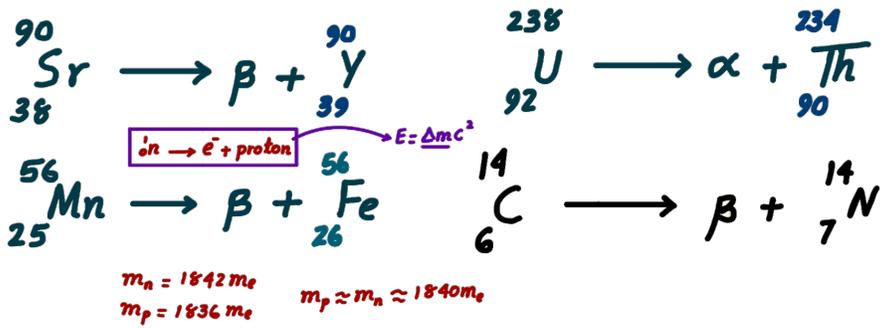Monday, March 13, 2023- Monday, March 21st, 2023.
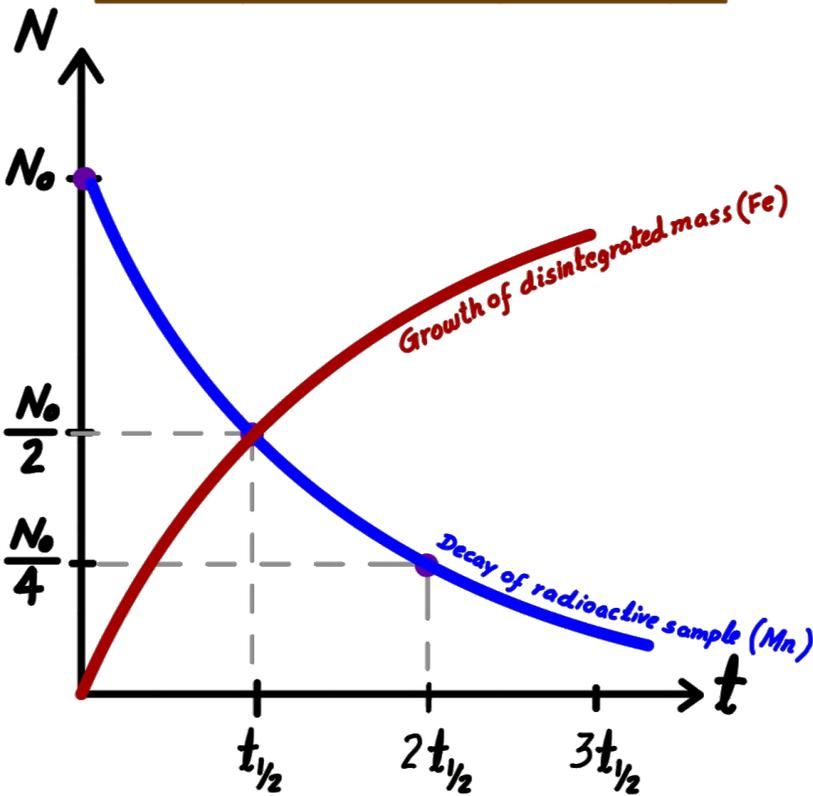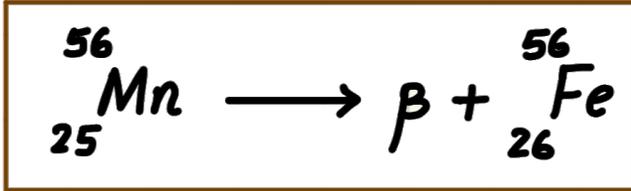
Nuclear Physics with Sir Yawar Abbas ☢️

Radioactive Decay

- Radioactive Nuclei are unstable due to a significant imbalance between nuclear attractive and repulsive forces among nucleons.

- Stable nuclei can also become radioactive.

    By gaining foreign neutrons which causes a significant increase in Imbalance of Forces. Called Radio-isotopes. E.g Co-40, I-131, Mn-56, C-14

- Radioactive nuclei decay/disintegrate by emitting radiations and particles to gain stability.

- The emission of an alpha or a beta particle causes decay of a nucleus

    One emission can be considered as one steps towards higher stability.

- The number of particles emitted per unit time is equal to the number of nuclei decaying in unit time.

    Rate of emission is equal to rate of decay.

- The rate of emission decreases exponentially but fluctuates with time

    Fluctuations are due to randomness of radioactive decay.

- Radioactive decay behavior is spontaneous.

    No dependence on physical variables like pressure, temperature.

- In a half life time, the quantities which reduce to their half are:-

    No of Active nuclei , Active Mass of Sample, Rate of emission, Rate of decay.

- During decay process of a radioactive sample:-

    No of Active nuclei decreases, the Active mass of sample decreases. The rate of decay (activity) decreases. The Decayed/disintegrated mass increases.

    The total mass of sample remains approximately same as emitting particles and radiations causes negligible change in mass.

$$^{90}_{38}Sr \longrightarrow \beta + ^{90}_{39}Y \qquad\qquad ^{238}_{92}U \longrightarrow \alpha + ^{234}_{90}Th$$

$$\boxed{^1_0n \longrightarrow e^- + proton} \longrightarrow E = \Delta mc^2$$

$$^{56}_{25}Mn \longrightarrow \beta + ^{56}_{26}Fe \qquad\qquad ^{14}_{6}C \longrightarrow \beta + ^{14}_{7}N$$

$$m_n = 1842\, m_e \qquad m_p \approx m_n \approx 1840\, m_e$$
$$m_p = 1836\, m_e$$

*

$$\boxed{^{56}_{25}Mn \longrightarrow \beta + ^{56}_{26}Fe}$$

Growth of disintegrated mass (Fe)

Decay of radioactive sample (Mn)

Mn	Fe
N_0	0
$\dfrac{N_0}{2}$	$\dfrac{N_0}{2}$
$\dfrac{N_0}{4}$	$\dfrac{3}{4}N_0$
$\dfrac{N_0}{8}$	$\dfrac{7}{8}N_0$

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Law of Decay/disintegration

The Law of Radioactive Decay

- The law of radioactive decay states that the rate of disintegration is directly proportional to the actual number of active stimuli

- The law suggests that the number of nuclei decaying per unit time is directly proportional to the actual number of active nuclei.

- Activity is the rate of disintegration measuring in Bq.

    It is defined as the number of nuclei decaying per unit time and it is equal to the number of emissions per unit time.

- The Decay constant is defined as the probability of decay with respect to time. It is measured in Hz or s^{*-1}

    Decay constant is a material's characteristic ; a sample of higher decay constant is more probable to disintegrate.

    Emits alpha or beta particles with greater kinetic energy.

    A sample of higher decay constant has a higher degree of instability. ; undergoes greater changes of mass $(E=mc^{*}2)$

$$\ominus \frac{\Delta N}{\Delta t} \propto N$$

Indicates decoy behaviour

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

Decay const.

Rate of decay (activity)

$$\boxed{A = \lambda N}$$

$$A = \lambda N$$

Activity   No. of active nuclei

$N \rightarrow$ number of active nuclei

$\Delta N \rightarrow$ number of nuclei decaying

$\dfrac{\Delta N}{\Delta t} \rightarrow$ number of nuclei decaying per unit time/Rate of decay

$N_o \rightarrow$ Initial No. of active nuclei

$\Delta N = N - N_o$

$\Delta N \rightarrow$ No. of nuclei decayed

$N \rightarrow$ Final no. of active nuclei

$N_o \rightarrow$ Initial no. of active nuclei

Always $-ve$ as final no. is always less

$$\dfrac{\Delta N}{\Delta t} = -\lambda N$$

$$\lambda = -\dfrac{\Delta N}{N \Delta t}$$

$$\lambda = \dfrac{\Delta N/N}{\Delta t}$$

$\Delta N/N \rightarrow$ Probability of decay

$A = \lambda N$

Activity

No. of active nuclei

Activity $A \longrightarrow$ Bequerrel (Bq)

$1\,Bq = 1$ nucleus decaying per sec

$1\,Bq = 1$ particle $(\alpha$ or $\beta)$ emitting per sec

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Decay equation

$$N_0 \xrightarrow{\quad t \quad} N$$

$$\frac{\Delta N}{\Delta t} = -\lambda N$$

$$\int \frac{\Delta N}{N} = -\lambda \int \Delta t$$

$$\int_{N_0}^{N} \frac{1}{N} \, dN = -\lambda \int_{0}^{t} dt$$

$$\ln N \Big]_{N_0}^{N} = -\lambda t$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\boxed{N = N_0 \, e^{-\lambda t}}$$

$N_0 \rightarrow$ Initial no. of active nuclei
$N \rightarrow$ No. of active nuclei after time t.
$t \rightarrow$ time of sample's decay.
$(N_0 - N) \rightarrow$ No. of decayed nuclei

This derivation is not included.

Half Life equation

$$N_0 \xrightarrow{\quad t_{1/2} \quad} \frac{N_0}{2}$$

$$N = N_0 \, e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 \, e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2} \quad \text{ln e}$$

$$-\ln 2 = -\lambda t_{1/2}$$

$$\boxed{t_{1/2} = \frac{\ln 2}{\lambda}}$$

$$\boxed{t_{1/2} = \frac{0.693}{\lambda}}$$

$$\int \frac{1}{x} \, dx = \ln x$$

$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

$$\ln\left(\frac{a}{b}\right) = -\ln\left(\frac{b}{a}\right)$$

$$\ln x^{y} = y \ln x$$

$$\ln a + \ln b = \ln(a \times b)$$

$$\left.\begin{array}{l} N = N_0 \, e^{-\lambda t} \\[4pt] m = m_0 \, e^{-\lambda t} \\[4pt] A = A_0 \, e^{-\lambda t} \end{array}\right\} \quad x = x_0 \, e^{-\lambda t}$$

$$m = m_0 \, e^{-\lambda t}$$

m — Final value of active mass
m_0 — Initial value of active mass

$$A = A_0 \, e^{-\lambda t}$$

A — Final activity
A_0 — Initial activity

$$n = \frac{N}{N_A} \qquad n = \frac{m}{m_r}$$

$$\frac{N}{N_A} = \frac{m}{m_r}$$

N → No. of nuclei
N_A → Avagadro's constant 6.02×10^{23}
m → mass of sample in grams.
m_r → Nucleon number

$$\boxed{N = \frac{m}{m_r} N_A}$$

$$\frac{\Delta N}{\Delta t} = - \lambda N$$
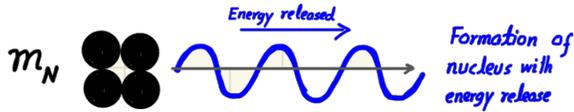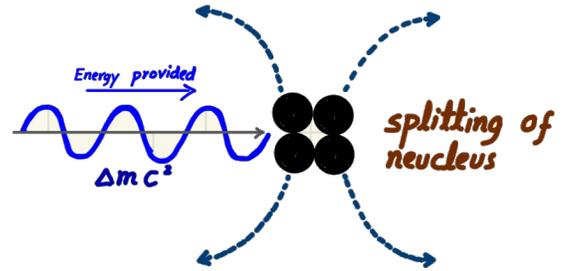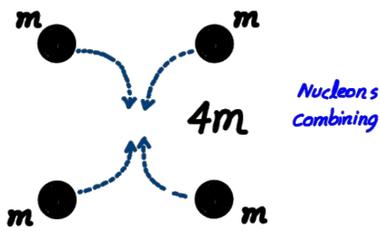
$$A = \lambda N$$

$$x = x_0 e^{-\lambda t}$$

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$N = \frac{m}{m_r} N_A$$

Binding Energy :-

The Mass Defect. Nuclear Stability.

- The mass of a nucleus is always less than the total mass of its nucleons. The difference is known as Mass Defect.

  • The mass defect/lost fraction converts into energy and released at the time of formation of nucleus.

- When nucleons are brought close to each other to form nuclei, a fraction of mass decays into energy and released during formation.

  • In order to split a nucleus, energy is required to be provided.

- The minimum amount of energy requited to split a nucleus into constituent particles (nucleons) to infinity is binding energy.

  • BE provides missing fraction of mass to a nucleus.

  • BE provides energy equivalence of mass defect.

  • BE is required to break the binding between nucleons.

- BE determines nuclear stability ; a nucleus of higher stability requires greater amount of energy for splitting.

  • BE per nucleon is a direct measure of nuclear stability.

  • Nuclear stability depends upon mass defect per nucleon.

Nucleons Combining

$4m$

Energy provided $\Delta m c^2$ — splitting of neucleus

Energy released — Formation of nucleus with energy release

m_N

$$\boxed{\text{Mass defect} = \text{Mass of nucleons} - \text{Mass of neucleus}}$$

$$\Delta m = 4m - m_N$$

$$E = m c^2$$

$$\text{Energy released} = \Delta m \cdot c^2$$

$$1u = 1.66 \times 10^{-27} kg$$

$$m_p = 1.00728 u \qquad m_n = 1.00867 u$$

$^{3}_{1}H$	3.01551 u ↗ 3.02462 u
$^{3}_{2}He$	3.01605 u ↗ 3.02323 u
$^{27}_{13}Al$	26.98153 u ↗ 27.21463 u
$^{97}_{40}Zr$	97.098 u ↗ 97.785 u
$^{234}_{90}Th$	234.04357 u ↗ 235.90368 u
$^{30}_{15}P$ →	29.97830 u ↗ 30.23925 u

$$BE = \Delta m \, c^2$$

Binding energy of a nucleus

→ Moss defect

BE per nucleon

$$\frac{BE}{n} = \frac{\Delta m}{n} c^2$$

→ Mass defect per nucleon

$E \to A$

$E \to B$
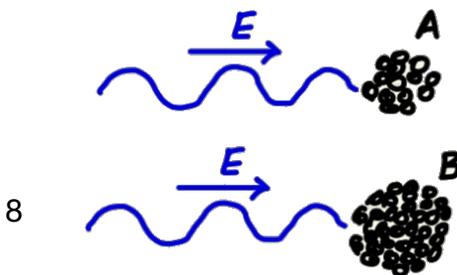
If nuclei A and B have same binding energy; nucleus A has higher stability as binding energy per nucleon of A is greater than that of B.

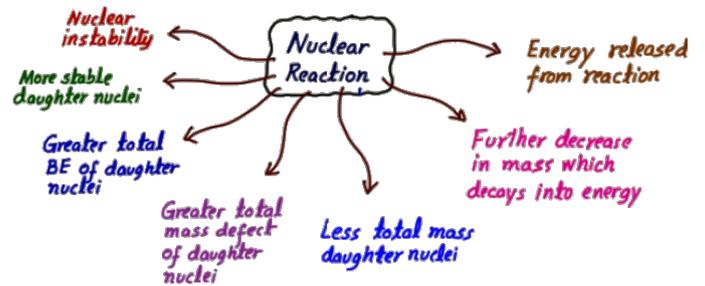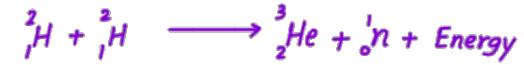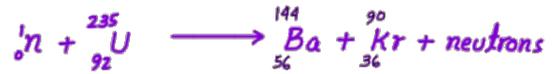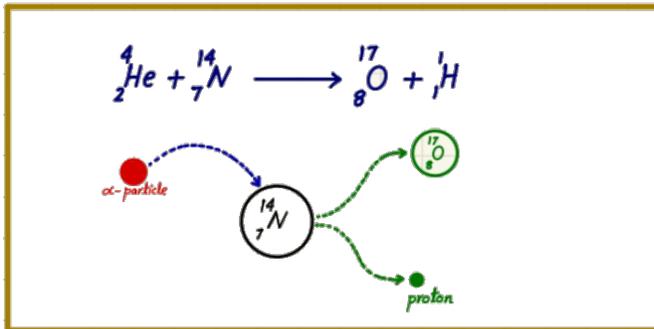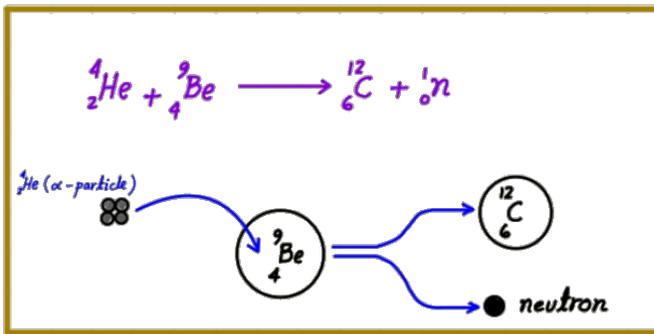- The graph shows variation of binding per nucleon against nucleon number/variation of nuclear stability against nucleon number.

  • On x-axis, it begins with 2 for Deutrium.

  • The graph is not consistent with any mathematical equation ; the values are experimentally determined.

  • The smooth curve of variation provides a close approximation as there are many fluctuations.

  • There are many fluctuations on the sketch, the smooth curve drawn is a close approximation

  • The peak of curve shows a nucleus with highest stability.
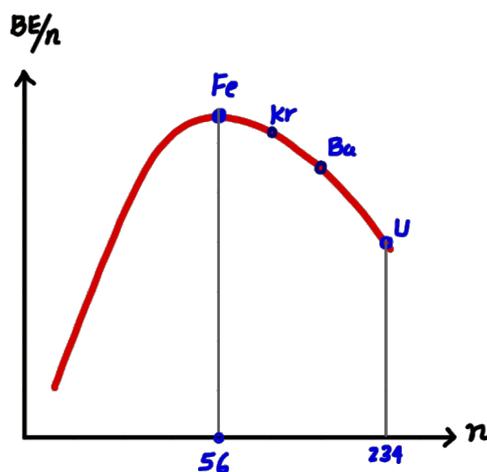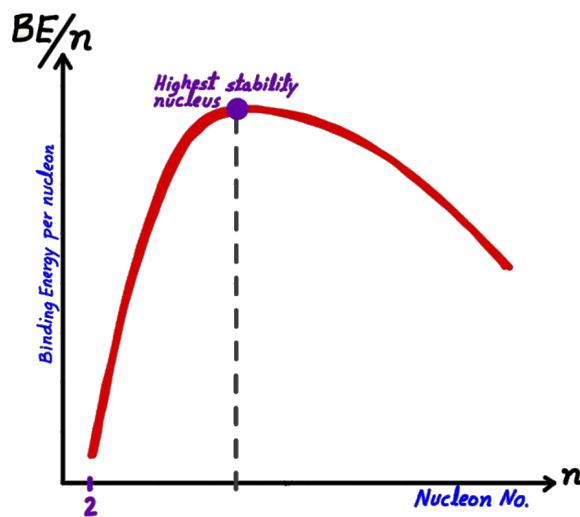
Nuclear Reactions :-0

- In a Nuclear reaction, change occurs inside the nucleus ; the change of proton number.

- The only reason to undergo a nuclear reaction is nuclear instability which maybe natural or externally caused by energy supply or by launching an external particle.

- The Daughter Nuclei must be:-

    • More stable than the parent nuclei.

    • Must have greater Binding Energy.

    • Must have less total mass.

- Hence a fraction of total mass converts into energy and is released during reaction.

    • The energy released from a nuclear reaction is an evidence of :-

        - Increase in mass defect.

        - Increase in binding energy.

        - Increase in stability.

- Nuclear reactions are always energy releasing transitions

    • Most of the energy released converts into kinetic energy of fragments, which subsequently converts into thermal energy. Hence nuclear reactions can cause temperature rise.

    • Reaction fragments exit at very high speeds.

    • Fraction of energy released in the form of gamma radiations is quite small.

- Energy released during nuclear reaction is the difference of binding energy between the daughter and parent nuclei.

$$^{4}_{2}He + ^{9}_{4}Be \longrightarrow ^{12}_{6}C + ^{1}_{0}n$$

$^{4}_{2}He$ (α-particle)

$^{9}_{4}Be$

$^{12}_{6}C$

● neutron

$$^{4}_{2}He + ^{14}_{7}N \longrightarrow ^{17}_{8}O + ^{1}_{1}H$$

α-particle

$^{14}_{7}N$

$^{17}_{8}O$

● proton

$$^{1}_{0}n + ^{235}_{92}U \longrightarrow ^{144}_{56}Ba + ^{90}_{36}Kr + neutrons$$

$$^{2}_{1}H + ^{2}_{1}H \longrightarrow ^{3}_{2}He + ^{1}_{0}n + Energy$$

$$^{4}_{2}He + ^{9}_{4}Be \longrightarrow ^{12}_{6}C + ^{1}_{0}n$$

$$^{1}_{1}H + ^{7}_{3}Li \longrightarrow 2^{4}_{2}He$$

$$^{222}_{86}Rn \longrightarrow ^{218}_{84}Po + ^{4}_{2}He$$

Nuclear Reaction

Nuclear instability

More stable daughter nuclei

Greater total BE of daughter nuclei

Greater total mass defect of daughter nuclei

Less total mass daughter nuclei

Energy released from reaction

Further decrease in mass which decays into energy

Nuclear reactions ⟶ More stable daughter nuclei ⟶ Increase in stability ⟶ Increase in Binding energy ⟶ Increase in mass defect ⟶ Decrease in total mass ⟶ $E = \Delta mc^2$ ⟶ Energy released.

$m_p > m_d$

$\boxed{m_d > m_p.}$

$m_p > m_d$

$\Delta m = m_p - m_d$
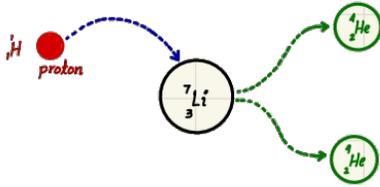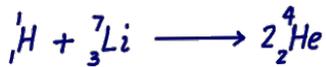
Increase in mass defect

Reaction is energetically possible; increase in defect decays into energy.

$m_p < m_d$

$(m_d - m_p) \times c^2 = KE_{min}$

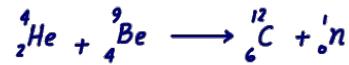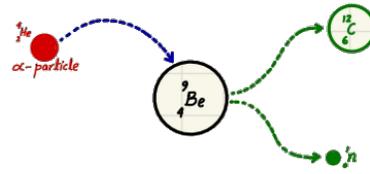$KE_{min} \longrightarrow$ Min KE of parent nuclei for reaction

If $m_p < m_d$; the reaction can proceed only if the parent nuclei have a large KE value which can provide the energy equivalence of difference in mass.

$${}^{1}_{1}H + {}^{7}_{3}Li \longrightarrow 2{}^{4}_{2}He$$

${}^{1}_{1}H$ ● proton

${}^{7}_{3}Li$

${}^{4}_{2}He$

${}^{4}_{2}He$

$$\text{Energy released} = \left[(m_P + m_L) - (2\,m_H) \right] c^2$$

$$\text{Energy released} = \left[(\Delta m_H + \Delta m_H) - \Delta m_L \right] c^2$$

$$\text{Energy released} = (BE_H + BE_H) - BE_L$$

${}^{4}_{2}He$ ● α-particle

${}^{9}_{4}Be$

${}^{12}_{6}C$

● ${}^{1}_{0}n$

$$\text{Energy released} = \left[(m_H + m_B) - (m_c + m_n) \right] \times c^2$$

$$\text{Energy released} = \left[\Delta m_c - (\Delta m_H + \Delta m_B) \right] \times c^2$$

$$\text{Energy released} = BE_c - (BE_H + BE_B)$$

$${}^{4}_{2}He + {}^{9}_{4}Be \longrightarrow {}^{12}_{6}C + {}^{1}_{0}n$$

$${}^{1}_{0}n + {}^{235}_{92}U \longrightarrow {}^{144}_{56}Ba + {}^{90}_{36}Kr + 2{}^{1}_{0}n$$
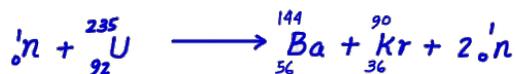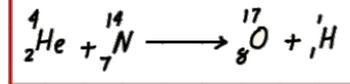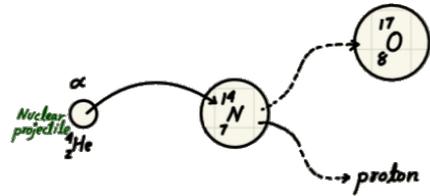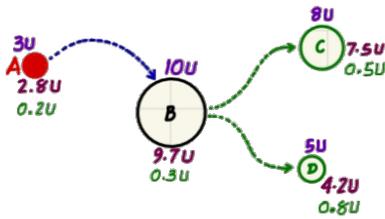
$$\text{Energy released from nuclear reaction} = \left[(m_n + m_u) - (m_B + m_K + 2m_n) \right] \times c^2$$

$$\text{Energy released from nuclear reaction} = \left[(\Delta m_B + \Delta m_K) - \Delta m_U \right] c^2$$

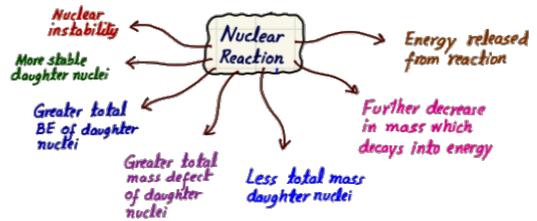$$\text{Energy released from nuclear reaction} = (BE_B + BE_K) - BE_U$$

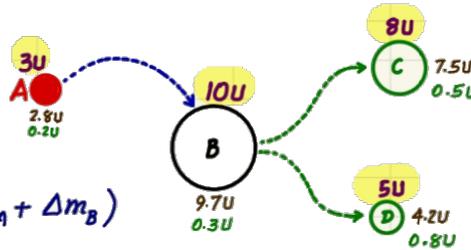**Increase in mass defect** = Mass defect of daughter nuclei − Mass defect of parent nuclei

$$(0.5 + 0.8) - (0.2 + 0.3)$$

Increase in mass defect = Total Mass of parent nuclei − Total Mass of daughter nuclei

$$(2.8 + 9.7) - (7.5 + 4.2)$$

$$^{4}_{2}He + {}^{14}_{7}N \longrightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

Energy released from nuclear reaction = Increase in mass defect × c^2

$$A + B \longrightarrow C + D$$

Increase in mass defect $= (\Delta m_C + \Delta m_D) - (\Delta m_A + \Delta m_B)$

Increase in mass defect $= (m_A + m_B) - (m_C + m_D)$

Energy released from nuclear reaction $= \left[(m_A + m_B) - (m_C + m_D) \right] c^2$

Energy released from nuclear reaction $= \left[(\Delta m_C + \Delta m_D) - (\Delta m_A + \Delta m_B) \right] c^2$

Energy released from nuclear reaction $= \underbrace{(\Delta m_C c^2 + \Delta m_D c^2)}_{\text{BE of daughter nuclei}} - \underbrace{(\Delta m_A c^2 + \Delta m_B c^2)}_{\text{BE of parent nuclei}}$

Energy released from nuclear reaction = Total BE of daughter nuclei − Total BE of Parent nuclei

The Case of Fission and Fusion

- Nuclear Fission is defined as splitting/breaking of a heavy nucleus into medium sized daughter nuclei;

- the fission of nuclei on the left hand side of the peak of binding energy per nucleon against nucleon number is Energetically Impossible

  - As daughter nuclei must be more stable/must have greater values of binding energy per nucleon.

  - The nuclei which can undergo fission must be on the right hand side of the peak.

- Nuclear Fusion is defined as the combination of a lighter nuclei to form a heavier daughter nucleus;

- The fusion of nuclei on the right hand side of the peak of Binding Energy per nucleon number graph is Energetically impossible

  - As the daughter nuclei must be more stable/must have greater value of binding energy per nucleon.

  - The nuclei which can undergo fusion must be on the left hand side of the peak.