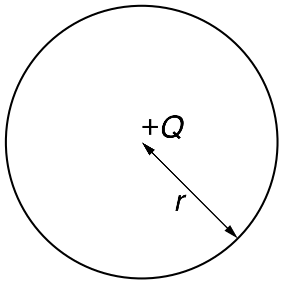
5 A solid metal sphere, of radius *r*, is insulated from its surroundings. The sphere has charge $+Q$.
This charge is on the surface of the sphere but it may be considered to be a point charge at its centre, as illustrated in Fig. 5.1.



$C = Q/V$

Fig. 5.1

(a) (i) Define *capacitance*.

Charge stored per unit potential difference. [1]

(ii) Show that the capacitance C of the sphere is given by the expression

$$C = 4\pi\varepsilon_0 r.$$

$C = \frac{q}{V}$ as $V = \frac{q}{4\pi\varepsilon_0 r}$

$C = q \div \frac{q}{4\pi\varepsilon_0 r}$

$C = \cancel{q} \times \frac{4\pi\varepsilon_0 r}{\cancel{q}}$

$$C = 4\pi\varepsilon_0 r$$

[1]

(b) The sphere has radius 36 cm.
Determine, for this sphere,

(i) the capacitance,

$$C = 4\pi\varepsilon_0 r$$

$$C = 4\pi\left(8.85 \times 10^{-12}\right)\left(36 \times 10^{-2}\right)$$

capacitance = 4.0×10^{-11} F [1]

(ii) the charge required to raise the potential of the sphere from zero to 7.0×10^5 V.

For Examiner's Use

$$q = CV$$

$$\Delta q = C\Delta V$$

$$\Delta q = (4 \times 10^{-11})(7 \times 10^{-5})$$

charge = 2.8×10^{-5} C [1]

(c) Suggest why your calculations in **(b)** for the metal sphere would not apply to a plastic sphere.

Most Spheres are conductors and charge in them is uniformly distributed so they can be treated as point charges. Plastic is an insulator. Its charges is either at surface or at centre. [3]

(d) A spark suddenly connects the metal sphere in **(b)** to the Earth, causing the potential of the sphere to be reduced from 7.0×10^5 V to 2.5×10^5 V.

Calculate the energy dissipated in the spark.

$$E = \frac{1}{2}CV^2$$

$$\Delta E = \frac{1}{2}C\left(V_F^2 - Vi^2\right)$$

energy = $8 \cdot 6$ J [3]

$$\frac{1}{2} \times 4 \times 10^{-11} \times \left[\left(2 \cdot 5 \times 10^5\right)^2 - \left(7 \times 10^5\right)^2\right]$$

5 (a) State two functions of capacitors in electrical circuits.

1. STORE ENERGY

2. USED IN TUNING CIRCUITS

[2]

(b) Three capacitors, each marked '30 μF, 6V max', are arranged as shown in Fig. 5.1.

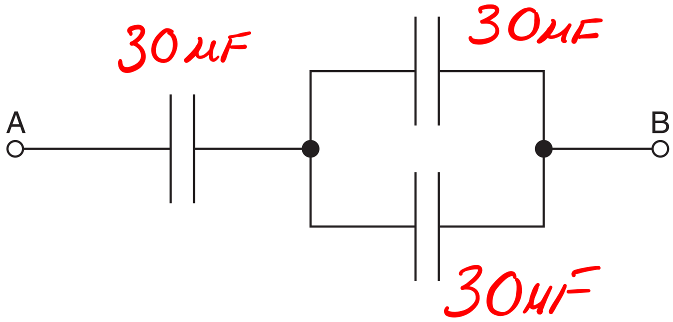
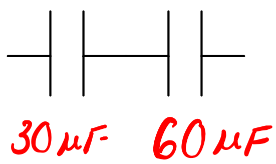


Fig. 5.1

Determine, for the arrangement shown in Fig. 5.1,

(i) the total capacitance,



$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{CT} = \frac{1}{30} + \frac{1}{60}$$

capacitance = 20 μF [2]

(ii) the maximum potential difference that can safely be applied between points A and B.

$q = CV$

q same in series

$\downarrow C \propto \frac{1}{V\uparrow}$ Smaller "C" more "V"

$q_1 = q_2$

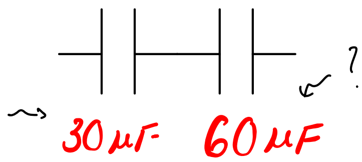
$C_1V_1 = C_2V_2$

$30 \times 6 = 60 \times V_2$

$V_2 = 3V$

max $V = 6 + 3$
$= 9V$

6V



We assume max 6V on smallest capacitor i.e 30μF

potential difference = 9 V [2]

Ratio :- $C_1V_1 = C_{TOTAL}V_{AB}$

$(30 \times 10^{-6})(6) = (20 \times 10^{-6})V_{AB}$

$V_{AB} = 9V$

For Examiner's Use

(c) A capacitor of capacitance 4700 μF is charged to a potential difference of 18 V. It is then partially discharged through a resistor. The potential difference is reduced to 12 V. Calculate the energy dissipated in the resistor during the discharge.

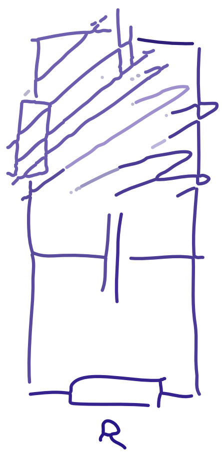
For Examiner's Use

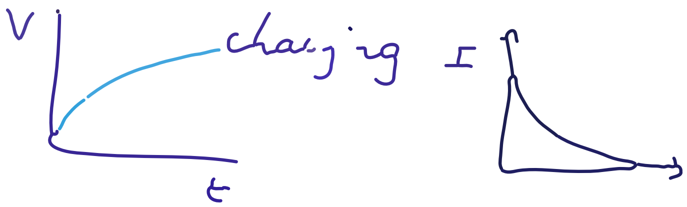
$$\Delta E = \frac{1}{2}C\left(V_f^2 - V_i^2\right)$$

$$\frac{1}{2}\left(4700 \times 10^{-6}\right)\left(12^2 - 18^2\right)$$

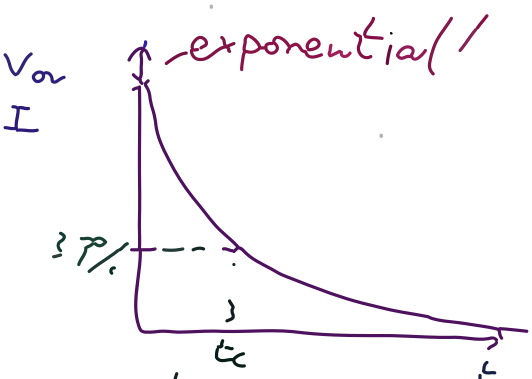
-0.423

energy = 0.42 J [3]





DISCHARGING



$$\frac{V}{V_0} = e^{-\frac{t}{RC}}$$

when $t = RC$, $e^{-1} = 0.37$ (37%)

RC = time constant! (τ_c)

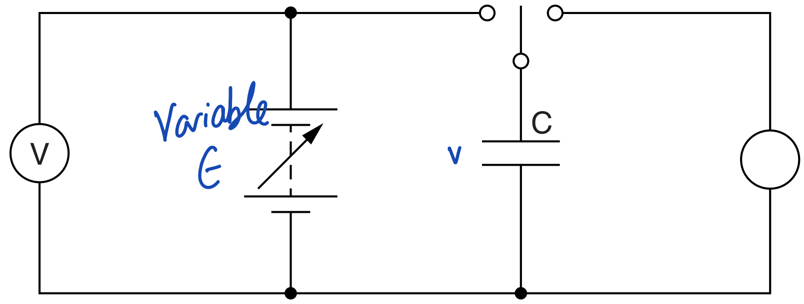
$$\ln\left(\frac{V}{V_0}\right) = \frac{-t}{RC}$$

[Turn over

Q

7 A student sets up the circuit shown in Fig. 7.1 to measure the charge on a capacitor C for different values of potential difference across the capacitor.

V

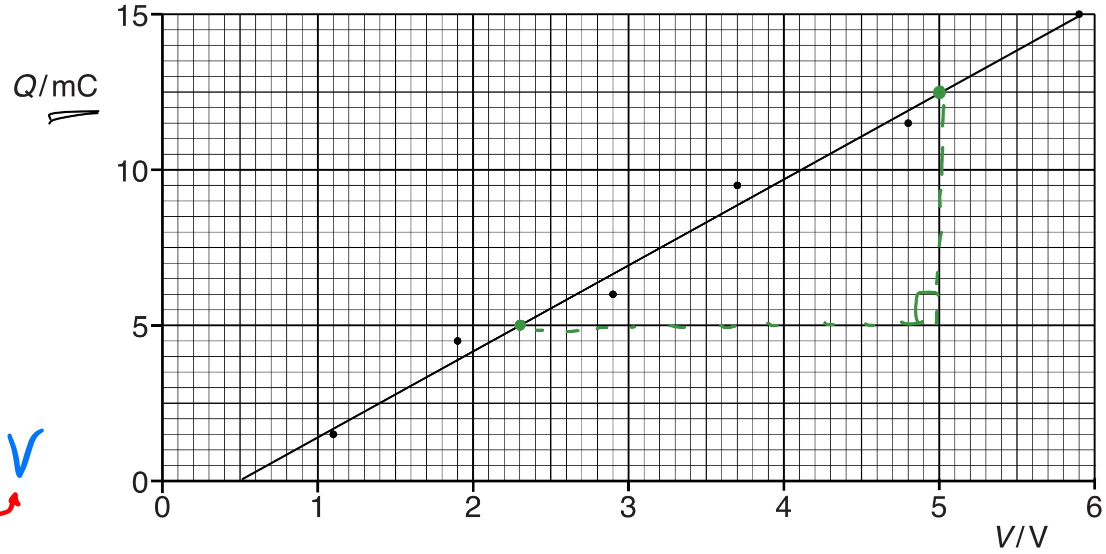


I

$Q = It$

Fig. 7.1

The variation with potential difference V of the charge Q stored on the capacitor is shown in Fig. 7.2.



$Q = CV$

$y = m\,x$

Fig. 7.2

(a) State and explain how Fig. 7.2 indicates that there is a systematic error in the readings of one of the meters.

Based on $Q = CV$, $Q \propto V$. But the graph does not pass through origin

[2]

(b) Use Fig. 7.2 to determine the capacitance, in μF, of capacitor C.

C = gradient

$$C = \frac{y_2 - y_1}{x_2 - x_1}$$

$$C = \frac{(12.5 - 5)\times 10^{-3}}{5 - 2.3} = 2780 \times 10^{-6}$$

capacitance = 2800 μF [3]

(c) Use your answer in **(b)** to determine the additional energy stored in the capacitor C when the potential difference across it is increased from 6.0 V to 9.0 V.

$$\Delta E = \frac{1}{2} C \{ V f^2 - V i^2 \}$$

$$\Delta E = \frac{1}{2} (2780 \times 10^{-6}) (6^2 - 9^2) = -0.06255$$

$$6.255 \times 10^{-2}$$

energy = 6.3×10^{-2} J [3]

[Total: 8]

$Q = CV \qquad C = Q/V$

For Examiner's Use

4 **(a)** Define *capacitance*.

Ratio of charge stored to potential difference across it. [1]

(b) An isolated metal sphere has a radius *r*. When charged to a potential *V*, the charge on the sphere is *q*.
The charge may be considered to act as a point charge at the centre of the sphere.

(i) State an expression, in terms of *r* and *q*, for the potential *V* of the sphere.

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ [1]

(ii) This isolated sphere has capacitance. Use your answers in **(a)** and **(b)(i)** to show that the capacitance of the sphere is proportional to its radius.

$q = CV$

$C = \frac{q}{V} = \frac{\cancel{q}}{\left(\frac{\cancel{q}}{4\pi\varepsilon_0 r}\right)}$

$C = 4\pi\varepsilon_0 r$

$4\pi\varepsilon_0$ is constant so,

$C \propto r$

[1]

(c) The sphere in **(b)** has a capacitance of 6.8 pF and is charged to a potential of 220 V.

Calculate

(i) the radius of the sphere,

$C = 4\pi\varepsilon_0 r$

$\frac{C}{4\pi\varepsilon_0} = r$

$$\frac{6 \cdot 8 \times 10^{-12}}{4\pi(8.85 \times 10^{-12})}$$

radius = 6.1×10^{-2} m [3]

(ii) the charge, in coulomb, on the sphere.

For Examiner's Use

$$Q = CV = 6.8 \times 10^{-12} \times 220$$

charge = $1..50 \times 10^{-9}$ C [1]

(d) A second uncharged metal sphere is brought up to the sphere in **(c)** so that they touch. The combined capacitance of the two spheres is 18 pF.

Calculate

(i) the potential of the two spheres,

$$Q = CV \qquad \frac{1.5 \times 10^{-9}}{18 \times 10^{-12}} = V$$

$$\frac{Q}{c} = V$$

potential = 83 V [1]

(ii) the change in the total energy stored on the spheres when they touch.

$$\Delta E = \frac{1}{2} Q \left(v_f - v_i \right)$$

$$\frac{1}{2} \times \left(1 \cdot 5 \times 10^{-9}\right) (83 - 220)$$

change = 1.03×10^{-7} J [3]

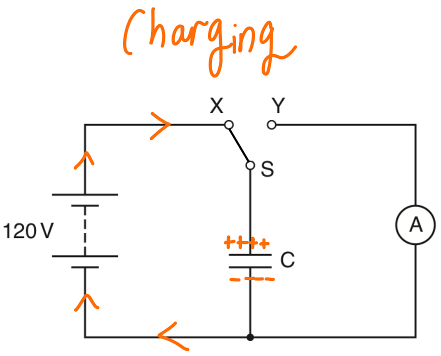
$Q = CV$ $C = Q/V$

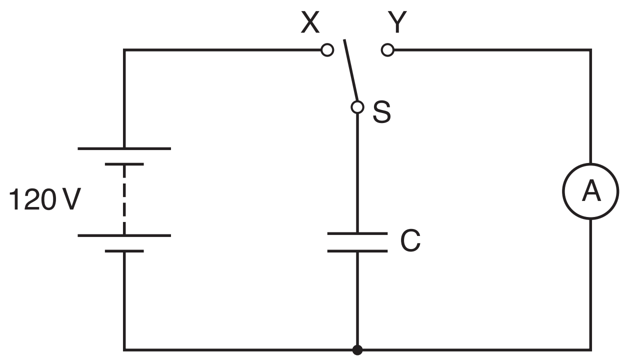
7 **(a)** Explain what is meant by the *capacitance* of a parallel plate capacitor.

Ratio of charge stored on one plate of capacitor to the p.d across plates.

....[3]

(b) A parallel plate capacitor C is connected into the circuit shown in Fig. 7.1.





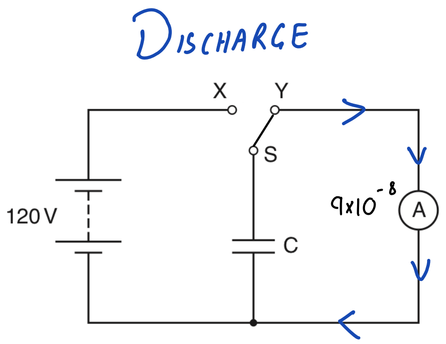


Fig. 7.1

When switch S is at position X, the battery of electromotive force 120 V and negligible internal resistance is connected to capacitor C.
When switch S is at position Y, the capacitor C is discharged through the sensitive ammeter.

The switch vibrates so that it is first in position X, then moves to position Y and then back to position X fifty times each second.

The current recorded on the ammeter is 4.5 μA. $\frac{1}{50} = 0.02\,s$

Determine

(i) the charge, in coulomb, passing through the ammeter in 1.0 s,

$$Q = It = 4.5 \times 10^{-6} \times 1$$

charge = 4.5×10^{-6} C [1]

(ii) the charge on one plate of the capacitor, each time that it is charged,

$Q_{TOTAL} = 4.5 \times 10^{-6}$

$Q = \dfrac{4.5 \times 10^{-6}}{50}$

charge = 9.0×10^{-8} C [1]

(iii) the capacitance of capacitor C.

$$C = \frac{Q}{V} \qquad C = \frac{9 \times 10^{-8}}{120}$$

capacitance = 7.5×10^{-10} F [2]

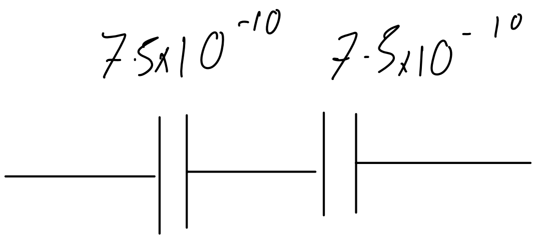
(c) A second capacitor, having a capacitance equal to that of capacitor C, is now placed in series with C.

Suggest and explain the effect on the current recorded on the ammeter.

Total Capacitance is halved (less total charge passing ammeter) so current halved.

.....[2]

[Total: 9]



$\downarrow C = \dfrac{Q}{V}\downarrow$

$C_{TOT} = 3.75 \times 10^{-10}$

A $\rightarrow \frac{1}{2}$ current!

6 **(a)** Explain what is meant by the *capacitance* of a parallel plate capacitor.

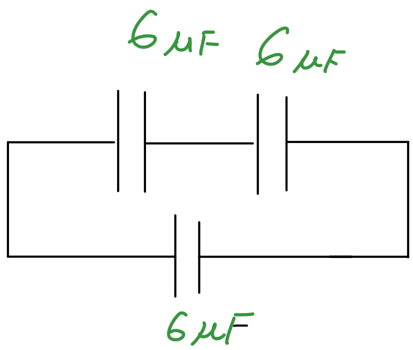
Capacitance is ratio of charge on a single plate of a capacitor to the potential difference b/w the plates of the capacitor.

[3]

(b) Three parallel plate capacitors each have a capacitance of 6.0 μF.

Draw circuit diagrams, one in each case, to show how the capacitors may be connected together to give a combined capacitance of

(i) 9.0 μF,

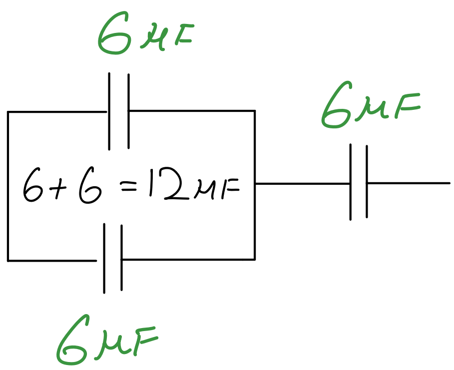


$$\left(\frac{1}{6}+\frac{1}{6}\right)^{-1}+6$$

$= 9\,\mu F$

[1]

(ii) 4.0 μF.



$$\left(\frac{1}{12}+\frac{1}{6}\right)^{-1} = 4\,\mu F$$

[1]

(c) Two capacitors of capacitances 3.0 μF and 2.0 μF are connected in series with a battery of electromotive force (e.m.f.) 8.0 V, as shown in Fig. 6.1.

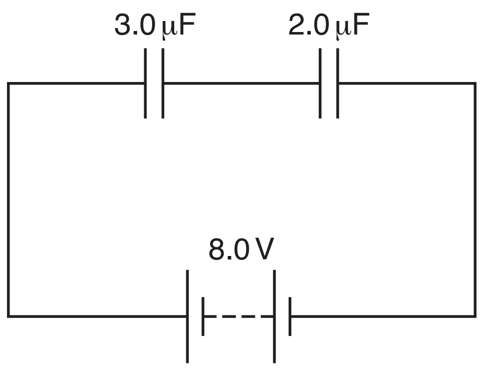


Fig. 6.1

(i) Calculate the combined capacitance of the capacitors.

$$C_T = \left(\frac{1}{3}+\frac{1}{2}\right)^{-1}$$

capacitance = ... $1 \cdot 2$ .... μF [1]

(ii) Use your answer in **(i)** to determine, for the capacitor of capacitance 3.0 μF,

1. the charge on one plate of the capacitor,

$$C = \frac{Q}{V};\, Q = CV$$

$$Q = (1 \cdot 2 \times 10^{-6})\,(8)$$

charge = ... $9 \cdot 6$ .... μC

2. the energy stored in the capacitor.

$$\therefore V = \frac{Q}{C} = \frac{9.6 \times \cancel{10^{-6}}}{3.0 \times \cancel{10^{-6}}} = 3.2\,V$$

$$E = \frac{1}{2}QV$$

$$\frac{1}{2}(9.6 \times 10^{-6})(3.2)$$

OR

$$E = \frac{1}{2} \cdot \frac{Q^2}{C}$$

$$= \frac{1}{2} \cdot \frac{(9.6 \times 10^{-6})^2}{3 \times 10^{-6}}$$

energy = ... 1.5×10^{-5} .... J [4]

$$\frac{1}{2}QV$$

$$\frac{1}{2}Q\frac{Q}{C} = \frac{1}{2}\frac{Q^2}{C}$$

[Total: 10]

6 Two capacitors P and Q, each of capacitance C, are connected in series with a battery of e.m.f. 9.0 V, as shown in Fig. 6.1.

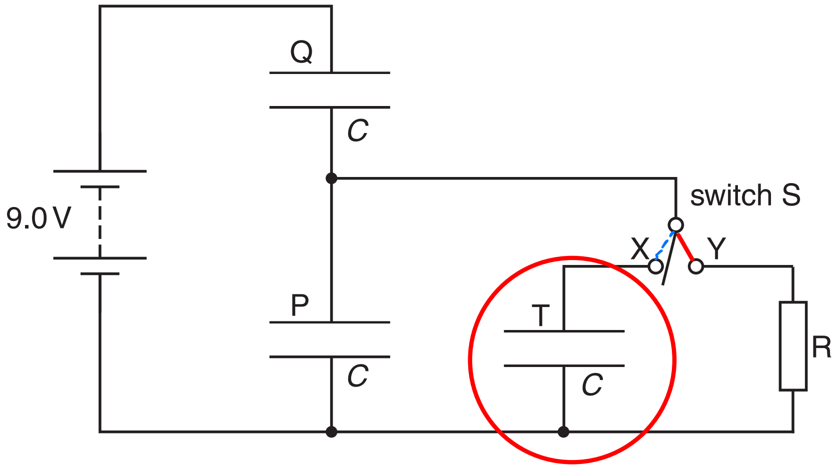


Fig. 6.1

A switch S is used to connect either a third capacitor T, also of capacitance C, or a resistor R, in parallel with capacitor P.

(a) Switch S is in position X.

Calculate

(i) the combined capacitance, in terms of C, of the three capacitors,

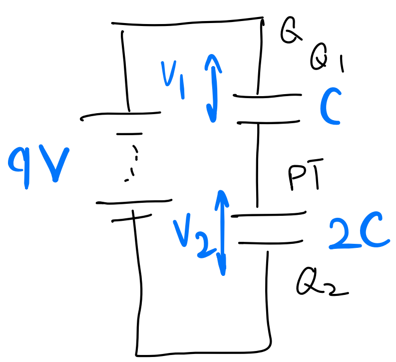
$$C_T = \left\{\frac{1}{C+C} + \frac{1}{C}\right\}^{-1}$$

$$\left(\frac{1}{2C} + \frac{1}{C}\right)^{-1}$$

$$\left(\frac{3}{2C}\right)^{-1}$$

capacitance = $\frac{2}{3}C$ [2]

(ii) the potential difference across capacitor Q. Explain your working.



$Q_1 = Q_2$

$C_1 V_1 = C_2 V_2$

$$\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{2\cancel{C}}{\cancel{C}}$$

$V_1 + V_2 = 9V$

$2V_2 + V_2 = 9V$

$3V_2 = 9V$

$V_2 = 3V$

potential difference = 6.0 V [2]

$$\frac{V_1}{V_2} = \frac{2}{1} \qquad V_1 = 2V_2$$

Charge stored in Q is equal to charge stored in combination of PT.

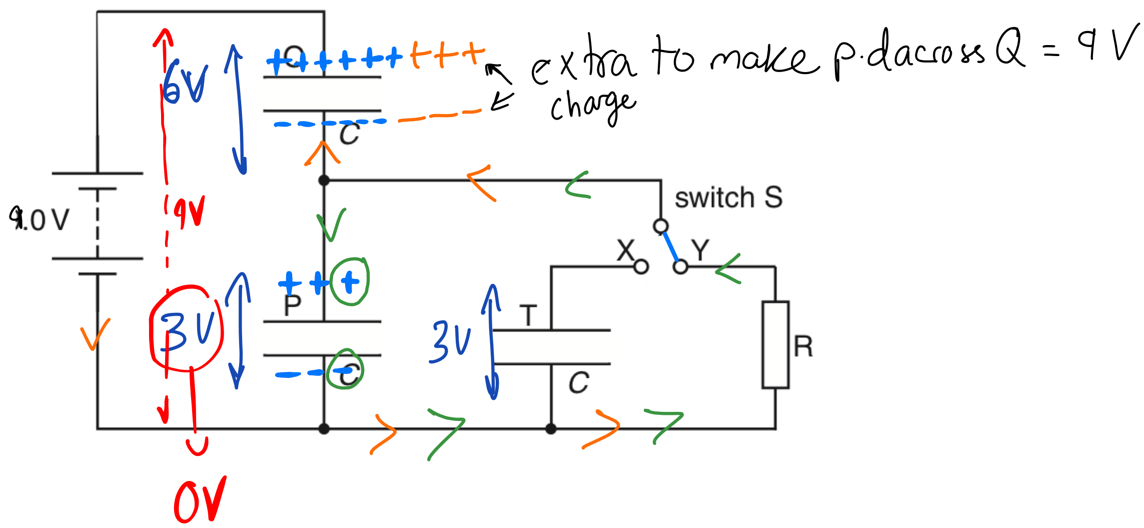
(b) Switch S is now moved to position Y.
State what happens to the potential difference across capacitor P and across capacitor Q.

capacitor P: p.d decreases from 3.0 V to 0V
* P discharges via R

capacitor Q: p.d increases from 6.0V to 9.0V
* to maintain p.d of 9.0V (power supply) across terminals of battery.

[4]

[Total: 8]



$C = Q/V$

7 **(a)** Define *capacitance*.

Ratio of charge stored on a single plate of a capacitor to the potential difference across the capacitor. [1]

(b) Three capacitors of capacitances C_1, C_2 and C_3 are initially uncharged. They are then connected in series to a battery, as shown in Fig. 7.1.

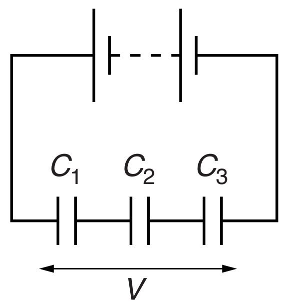


Fig. 7.1

The battery applies a potential difference V across the three capacitors.

Show that the combined capacitance C of the capacitors is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$V = V_1 + V_2 + V_3$; $Q = CV$; $V = Q/C$

$\frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$ $\qquad Q = Q_1 = Q_2 = Q_3$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ [2]

(c) A battery of e.m.f. 12 V and negligible internal resistance is connected to a network of two capacitors and a resistor, as shown in Fig. 7.2.

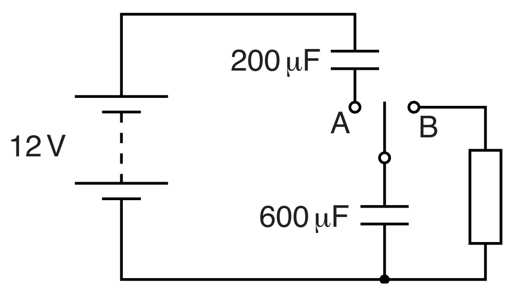
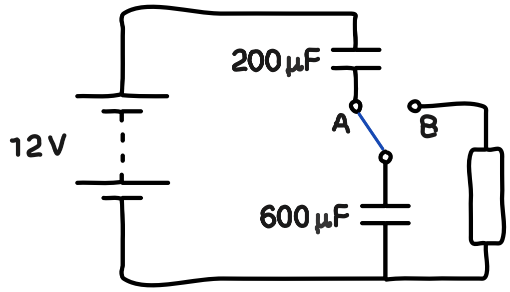


Fig. 7.2

The capacitors have capacitances of 200 μF and 600 μF. The switch has two positions, A and B.



(i) The switch is moved to position A.

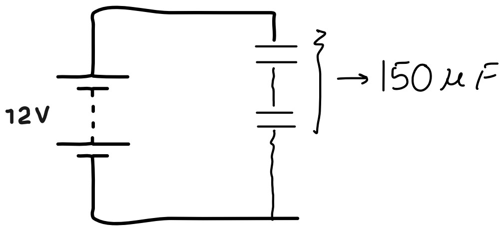
Calculate

1. the combined capacitance of the two capacitors,

$$\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}=\left(\frac{1}{200}+\frac{1}{600}\right)^{-1}$$

combined capacitance = ...150........ μF [1]

2. the charge on the 600 μF capacitor,



$Q = CV$

$Q = (150\times10^{-6})(12)$

$Q = 1.8\times10^{-3}\,C$

charge = ...1.8×10^{-3}........ C [1]

Charge same in series!

3. the potential difference across the 600 μF capacitor.

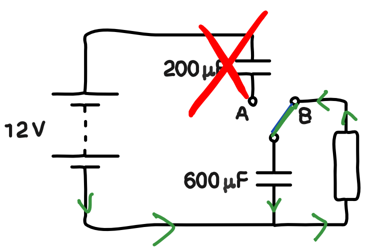
$$V=\frac{Q}{C}=\frac{1.8\times10^{-3}}{600\times10^{-6}}$$

potential difference = ...3.0........ V [1]

(ii) The switch is now moved from position A to position B.

Calculate the potential difference across the 600 μF capacitor when it has discharged $\boxed{\text{50\% of its initial energy.}}$

$E=\frac{1}{2}CV^2$ $\quad E \propto V^2$



$$\frac{E_1}{E_2}=\left(\frac{V_1}{V_2}\right)^2$$

potential difference = ...2.1........ V [3]

$$\frac{1}{0.5}=\left(\frac{3.0}{V_2}\right)^2 \qquad 2=\frac{9}{V_2^{\,2}}$$

$$V_2=\sqrt{\frac{9}{2}}$$

[Total: 9]

6 **(a)** Define the *capacitance* of a parallel-plate capacitor.

Ratio of charge stored on one plate of the capacitor to the potential difference across plates.

.... [2]

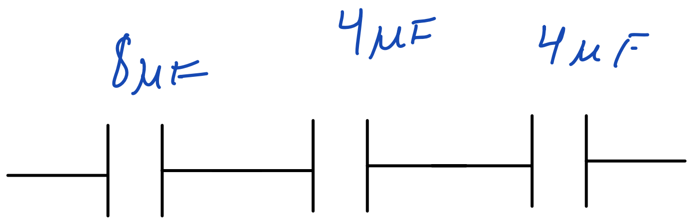
(b) A student has three capacitors. Two of the capacitors have a capacitance of 4.0 μF and one has a capacitance of 8.0 μF.

Draw labelled circuit diagrams, one in each case, to show how the three capacitors may be connected to give a total capacitance of:

(i) 1.6 μF

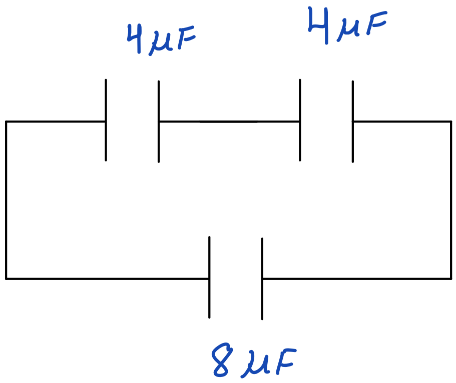
$$\left(\frac{1}{8}+\frac{1}{4}+\frac{1}{4}\right)^{-1}$$

$$= 1.6\,\mu F$$



[1]

(ii) 10 μF.



$$\left(\frac{1}{4}+\frac{1}{4}\right)^{-1}+8 = 10\,\mu F$$

[1]

(c) A capacitor C of capacitance 47 μF is connected across the output terminals of a bridge rectifier, as shown in Fig. 6.1.

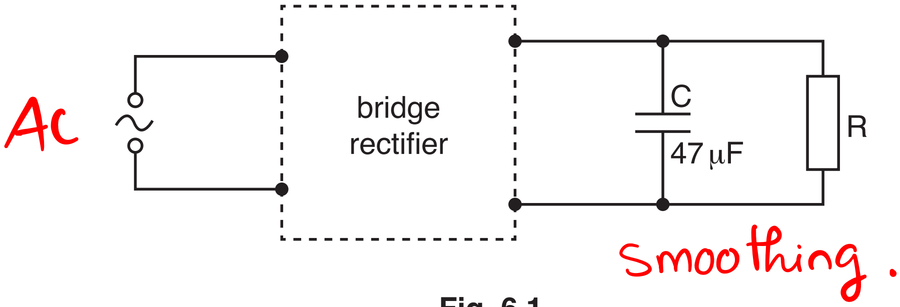


Fig. 6.1

The variation with time t of the potential difference V across the resistor R is shown in Fig. 6.2.

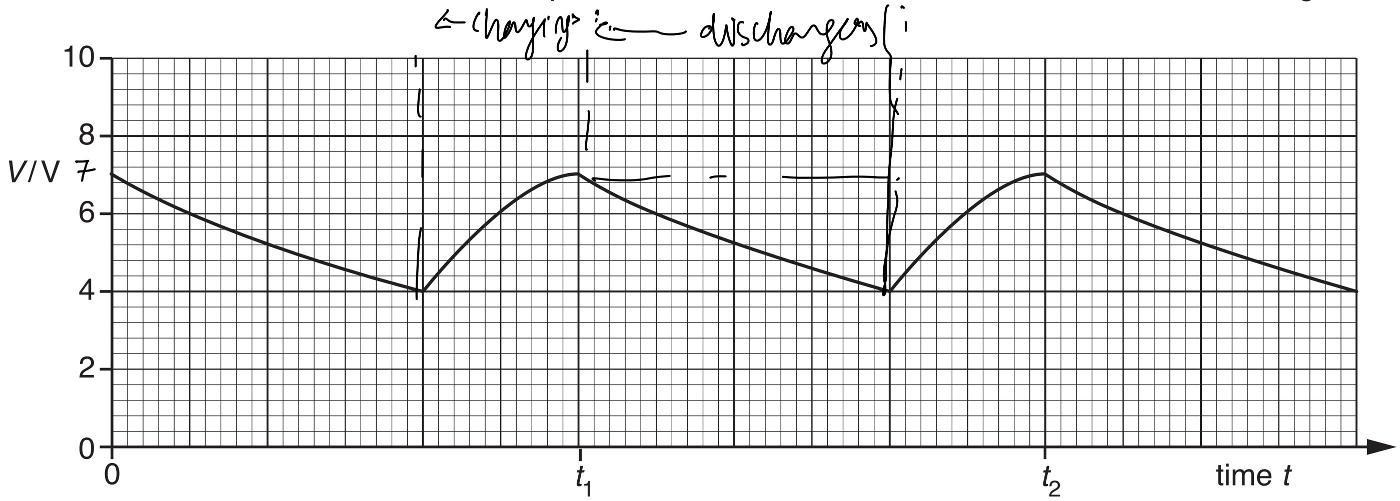


Fig. 6.2

Use data from Fig. 6.2 to determine the energy transfer from the capacitor C to the resistor R between time t_1 and time t_2.

$$\Delta E = \frac{1}{2} C (V_f^2 - V_i^2)$$

$$\frac{1}{2}(47 \times 10^{-6})(7^2 - 4^2)$$

energy = 7.8×10^{-4} J [3]

[Total: 7]

4 **(a)** State two functions of capacitors in electrical circuits.

1. Store energy // smoothing rectified AC

2. Seperate charges // preventing sparks.

[2]

(b) Three uncharged capacitors of capacitance C_1, C_2 and C_3 are connected in series, as shown in Fig. 4.1.

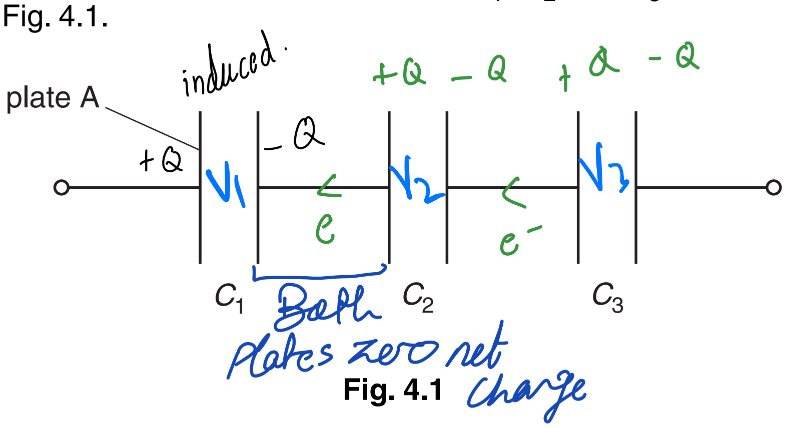


Fig. 4.1

A charge of $+Q$ is put on plate A of the capacitor of capacitance C_1.

(i) State and explain the charges that will be observed on the other plates of the capacitors.
You may draw on Fig. 4.1 if you wish.

– Q is induced on plate of C1 opposite to plate A.
– by conversation of charge, the charges on each plate, starting from plate A are
$+Q, -Q, +Q, -Q, +Q, -Q$ [2]

(ii) Use your answer in (i) to derive an expression for the combined capacitance of the capacitors.

$Q = CV$

$V = \frac{Q}{C}$

C in series =>

$V_T = V_1 + V_2 + V_3$

$\frac{Q}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$; since $Q_1 = Q_2 = Q_3 = Q$

[2]

$\frac{1}{C_{Total}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

For Examiner's Use

(c) A capacitor of capacitance 12 μF is charged using a battery of e.m.f. 9.0 V, as shown in Fig. 4.2.

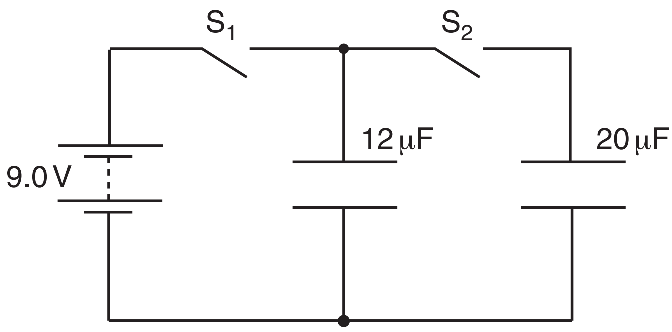
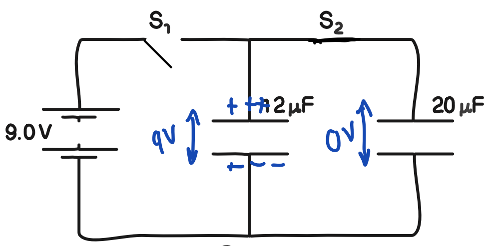


Fig. 4.2

Switch S_1 is closed and switch S_2 is open.

(i) The capacitor is now disconnected from the battery by opening S_1. Calculate the energy stored in the capacitor.



$E = \frac{1}{2} C V^2$

$= \frac{1}{2} (12 \times 10^{-6})(9)^2$

q flow to 20 μF $Q = CV$

$= 12 \times 9$

$V_{12\mu F} = V_{20\,\mu F}$ $= 108\,\mu C$

energy = 4.86×10^{-4} J [2]

(ii) The 12 μF capacitor is now connected to an uncharged capacitor of capacitance 20 μF by closing S_2. Switch S_1 remains open. The total energy now stored in the two capacitors is 1.82×10^{-4} J.

Suggest why this value is different from your answer in **(i).**

There is energy dissipitated due to resistance of wires / as sparks. [1]

5 The variation with potential difference V of the charge Q on one of the plates of a capacitor is shown in Fig. 5.1.

$Q = CV$

$y = mx$

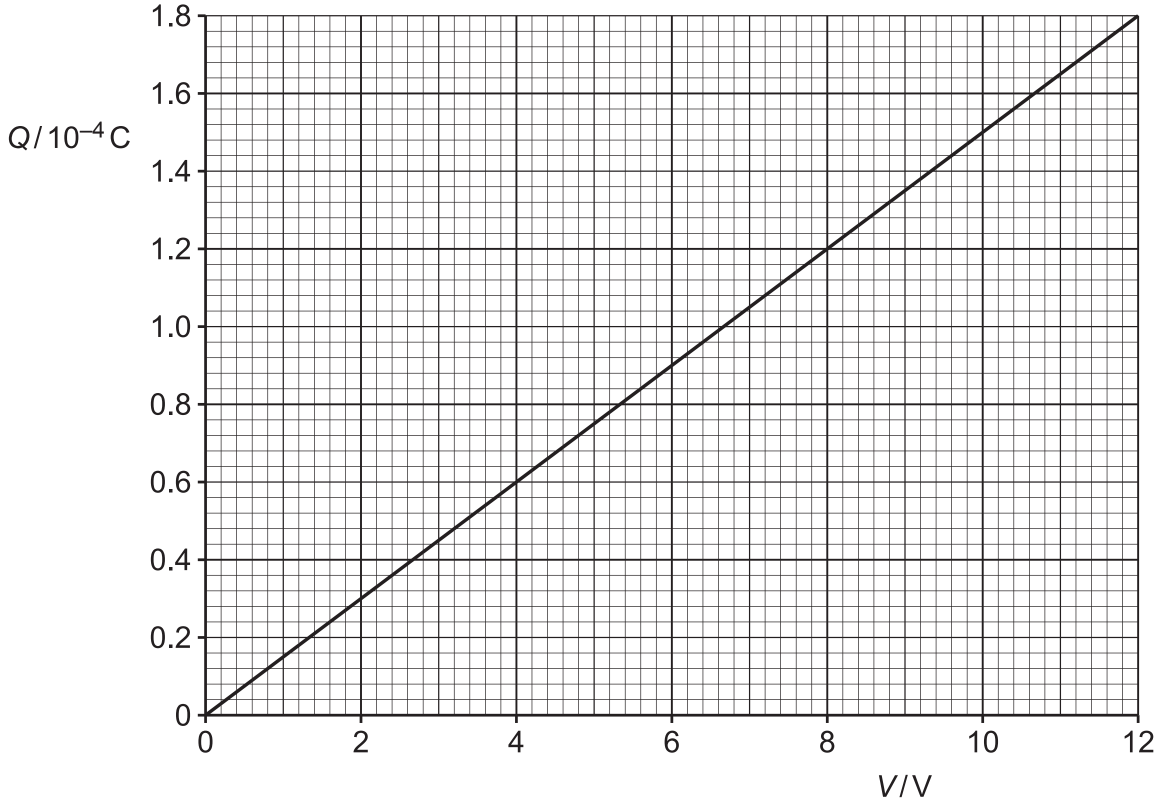


Fig. 5.1

The capacitor is connected to an 8.0 V power supply and two resistors R and S as shown in Fig. 5.2.

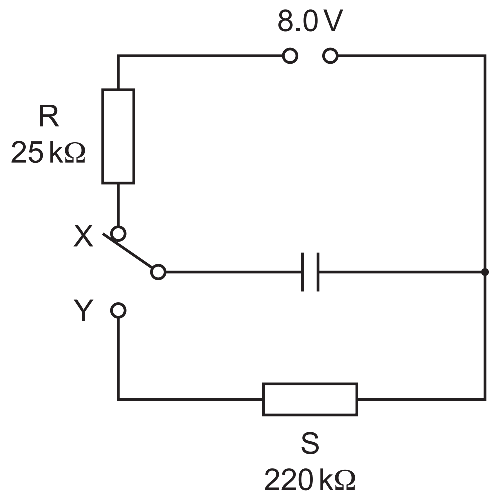


Fig. 5.2

The resistance of R is 25 kΩ and the resistance of S is 220 kΩ.

The switch can be in either position X or position Y.

(a) The switch is in position X so that the capacitor is fully charged.

Calculate the energy E stored in the capacitor.

$E = \frac{1}{2}QV$

$= \frac{1}{2}(1.2 \times 10^{-4})(8)$

$E =$ ....4.8×10^{-4}.... J [2]

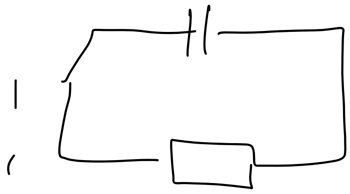
(b) The switch is now moved to position Y.

(i) Show that the time constant of the discharge circuit is 3.3 s.

$\tau = RC$

$= (220 \times 10^{3})(1.5 \times 10^{-5})$

$= 3.3\,s$



$Q = CV$

$C = \frac{Q}{V} = \frac{1.2 \times 10^{-4}}{8}$

$C = 1.5 \times 10^{-5}$

[2]

(ii) The fully charged capacitor in **(a)** stores energy E.

Determine the time t taken for the stored energy to decrease from E to $E/9$.

discharge equation.

From $E = \frac{1}{2}CV^2$

$E \propto V^2$

$\frac{E_1}{E_2} = \left(\frac{V_1}{V_2}\right)^2$

$\frac{E}{E/9} = \left(\frac{8.0}{V}\right)^2$

$9 = \left(\frac{8}{V}\right)^2$

$V = \frac{8}{3}$; $\frac{V_0}{3}$

$V = V_0 e^{-t/RC}$

$\frac{8}{3} = \cancel{8} e^{-t/RC}$ or $\cancel{\frac{V_0}{3} = V_0}$

$\frac{1}{3} = e^{-t/RC}$

$\ln\left(\frac{1}{3}\right) = -\frac{t}{3.3}$

$t = 3.63\,s$

$E = \frac{1}{2}CV^2$; C: constant

$E \propto V^2$

$t =$ ....3.6.... s [4]

$Q = CV$ $Q = Q_0 e^{-t/RC}$

$Q \propto V$ $V = V_0 e^{-t/RC}$

$V = IR$ $I = I_0 e^{-t/RC}$

$V \propto I$

Total Capacitance ↑

(c) A second identical capacitor is connected in parallel with the first capacitor.

State and explain the change, if any, to the time constant of the discharge circuit.

Total capacitance is doubled, hence time constant is also doubled.

.... [2]

[Total: 10]

$$\uparrow \tau = RC \uparrow$$

$\times 2$ $\times 2$