PROSPERITY ACADEMY

A2 PHYSICS 9702
Crash Course

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RADIOACTIVITY

COMPLETE NOTES

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Nuclear Radioactivity :-

Mass defect (Δm) :-

The difference in mass of a nucleus bounded atom and the mass of its individual component

We will use this for reactions:-

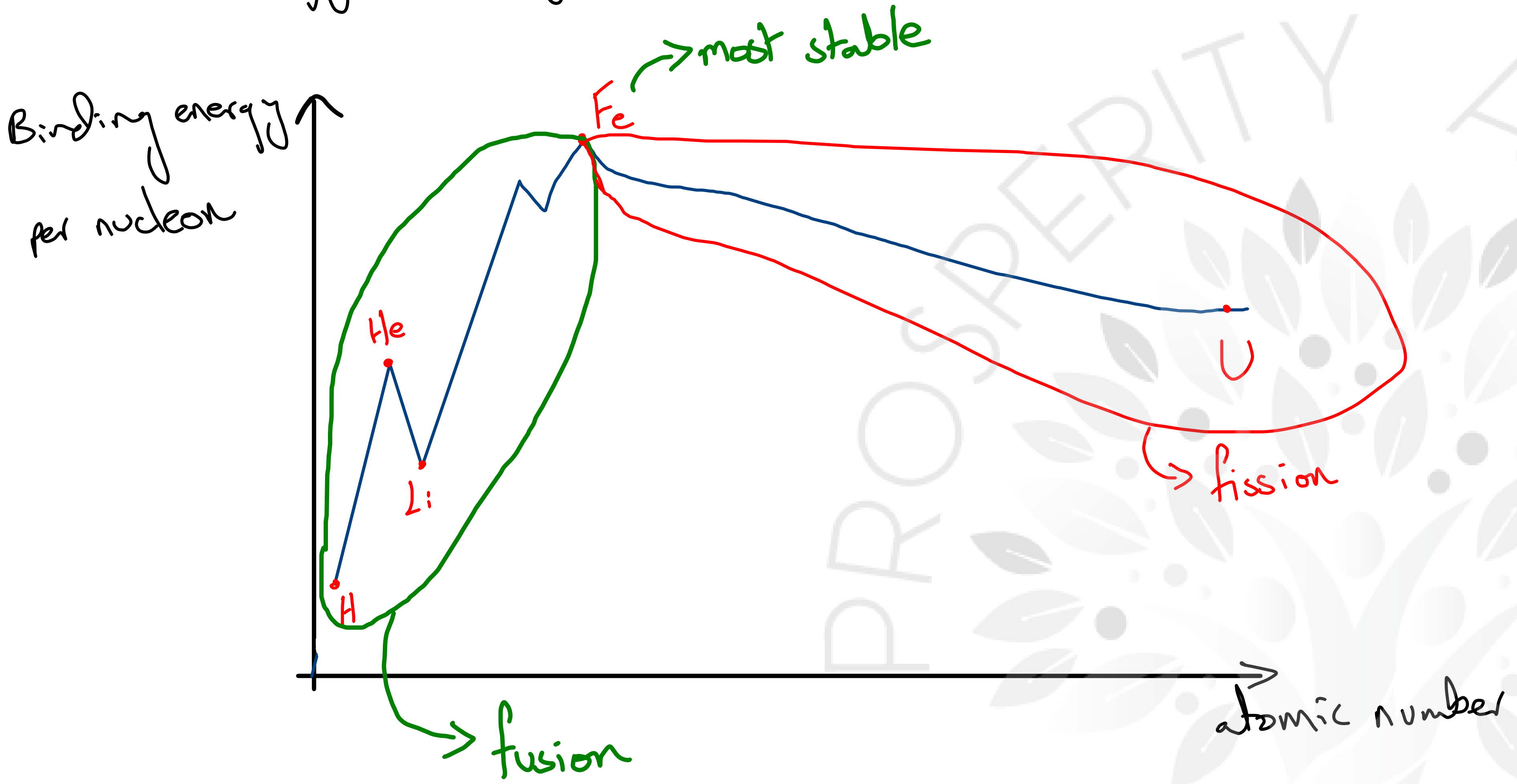
$$\Delta m = m_{products} - m_{reactants}$$

Binding energy :-

Minimum energy required to break a nucleus and separate its individual components at infinity.

$$\boxed{E = \Delta m c^2}$$

Binding energy tells you how stable a species is



Fusion :- Combination of smaller unstable nuclei to form bigger more stable nucleus.

Fission :- Breaking of larger unstable nuclei into more stable daughter nuclei.

8 (a) State what is meant by the *binding energy* of a nucleus.

It is the minimum energy required to break the nucleus into its seperate components and seperate them at infinity. [2]

*(b) Show that the energy equivalence of 1.0u is 930 MeV.

$E = mc^2$

$E = 1 \times (1.66 \times 10^{-27}) \times (3 \times 10^8)^2$

$E = 1.494 \times 10^{-10}$ J

1.6×10^{-13} J : 1 MeV

1.494×10^{-10} J : x MeV

$x = \frac{1.494 \times 10^{-10}}{1.6 \times 10^{-19}}$

$x = 933.75$ MeV

$\boxed{930 \text{ MeV}}$

[3]

(c) Data for the masses of some particles and nuclei are given in Fig. 8.1.

		mass/u
	proton	1.0073u
	neutron	1.0087u
1p, 1n ←	deuterium (^2_1H)	2.0141u
40p, 57n	zirconium ($^{97}_{40}Zr$)	97.0980u

Fig. 8.1

Use data from Fig. 8.1 and information from **(b)** to determine, in MeV,

(i) the binding energy of deuterium,

$\Delta m = (2.0141)u - (1.0087)u - (1.0073)u$

$\Delta m = -1.9 \times 10^{-3}$ u

$E = (1.9 \times 10^{-3}) \times (930) = 1.767$

binding energy = 1.8 MeV [2]

(ii) the binding energy **per nucleon** of zirconium.

$\Delta m = 97.0980u - 40(1.0073u) - 57(1.0087)$

$\Delta m = -0.6899u$

$E = \frac{0.6899 \times 930}{97} = 6.615$ MeV

binding energy per nucleon = 6.6 MeV [3]

Q. Will Aluminium decay into sodium via an emission of an α particle?

$${}^{27}_{13}Al \longrightarrow {}^{23}_{11}Na + {}^{4}_{2}\alpha^{2+}$$

(This reaction is not energetically feasible as Aluminium has more binding energy than the products.)

Masses of :-

Al :- $26.9815\,u$ proton :- $1.0073\,u$

Na :- $22.9878\,u$ neutron :- $1.0087\,u$

He :- $4.00260\,u$

Binding energies (in terms of u) (Basically Mass defect) :-

Al :- $26.9815u - 13(1.0073u) - 14(1.0087u) = 0.2352\,u$ (Binding energy of reactant)

Na :- $22.9878u - 11(1.0073u) - 12(1.0087u) = 0.1969\,u$

He :- $4.00260u - 2(1.0073u) - 2(1.0087u) = 0.0294\,u$

$\longrightarrow$ $0.2263u$ (Binding energy of products)

Q. What is the energy required for the reaction?

$\Delta m = m_{products} - m_{reactants}$ (for a reaction)

$(22.9878u + 4.00260u) - 26.9815u = \underline{\underline{+8.9 \times 10^{-3}\,u}}$

↳ this reaction is energertically unfeasible

In a reaction:-

1) A +ve Δm means the reaction requires energy and is unfeasible

2) A -ve Δm means the reaction gives off energy and is feasible

$$E = \Delta m c^2 \Rightarrow E = (8.9 \times 10^{-3} \times 1.66 \times 10^{-27})(3 \times 10^{8})^2$$

$$E = 1.3297 \times 10^{-12}\,J$$

$1.6 \times 10^{-13}\,J$: $1\,MeV$

1.3297×10^{-12} : $x\,MeV$

$$x = \frac{1.3297 \times 10^{-12}}{1.6 \times 10^{-13}} = 8.31\,MeV$$

① $236 = 95 + 139 + x(1) + 7(0)$

$236 - 95 - 139 = x$

$x = 2$

8 When a neutron is captured by a uranium-235 nucleus, the outcome may be represented by the nuclear equation shown below.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + x^{1}_{0}n + 7^{0}_{-1}e$$

(a) **(i)** Use the equation to determine the value of *x*.

x = 2 [1]

(ii) State the name of the particle represented by the symbol $^{0}_{-1}e$.

β^- particle, electron [1]

(b) Some data for the nuclei in the reaction are given in Fig. 8.1.

	mass/u	binding energy per nucleon /MeV
uranium-235 ($^{235}_{92}U$)	235.123	7.18
molybdenum-95 ($^{95}_{42}Mo$)	94.945	8.09
lanthanum-139 ($^{139}_{57}La$)	138.955	7.92
proton ($^{1}_{1}p$)	1.007	0
neutron ($^{1}_{0}n$)	1.009	0

Fig. 8.1

Use data from Fig. 8.1 to

(i) determine the binding energy, in u, of a nucleus of uranium-235,

Δm

$\Delta m = 235.123u - 92(1.007) - 143(1.009)$

$\Delta m = -1.808u$

binding energy = 1.808 u [3]

(ii) show that the binding energy per nucleon of a nucleus of uranium-235 is 7.18 MeV.

$E = \dfrac{\Delta mc^2}{\text{no. of nucleons}} \Rightarrow \dfrac{[1.808 \times 1.66 \times 10^{-27}] \times (3 \times 10^8)^2}{235}$

$= 1.149 \times 10^{-12}\ J$

$1.6 \times 10^{-13}\ J = 1\ MeV$

$1.149 \times 10^{-12}\ J = x\ MeV \Rightarrow \boxed{x = 7.18\ MeV}$

[3]

(c) The kinetic energy of the neutron before the reaction is negligible.
Use data from **(b)** to calculate the total energy, in MeV, released in this reaction.

$\Delta E = E_{products} - E_{reactant}$

$= [8.09(95) + 7.92(139)] - 7.18(235)$

$= 182.13\ MeV$

energy = 182 MeV [2]

8 **(a)** State what is meant by *nuclear binding energy*.

....

....

.... [2]

(b) The variation with nucleon number A of the binding energy per nucleon B_E is shown in Fig. 8.1.

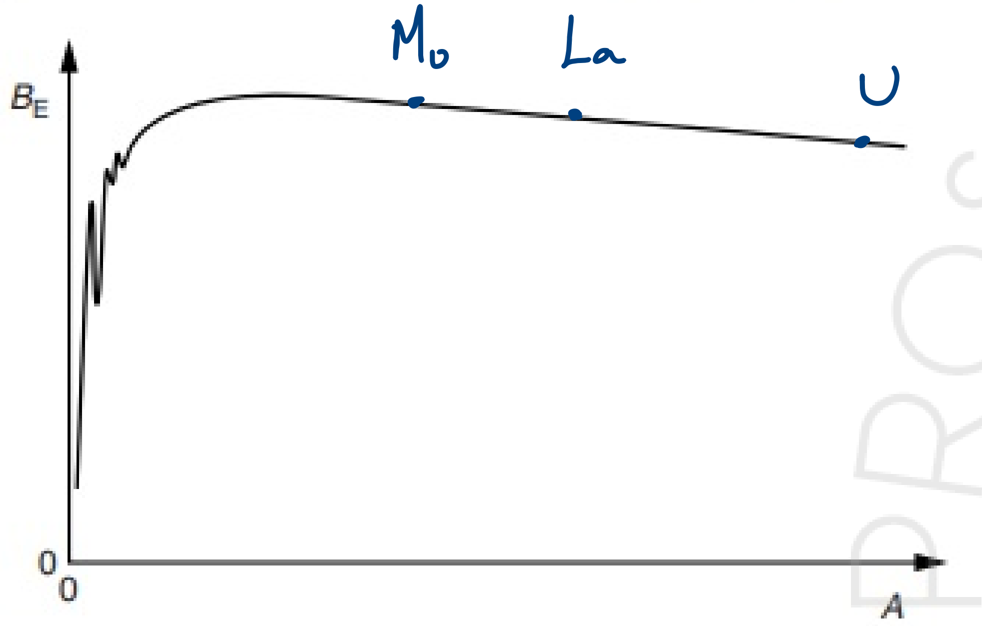


Fig. 8.1

When uranium-235 ($^{235}_{92}U$) absorbs a slow-moving neutron, one possible nuclear reaction is

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + 2^{1}_{0}n + 7^{0}_{-1}\beta + \text{energy}.$$

(i) State the name of this type of nuclear reaction.

fission .... [1]

(ii) On Fig. 8.1, mark the position of

1. the uranium-235 nucleus (label this position U), [1]

2. the molybdenum-95 ($^{95}_{42}Mo$) nucleus (label this position Mo), [1]

3. the lanthanum-139 ($^{139}_{57}La$) nucleus (label this position La). [1]

(iii) The masses of some particles and nuclei are given in Fig. 8.2.

	mass / u
β-particle	5.5×10^{-4}
neutron	1.009
proton	1.007
uranium-235	235.123
molybdenum-95	94.945
lanthanum-139	138.955

Fig. 8.2

Calculate, for this reaction,

1. the change, in u, of the rest mass,

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + 2^{1}_{0}n + 7^{0}_{-1}\beta + \text{energy}.$$

$\Delta m = m_{products} - m_{reactants}$

$\Delta m = \left[(94.945) + (138.955) + 2(1.009) + 7(5.5 \times 10^{-4})\right] - 235.123 - 1.009$

$\Delta m = -0.21015$

change in mass = ...0.210........ u [2]

2. the energy released, in MeV, to three significant figures.

$E = \Delta m c^2$

$E = (0.210 \times 1.66 \times 10^{-27}) \times (3 \times 10^8)^2 = 3.1374 \times 10^{-11}$ J

$1.6 \times 10^{-13} = 1$ MeV

$3.1374 \times 10^{-11} = x$ MeV $\Rightarrow x = 196.0875$

energy = ...196........ MeV [3]

Radioactivity:- It is the random and spontaneous decay of unstable nuclei emitting β^-, β^+, γ or α particles.

Random:- Equally likely probability of any unstable nucleus decaying in the sample.

- Experienced by fluctuation in readings

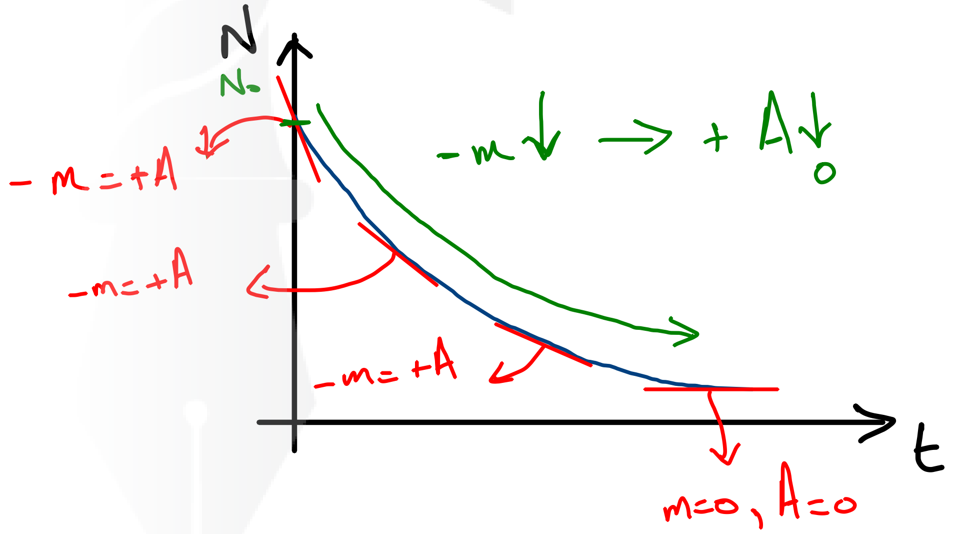
Spontaneous:- The rate of decay of a radioactive sample does not depend on external factors

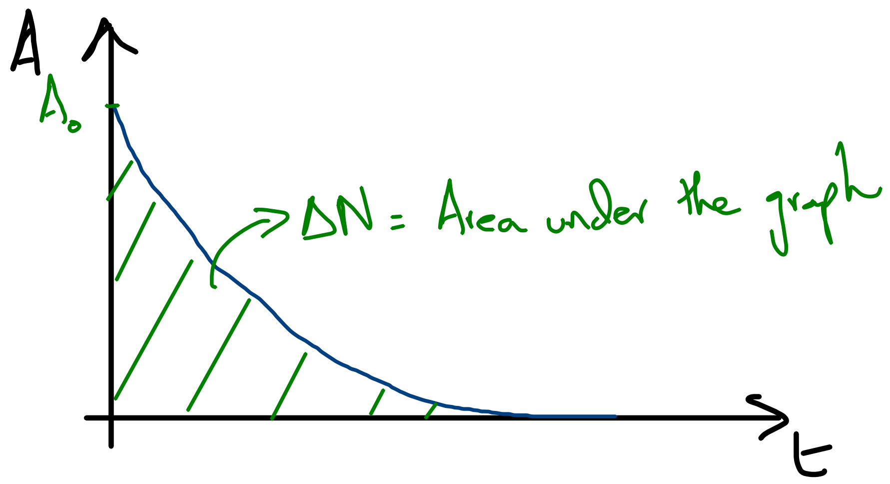
- Experienced by no change as temp/pressure is increased/decreased.

Activity:- (Measured in Becquerels (Bq))

Rate of disintegrations per unit time

$$A = \frac{-dN}{dt} = \frac{-\Delta N}{\Delta t} \quad (A \times \Delta t = -\Delta N)$$





ΔN: Number of nuclei that have decayed
N: number of nuclei remaining
N_0: initial number of nuclei

$$\boxed{N_0 - \Delta N = N}$$

Activity $\propto$ Number of unstable nuclei remaining

$$A \propto N$$

$$A = \underbrace{\lambda}_{\text{decay constant}} N$$

$$-\frac{\Delta N}{\Delta t} = \lambda \times N$$

$$-\frac{\Delta N}{\Delta t} \times \frac{1}{N} = \lambda$$

$$-\frac{\Delta N}{N} \times \frac{1}{\Delta t} = \lambda$$

$-\frac{\Delta N}{N}$ → probability of an unstable nucleus decaying

Δt → change in time

$$\boxed{A = \lambda N}$$

$$\boxed{A_0 = \lambda N_0}$$

Decay constant:-

It is the probability of an unstable nucleus decaying per unit time

$$\lambda = \frac{-\Delta N}{N t}$$

Decay Equations:-

- $\boxed{N = N_o e^{-\lambda t}}$

$$\frac{A}{\cancel{\lambda}} = \frac{A_o}{\cancel{\lambda}} e^{-\lambda t}$$

- $\boxed{A = A_o e^{-\lambda t}}$

N: number of unstable nuclei remaining
N_o: initial number of unstable nuclei
λ: decay constant
t: time elapsed
A: activity
A_o: initial activity

Half life:- It is the time required for half the nuclei in the radioactive sample to decay.

$$N = N_o e^{-t\lambda}$$

$$\frac{\cancel{N_o}}{2} = \cancel{N_o} e^{-t_{\frac{1}{2}}\lambda}$$

$$\frac{1}{2} = \frac{1}{e^{t_{\frac{1}{2}} \times \lambda}}$$

$$\ln\left(e^{+\frac{1}{2}\lambda}\right) = \ln(2)$$

$$t_{\frac{1}{2}} \times \lambda = \ln 2 \Rightarrow \boxed{t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}}$$

9 (a) A sample of a radioactive isotope contains N nuclei at time t. At time $(t + \Delta t)$, it contains $(N - \Delta N)$ nuclei of the isotope.

N_o

For the period Δt, state, in terms of N, ΔN and Δt,

(i) the mean activity of the sample,

activity = $-\dfrac{\Delta N}{\Delta t}$ [1]

(ii) the probability of decay of a nucleus.

probability = $\dfrac{\Delta N}{N}$ [1]

(b) A cobalt-60 source having a half-life of 5.27 years is calibrated and found to have an activity of 3.50×10^5 Bq. The uncertainty in the calibration is ±2%.

λ A_o 8%

Calculate the length of time, in days, after the calibration has been made, for the stated activity of 3.50×10^5 Bq to have a maximum possible error of 10%.

We want to find the time when the activity will decrease by 8%.

$A = 92\% \times A_o$ $\quad t_{1/2} = 5.27$ years

$= \dfrac{92}{100} \times 3.5 \times 10^5$ $\quad 5.27 \times 365 = \dfrac{\ln 2}{\lambda}$

$\lambda = \dfrac{\ln 2}{5.27 \times 365} = 3.603 \times 10^{-4}$

$A = 3.22 \times 10^5$

time = 231 days [4]

$A = A_o e^{-t \times \lambda}$

$(3.22 \times \cancel{10^5}) = (3.5 \times \cancel{10^5}) \times e^{-t \times (3.603 \times 10^{-4})}$

$\ln\left(\dfrac{3.22}{3.5}\right) = \ln\left(e^{-t \times (3.603 \times 10^{-4})}\right)$

$\ln\left(\dfrac{3.22}{3.5}\right) = -t \times (3.603 \times 10^{-4})$

$t = 231.4$ days

8 The element strontium has at least 16 isotopes. One of these isotopes is strontium-89. This isotope has a half-life of 52 days.

(a) State what is meant by *isotopes*.

Atoms of the same element having the same number of protons but different number of neutrons. [2]

(b) Calculate the probability per second of decay of a nucleus of strontium-89.

λ

$N = N_o e^{-t\lambda}$

$\dfrac{\cancel{N_o}}{2} = \cancel{N_o} e^{-t\lambda}$

$\dfrac{1}{2} = \dfrac{1}{e^{t\lambda}}$

$\ln(e^{t\lambda}) = \ln(2)$

$t\lambda = \ln 2$

$t_{\frac{1}{2}} = \dfrac{\ln 2}{\lambda}$

$52 \times 24 \times 60 \times 60 = \dfrac{\ln 2}{\lambda}$

$\lambda = 1.5428 \times 10^{-7}$

$M_r / A_r = 89$ probability = 1.5×10^{-7} s^{-1} [3]

(c) A laboratory prepares a strontium-89 source. t
The activity of this source is measured 21 days after preparation of the source and is found to be 7.4×10^6 Bq. $= A$

$t = 0$

Determine, for the strontium-89 source at the time that it was prepared,

(i) the activity, A_o

$A = A_o e^{-t\lambda}$

$(7.4 \times 10^6) = A_o \times e^{-(21 \times 24 \times 60 \times 60)(1.5 \times 10^{-7})}$

$A_o = 9714698 \Rightarrow 9.7 \times 10^6$

activity = 9.7×10^6 Bq [2]

(ii) the mass of strontium-89.

$A_o = \lambda N_o$

$9.7 \times 10^6 = (1.5 \times 10^{-7}) N_o$

$N_o = 6.4667 \times 10^{13}$

(number of particles)

$\text{mol} \times 6.02 \times 10^{23} = N_o$

$\dfrac{6.4667 \times 10^{13}}{6.02 \times 10^{23}} = \text{mol}$

$\text{mol} = 1.0741 \times 10^{-10}$

$\text{Mass} = \text{mol} \times M_r / A_r$

$\text{Mass} = (1.0741 \times 10^{-10}) \times 89$

$= 9.56 \times 10^{-9}$

$\approx 9.6 \times 10^{-9}$ g

9 During the de-commissioning of a nuclear reactor, a mass of 2.5×10^6 kg of steel is found to be contaminated with radioactive nickel-63 ($^{63}_{28}Ni$).
The total activity of the steel due to the nickel-63 contamination is 1.7×10^{14} Bq.

(a) Calculate the activity per unit mass of the steel.

$$\frac{A}{m} = \frac{1.7\times10^{14}\ \text{Bq}}{2.5\times10^{6}\ \text{Kg}} = 6.8\times10^{7}$$

activity per unit mass = 6.8×10^7 Bq kg^{-1} [1]

(b) Special storage precautions need to be taken when the activity per unit mass due to contamination exceeds 400 Bq kg^{-1}.
Nickel-63 is a β-emitter with a half-life of 92 years.
The maximum energy of an emitted β-particle is 0.067 MeV.

(i) Use your answer in **(a)** to calculate the energy, in J, released per second in a mass of 1.0 kg of steel due to the radioactive decay of the nickel.

$$A = \frac{-\Delta N}{\Delta t} \Rightarrow A \times \Delta t = \Delta N$$

$$6.8\times10^{7} \times 1 = \Delta N$$

in 1sec, 6.8×10^7 nuclei decay, so 6.8×10^7 β^- particles

$$E_T = 6.8\times10^{7} \times 0.067 = 4.556\times10^{6}\ \text{MeV}$$

$$E\ \text{in J} = 4.556\times10^{6} \times (1.6\times10^{-13}) = 7.2896\times10^{-7}\ \text{J}$$

energy = 7.3×10^{-7} J [1]

(ii) Use your answer in **(i)** to suggest, with a reason, whether the steel will be at a high temperature.

Very low amount of energy is released per second so no change in temperature.

[1]

(iii) Use your answer in **(a)** to determine the time interval before special storage precautions for the steel are not required.

$$A = A_0 e^{-\lambda t}$$

$$400 = (6.8\times10^{7}) \times e^{-\left(\frac{\ln 2}{92}\right)t}$$

$$\ln\left(\frac{400}{6.8\times10^{7}}\right) = \ln\left(e^{-\left(\frac{\ln 2}{92}\right)t}\right) \Rightarrow \ln\left(\frac{400}{6.8\times10^{7}}\right) = -\left(\frac{\ln 2}{92}\right)t$$

$t = 1598$ years

$t_{\frac{1}{2}} = 92$

$\frac{\ln 2}{\lambda} = 92$ years

$\lambda = \frac{\ln 2}{92}$

time = 1600 years [3]

9 **(a)** An isotope of an element is radioactive. Explain what is meant by *radioactive decay*.

It is the random and spontaneous decay of unstable nuclei releasing α, β^-, β^+, γ radiation. By random we mean the equally likely probability of any nucleus disintegrating and by spontaneous we mean that the rate of decay is not affected by external factors. [3]

(b) At time t, a sample of a radioactive isotope contains N nuclei. In a short time Δt, the number of nuclei that decay is ΔN.

N_0

State expressions, in terms of the symbols t, Δt, N and ΔN for

(i) the number of undecayed nuclei at time $(t + \Delta t)$,

number = $N - \Delta N$ [1]

(ii) the mean activity of the sample during the time interval Δt,

mean activity = $-\dfrac{\Delta N}{\Delta t}$ [1]

(iii) the probability of decay of a nucleus during the time interval Δt,

probability = $\dfrac{\Delta N}{N}$ [1]

(iv) the decay constant.

decay constant = $-\dfrac{\Delta N}{N \Delta t}$ [1]

(c) The variation with time t of the activity A of a sample of a radioactive isotope is shown in Fig. 9.1.

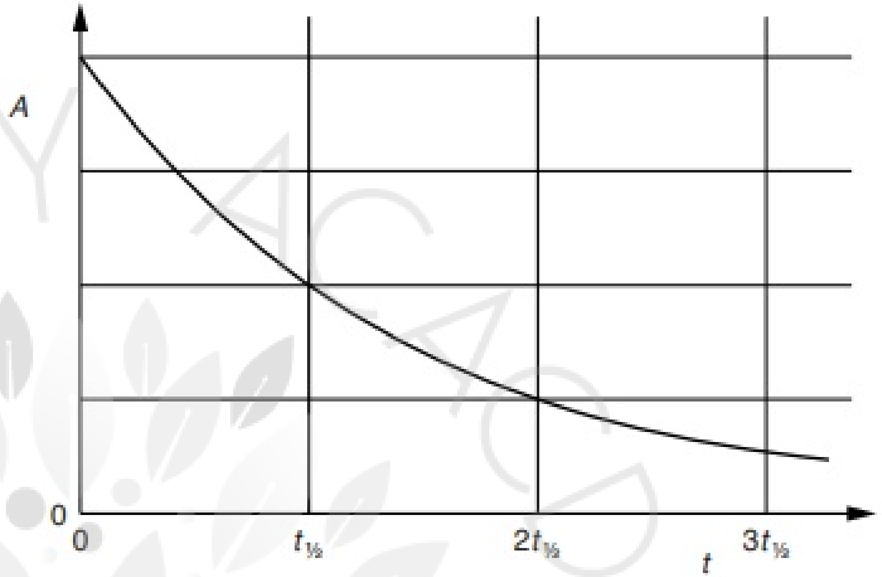


Fig. 9.1

$$A = -\frac{\Delta N}{\Delta t}$$

The radioactive isotope decays to form a stable isotope S. At time $t = 0$, there are no nuclei of S in the sample.

On the axes of Fig. 9.2, sketch a graph to show the variation with time t of the number n of nuclei of S in the sample.

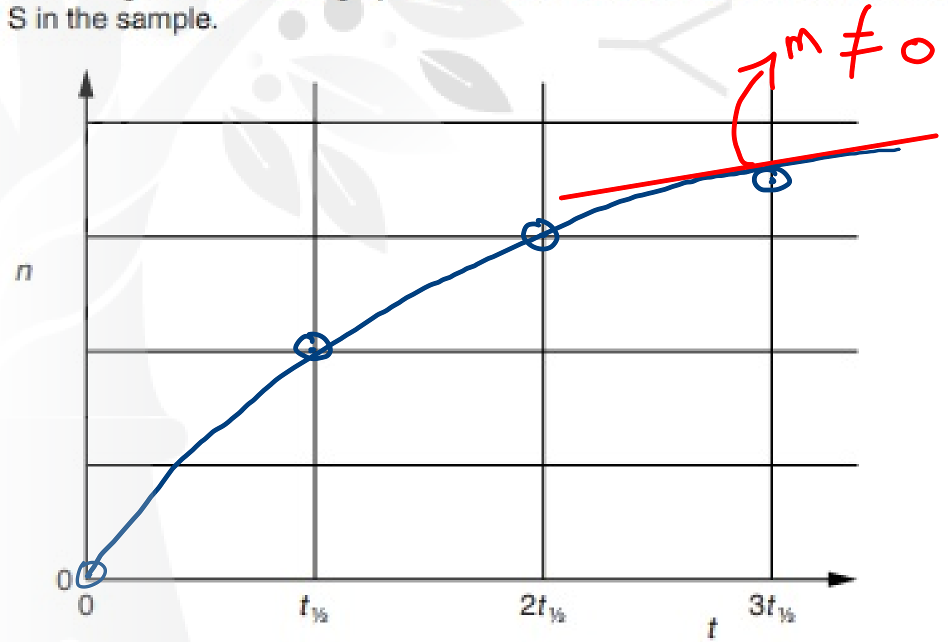


Fig. 9.2

① $t_{\frac{1}{2}} \Rightarrow X_0 \rightarrow \frac{1}{2}X_0 \rightarrow \frac{1}{2}S$

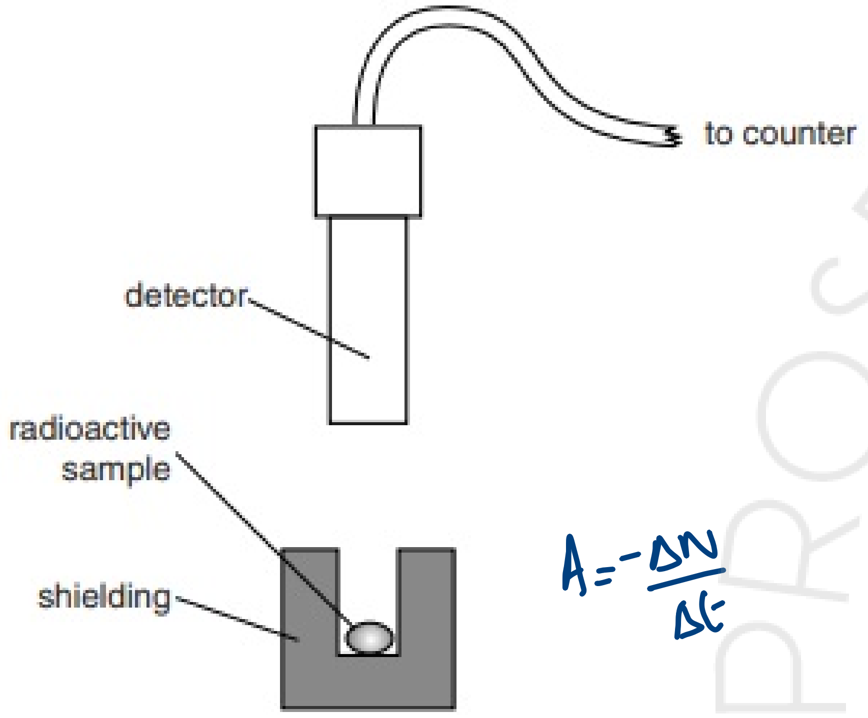
② $t_{\frac{1}{2}} \Rightarrow \frac{1}{2}X_0 \rightarrow \frac{1}{4}X_0 \rightarrow \frac{1}{4}S$

[2]

13 Copper-66 is a radioactive isotope.

When a nucleus of copper-66 decays, the emissions include a β^- particle and a γ-ray photon.

The count rate produced from a sample of the isotope copper-66 is measured using a detector and counter, as illustrated in Fig. 13.1.



$\frac{\Delta N}{\Delta t}$

$A = -\frac{\Delta N}{\Delta t}$

Fig. 13.1

(a) State three reasons why the activity of the sample of copper-66 is not equal to the measured count rate.

1. Maybe there is background radiation

2. Radiation is emitted in all directions and may not be detected by the detector above

3. Maybe the products of the decay of copper-66 are radioactive

[3]

Machine errors :-

- dead time of a counter
- there are multiple counts for 1 decay.

(b) In a time of 42.0 minutes, the count rate from the sample of copper-66 is found to decrease from 3.62×10^4 Bq to 1.21×10^2 Bq.

Calculate the half-life of copper-66.

$A = A_o e^{-\lambda t}$

$1.21 \times 10^2 = (3.62 \times 10^4) e^{-\lambda(42)}$

$\ln\left(\frac{1.21 \times 10^2}{3.62 \times 10^4}\right) = \ln\left(e^{-\lambda(42)}\right)$

$\ln\left(\frac{1.21 \times 10^2}{3.62 \times 10^4}\right) = -\lambda(42) \Rightarrow \lambda = 0.1357$

$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

$t_{\frac{1}{2}} = 5.106\text{min}$

half-life = 5.11 minutes [2]

(c) The γ-ray photons emitted from radioactive nuclei have specific energies, dependent on the nucleus emitting the photons.

By comparison with emission line spectra, suggest what can be deduced about energy levels in nuclei.

discrete energy levels in nucleus

[1]

[Total: 6]

m22 qp42 q9

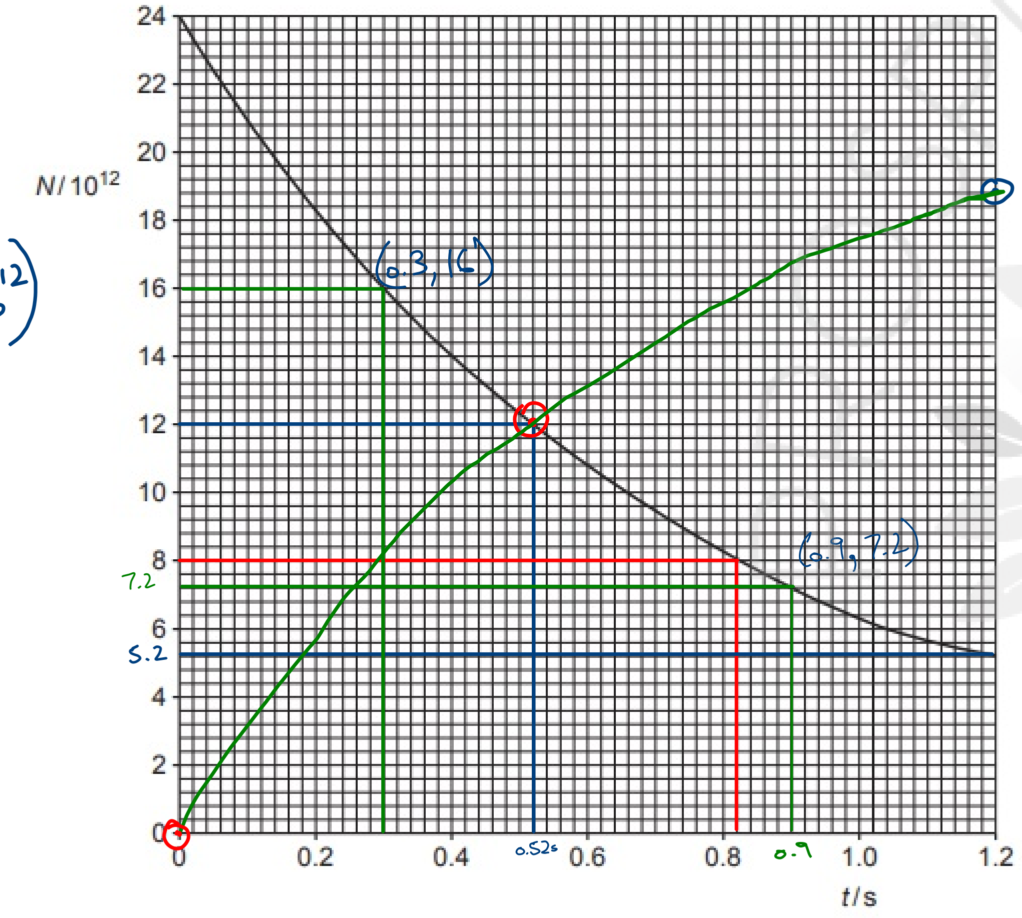
9 Polonium-211 ($^{211}_{84}$Po) decays by alpha emission to form a stable isotope of lead (Pb).

(a) Complete the equation for this decay.

$$^{211}_{84}\text{Po} \rightarrow {}^{207}_{82}\text{Pb} + {}^{4}_{2}\alpha$$

[2]

(b) The variation with time t of the number of unstable nuclei N in a sample of polonium-211 is shown in Fig. 9.1.



$N_0 - N$

$\Delta N = (24\times10^{12} - 5.2\times10^{12})$

18.8×10^{12}

At time $t = 0$, the sample contains only polonium-211.

(i) Use Fig. 9.1 to determine the decay constant λ of polonium-211. Give a unit with your answer.

① $t_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$

$\lambda = \frac{\ln 2}{0.52}$

② $N = N_0 e^{-\lambda t}$

$\lambda = \frac{\Delta N}{N \Delta t} = s^{-1}$

λ = 1.3 unit s^{-1} [2]

(ii) Use your answer in **(b)(i)** to calculate the activity at time $t = 0$ of the sample of polonium-211.

$A_0 = \lambda N_0$

$A_0 = 1.3 \times (24\times10^{12}) = 3.12\times10^{13}$

activity = 3.1×10^{13} Bq [1]

(iii) On Fig. 9.1, sketch a line to show the variation with t of the number of lead nuclei in the sample. [2]

(c) Each decay releases an alpha particle with energy 6900 keV.

(i) Calculate, in J, the total amount of energy given to alpha particles that are emitted between time $t = 0.30$ s and time $t = 0.90$ s.

$\Delta N = (16 - 7.2)\times10^{12}$

$\Delta N = 8.8\times10^{12}$ nuclei decayed = α particles

$E_T = 8.8\times10^{12} \times 6900 \times (1.6\times10^{-19}\times10^{3})$

$E_T = 9.7152$ J

energy = 9.7 J [3]

(ii) Suggest why the total amount of energy released by the decay process between time $t = 0.30$ s and time $t = 0.90$ s is greater than your answer in **(c)(i)**.

The lead nuclei will also have kinetic energy

The energy is released to the surroundings in the form of γ radiation [1]

[Total: 11]