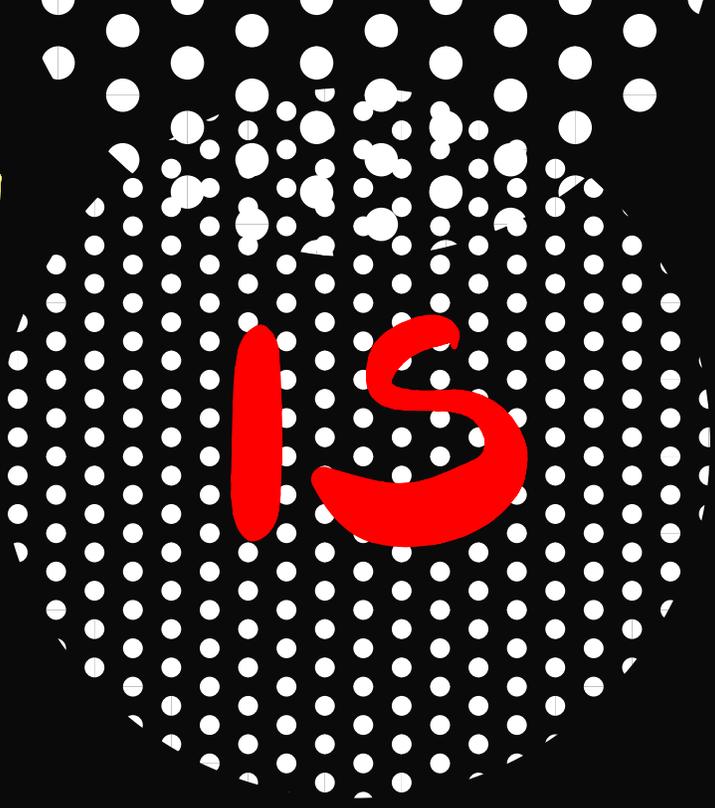


LEARNING

FUN!



CAPACITANCE

With



19

Learning outcomes

By the end of this topic, you will be able to:

19.1 Capacitors and capacitance

- 1 define capacitance, as applied to both isolated spherical conductors and to parallel plate capacitors
- 2 recall and use $C = Q/V$
- 3 derive, using $C = Q/V$, formulae for the combined capacitance for capacitors in series and in parallel
- 4 use the capacitance formulae for capacitors in series and in parallel

19.2 Energy stored in a capacitor

- 1 determine the electric potential energy stored in a capacitor from the area under the potential-charge graph
- 2 recall and use $W = \frac{1}{2}QV$ and hence $W = \frac{1}{2}CV^2$

19.3 Discharging a capacitor

- 1 analyse graphs of the variation with time of potential difference, charge and current for a capacitor discharging through a resistor
- 2 recall and use $\tau = RC$ for the time constant for a capacitor discharging through a resistor
- 3 use equations of the form $x = x_0 e^{-(t/RC)}$ where x could represent current, charge or potential difference for a capacitor discharging through a resistor

-) Capacitors are electrical component used in circuits.
-) They are designed to store electrostatic charges on their plates.
-) Net charge stored by a capacitor is zero as the two plates store equal amounts of charge with opposite signs.
-) Capacitors stored energy in the form of Electric potential energy of separated charges
-) When capacitors are charged the voltage across the plates increases, and the net driving voltage in a circuit decreases.
-) The charging takes place at the gradually decreasing rate.
-) During discharge of a capacitor, the voltage across plates decreases. Therefore the rate of flow decreases with time and capacity discharges a decreasing rate.

$$Q \propto V$$

$$Q = CV$$

$C = \frac{Q}{V}$

Capacitance

Amount of charge stored on plates per unit voltage

Capacitance \rightarrow Farads (F)

$$1F = 1CV^{-1}$$

CAPACITANCE of a CAPACITOR \Rightarrow

-) Amount of charge stored by capacitor's plate depends upon the potential difference applied across the plate.
-) The capacitance of a capacitor is defined as the amount of charge stored on plates per unit voltage.

The capacitance of a parallel plate capacitor depends upon :-

- Interfacing area of the plates
- Separation between the plates
- Permittivity of the medium between the plates

$$\therefore \epsilon_r = \frac{\epsilon}{\epsilon_0}$$

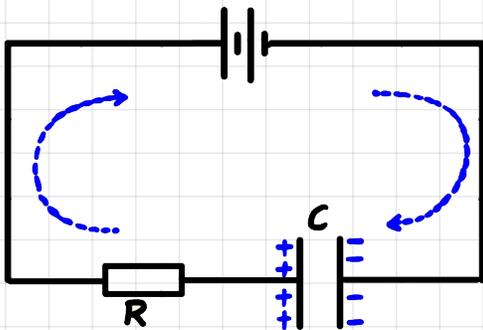
$$C = \frac{\epsilon A}{d} \rightarrow C = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$\epsilon = \epsilon_0 \epsilon_r$$

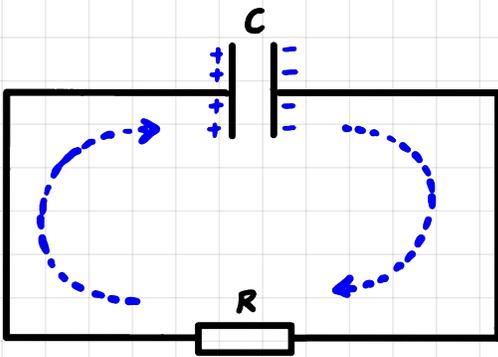
CHARGING / DISCHARGING Time & Behavior.

The charging or discharging time of a capacitor depends upon :-

-) Capacitance as it determines the amount of charge to be stored
-) The resistance of a resistor connected in series as it determines the rate of flow.



When a capacitor is charged up, the potential difference across plates gradually increases that opposes the subsequent flow; the net driving voltage decreases during charging process. Therefore, the amount of charge on plates and potential difference across plate increases exponentially while the amount of current in the circuit decreases exponentially.



When a capacitor discharges, the potential difference across the plates decreases with the simultaneous decrease in the rate of flow of charge. Therefore, the amount of charge accumulated on the plates, potential difference across plates and the current flowing during discharge process decreases exponentially.

Energy Stored →

When a capacitor is charged, a fraction of work done by battery is lost as heat in overcoming gradually increasing hindrance on the plate of capacitor for subsequent transfer.

During charging of a capacitor, half of the work done by supply is lost as heat, and the rest is stored by electrical potential energy of separated charges.

$$V = \frac{W}{Q} \rightarrow W = VQ$$

Work done by Battery → Electrical energy Stored

$$E = VarQ \rightarrow \frac{V_{max} + V_{min}}{2} \cdot Q$$

$$\rightarrow \frac{V+0}{2} \cdot Q \rightarrow E = \frac{1}{2} VQ$$

$$\frac{1}{2} \cdot V \cdot CV$$

$$\frac{1}{2} \cdot Q \cdot \frac{Q}{C}$$

$$\frac{1}{2} CV^2$$

$$\frac{1}{2} \frac{Q^2}{C}$$

≈ SAFE MAX VOLTAGE ≈

Capacitors are labelled / rated with the same max voltage to be applied across plates. If applied voltage exceeds the safe max value, the electrons start hopping from negatively to positively charged plate. Sparks are observed.



The capacitor is designed to store $30\mu\text{C}$ charge for a manual to cross the plates and the voltage of glass plate must not exceed 6V

CAPACITORS IN SERIES

All capacitors in series store same amount of charge equal to that delivered by battery.

Sum of potential differences across capacitors is equal to source EMF. (Energy conserved)

The net Capacitance of a series is less than the smallest value of capacitance in series

Voltage share of capacitors in series, depends upon the capacitance. The higher the capacitance the smaller the voltage share

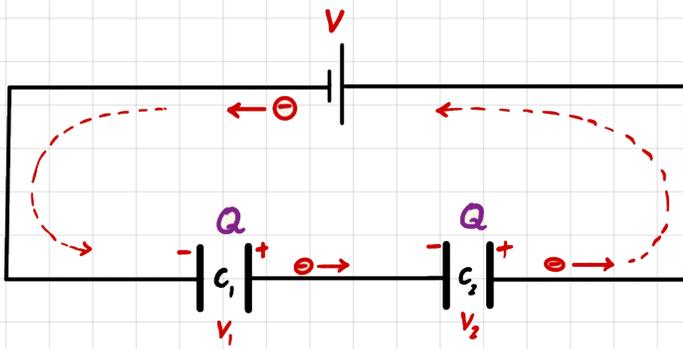
The total charge stored by a Series combination is equal to charge stored by one of the capacitors.

$$Q = CV$$

$$V = \frac{Q}{C}$$

Lower \leftarrow \leftarrow higher

$$Q = Q_1 = Q_2$$



$$V = V_1 + V_2 \rightsquigarrow \text{Principle of Energy Conservation}$$

$$\frac{Q}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\frac{Q}{C_T} = Q \left(\frac{1}{C_1} + \frac{1}{C_2} \right)$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

$$Q_1 = Q_2$$

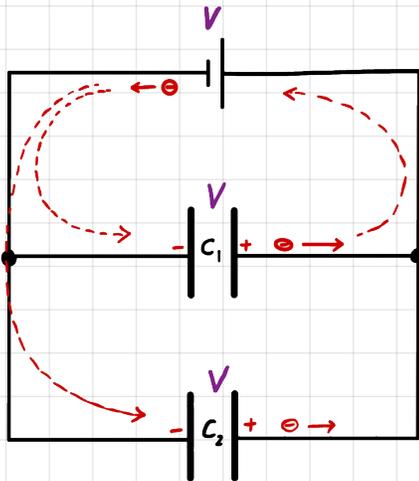
$$C_1 V_1 = C_2 V_2$$

$$\frac{C_1}{C_2} = \frac{V_2}{V_1}$$

CAPACITORS IN PARALLEL

Same voltage across all the capacitors in parallel irrespective of the difference between the capacitance.

The charge delivered by source divides between capacitors, according to their capacitances.



$$Q = Q_1 + Q_2 \rightarrow \text{Kirchoff's 1st Law!}$$

$$C_T V_T = C_1 V_1 + C_2 V_2$$

$$C_T V = V(C_1 + C_2)$$

$$C_T = C_1 + C_2$$

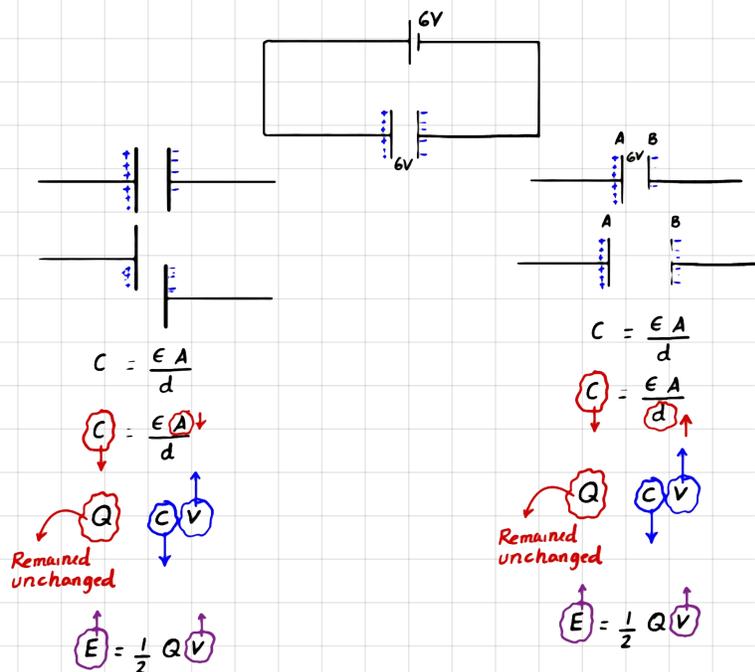
$$V_1 = V_2$$

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2}$$

Mechanical Work by Capacitor :-

If a capacitor is charged, then isolated from the circuit, the amount of energy it stores can be increased by doing mechanical work against electrical forces. The amount of energy stored by a previously charged and isolated capacitor can be increased by doing mechanical work through increasing the separation between plates or by reducing the interfacing area.



SPHERICAL CAPACITOR

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R}$$

$$Q = CV$$
$$C = \frac{Q}{V}$$

$$C = \frac{Q}{V - 0}$$

$$C = \frac{Q}{\frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{R} - 0}$$

$$C = \frac{\cancel{Q}}{\frac{1}{4\pi\epsilon_0} \cdot \frac{\cancel{Q}}{R}} = 4\pi\epsilon R$$

