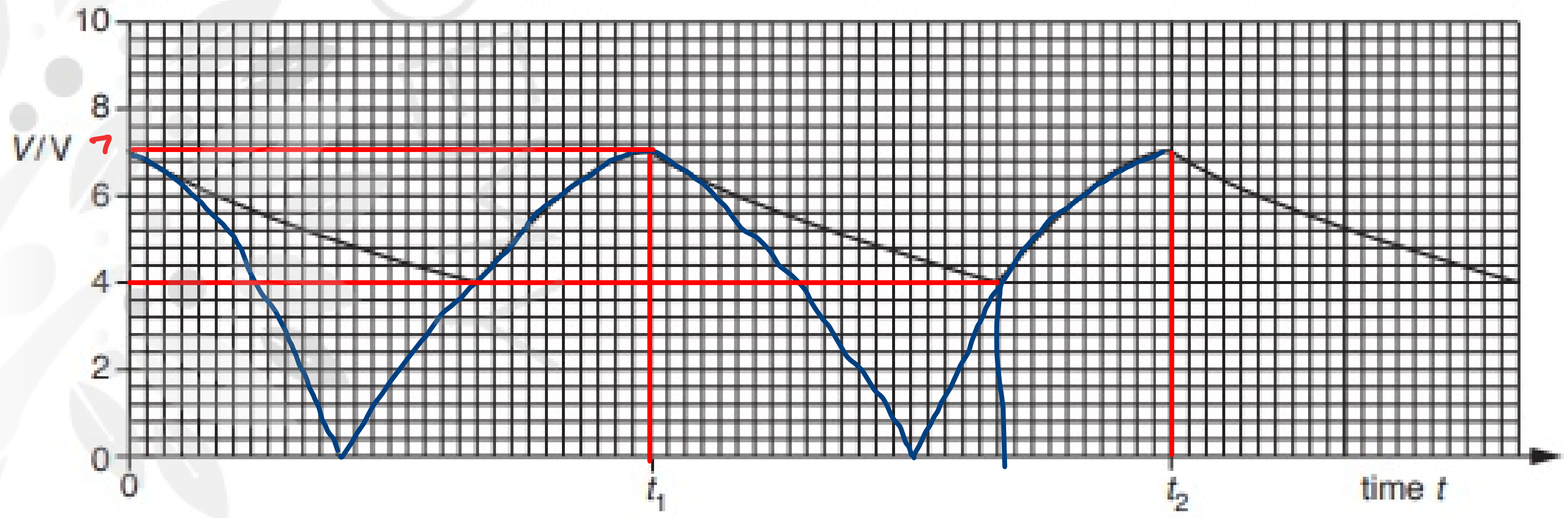



**Fig. 6.2**

Use data from Fig. 6.2 to determine the energy transfer from the capacitor C to the resistor R between time $t_1$ and time $t_2$.

$$\Delta E = \frac{1}{2}C(\Delta V^2)$$

$$= \frac{1}{2}(47\times10^{-6})(7^2-4^2)$$

$$= 7.755\times10^{-4}$$

energy = $7.8\times10^{-4}$ J [3]

[Total: 7]

10 A bridge rectifier using four ideal diodes is shown in Fig. 10.1.

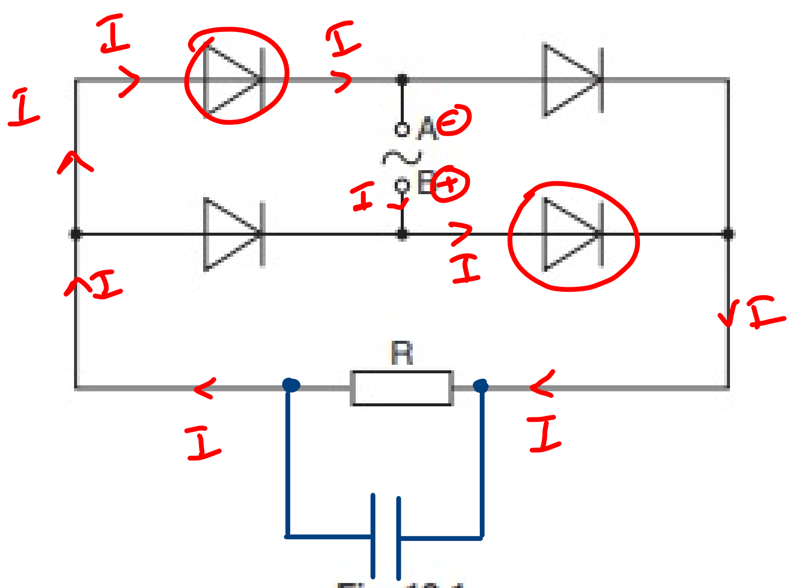



Fig. 10.1

The sinusoidal alternating electromotive force (e.m.f.) applied between points A and B has a root-mean-square (r.m.s.) value of 7.0 V.

(a) (i) On Fig. 10.1, circle the diodes that conduct when point B is positive with respect to point A. [1]

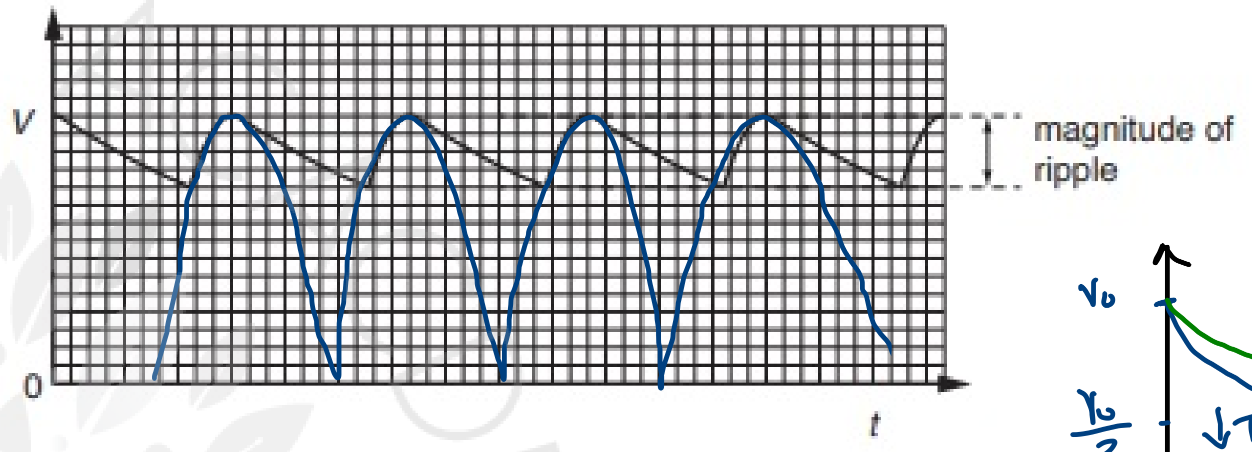
(ii) Calculate the maximum potential difference $V_{MAX}$ across resistor R.

$$V_{r.m.s} = \frac{V_0}{\sqrt{2}} \Rightarrow 7 \times \sqrt{2} = V_0 = 9.899$$

$V_{MAX}$ = ........9.9........ V [1]

(b) A capacitor is connected into the circuit to produce smoothing of the potential difference across resistor R.

The variation with time $t$ of the potential difference $V$ across resistor R is shown in Fig. 10.2.



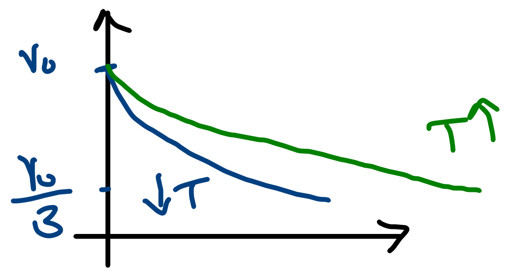




Fig. 10.2

(i) On Fig. 10.1, draw the symbol for a capacitor, connected so as to produce smoothing. [1]

(ii) State the effect, if any, on the magnitude of the ripple on $V$ when, separately: $\uparrow T = R \times C\uparrow$

1. a capacitor of larger capacitance is used

decreases

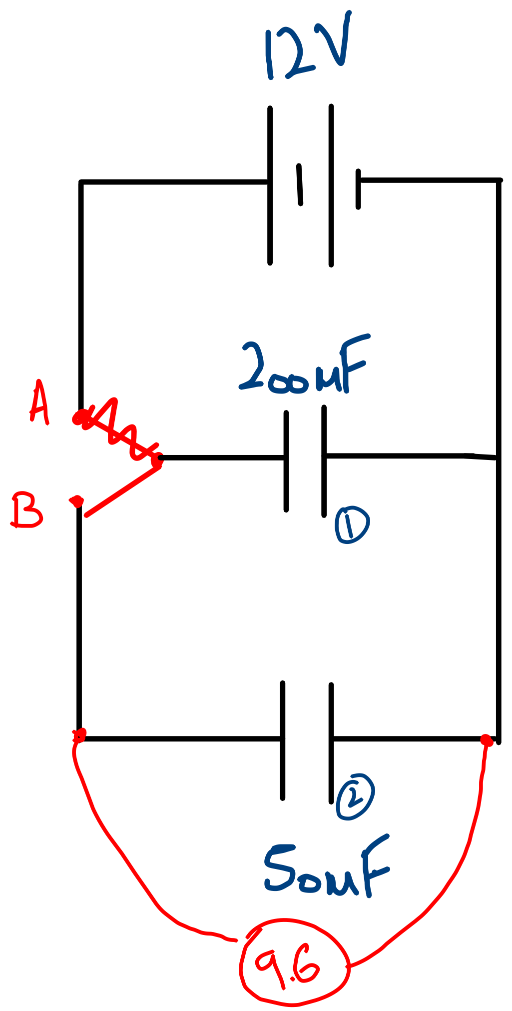
2. the resistor R has a smaller resistance. $\downarrow T = \downarrow R \times C$

increases

[2]

[Total: 5]

# Capacitors charging other capacitors:- Work with charge




Q. First the switch was connected to end A and the 200μF capacitor was completely charged up. Then the switch was connected to end B. What is the final settling voltage of the capacitors? Also calculate the charge on each capacitor.

$$C = \frac{Q}{V} \Rightarrow Q_T = C \times V$$

$$Q_T = (200\times10^{-6}) \times (12)$$

$$Q_T = 2400\times10^{-6}\,C$$

$$C_T = 200 + 50 = 250\,\mu F$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}$$

$$V = \frac{2400\times\cancel{10^{-6}}}{250\times\cancel{10^{-6}}}$$

settling $V = 9.6V$

$$C = \frac{Q}{V} \Rightarrow Q = C \times V$$

$$Q_1 = (200\times10^{-6}) \times 9.6 = 1.92\times10^{-3}\,C$$

$$Q_2 = (50\times10^{-6}) \times 9.6 = 4.8\times10^{-4}\,C$$

Q5.

Fig. 1 shows a football balanced above a metal bench on a length of plastic drain pipe. The surface of the ball is coated with a smooth layer of an electrically conducting paint. The pipe insulates the ball from the bench.

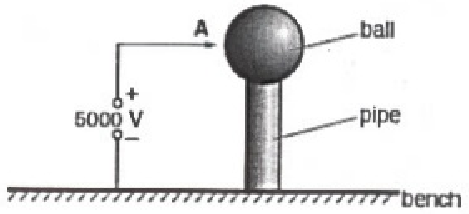



Fig. 1

(a) The ball is charged by touching it momentarily with a wire **A** connected to the positive terminal of a 5000 V power supply. The capacitance $C$ of the ball is $1.2 \times 10^{-11}$ F. Calculate the charge $Q_o$ on the ball. Give a suitable unit for your answer.

$$C = \frac{Q}{V} \Rightarrow Q_o = C \times V$$
$$= (1.2\times10^{-11})(5000)$$
$$= 6\times10^{-8}\ C$$

(b) The charge on the ball leaks slowly to the bench through the plastic pipe, which has a resistance $R$ of $1.2 \times 10^{15}\ \Omega$.

(i) Show that the time constant for the ball to discharge through the pipe is about $1.\cancel{5}^{44} \times 10^4$ s.

$$T = R \times C$$
$$= (1.2\times10^{15})(1.2\times10^{-11})$$
$$T = 14400 \simeq 1.44 \times 10^4\ s$$

(ii) Show that the initial value of the leakage current is about $4 \times 10^{-12}$ A.

$$V = IR$$
$$5000 = I(1.2\times10^{15})$$
$$I = 4.167\times10^{-12} \simeq 4\times10^{-12}$$

$$I = \frac{dQ}{dt}$$

(iii) Suppose that the ball continues to discharge at the constant rate calculated in (ii). Show that the charge $Q_o$ would leak away in a time equal to the time constant.

$$I = \frac{dQ}{dt} \Rightarrow 4\times10^{-12} = \frac{6\times10^{-8}}{\Delta t}$$
$$\Delta t = 1.5\times10^{4}$$

(Yes, in time T, $Q_o$ would leak away)

(iv) Using the equation for the charge $Q$ at time $t$

$$Q = Q_o e^{-t/RC}$$

show that, in practice, the ball only loses about 2/3 of its charge in a time equal to one time constant.

$$Q = Q_o e^{-t/RC \to T}$$
$$\frac{1}{3}\cancel{Q_o} = \cancel{Q_o} e^{-t/T}$$
$$\ln\left(\frac{1}{3}\right) = \ln\left(e^{-t/T}\right)$$

$$\ln\frac{1}{3} = \frac{-t}{T}$$
$$\cancel{+}1.0986T = \cancel{+}t$$
$$1.1T \simeq t$$
$$T \simeq t$$

(c) The ball is recharged to 5000 V by touching it momentarily with wire **A**. The ball is now connected in parallel via wire **B** to an uncharged capacitor of capacitance $1.2 \times 10^{-8}$ F and a voltmeter as shown in Fig. 2.

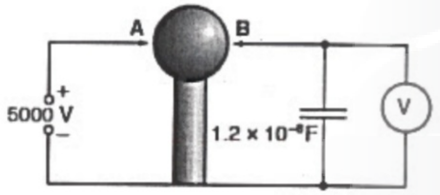



Fig. 2

(i) The ball and the uncharged capacitor act as two capacitors in parallel. The total charge $Q_o$ is shared instantly between the two capacitors. Explain why the charge left on the ball is $Q_o/1000$.

$$Q_T = 6\times10^{-8}\ C \qquad C_T = (1.2\times10^{-11}) + (1.2\times10^{-8}) = 1.2\times10^{-8}$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C} = \frac{6\times\cancel{10^{-8}}}{1.2\times\cancel{10^{-8}}} = 5V$$

$$C = \frac{Q}{V} \Rightarrow Q = C\times V \Rightarrow (1.2\times10^{-11})\times5 = 6\times10^{-11}\ C$$

$$Q_o = 6\times10^{-8} \qquad 6\times10^{-11} = \frac{Q_o}{1000}$$