NUCLEAR PHYSICS

23 Nuclear physics

23.1 Mass defect and nuclear binding energy

Candidates should be able to:

1 understand the equivalence between energy and mass as represented by $E = mc^2$ and recall and use this equation

2 represent simple nuclear reactions by nuclear equations of the form ${}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{17}_{8}\mathrm{O} + {}^{1}_{1}\mathrm{H}$

3 define and use the terms mass defect and binding energy

4 sketch the variation of binding energy per nucleon with nucleon number

5 explain what is meant by nuclear fusion and nuclear fission

6 explain the relevance of binding energy per nucleon to nuclear reactions, including nuclear fusion and nuclear fission

7 calculate the energy released in nuclear reactions using $E = c^2 \Delta m$

Einstein's mass-energy equivalence:-

$$\Delta E = (\Delta m)\, c^2$$

Here, ΔE is the change in Energy corresponding to the change in mass Δm. 'c' is the speed of light in vacuum. So,

(i) mass of a system increases, if energy is supplied to it and

(ii) mass of a system decreases, if energy is released by it.

(iii) Sun is a big source of energy because it converts its mass into energy

Example: Bodies with higher energy state have more mass than in a lower energy state. i.e A bucket of water at the top of hill will have more mass than when it is at the bottom because energy has been transferred to it in carrying it up the hill.

1- A person who runs, and thus gains K.E., gains mass.
2- Lifting a pen to a higher position and thus increasing its potential energy also increases its mass.

Nuclear reactions:-

Note: In any Nuclear reaction, there is no change in
(i) Mass no. (A), (ii) charge no. (Z), (iii) momentum
(iv) mass and energy

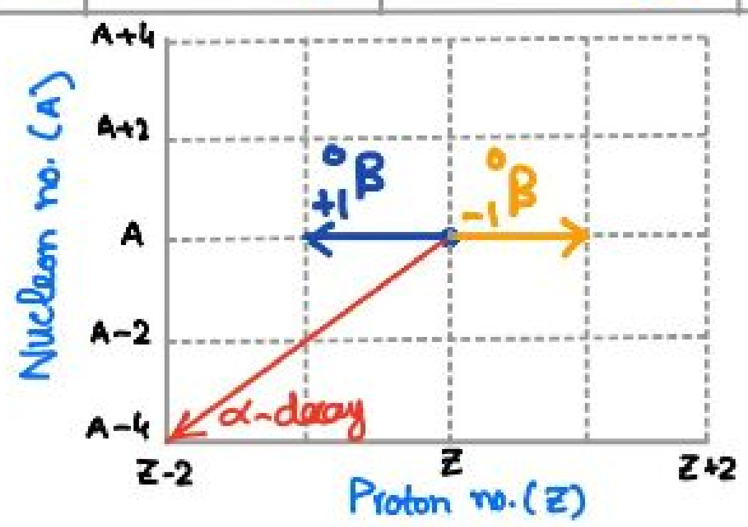
All are conserved in a nuclear reaction.

General reaction:-

$$^{14}_{7}N + ^{4}_{2}He \longrightarrow ^{\boxed{17}}_{\boxed{8}}O + ^{1}_{1}H$$

Example:

S.No.	Decay reaction	Daughter Nuclide	Change in Mass no. (A)	Change in charge no. (Z)	Any new Particle emitted
1.	Alpha decay	New Nuclide is obtained	decreases by 4	decreases by 2	—
2.	Electron decay ($^{0}_{-1}\beta$)	New Nuclide is obtained	Unchanged	Increased by 1	Anti-neutrino ($\bar{\nu}$)
3.	Positron decay ($^{0}_{+1}\beta$)	New Nuclide is obtained	Unchanged	Decreased by 1	Neutrino (ν)
4.	Gamma decay	No change	No change	No change	—



For Examiner's Use

8 **(a)** Explain why the mass of an α-particle is less than the total mass of two individual protons and two individual neutrons.

Since mass is reduced, so energy is released by $\Delta E = \Delta mc^2$. Therefore, α-particle is formed from its component particle. [2]

(b) An equation for one possible nuclear reaction is

$$^{4}_{2}He + ^{14}_{7}N \rightarrow ^{17}_{8}O + ^{1}_{1}p.$$

Data for the masses of the nuclei are given in Fig. 8.1.

		mass / u
proton	$^{1}_{1}p$	1.00728
helium-4	$^{4}_{2}He$	4.00260
nitrogen-14	$^{14}_{7}N$	14.00307
oxygen-17	$^{17}_{8}O$	16.99913

Fig. 8.1

(i) Calculate the mass change, in u, associated with this reaction.

Total mass of He and N = 4.00260 + 14.00307 = 18.00567u

Total mass of O and $^{1}_{1}p$ = 16.99913 + 1.00728 = 18.00641u

$\Delta m = 18.00641 - 18.00567$
$= 7.4 \times 10^{-4}$

mass change = 7.4×10^{-4} u [2]

(ii) Calculate the energy, in J, associated with the mass change in **(i)**.

$\Delta E = \Delta m c^2$
$= [(7.4 \times 10^{-4})(1.66 \times 10^{-27})][3.00 \times 10^{8}]^2$
$= 1.1 \times 10^{-13}$ J

energy = 1.1×10^{-13} J [2]

For Examiner's Use

(iii) Suggest and explain why, for this reaction to occur, the helium-4 nucleus must have a minimum speed.

Since both nuclei ($^{14}_{7}N$ and $^{4}_{2}He$) are positively charged, so they must have some kinetic energy to overcome electric repulsive force and initiate this reaction. [2]

Q) Find the increase in mass when 4200J of heat are absorbed by 1 kg of water to cause a temperature rise of 1 K.

Since,

$$\Delta E = \Delta m \; c^2$$

$$4200 = (\Delta m)\,(3.00 \times 10^{8})^2$$

$$\Delta m = 4.67 \times 10^{-14}\ \text{kg}$$

This is a negligibly small increase in mass.

8 A positron ($^{0}_{+1}e$) is a particle that has the same mass as an electron and has a charge of $+1.6 \times 10^{-19}$ C.

A positron will interact with an electron to form two γ-ray photons.

$P = mv$

$E_k = \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2)$

So $E_k = \frac{p^2}{2m}$

$$^{0}_{+1}e + ^{0}_{-1}e \rightarrow 2\gamma$$

Assuming that the kinetic energy of the positron and the electron is negligible when they interact,

(a) suggest why the two photons will move off in opposite directions with equal energies,

Since the initial momentum is zero. So the final total momentum must also be zero to keep momenta conserved. Therefore photons of Gamma rays after interaction move in opposite directions with same magnitude of momentum and kinetic energy.

.... [3]

(b) calculate the energy, in MeV, of one of the γ-ray photons.

$E = mc^2$

$= (9.11 \times 10^{-31})(3.00 \times 10^{8})^2$

$= 8.199 \times 10^{-14}$ J

1 MeV $= (10^6)(1.60 \times 10^{-19}) = 1.60 \times 10^{-13}$ J

$E = \frac{8.199 \times 10^{-14}}{1.60 \times 10^{-13}} =$

energy = 0.512 MeV [3]

MASS DEFECT/ MASS DEFICIT:-

Concept:- When protons and Neutrons are combined together to form a nucleus, then it is observed that mass of nucleus is always less than sum of masses of its nucleons. This discrepancy in mass is called mass defect/ mass deficit.

Def. It is the difference of sum of masses of nucleons and the nucleus.

Symbol: Δm

Formula:

(i) Atom:

$$\Delta m = (\Sigma m_p + \Sigma m_n + \Sigma m_e) - m_{atom}$$

(ii) Nucleus:

$$\Delta m = (\Sigma m_p + \Sigma m_n) - m_{nucleus}$$

Example: Mass defect of C-12 isotope

mass of a proton: $m_p = 1.672623 \times 10^{-27}$ kg

mass of a neutron: $m_n = 1.674929 \times 10^{-27}$ kg

mass of ${}^{12}_{6}C$ nucleus: $m_c = 19.926483 \times 10^{-27}$ kg

Sol. ${}^{12}_{6}C$

$$\Delta m = (6m_p + 6m_n) - m_{nucleus}$$

$$\Delta m = \{[6(1.672623) + 6(1.674929)] - 19.926483\} 10^{-27}$$

$$\Delta m = 0.158829 \times 10^{-27} \text{ kg}$$

Significance: This loss of mass implies that energy is released in this process by Einstein's mass-energy equation ie $\Delta E = \Delta m c^2$

Binding Energy:

Def. The minimum energy required to completely separate nucleons in a nucleus to an infinite distance.

OR

The amount of energy released when a nucleus is formed from its component nucleons.

Symbol: ΔE

Formula: $\Delta E = (\Delta m) c^2$

Δm - Mass defect in kg

c - $3.00 \times 10^8 ms^{-2}$

Example: Binding energy of Helium nucleus (^4_2He)

mass of a proton: $m_p = 1.007276u$

mass of a neutron: $m_n = 1.008665u$

mass of ^4_2He nucleus: $m_{He} = 4.002604u$

$\Delta m = (2m_p + 2m_n) - m_{nucleus}$

$= 2(1.007276) + 2(1.008665) - 4.002604$

$=$ ______________ u

But $1u = 1.6605389 \times 10^{-27}$ kg

$\Delta m = (\qquad\qquad)(1.6605389 \times 10^{-27}) =$ __________ kg

$\Delta E = (\Delta m) c^2 \Rightarrow \Delta E = (\qquad\qquad)(3.00 \times 10^8)^2$

$\Delta E =$ ______________ J

NUCLEAR STABILITY – BINDING ENERGY PER NUCLEON:-

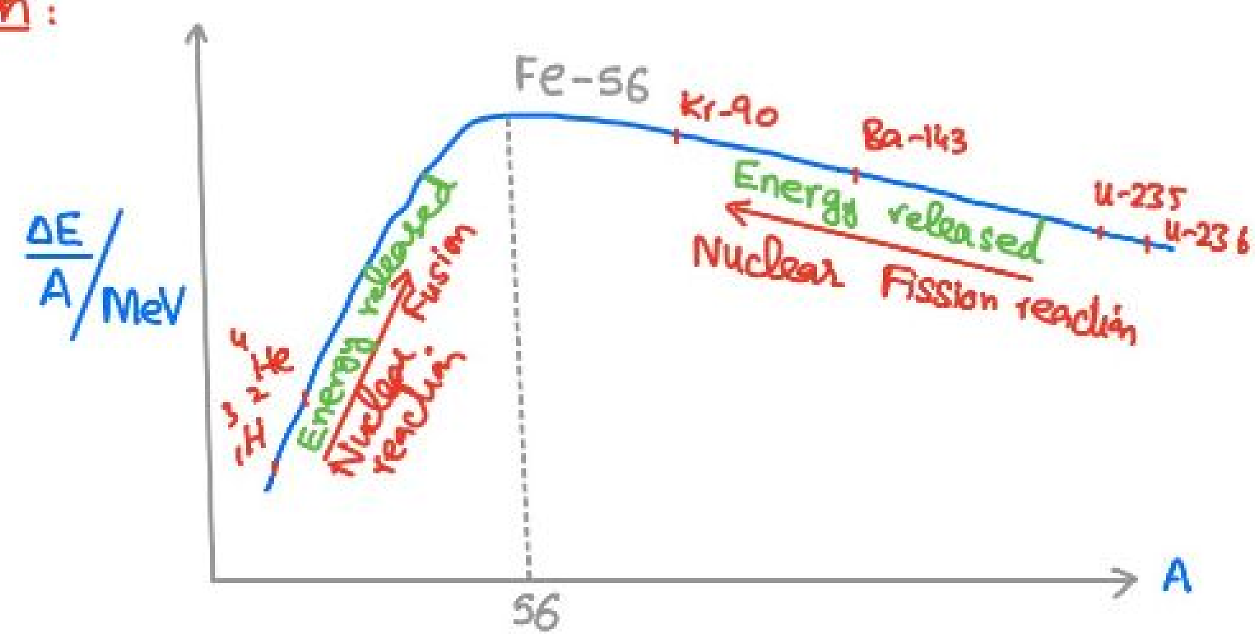
Def. It is the ratio of minimum energy required to completely separate nucleons in a nucleus to an infinite distance and its nucleon number.

Symbol: $\frac{\Delta E}{A}$

Formula: $\frac{\Delta E}{A} = \frac{(\Delta m) c^2}{Z + N}$

Significance:- Binding energy per nucleon is a measure of Nuclear stability i.e Nuclide with greater binding energy per Nucleons are more stable and are least likely to decay.

Graph:



Note:

1- The peak of the curve corresponds to Iron-56 which is the most stable nuclide that excist in nature.

2- In Nuclear Fission reaction, a slow neutron is injected in U-235 nucleus. Neutron must be slow so that it can penetrate into U-235 nucleus because it does not experience any electric force. Injection of slow neutron increasing the unstability by decreasing binding energy per nucleon so that it splits up quickly.

$$ {}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{236}_{92}U \longrightarrow {}^{143}_{56}Ba + {}^{90}_{36}Kr + 3\,{}_{0}n + E $$

(3) The products of Fission reaction have greater binding energy per nucleon and therefore are more stable as compared to their parent nuclide.

(4) In Nuclear Fusion reaction, two light nuclei with lesser binding energy per nucleon are combined/ fused together to form Helium nucleus with greater stability due to its greater binding energy per nucleon.

$$ {}^{3}_{1}H + {}^{3}_{1}H \longrightarrow {}^{4}_{2}He + 2\,{}^{1}_{0}n + E $$

5- In both Fission and Fusion reactions, products are more stable due to their greater binding energy per nucleon as compared to their parent nuclides.

6- Neutron is a simple particle and does not have any binding energy per nucleon.

Q) Calculate the binding energy per nucleon of Iron - 56 ($^{56}_{26}Fe$) in MeV.

Mass of neutron : $m_n = 1.675 \times 10^{-27}$ kg

Mass of proton : $m_p = 1.673 \times 10^{-27}$ kg

Mass of $^{56}_{26}Fe$ nucleus : $m_{Fe} = 9.288 \times 10^{-26}$ kg

<u>Step 1</u>:

$$\Delta m = (30 m_n + 26 m_p) - m_{Fe}$$
$$= [30(1.675) + 26(1.673)]10^{-27} - 9.288 \times 10^{-26}$$
$$= 8.680 \times 10^{-28} \text{ kg}$$

<u>Step 2</u>:

$$\Delta E = \Delta m c^2$$
$$= (8.680 \times 10^{-28})(3.00 \times 10^{8})^2$$
$$= 7.812 \times 10^{-11} \text{ J}$$

<u>Step 3</u>:

$$\frac{\Delta E}{A} = \frac{7.812 \times 10^{-11}}{56} = 13.95 \times 10^{-13} \text{ J}$$

<u>Step 4</u>:

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J} \Rightarrow 1 \text{ MeV} = 1.60 \times 10^{-13} \text{ J}$$

$$\frac{\Delta E}{A} = \frac{13.95 \times 10^{-13}}{1.60 \times 10^{-13}} = 8.72 \text{ MeV}$$

For Examiner's Use

8 **(a)** State what is meant by *nuclear binding energy*.

The minimum energy required to completely separate nucleons in a nucleus to an infinite distance. [2]

(b) The variation with nucleon number A of the binding energy per nucleon B_E is shown in Fig. 8.1.

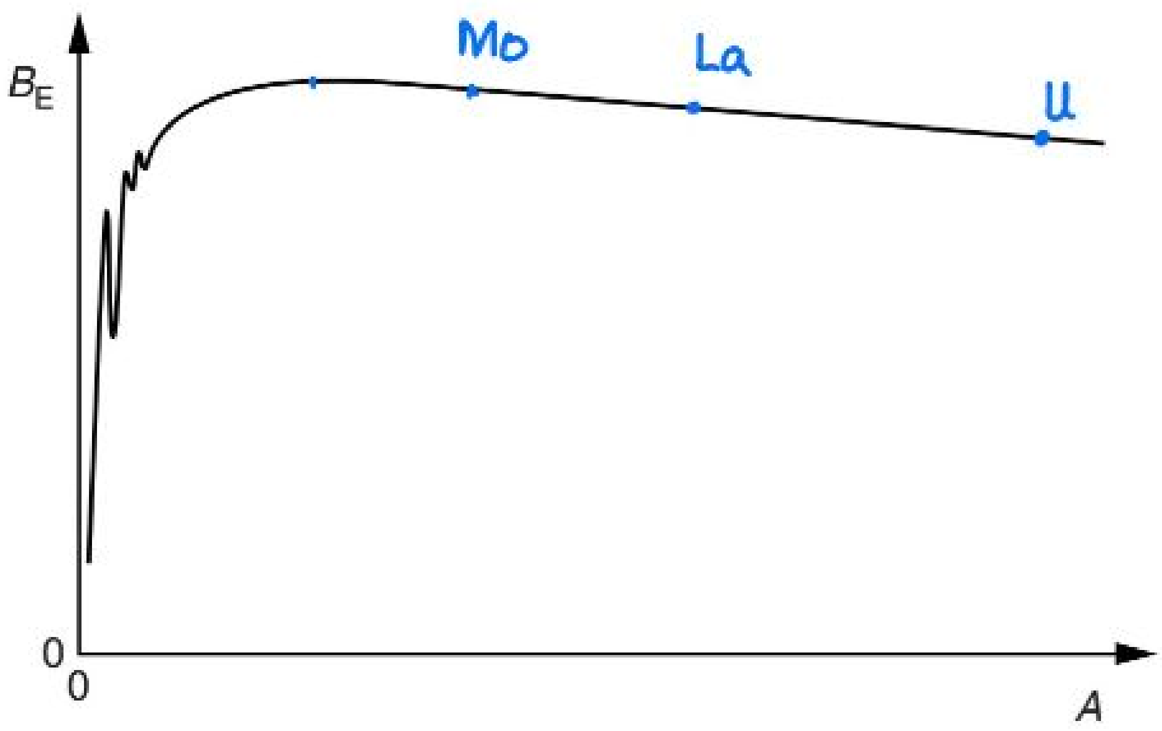


Fig. 8.1

When uranium-235 ($^{235}_{92}U$) absorbs a slow-moving neutron, one possible nuclear reaction is

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{95}_{42}Mo + ^{139}_{57}La + 2^{1}_{0}n + 7^{0}_{-1}\beta + \text{energy}.$$

(i) State the name of this type of nuclear reaction.

Nuclear Fision reaction [1]

(ii) On Fig. 8.1, mark the position of

1. the uranium-235 nucleus (label this position U), [1]

2. the molybdenum-95 ($^{95}_{42}Mo$) nucleus (label this position Mo), [1]

3. the lanthanum-139 ($^{139}_{57}La$) nucleus (label this position La). [1]

(iii) The masses of some particles and nuclei are given in Fig. 8.2.

For Examiner's Use

	mass / u
β-particle	5.5×10^{-4}
neutron	1.009
proton	1.007
uranium-235	235.123
molybdenum-95	94.945
lanthanum-139	138.955

Fig. 8.2

Calculate, for this reaction,

1. the change, in u, of the rest mass,

$$\Delta m = (m_u + m_n) - (m_{Mo} + m_{La} + 2m_n + 7m_\beta)$$

$$= [(235.123 + 1.009) - (94.945 + 138.955 + 2(1.009) + 7(5.5\times10^{-4}))]$$

$$= 236.132 - 235.922$$

$$= 0.210$$

change in mass = ...0.210........ u [2]

2. the energy released, in MeV, to three significant figures.

$$\Delta E = (\Delta m)c^2$$

$$= [(0.210)(1.66\times10^{-27})][3.00\times10^{8}]^2$$

$$= 3.1374\times10^{-11}\,J$$

As $1\,MeV = (10^6)(1.60\times10^{-19}) = 1.60\times10^{-13}\,J$

$$\Delta E = \frac{3.1374\times10^{-11}}{1.60\times10^{-13}} = 196\,MeV$$

energy = ...196........ MeV [3]

23.2	**Radioactive decay**	

Candidates should be able to:

1 understand that fluctuations in count rate provide evidence for the random nature of radioactive decay

2 understand that radioactive decay is both spontaneous and random

3 define activity and decay constant, and recall and use $A = \lambda N$

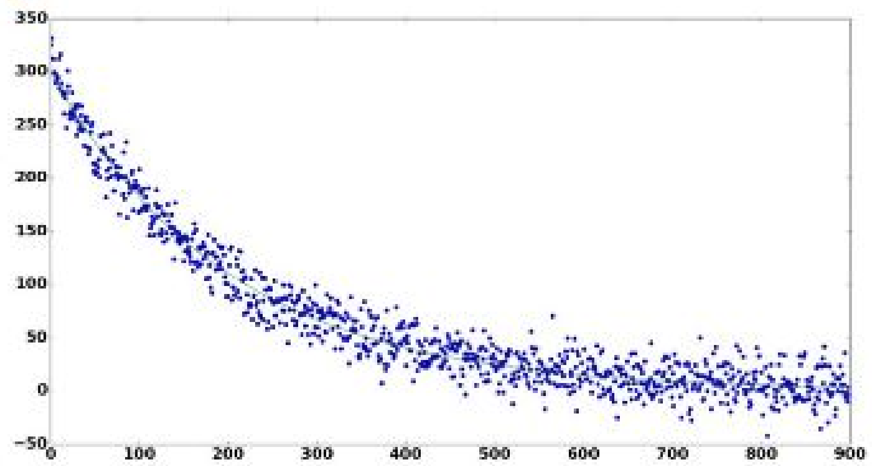
4 define half-life

5 use $\lambda = 0.693 / t_{\frac{1}{2}}$

6 understand the exponential nature of radioactive decay, and sketch and use the relationship $x = x_0 e^{-\lambda t}$, where x could represent activity, number of undecayed nuclei or received count rate

Radioactive decay:-

The spontaneous and random disintegration of particles from the nucleus of unstable nuclide is radioactivity.



(a) Nuclear decay is spontaneous because

(i) the decay of a particular nucleus is not affected by the presence of other nuclei.

(ii) the decay is not affected by any external conditions such as high or low temperatures, high or low pressures, strong or weak magnetic or electric fields.

(b) Nuclear decay is random because

(i) it is impossible to predict when a particular nucleus in a sample is going to decay.

(ii) each nucleus in a sample has the same chance of decaying per unit time i.e constant probability of decay of a nucleus.

(c) Evidence from graph:-

(i) Graph has same trend i.e same half life or same count rate at different external factors (temperature or pressure) show spontaneous nature.

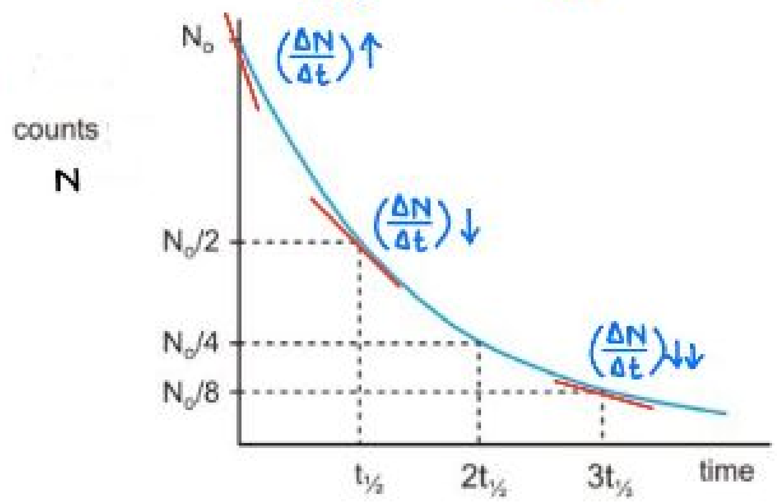
(ii) The fluctuations in graph show the random nature of radioactive decay.

Rate of decay / Activity:-

Def. Number of nuclei decay or disintegrate per unit time is Activity.

Symbol: A or $\frac{\Delta N}{\Delta t}$ or $\frac{dN}{dt}$

Concept:

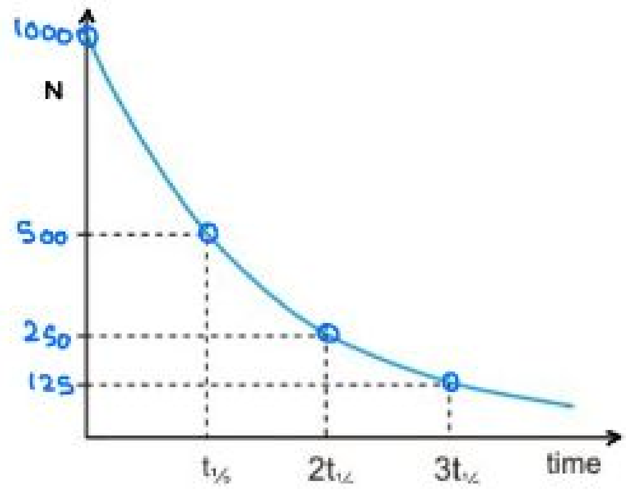


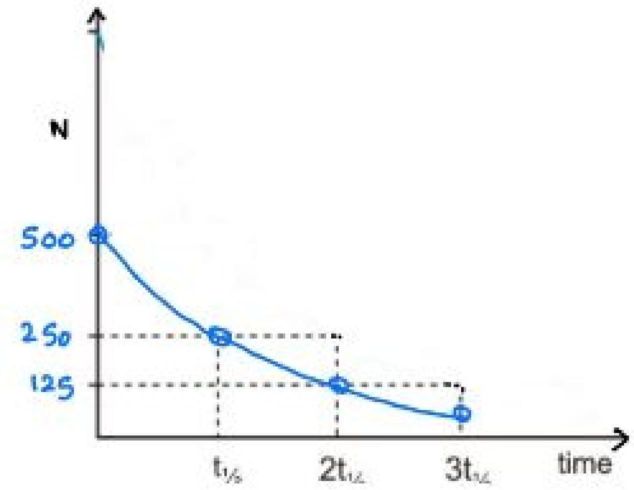
Activity = Gradient of decay graph at any time

Dependance:

Activity $\propto$ (No. of undecayed nuclei in the beginning i.e. at time, t=0)

Example:- Decay graphs of same sample/nuclide with different no. of undecayed nuclei are studied on the same scaled grid / graph.





At any time i.e. at any half life
(Steepness of left side graph) > (Steepness of right side graph)

If −ve sign is neglected,
(Gradient of left side graph) > (Gradient of right side graph)

$$A \propto -N$$

$$A = \lambda(-N)$$

−ve sign can be neglected (as per CAIE syllabus) and this shows that no. of undecayed nuclei decreases with time.

P.S.: Scalar

Units: s^{-1} or Becquerel (Bq)

DECAY CONSTANT:-

$$A = -\lambda N$$

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$\frac{\frac{\Delta N}{N}}{\Delta t} = \lambda \qquad [\text{−ve sign is neglected}]$$

$$\frac{\text{Probability of decay of a nucleus}}{\text{time}} = \text{Decay constant}$$

Def. Probability of decay of a nucleus per unit time is decay constant.

Symbol: λ

Formula: $A = -\lambda N$ or $\lambda = \frac{A}{N} \Rightarrow \lambda = \frac{\text{Activity}}{\text{No. of undecayed nuclei at time } t=0}$

P.S. Scalar

Units: Becquerel (Bq) or s^{-1}

Dependance:- Nature of nuclide i.e its half life ($\lambda\downarrow$ if half life$\uparrow$ as $\lambda \propto \frac{1}{t_{\frac{1}{2}}}$)

General eq. of Decay graph:-

$$\boxed{N = N_0 e^{-\lambda t}}$$

Here,

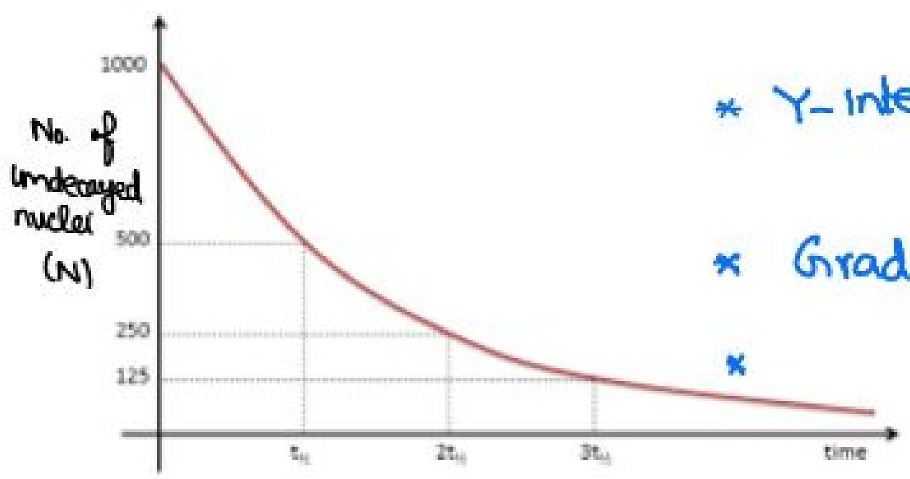
N - No. of undecayed nuclei after time 't'

N_0 - Original number of undecayed nuclei at time $t = 0$.

λ - Decay constant

t - time

e - exponential function



* Y-intercept $\longrightarrow$ original no. of undecayed nuclei in the beginning

* Gradient $\longrightarrow$ Activity (A)

* $\frac{\text{Gradient}}{N_0}$ = Decay constant (λ)

2nd form: $N = N_0 e^{-\lambda t}$

Divide both sides by time t

$$\frac{N}{t} = \frac{N_o}{t} e^{-\lambda t}$$

$$\boxed{A = A_o e^{-\lambda t}}$$

HALF LIFE:-

Def. The time in which half of the original number of undecayed nuclei decay is half life.

OR

The time in which no. of undecayed nuclei are reduced to their half value.

Symbol :- $t_{\frac{1}{2}}$

Units :- second (s)

P.S : Scalar

N-2009)

Formula Analysis :-

(1) Since general eq. of decay curve is

$$N = N_o e^{-\lambda t}$$

After first half life, $t = t_{\frac{1}{2}}$ and $N = \frac{N_o}{2}$

$$\frac{\cancel{N_o}}{2} = \cancel{N_o} e^{-\lambda t_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\lambda t_{\frac{1}{2}}}$$

Taking natural log (ln) to both sides

$$\ln\left(\tfrac{1}{2}\right) = \ln\left(e^{-\lambda t_{\frac{1}{2}}}\right)$$

$$\ln 1 - \ln 2 = -\lambda t_{\frac{1}{2}} \ln e$$

$$0 - \ln 2 = -\lambda t_{\frac{1}{2}} (1)$$

$$t_{\frac{1}{2}} = \frac{\ln 2}{\lambda} \quad \text{or} \quad \boxed{t_{\frac{1}{2}} = \frac{0.693}{\lambda}}$$

(2)

At time $t = 0$ ie $t = \boxed{0} t_{\frac{1}{2}}$, no. of undecayed nuclei: $N = N_0 = \frac{N_0}{2^0}$

After first half life, $t = \boxed{1} t_{\frac{1}{2}}$,

no. of undecayed nuclei: $N = \frac{N_0}{2} = \frac{N_0}{2^1}$

After 2^{nd} half life, $t = \boxed{2} t_{\frac{1}{2}}$,

no. of undecayed nuclei: $N = \frac{\frac{N_0}{2}}{2} = \frac{N_0}{2^{\boxed{2}}}$

After 3^{rd} half life, $t = \boxed{3} t_{\frac{1}{2}}$,

no. of undecayed nuclei: $N = \frac{\frac{N_0}{2^2}}{2} = \frac{N_0}{2^{\boxed{3}}}$

After n - half lives,

no. of undecayed nuclei: $N = \frac{N_0}{2^n}$

$$N = \frac{N_0}{2^n}$$

$$\frac{N}{N_0} = \frac{1}{2^n} \Rightarrow \boxed{\frac{N}{N_0} = \left(\frac{1}{2}\right)^n}$$

i.e.

$$\frac{\text{No. of undecayed nuclei after n half lives}}{\text{No. of undecayed nuclei in the beginning}} = \left(\frac{1}{2}\right)^{\text{no. of half lives}}$$

Also, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{\frac{N}{t}}{\frac{N_0}{t}} = \left(\frac{1}{2}\right)^n \Rightarrow \boxed{\frac{A}{A_0} = \left(\frac{1}{2}\right)^n}$

Q) Calculate the time interval in which activity of a radioactive source is reduced to $\left(\frac{1}{10}\right)$ of its initial value if its half life is 12 days.

$A = A_0 e^{-\lambda t}$ Also $\lambda = \frac{0.693}{t_{\frac{1}{2}}}$

$$A = A_0 e^{-\left(\frac{0.693}{t_{\frac{1}{2}}}\right)t}$$

$$\frac{\cancel{A_0}}{10} = \cancel{A_0}\, e^{-\left(\frac{0.693}{12}\right)t}$$

$$\frac{1}{10} = e^{-\left(\frac{0.693}{12}\right)t}$$

Taking natural log (ln) to both sides

$$\ln 1 - \ln 10 = -\left(\frac{0.693}{12}\right)t \ln e$$

$$0 - \ln 10 = -\left(\frac{0.693}{12}\right)t \ (1)$$

$$t = \frac{12 \ln 10}{0.693} = \underline{39.8} \text{ days}$$

Method 2:

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^n$$

$$\frac{\frac{\cancel{A_0}}{10}}{\cancel{A_0}} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{1}{10} = \left(\frac{1}{2}\right)^n$$

$$\ln 1 - \ln 10 = n\left[\ln 1 - \ln 2\right] \Rightarrow n = \frac{\ln 10}{\ln 2}$$

$$n = \underline{3.32}$$

$$\text{Total Time} = \left(\begin{matrix}\text{no. of half}\\\text{lives}\end{matrix}\right)\left(\begin{matrix}\text{time for single}\\\text{half life}\end{matrix}\right)$$

$$= (3.32)(12)$$

$$= \underline{39.8} \text{ days}$$

Q) At the start of an experiment 1.0×10^{15} undecayed nuclei of an isotope for which decay constant is 0.02 Bq are present. Calculate the number of undecayed nuclei after 20 s.

$$N = N_0 e^{-\lambda t}$$

$$N = (1.0 \times 10^{15}) e^{-(0.02)(20)}$$

$$N = \_\_\_\_\_\_\_\_\_\_$$

12 The isotope iodine-131 ($^{131}_{53}I$) is radioactive with a decay constant of 8.6×10^{-2} day^{-1} $= \lambda$

β^- particles are emitted with a maximum energy of 0.61 MeV.

(a) State what is meant by

(i) *radioactive*,

The spontaneous and random emission of particles from the nucleus of an unstable nuclide. [2]

(ii) *decay constant*.

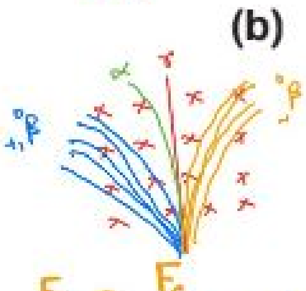
$A = \lambda N$

$\frac{\Delta N}{\Delta t} = \lambda N$

$\frac{\frac{\Delta N}{N}}{\Delta t} = \lambda$

Probability of decay of a nucleus per unit time. [2]

(b) Explain why the emitted β^- particles have a range of energies.



In β^- decay, energy is shared between -ve β particle and anti-neutrino. β-particles have maximum energy if energy of antin-neutrino is zero or via versa. [2]

$F_B = F_C$

$BQv \sin 90° = \frac{mv^2}{r}$

$r = \frac{BQ}{mv}$

$r = \frac{BQ}{P}$

(c) A sample of blood contains 1.2×10^{-9} g of iodine-131.

Determine, for this sample of blood,

(i) the activity of the iodine-131,

No. of particle in 1.2×10^{-9} g of Iodine $= \left(\frac{1.2 \times 10^{-9}}{131}\right)(6.02 \times 10^{23})$

$N = 5.51 \times 10^{12}$

$A = \lambda N$

$= \left(\frac{8.6 \times 10^{-2}}{24 \times 60 \times 60}\right)(5.51 \times 10^{12})$

activity = 5.48×10^{6} Bq [3]

(ii) the time for the activity of the iodine-131 to be reduced to 1/50 of the activity calculated in **(i)**.

$$A = A_0 e^{-\lambda t}$$

$$\left(\frac{1}{50}\right)\cancel{A_0} = \cancel{A_0}\, e^{-\lambda t}$$

$$\frac{1}{50} = e^{-\lambda t}$$

$$\ln 1 - \ln 50 = -\lambda t \ln e$$

$$t = \frac{\ln 50}{(8.6 \times 10^{-2})}$$

time = ...45.......... days [2]

[Total: 11]

A-2 NUCLEAR PHYSICS (CIE Past Paper Questions)

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

Data & Formulae provided in CAIE paper:

unified atomic mass unit	$1\,u = 1.66 \times 10^{-27}\,kg$
rest mass of electron	$m_e = 9.11 \times 10^{-31}\,kg$
rest mass of proton	$m_p = 1.67 \times 10^{-27}\,kg$
radioactive decay	$x = x_0 \exp(-\lambda t)$
decay constant	$\lambda = \dfrac{0.693}{t_{\frac{1}{2}}}$

Q.1 /{*Q.12/June 2020/41*}

(a) The decay of a sample of a radioactive isotope is said to be random and spontaneous.

Explain what is meant by the decay being:

(i) *random*

....

.... [1]

(ii) *spontaneous*.

....

.... [1]

(b) A radioactive isotope X has a half-life of 1.4 hours.

Initially, a pure sample of this isotope X has an activity of 3.6×10^5 Bq.

Determine the activity of the isotope X in the sample after a time of 2.0 hours.

activity = ... Bq [3]

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

(c) The variation with time *t* of the actual activity *A* of the sample in **(b)** is shown in Fig. 12.1.

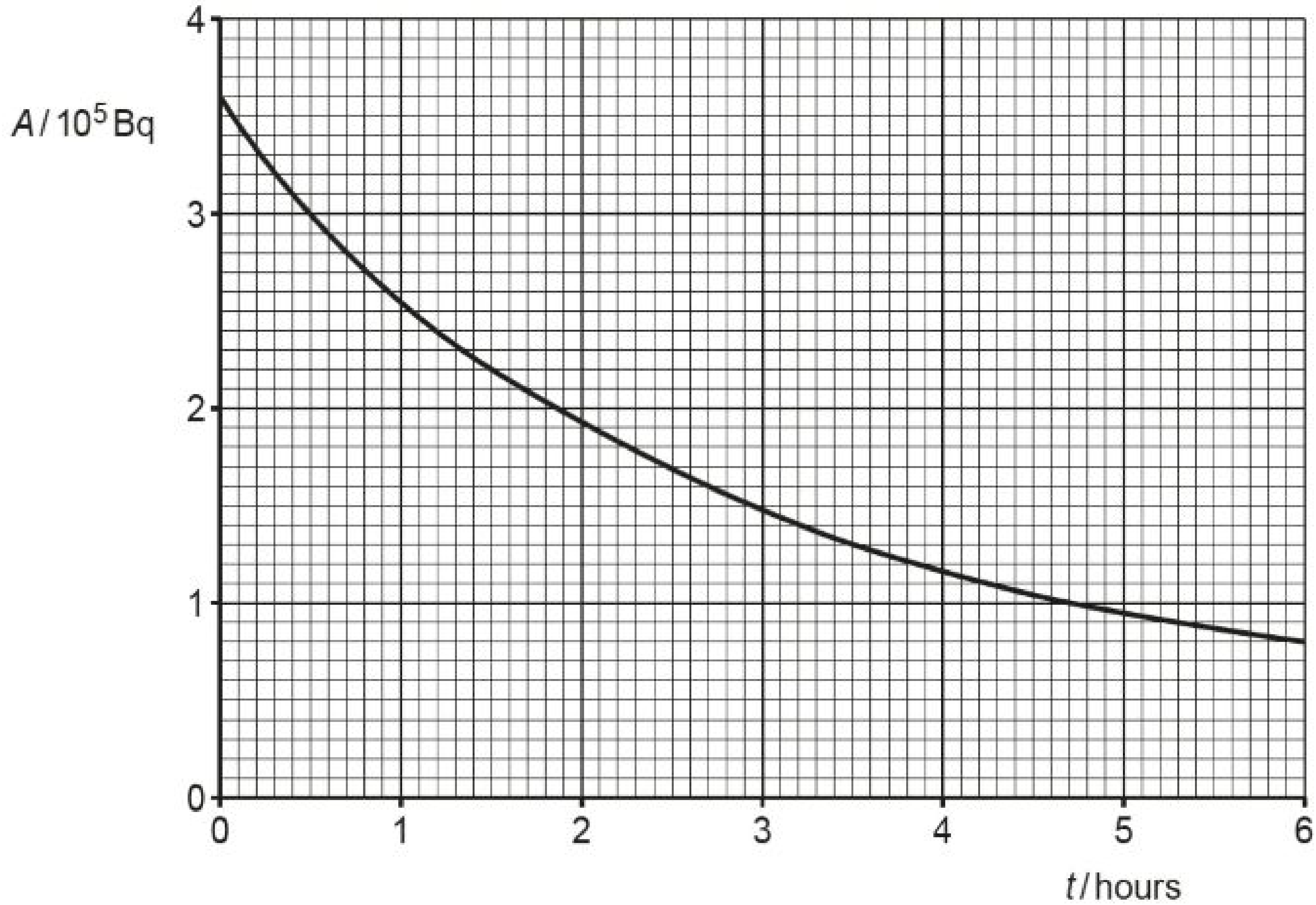


Fig. 12.1

(i) The initial activity of isotope X in the sample is 3.6×10^5 Bq.

Use information from **(b)** to sketch, on the axes of Fig. 12.1, the variation with time *t* of the activity of a pure sample of isotope X. [1]

(ii) Suggest an explanation for any difference between the actual activity of the sample shown in Fig. 12.1 and the curve you have drawn for the activity of isotope X.

....

....

.... [2]

Q.2 ***{Q.12/June 2020/42}***

(a) State what is meant by the *mass defect* of a nucleus.

....

....

.... [2]

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

(b) Some masses are shown in Table 12.1.

Table 12.1

	mass / u
proton $^{1}_{1}p$	1.007 276
neutron $^{1}_{0}n$	1.008 665
helium-4 ($^{4}_{2}He$) nucleus	4.001 506

Show that:

(i) the energy equivalence of 1.00 u is 934 MeV

[2]

(ii) the binding energy per nucleon of a helium-4 nucleus is 7.09 MeV.

[2]

(c) Isotopes of hydrogen have binding energies per nucleon of less than 3 MeV.

Suggest why a nucleus of helium-4 does not spontaneously break down to become nuclei of hydrogen.

....

....

.... [2]

Akhtar Mahmood (0333-4281759)
M.Sc (Physics), MCS, MBA-IT, B.Ed, MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

Q.3 {***Q.12/March 2020/42***}

(a) Explain what is meant by the *binding energy* of a nucleus.

....

....

.... [2]

(b) The following nuclear reaction takes place:

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{55}Cs + {}^{90}_{x}Rb + y\,{}^{1}_{0}n$$

(i) Determine the values of x and y.

x = ...

y = ...
[1]

(ii) State the name of this type of nuclear reaction.

.... [1]

(iii) Compare the binding energy per nucleon of uranium-235 with the binding energy per nucleon of caesium-144.

....

.... [1]

(c) Yttrium-90 decays into zirconium-90, a stable isotope.

A sample initially consists of pure yttrium-90.

Calculate the time, in days, when the ratio of the number of yttrium-90 nuclei to the number of zirconium-90 nuclei would be 2.0.

The half-life of yttrium-90 is 2.7 days.

time = ... days [3]

Q.4{ ***Q.12/June 2019/41***}

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

(a) A sample of a radioactive isotope contains *N* nuclei of the isotope at time *T*.
At time ($T + \Delta T$), the sample contains ($N - \Delta N$) nuclei of the isotope. The time interval ΔT is short.

Use the symbols N, ΔN, T and ΔT to give expressions for:

(i) the average activity of the sample during the time ΔT

.... [1]

(ii) the probability of decay of a nucleus in the time ΔT

.... [1]

(iii) the decay constant λ of the isotope.

.... [1]

(b) The isotope polonium-208 ($^{208}_{84}Po$) is radioactive and decays to form lead-204 ($^{204}_{82}Pb$).
The nuclear equation for this decay is

$$^{208}_{84}Po \rightarrow {}^{204}_{82}Pb + {}^{4}_{2}He.$$

Data for nuclear masses are given in Fig. 12.1.

	mass / u
$^{4}_{2}He$	4.002 603
$^{204}_{82}Pb$	203.973 043
$^{208}_{84}Po$	207.981 245

Fig. 12.1

(i) Determine, for the decay of one nucleus of polonium-208:

1. the change, in u, of the mass

mass change = ... u [1]

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

2. the total energy, in pJ, released.

energy = ... pJ [3]

(ii) The polonium-208 nucleus is initially stationary. The initial kinetic energy of the $^{4}_{2}He$ nucleus (α-particle) is found to be less than the energy calculated in **(i) part 2**.

Suggest **two** possible reasons for this difference.

1. ....

....

2. ....

....
[2]

Q.5 {*Q.12/June 2018/42*}

(a) State what is meant by *radioactive decay*.

....

....

....

....[2]

(b) An unstable nuclide P has decay constant λ_P and decays to form a nuclide D.
This nuclide D is unstable and decays with decay constant λ_D to form a stable nuclide S. The decay chain is illustrated in Fig. 12.1.

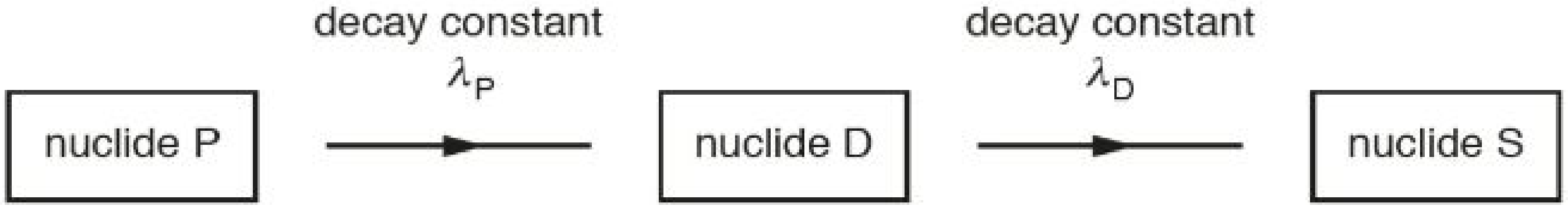


Fig. 12.1

The symbols P, D and S are not the nuclide symbols.

Initially, a radioactive sample contains only nuclide P.

The variation with time t of the number of nuclei of each of the three nuclides in the sample shown in Fig. 12.2.

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

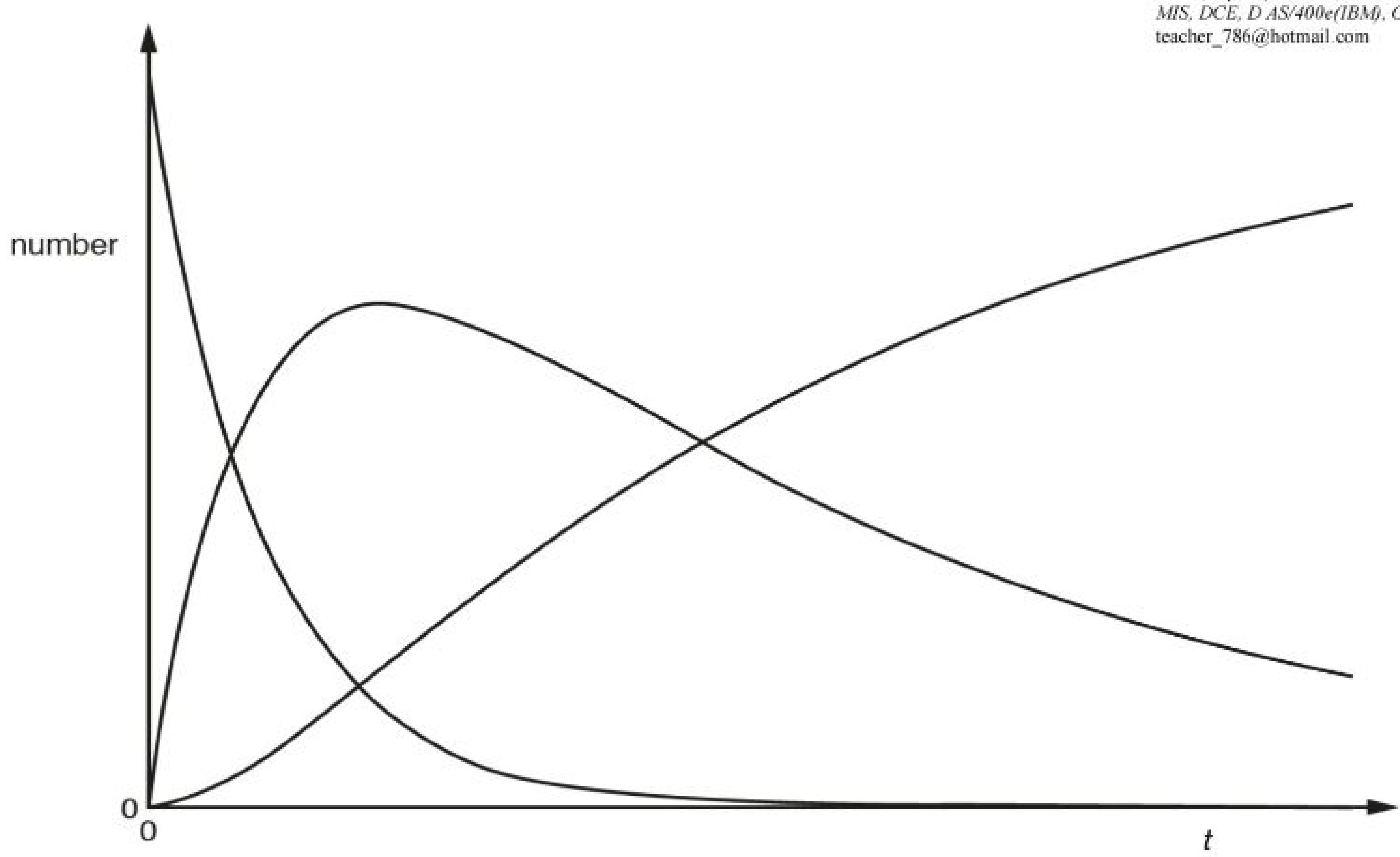


Fig. 12.2

(i) On Fig. 12.2, use the symbols P, D and S to identify the curve for each of the three nuclides. [2]

(ii) The half-life of nuclide P is 60.0 minutes.

Calculate the decay constant λ_P, in s^{-1}, of this nuclide.

λ_P = ... s^{-1} [2]

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

(c) In the decay chain shown in Fig. 12.1, λ_P is approximately equal to $5\lambda_D$.

The decay chain of a different nuclide E is illustrated in Fig. 12.3.

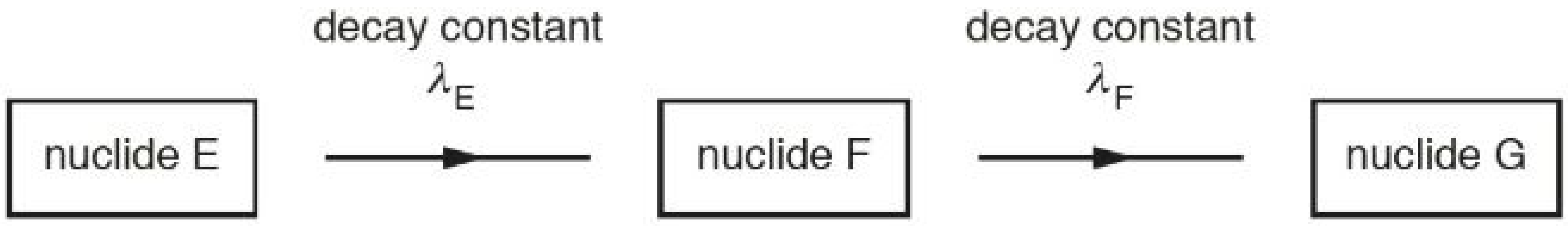


Fig. 12.3

The decay constant λ_F of nuclide F is very much larger than the decay constant λ_E of nuclide E.

By reference to the half-life of nuclide F, explain why the number of nuclei of nuclide F in the sample is always small.

....

....

....[2]

Q.6 ***{Q.7/Nov. 2007/4}***

(a) Explain what is meant by the *binding energy* of a nucleus.

....

....[1]

(b) Fig. 7.1 shows the variation with nucleon number (mass number) *A* of the binding energy per nucleon E_B of nuclei.

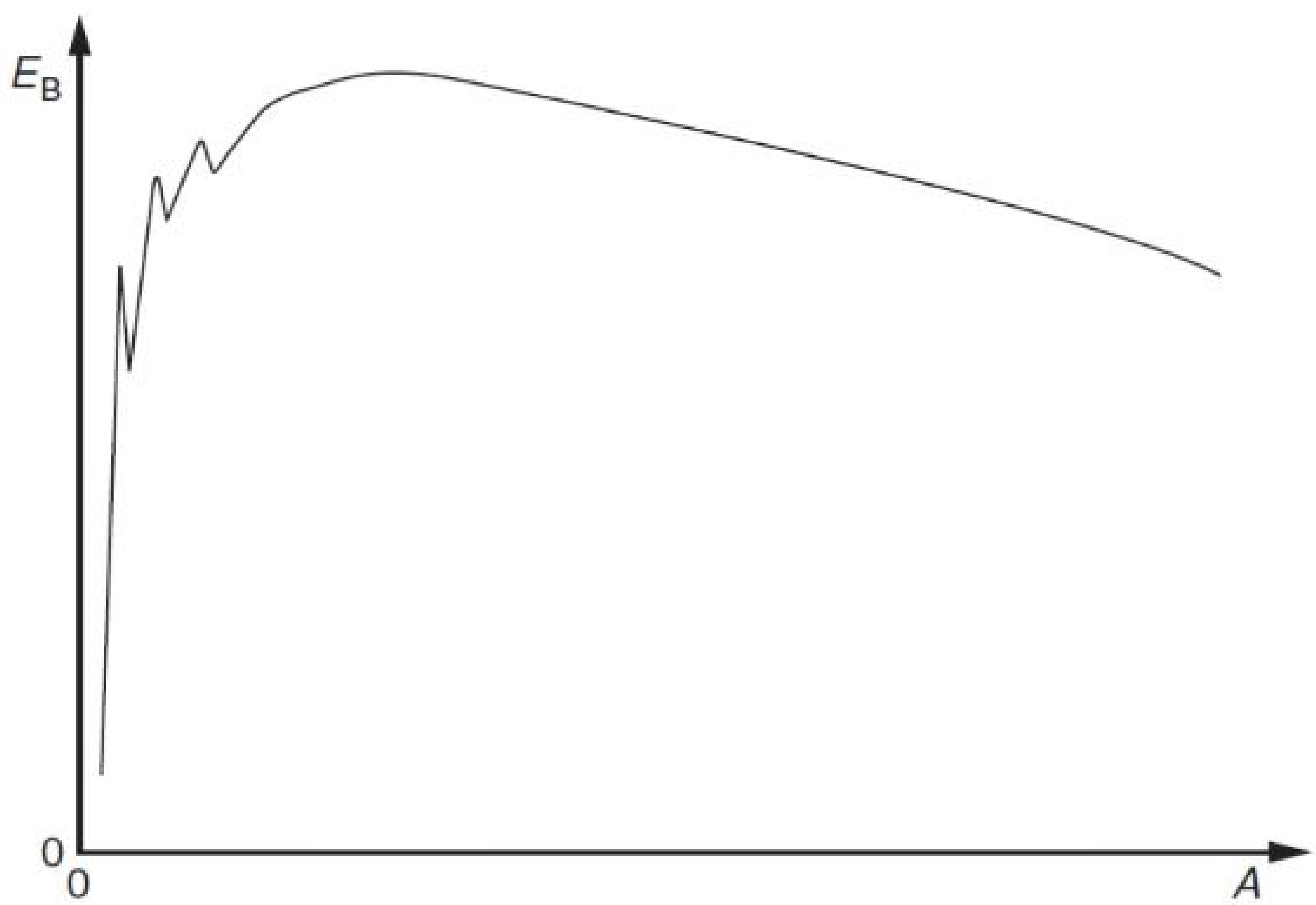


Fig. 7.1

One particular fission reaction may be represented by the nuclear equation

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

$$^{235}_{92}\text{U} + ^{1}_{0}\text{n} \rightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3^{1}_{0}\text{n}.$$

(i) On Fig. 7.1, label the approximate positions of

1. the uranium ($^{235}_{92}$U) nucleus with the symbol U,

2. the barium ($^{141}_{56}$Ba) nucleus with the symbol Ba,

3. the krypton ($^{92}_{36}$Kr) nucleus with the symbol Kr. [2]

(ii) The neutron that is absorbed by the uranium nucleus has very little kinetic energy. Explain why this fission reaction is energetically possible.

....

....

....[2]

(c) Barium-141 has a half-life of 18 minutes. The half-life of Krypton-92 is 3.0 s.
In the fission reaction of a mass of Uranium-235, equal numbers of barium and krypton nuclei are produced.
Estimate the time taken after the fission of the sample of uranium for the ratio

$$\frac{\text{number of Barium-141 nuclei}}{\text{number of Krypton-92 nuclei}}$$

to be approximately equal to 8.

time = ... s [3]

Q.7

(a) Define the term radioactive decay constant.

....

....

.... [2]

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

(b) State the relation between the activity A of a sample of a radioactive isotope containing N atoms and the decay constant λ of the isotope.

……………………………………………………………………………………………[1]

(c) Radon is a radioactive gas with half-life 56s. For health reasons,the maximum permissible level of radon in air in a building is set at 1 radon atom for every 1.5×10^{21} molecules of air. 1 mol of air in the building is contained in $0.024 m^3$.

Calculate, for this building,

(i) the number of molecules of air in $1.0\ m^3$,

number: ……………………………

(ii) the maximum permissible number of radon atoms in $1.0\ m^3$ of air,

number: ……………………………

(iii) the maximum permissible activity of radon per cubic metre of air.

Activity: …………………………… Bq [5]

Q.8 Strontium-90 decays with the emission of a β-particle to form Yttrium-90. The reaction is represented by the equation

$$^{90}_{38}Sr \rightarrow {}^{90}_{39}Y + {}^{0}_{-1}e + 0.55\ MeV.$$

The decay constant is $0.025\ year^{-1}$.

(a) Suggest, with a reason, which nucleus, $^{90}_{38}Sr$ or $^{90}_{39}Y$, has the greater binding energy.

……………………………………………………………………………………………

……………………………………………………………………………………………

…………………………………………………………………………………………… [2]

(b) Explain what is meant by the decay constant.

……………………………………………………………………………………………

……………………………………………………………………………………………

…………………………………………………………………………………………… [2]

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

(c) At the time of purchase of a Strontium-90 source, the activity is 3.7×10^6 Bq.

(i) Calculate, for this sample of strontium,

1. the initial number of atoms,

number: ………………………….. [3]

2. the initial mass.

mass: ………………………………….. Kg [2]

(ii) Determine the activity A of the sample 5.0 years after purchase, expressing the answer as a fraction of the initial activity A_0. That is, calculate the ratio $\frac{A}{A_0}$.

ratio = ………………………………….. [2]

ANSWER KEYS

Q.1	***Q.12/June 2020/41***		
	12(a)(i)	time at which a nucleus will decay cannot be predicted **or** constant probability of decay of a nucleus	**B1**
	12(a)(ii)	decay (of a nucleus) not affected by environmental factors	**B1**
	12(b)	$A = A_0 e^{-\lambda t}$ **and** $\lambda = \ln 2 / t_{½}$	**C1**
		$= 3.6 \times 10^5 \times \exp[-(2 \times \ln 2) / 1.4]$	**C1**
		or	
		$A = A_0 \times 0.5^N$	**(C1)**
		$= 3.6 \times 10^5 \times 0.5^N$ where $N = 2 / 1.4$	**(C1)**
		$A = 1.3 \times 10^5$ Bq	**A1**
	12(c)(i)	smooth curve, starting at (0, 3.6×10^5) and passing through (1.4, 1.8×10^5) and (2.0, 1.3×10^5)	**B1**
	12(c)(ii)	(activity of sample is greater than activity of X so) there must be an additional source of activity	**C1**
		the decay product (of isotope X) is radioactive	**A1**
Q. 2	***Q.12/June 2020/42***		
	12(a)	difference between mass of nucleus and mass of (constituent) nucleons	**M1**
		where nucleons are separated to infinity	**A1**
	12(b)(i)	$E = mc^2$	**C1**
		$= 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2 / (1.60 \times 10^{-13}) = 934$ MeV	**A1**
	12(b)(ii)	mass defect = $2 \times (1.007276 + 1.008665) - 4.001506$ (= 0.030376)	**B1**
		binding energy per nucleon = $(0.030376 \times 934) / 4 = 7.09$ MeV	**A1**
	12(c)	binding energy per nucleon is much greater	**M1**
		so would require a large amount of energy to separate the nucleons in helium	**A1**
		or	
		amount of energy released in forming hydrogen isotopes	**(M1)**
		is less than energy required to break apart helium nucleus	**(A1)**
Q. 3	***Q.12/March 2020/42***		
	12(a)	(minimum) energy required to separate the nucleons	**M1**
		to infinity	**A1**
	12(b)(i)	**37** **2**	**B1**
	12(b)(ii)	fission	**B1**
	12(b)(iii)	binding energy per nucleon smaller for U than for Cs	**B1**
	12(c)	Current ratio 2 Y to 1 Zr, so initially 3 Y $2 = 3e^{-\lambda t}$ $\lambda = 0.693 / 2.7$	**C1**
		$\ln(2/3) = -(\ln 2 / 2.7)t$	**C1**
		t = **1.6** days	**A1**
		or	
		$(½)^n = 2/3$	**(C1)**
		n = 0.585	**(C1)**
		time = 0.585×2.7 = **1.6** days	**(A1)**

Q. 4	***Q.12/June 2019/41***		
	12(a)(i)	$\Delta N / \Delta T$	B1
	12(a)(ii)	$\Delta N / N$	B1
	12(a)(iii)	$\Delta N / (N \Delta T)$	B1
	12(b)(i)	1. mass change = 5.60×10^{-3} u	A1
		2. energy = $(\Delta)mc^2$	C1
		$= 5.6 \times 10^{-3} \times 1.66 \times 10^{-27} \times (3.0 \times 10^8)^2$ $(= 8.36 \times 10^{-13}$ J)	C1
		= 0.84 pJ	A1
	12(b)(ii)	kinetic energy (of recoil) of lead (nucleus)	B1
		energy of γ-ray photon	B1
Q. 5	***Q.12/June 2018/42***		
	12(a)	emission of particles/radiation by <u>unstable nucleus</u>	B1
		spontaneous emission	B1
	12(b)(i)	P – the curve that starts with a high number D – the curve with the peak S – the curve that increases from zero throughout *(one correct 1 mark, all three correct 2 marks)*	B2
	12(b)(ii)	$\lambda t_{½} = 0.693$ $\lambda = 0.693 / (60.0 \times 60)$	C1
		$= 1.93 \times 10^{-4}$ s^{-1}	A1
	12(c)	half-life of F is much shorter than half-life of E	B1
		<u>nuclei</u> of F decay (almost) as soon as they are produced	B1

Q.6 ***Q.7/Nov. 2007/4***

(a) energy required to (completely) separate the nucleons (in a nucleus) .... B1 [1]

(b) (i) U labelled near right-hand end of line B1
Ba and Kr in approximately correct positions .... B1 [2]

(ii) binding energy is $A \times E_B$ B1
either binding energy of U < binding energy of (Ba + Kr)
or E_B of U < E_B of (Ba + Kr) .... B1 [2]

(c) Krypton-92 reduced to 1/8 in 9 s M1
in 9 s, very little decay of Barium-141 .... M1
so, approximately 9 s .... A1 [3]
OR
$\lambda_{Kr} = 0.231$ or $\lambda_{Ba} = 6.42 \times 10^{-4}$ (M1)
$8 = e^{-\lambda B \times t} / e^{-\lambda K \times t}$ (C1)
$t = 9.0$ s (A1)