

MEDICAL PHYSICS

Achkar

Ultra sound :-

Above a certain value

Max value of audible frequency of 20,000 Hz

Sound waves having frequency of the order of MHz are used for scanning purpose.

Longitudinal wave

Needs a state of matter to propagate / travel

Speed is dependant upon material medium i.e at 0°C

air $\rightarrow 330 \text{ m s}^{-1}$

water $\rightarrow 1500 \text{ m s}^{-1}$

Concrete $\rightarrow 5000 \text{ m s}^{-1}$

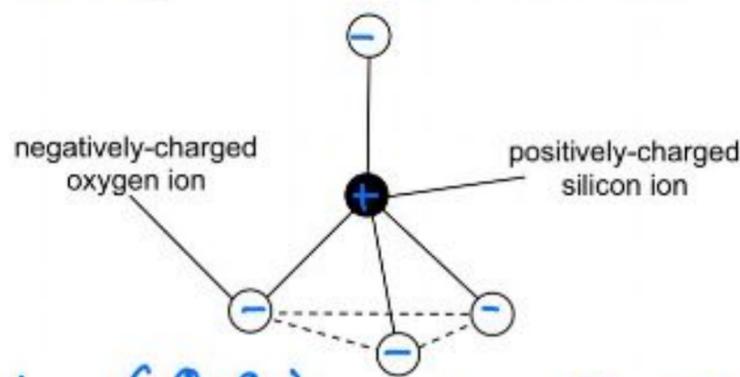
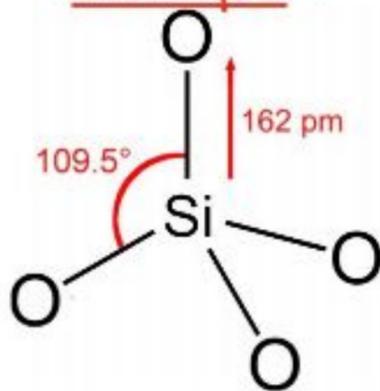
Steel $\rightarrow 6000 \text{ m s}^{-1}$

Soft tissue $\rightarrow 1500 \text{ m s}^{-1}$

Piezoelectric effect:

Statement: An Alternating p.d. is developed across a piezoelectric crystal (Quartz - SiO_2) when separation between its ions is varied.

Example: Quartz - Silicon Dioxide (SiO_2)



tetrahedral structure

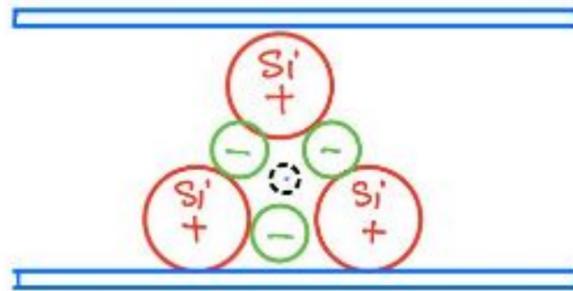
$$E_p = -\frac{1}{4\pi\epsilon_0} \left(\frac{Q+Q-}{r} \right) \Rightarrow E_p \propto - \left(\frac{Q+Q-}{r} \right)$$

If r vary, E_p is developed

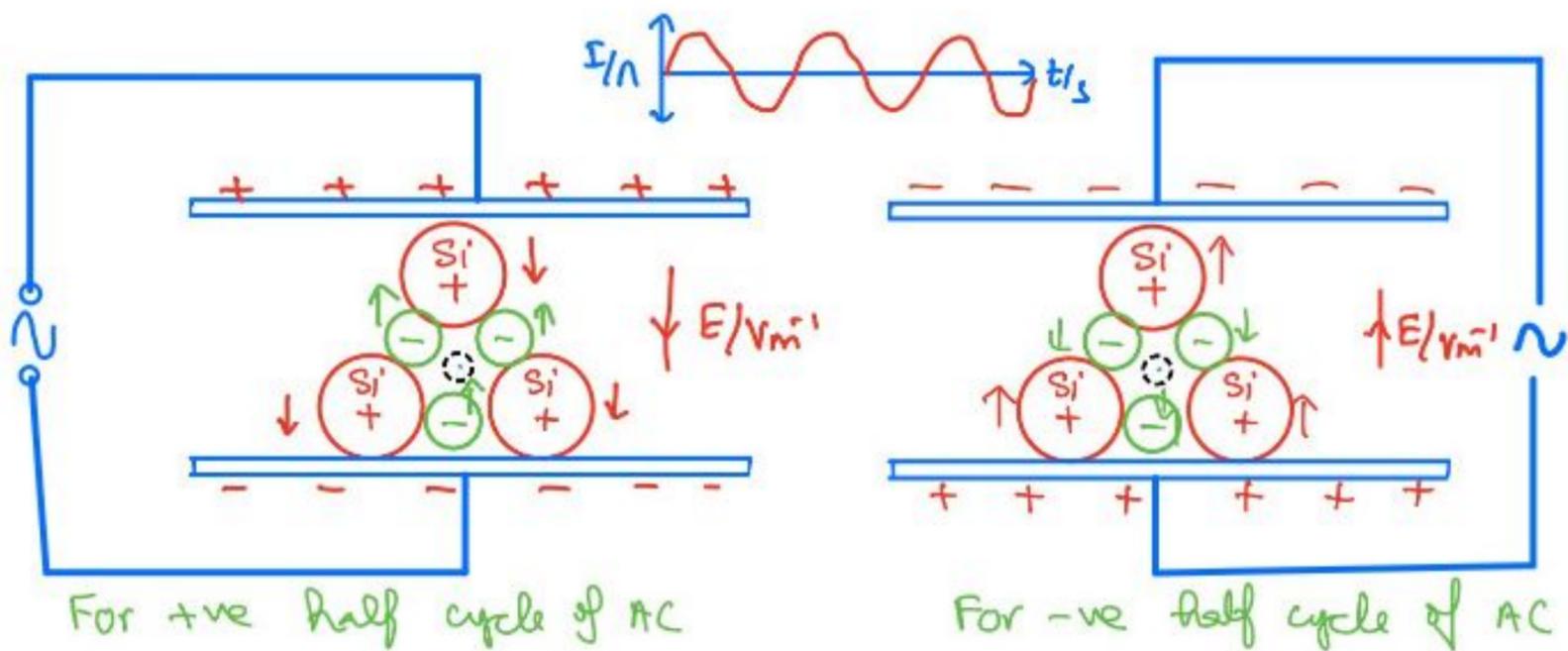
$$V = \frac{W}{Q} \Rightarrow V = \frac{E_p}{Q}$$

Production Principle:-

Resonance of piezo-electric crystal / Quartz / SiO_2 .



Unstretched / Uncompressed
crystal in equilibrium
position between two silvered
electrodes.

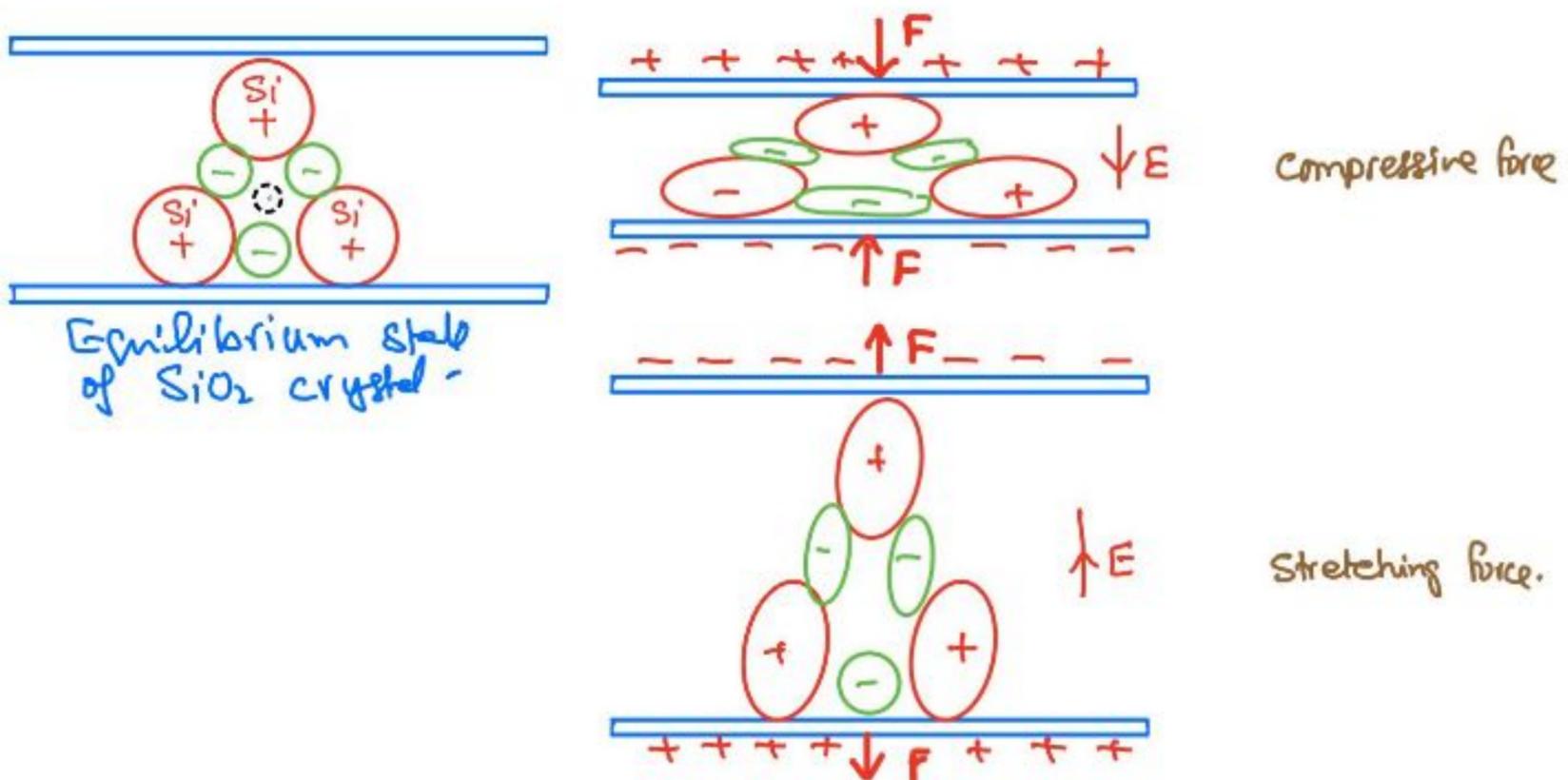


An AC pulse is provided across silvered electrodes so that silicon and oxygen ions experience electric force in opposite ^{directions} and displace crystal so that it can vibrate with its natural frequency.

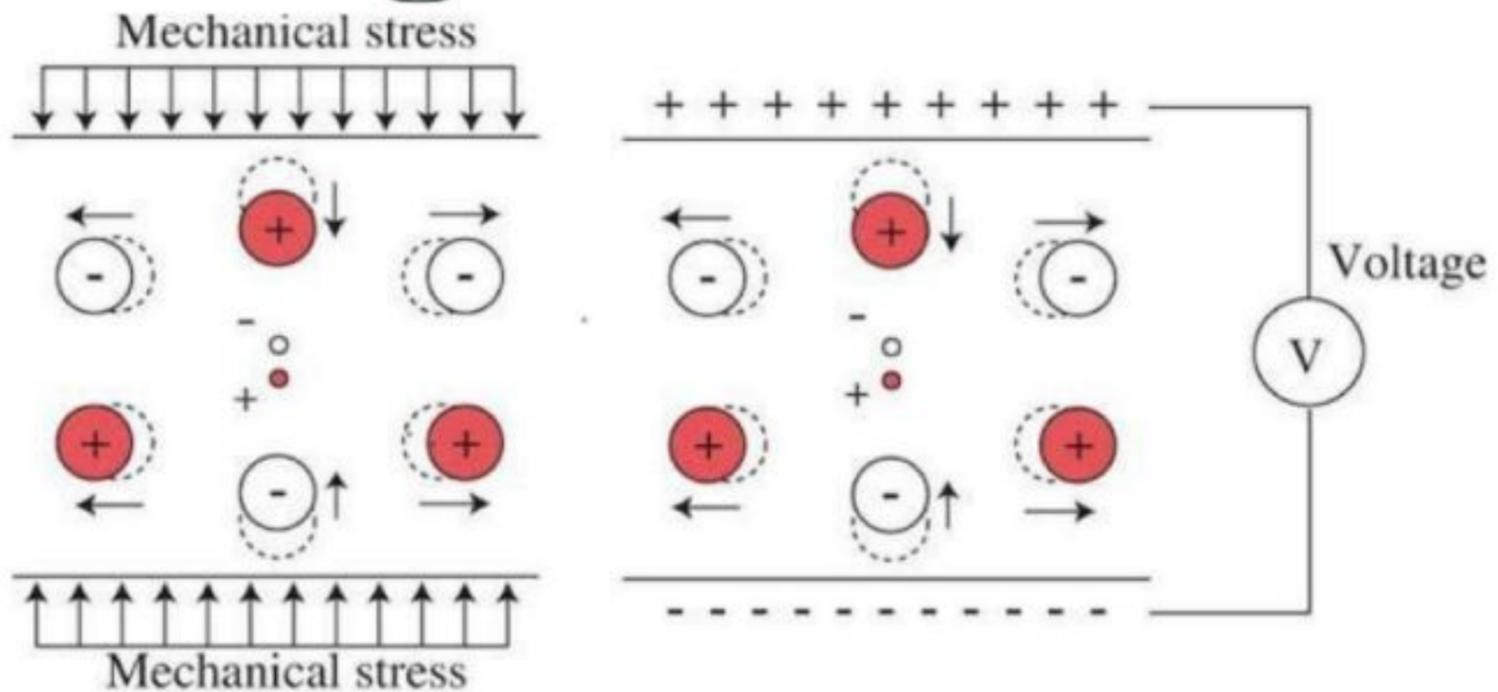
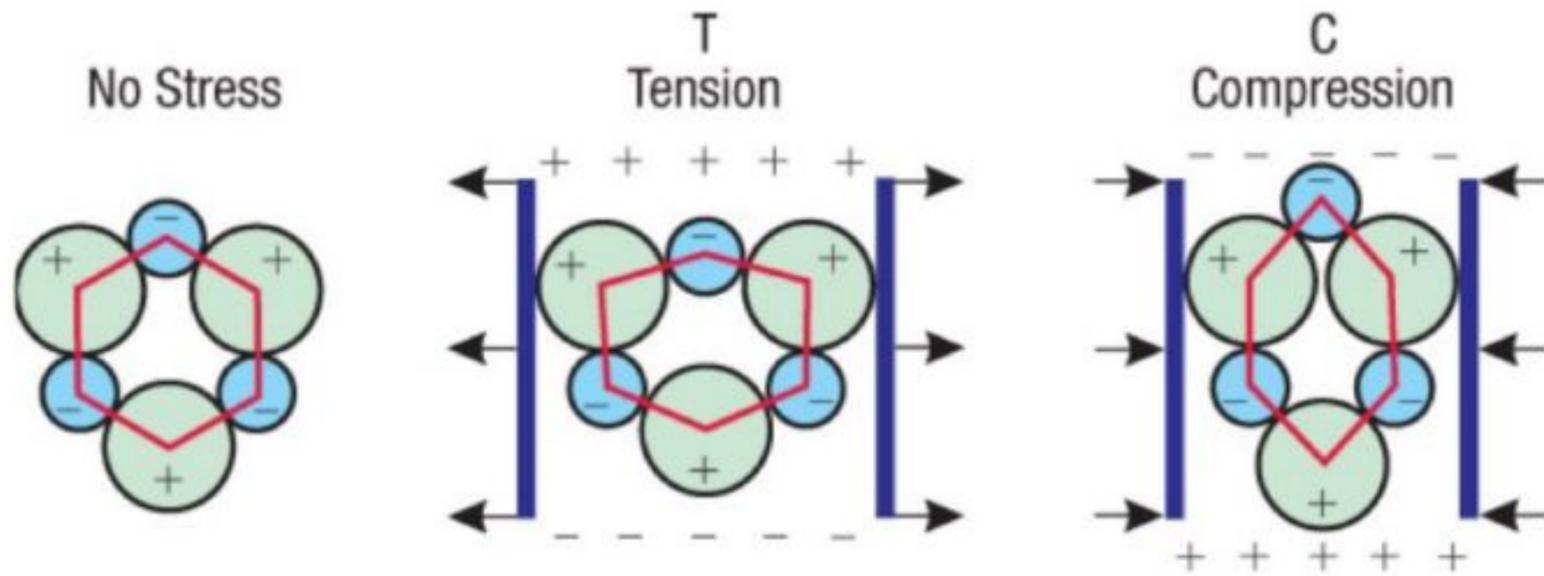
Another AC pulse is again provided to vibrating Quartz crystal as forced or driving frequency. Resonance occur if natural frequency of Quartz crystal is equal to the driving frequency of AC pulse and ultrasonic are emitted from the vibrating crystal with larger amplitude.

Reception Principle:- Piezoelectric effect i.e Alternating p.d. is developed across silvered electrodes when separation b/w ions of Quartz crystal is varied.

Compressions and rarefactions of sound waves compress and stretch the Quartz crystal to produce alternating p.d which is processed and displayed on screen.



Piezoelectric Effect in Quartz



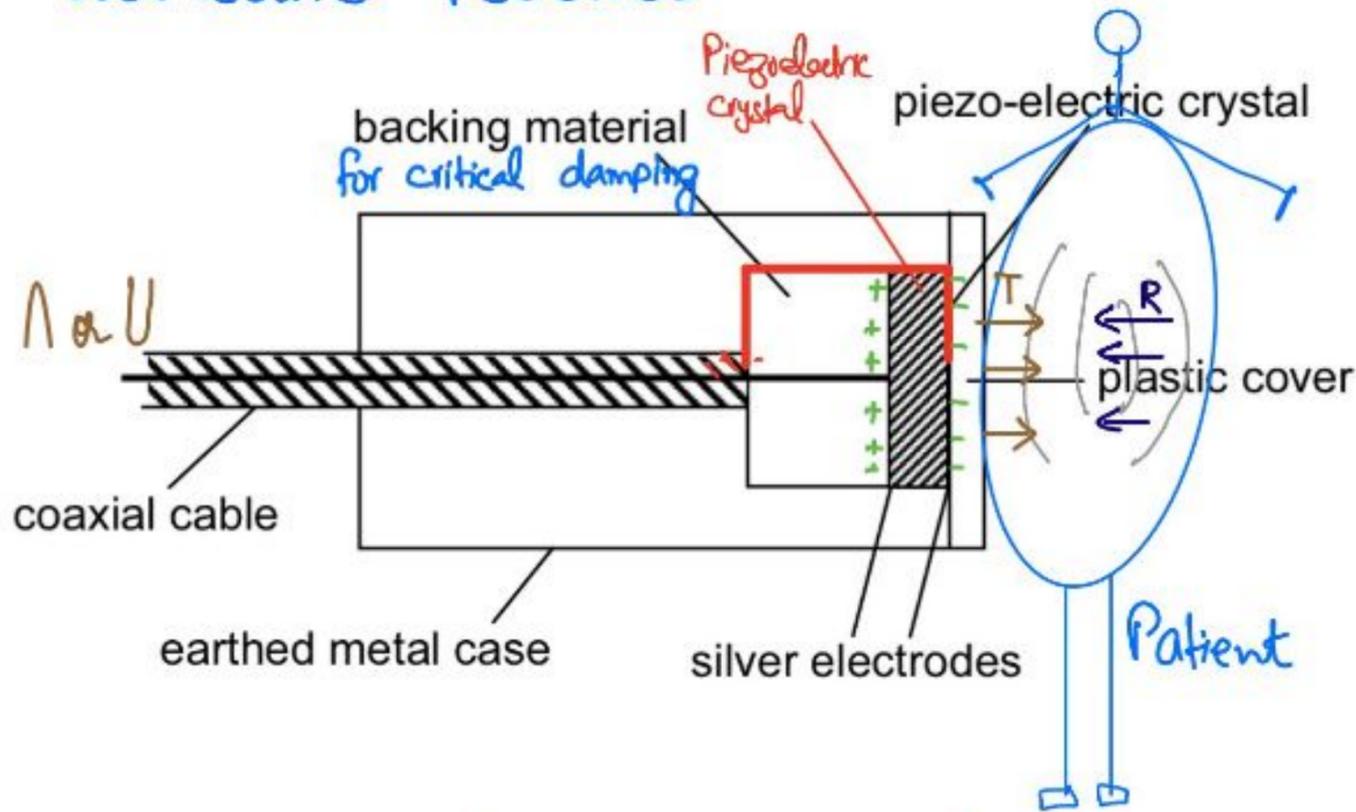
Mechanical-to-electrical
(Sensing ability)

(a)

Electrical-to-mechanical
(Actuating ability)

(b)

Ultrasound Probe:- Ultrasound transmitter and ultrasound receiver.



An A.C. pulse is provided to set up mechanical vibrations in the crystal. Another pulse is provided to cause Resonance and crystal start producing ultrasonics in the surrounding medium.

The crystal is brought to rest immediately due to critical damped backing material so that it can no longer further produce ultrasonics and can act as a receiver.

These ultrasonics are reflected back from different organs / tissues and exert pressure variation to change position of +ve and -ve ions so that Alternating p.d. is produced. This varying p.d. can then be

amplified and processed to display on screen.

Important terms:-

1 - Specific Acoustic impedance:-

depends upon
matter (density)

Sound
waves

properties of a
material medium

Def. Product of density of a medium and the speed of sound wave in it.

Symbol: Z

Formula: $Z = \rho c$

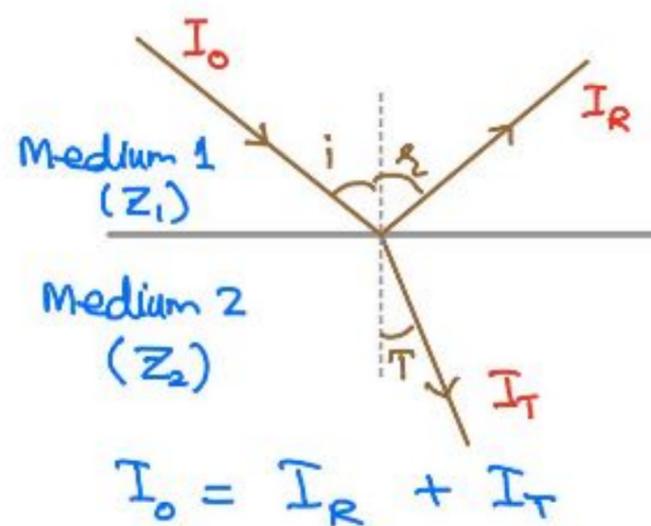
ρ - Density of medium, c - Speed of sound

Units: $(\text{kg m}^{-3})(\text{m s}^{-1}) = \text{kg m}^{-2} \text{s}^{-1}$

P.S: Scalar

Dependence: $Z \uparrow$ if $\rho \uparrow$ or vice versa

2 - Conservation of Intensity Principle:-



Note:

- (i) - $(I_R > I_T)$ if $(Z_2 > Z_1)$ i.e. 2nd medium is denser.
- (ii) - $(I_R < I_T)$ if $(Z_2 < Z_1)$ i.e. 2nd medium is rarer.

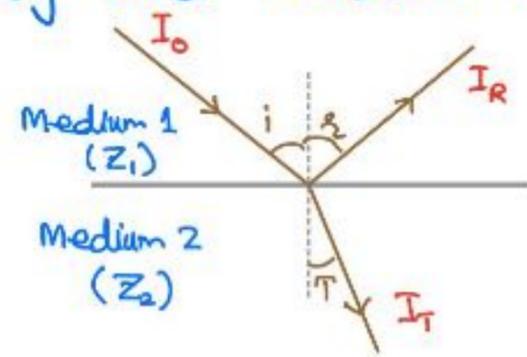
3 - Intensity Reflection Co-efficient:

Def: Ratio of reflected intensity and incident intensity.

Symbol: α

Formula: $\alpha = \frac{I_R}{I_0}$

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$



Here Z_1 and Z_2 are the specific acoustic impedances of medium 1 and 2 respectively.

Units: No units as it is the ratio b/w two similar quantities.

P.S. Scalar

Note:

- (i) If $(Z_2 \gg Z_1)$ i.e. Z_1 is neglected in comparison to Z_2

$$\alpha = \frac{I_R}{I_0} = \frac{(Z_2 - 0)^2}{(Z_2 + 0)^2} \Rightarrow \frac{I_R}{I_0} = \frac{Z_2^2}{Z_2^2} = 1$$

$$\frac{I_R}{I_0} = 1 \Rightarrow \boxed{I_R = I_0}$$

i.e. No transmission and incident intensity is totally

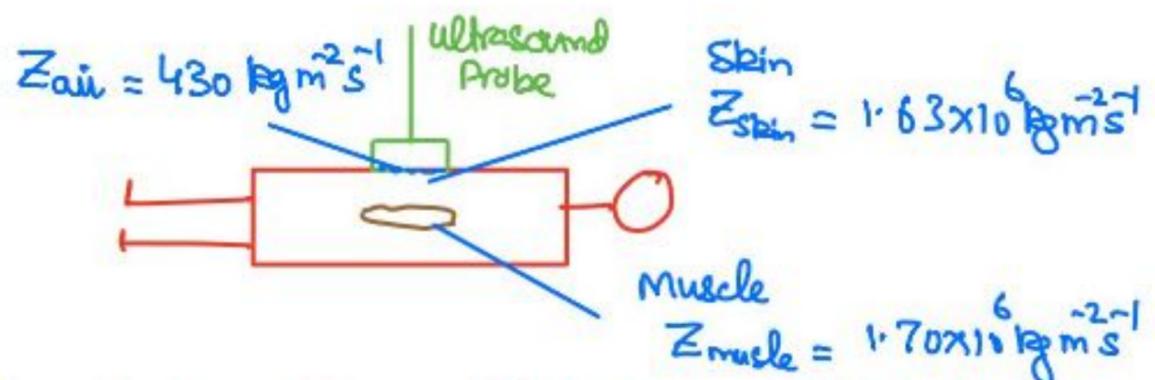
reflected back.

$$(ii) \text{ If } Z_2 \approx Z_1 \\ \alpha = \frac{I_R}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} = 0$$

$$\frac{I_R}{I_0} = 0 \Rightarrow \boxed{I_R = 0}$$

i.e. No reflection and all ultrasonics are transmitted into the second medium.

a)



(a) Calculate the intensity reflection coefficient b/w

(i) air and skin:

$$\alpha = \frac{I_R}{I_0} = \frac{(Z_s - Z_a)^2}{(Z_s + Z_a)^2} = \frac{(1.63 \times 10^6 - 430)^2}{(1.63 \times 10^6 + 430)^2} = 0.999$$

$$\frac{I_R}{I_0} \approx 1 \Rightarrow \boxed{I_R = I_0}$$

Analysis: No transmission as all ultrasonics are reflected back. Hence there must be no air gap b/w ultrasound probe and skin. This air gap is reduced/eliminated by a gel, called as "coupling medium".

(ii) Skin to muscle:

$$\alpha = \frac{I_R}{I_0} = \frac{(Z_m - Z_s)^2}{(Z_m + Z_s)^2} = \frac{[(1.70 - 1.63)10^6]^2}{[(1.70 + 1.63)10^6]^2} = 0.004$$

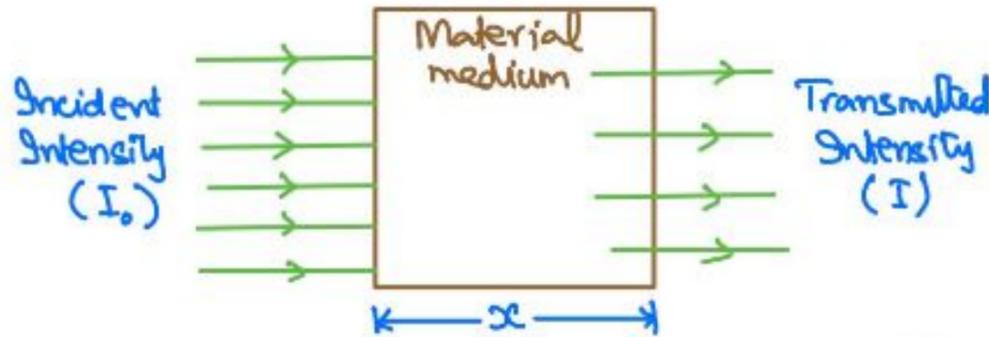
$$\frac{I_R}{I_0} \approx 0 \Rightarrow \boxed{I_R = 0}$$

Analysis:-

All is transmitted into muscle and negligible is reflected back.

4- Linear attenuation or Absorption coefficient:-

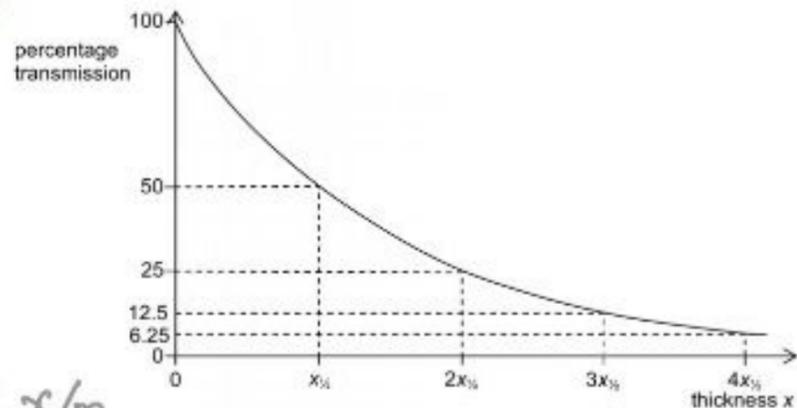
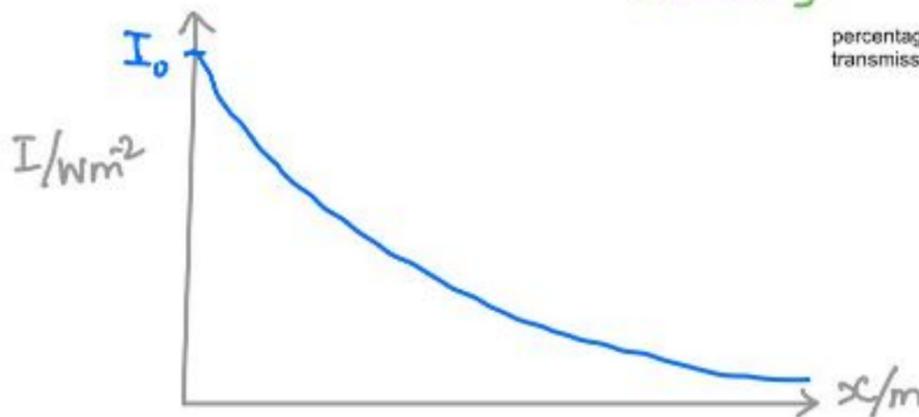
Decrease in amplitude / energy / Power / Intensity of a wave due to loss of energy in passing through a medium.



$$I = I_0 e^{-\mu x}$$

Labels for the equation:

- I : Transmitted Intensity
- I_0 : Incident Intensity
- μ : Linear attenuation / absorption coefficient
- x : Thickness of material medium.



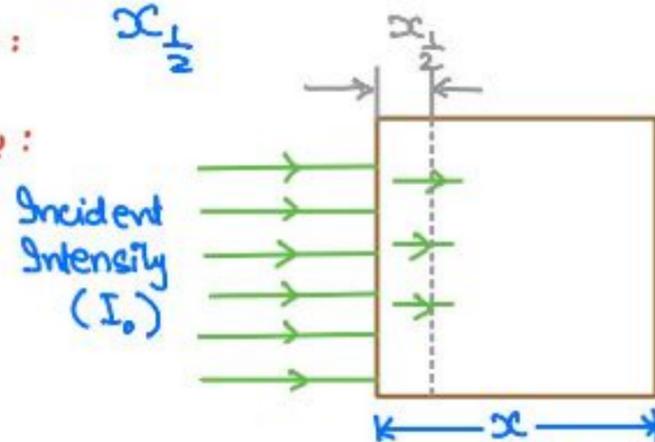
Condition: To apply $I = I_0 e^{-\mu x}$, incident wave must be perpendicular to the surface of material medium i.e. angle of incidence is zero.

5- Half value thickness: (HVT)

This is the thickness/width of a material medium which reduces the incident intensity to its half value.

Symbol: $x_{\frac{1}{2}}$

Concept:



Formula:

$$I = I_0 e^{-\mu x}$$

For HVT, $I = \frac{I_0}{2}$ and $x = x_{\frac{1}{2}}$

$$\frac{I_0}{2} = I_0 e^{-\mu x_{\frac{1}{2}}}$$

$$\frac{1}{2} = e^{-\mu x_{\frac{1}{2}}}$$

Taking natural log (ln) to both sides

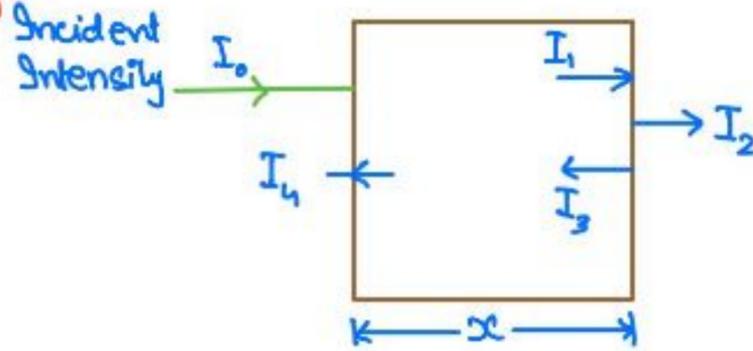
$$\ln 1 - \ln 2 = -\mu x_{\frac{1}{2}} \ln(e)$$

$$0 - \ln 2 = -\mu x_{\frac{1}{2}} \quad (1)$$

$$x_{\frac{1}{2}} = \frac{\ln 2}{\mu}$$

$$x_{\frac{1}{2}} = \frac{0.693}{\mu}$$

6- Relationship b/w incident intensity and received intensity:-



Relationship b/w I_0 and I_1 :

$$I_1 = I_0 e^{-\mu x} \dots\dots\dots(1)$$

Relation b/w I_1 and I_3 :

$$\alpha = \frac{I_3}{I_0} \Rightarrow \alpha = \frac{I_3}{I_1}$$

$$I_3 = \alpha I_1 \dots\dots\dots(2)$$

Put eq. (1) into eq. (2)

$$I_3 = \alpha I_0 e^{-\mu x}$$

Relationship b/w I_4 and I_3 :

$$I_4 = I_3 e^{-\mu x}$$

$$I_4 = (\alpha I_0 e^{-\mu x}) e^{-\mu x}$$

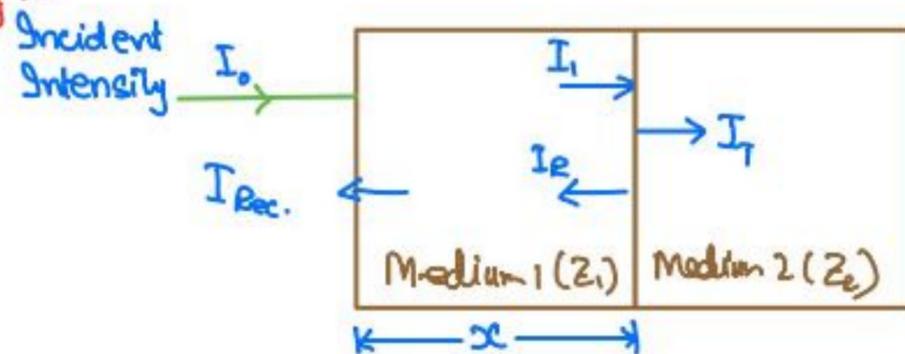
$$I_4 = \alpha I_0 e^{-2\mu x}$$

$$\boxed{\frac{I_4}{I_0} = \alpha e^{-2\mu x}}$$

Formula:

$$\frac{I_{\text{Received}}}{I_{\text{Incident}}} = \alpha e^{-2\mu x} \quad \text{Here } \alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

7 - Relationship b/w incident intensity and transmitted intensity :-



Relationship b/w I_0 and I_1 :

$$I_1 = I_0 e^{-\mu x} \dots\dots\dots (1)$$

Relation b/w I_1 and \bar{I} :

$$\alpha = \frac{I_R}{I_1} \Rightarrow \alpha = \frac{I_R}{I_1}$$

$$I_R = \alpha I_1 \Rightarrow I_R = \alpha I_0 e^{-\mu x} \dots\dots\dots (2)$$

By Principle of conservation of Intensity

$$I_1 = I_R + I_T$$

$$I_0 e^{-\mu x} = \alpha I_0 e^{-\mu x} + I_T$$

$$I_0(1 - \alpha) e^{-\mu x} = I_T$$

$$\frac{I_T}{I_0} = (1 - \alpha e^{-\mu x})$$

Formula :

$$\frac{I_{\text{Transmitted}}}{I_{\text{Incident}}} = [1 - \alpha e^{-\mu x}]$$

March 20/42/Q.no.4

- 4 (a) (i) Explain why ultrasound used in medical diagnosis is emitted in pulses.

.....
.....
..... [2]

- (ii) Explain the principles of the **detection** of ultrasound waves used in medical diagnosis.

.....
.....
.....
..... [3]

- (b) The specific acoustic impedances Z of some media are given in Table 4.1.

Table 4.1

media	$Z/\text{kg m}^{-2}\text{s}^{-1}$
air	4.3×10^2
gel	1.5×10^6
soft tissue	1.6×10^6

- (i) The specific acoustic impedances of two media are Z_1 and Z_2 . The intensity reflection coefficient α for the boundary of these two media is given by:

$$\alpha = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

Calculate, to three significant figures, the fraction of the ultrasound intensity that is reflected at a boundary between air and soft tissue.

$$\alpha = \frac{(Z_{\text{air}} - Z_{\text{tissue}})^2}{(Z_{\text{air}} + Z_{\text{tissue}})^2} = \frac{(4.3 \times 10^2 - 1.6 \times 10^6)^2}{(4.3 \times 10^2 + 1.6 \times 10^6)^2}$$
$$\alpha = 0.999$$

$\alpha =$ [1]

- (ii) Use your value in (b)(i) to explain why gel is applied to the surface of the skin during an ultrasound scan.

Intensity reflection coefficient from gel to tissue:

$$\alpha = \frac{(Z_{\text{gel}} - Z_{\text{tissue}})^2}{(Z_{\text{gel}} + Z_{\text{tissue}})^2} = \frac{[(1.5 - 1.6) \times 10^6]^2}{[(1.5 + 1.6) \times 10^6]^2}$$

Without gel, most of ultrasonics are reflected back from skin and can not transmit through it. Gel reduces the air gap b/w ultrasound probe and skin due to reduced 'α' so that maximum can transmit into the skin. [2]

[Total: 8]

June 20/41/Q.No 4)

- 4 (a) (i) By reference to an ultrasound wave, explain what is meant by *specific acoustic impedance*.

$$Z = \rho c$$

Product of density of a medium and speed of sound wave in it. [2]

- (ii) An ultrasound wave is incident normally on the boundary between two media. The media have specific acoustic impedances Z_1 and Z_2 .

State how the ratio

$$\alpha = \frac{\text{intensity of ultrasound reflected from boundary}}{\text{intensity of ultrasound incident on boundary}} = \frac{I_R}{I_0}$$

depends on the relative magnitudes of Z_1 and Z_2 .

→ $I_R > I_T$ if difference between Z_1 and Z_2 is large because $\alpha \approx 1$ i.e. $I_R \approx I_0$

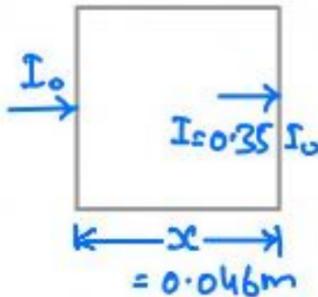
→ $I_T > I_R$ if difference between Z_1 and Z_2 is small because $\alpha \approx 0$ i.e. $I_R \approx 0$. So $I_0 \approx I_T$. [2]

(b) (i) State what is meant by the *attenuation* of an ultrasound wave.

Decrease in amplitude/energy/power/intensity of a wave in passing through a medium. [1]

(ii) A parallel beam of ultrasound is passing through a medium. The incident intensity I_0 is reduced to $0.35I_0$ on passing through a thickness of 0.046 m of the medium.

Calculate the linear attenuation coefficient μ of the ultrasound beam in the medium.



$$I = I_0 e^{-\mu x}$$
$$0.35 I_0 = I_0 e^{-\mu(0.046)}$$

Taking natural log (\ln) to both sides

$$\ln(0.35) = - (0.046) \mu \ln e$$
$$\mu = 23$$

$\mu = \dots\dots\dots \text{ m}^{-1}$ [2]

June 2011/41/Q.No.10

- 10 (a) State what is meant by the *acoustic impedance* Z of a medium.

$$Z = \rho c$$

ρ - Density of medium, c - Speed of sound [1]

- (b) Two media have acoustic impedances Z_1 and Z_2 . The intensity reflection coefficient α for the boundary between the two media is given by

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

Describe the effect on the transmission of ultrasound through a boundary where there is a large difference between the acoustic impedances of the two media.

$$\alpha \approx 1 \Rightarrow \frac{I_R}{I_0} \approx 1 \Rightarrow I_R \approx I_0$$

i.e. Reflected intensity is nearly equal to the incident intensity and negligible is transmitted deep into the skin. [3]

- (c) Data for the acoustic impedance Z and the absorption coefficient μ for fat and for muscle are shown in Fig. 10.1.

	$Z/\text{kg m}^{-2} \text{s}^{-1}$	μ/m^{-1}
fat	1.3×10^6	48
muscle	1.7×10^6	23

Fig. 10.1

The thickness x of the layer of fat on an animal, as illustrated in Fig. 10.2, is to be investigated using ultrasound.

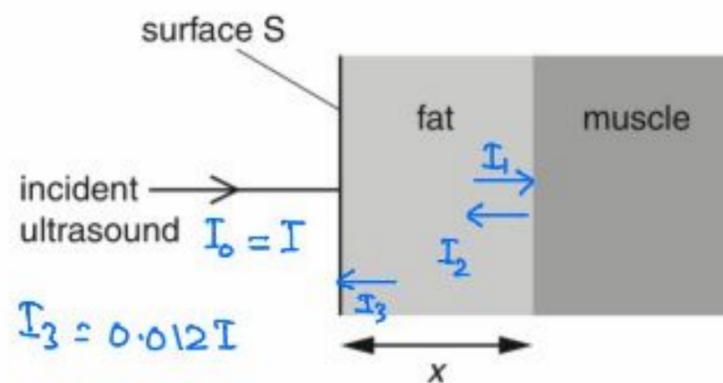


Fig. 10.2

The intensity of the parallel ultrasound beam entering the surface S of the layer of fat is I . The beam is reflected from the boundary between fat and muscle. The intensity of the reflected ultrasound detected at the surface S of the fat is $0.012 I$. Calculate

- (i) the intensity reflection coefficient at the boundary between the fat and the muscle,

$$\alpha = \frac{(Z_m - Z_f)^2}{(Z_m + Z_f)^2} = \frac{[(1.7 - 1.3)10^6]^2}{[(1.7 + 1.3)10^6]^2}$$

$$\alpha = 0.0178$$

coefficient = [2]

- (ii) the thickness x of the layer of fat.

$$\frac{I_{\text{received}}}{I_{\text{incident}}} = \alpha e^{-2\mu x}$$

$$\frac{0.012I}{I} = (0.0178) e^{-2(48)x}$$

$x = \dots\dots\dots$ cm [3]

ULTRASOUND SCANNING TECHNIQUES:-



Scanning Principle:-

A pulse of ultrasonics is transmitted into the skin through a coupling medium. This sound

wave/pulse is reflected back from boundaries of different mediums deep in the skin and is received back. The received reflected pulses are processed and displayed on the screen.

The depth of the reflecting surface is obtained from the time interval between transmitting and reception of echo. Also the nature of second medium i.e. its density is obtained from the amplitude of the reflected received sound.

Depth: $\text{Speed} = \frac{\text{Distance travelled}}{\text{time}}$

$$v = \frac{2d}{t} \Rightarrow d = \frac{vt}{2}$$

Here time 't' is determined by using

x-plate sensitivity of C.R.O.

$$t = \left(\frac{\text{no. of divisions}}{\text{along x-axis}} \right) \left(\frac{\text{x-plate}}{\text{sensitivity}} \right)$$

Nature: Larger amplitude of received pulse shows that reflecting medium has high density.

Note: Higher is the frequency of ultrasonics, smaller is the wavelength and is used to get better information between layers within the skin i.e. provides better information.

In order for ultrasound to 'see' detail, say in

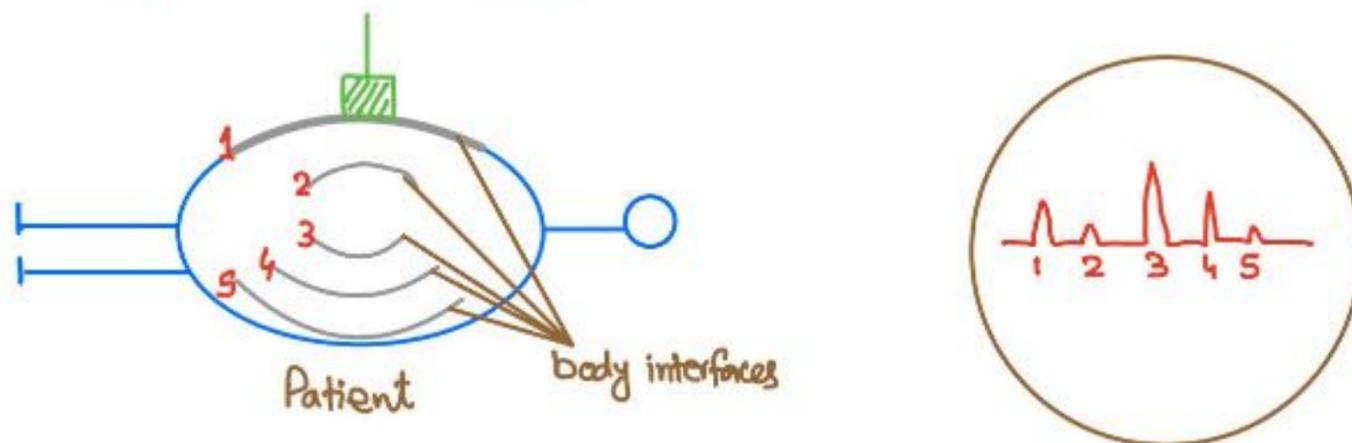
soft tissue, down to linear dimension x , the wavelength λ used must be of the same order. Thus, in soft tissue, where the velocity of sound c is $1.54 \times 10^3 \text{ m s}^{-1}$, to observe detail down to 1mm would require an ultrasound frequency.

$$f = \frac{c}{\lambda} = \frac{c}{x} = \frac{1.54 \times 10^3}{10^{-3}} = 1.54 \times 10^6 \text{ Hz}$$

$$f = 1.54 \text{ MHz}$$

(a) Ultrasound A-scan:-

In A-scan, position of ultrasound probe to transmit and receive signal must be fixed. At each interface, some energy is reflected and some is transmitted. Each reflection will give rise to an echo. The echoes become weaker, the deeper the ultrasound penetrates into the body. The echoes will be received by the probe amplified appropriately and displayed on an oscilloscope screen. Each interface will be represented by a pulse as shown.

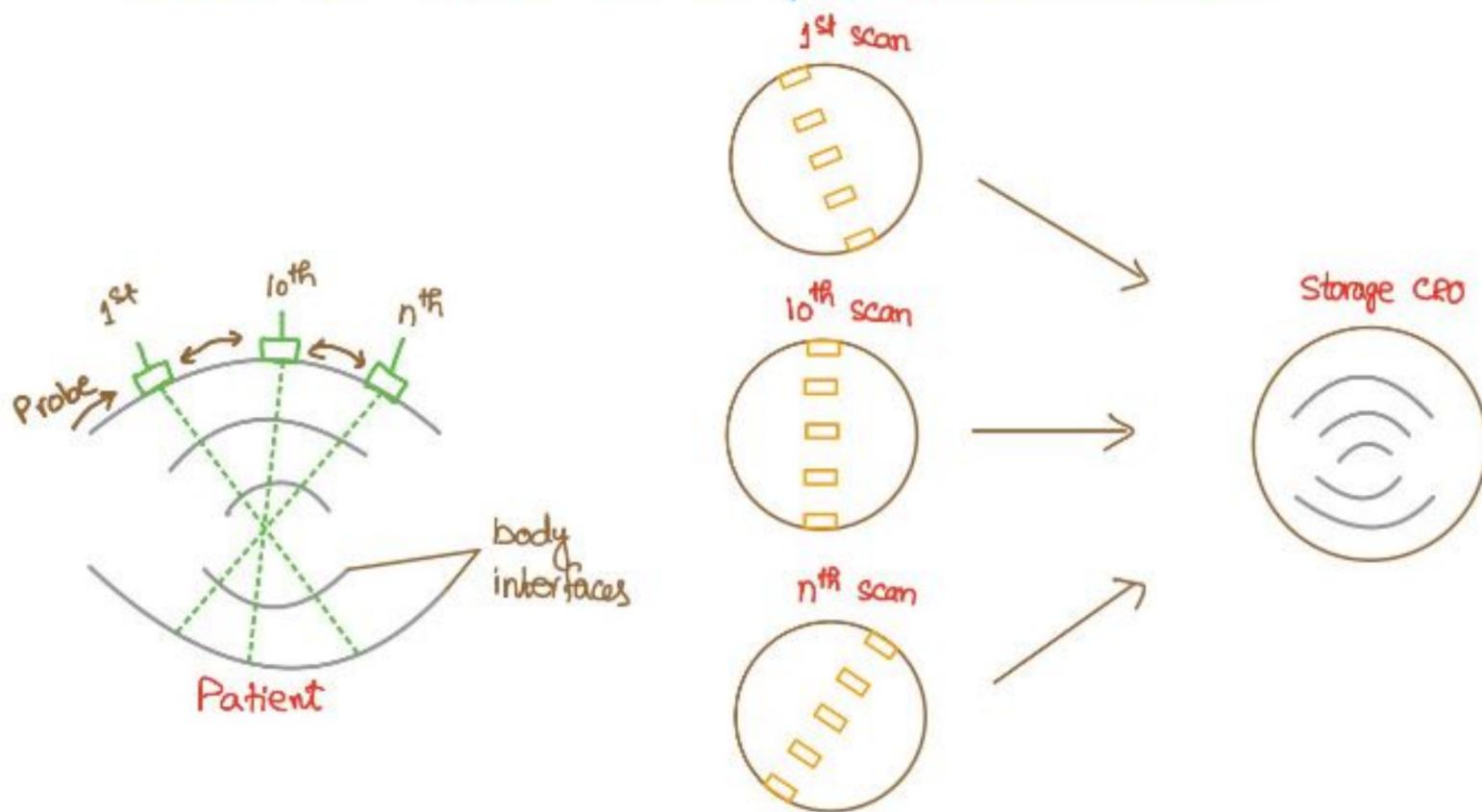


Application: It is used

- (i) to diagnose whether a tumour is present in the brain.
- (ii) in Ophthalmology to diagnose eye diseases.
- (iii) in Biometry to get distances in the eye.

(b) Ultrasound B-scan:-

Multiple A-scans from different angles/positions. Each scan position will produce a spot whose brightness indicates the strength of the echo. If the scans corresponding to all the scan positions are taken together to get body interfaces on a screen. Polaroid photographs of the images are taken to serve as a permanent record.



$$I = I_0 e^{-\mu x}$$

- 6 A parallel beam of ultrasound is incident normally on the surface of a layer of fat of thickness 1.1 cm, as shown in Fig. 6.1.

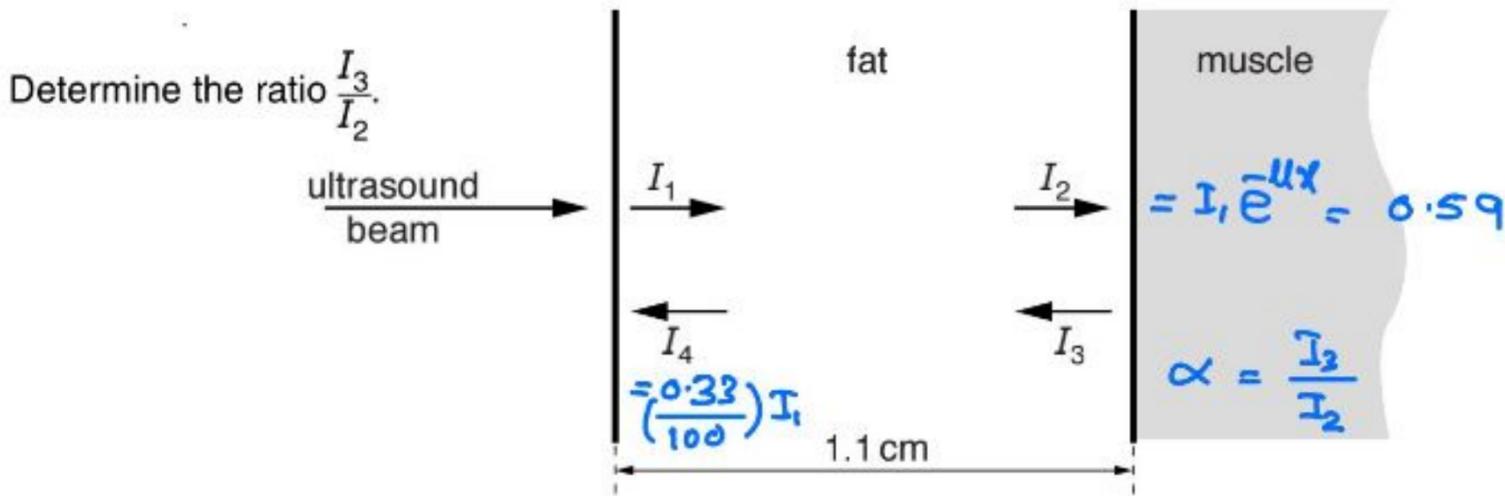


Fig. 6.1

For the ultrasound,

- I_1 is the intensity just after entering the surface of the fat layer,
- I_2 is the intensity incident on the fat-muscle boundary,
- I_3 is the intensity reflected from the fat-muscle boundary,
- I_4 is the intensity received back at the surface of the fat layer.

Some data for the fat are given in Fig. 6.2.

specific acoustic impedance Z	$1.4 \times 10^6 \text{ kg m}^{-2} \text{ s}^{-1}$
density ρ	940 kg m^{-3}
absorption (attenuation) coefficient μ	48 m^{-1}

Fig. 6.2

- (a) Calculate the time interval between a short pulse of ultrasound initially entering the layer of fat and then returning back to the surface of the fat layer.

Initially calculate speed: $Z = \rho c$
 $1.4 \times 10^6 = (940) (c)$
 $c = 1.4 \times 10^3 \text{ m s}^{-1}$

Speed = $\frac{\text{Distance}}{\text{time}}$
 $1.4 \times 10^3 = \frac{2(1.1 \times 10^{-2})}{t}$

$t = 1.47 \times 10^{-5}$

time = 1.47×10^{-5} s [3]

- (b) Calculate the ratio $\frac{I_2}{I_1}$.

$$I_2 = I_1 e^{-\mu x}$$

$$\frac{I_2}{I_1} = e^{-(48)(1.1 \times 10^{-2})}$$

$$= 0.59$$

ratio =[2]

- (c) Intensity I_4 is 0.33% of intensity I_1 .

Determine the ratio $\frac{I_3}{I_2}$.

$$I_4 = I_3 e^{-\mu x}$$

$$I_4 = \alpha I_2 e^{-\mu x}$$

$$I_4 = \alpha (I_1 e^{-\mu x}) e^{-\mu x}$$

$$I_4 = \left(\frac{I_3}{I_2}\right) I_1 e^{-2\mu x}$$

$$\left(\frac{0.33}{100}\right) I_1 = \left(\frac{I_3}{I_2}\right) I_1 e^{-2(48)(1.1)}$$

$$\frac{0.33}{100} = \left(\frac{I_3}{I_2}\right) e^{-2(48)(1.1)}$$

$$\frac{I_3}{I_2} = 9.5 \times 10^{-3}$$

ratio =[2]

- (d) The specific acoustic impedance of the muscle is greater than that of the fat.

State the effect, if any, on the value of the ratio $\frac{I_3}{I_2}$ of an increase in the difference between the specific acoustic impedance of the muscle and that of the fat.

$$\alpha = \frac{I_3}{I_0} = \frac{(Z_m - Z_f)^2}{(Z_m + Z_f)^2} \Rightarrow \frac{I_3}{I_0} \approx 1 \Rightarrow I_3 \approx I_0$$

i.e. Reflected intensity is greater than transmitted intensity i.e. transmitted intensity into muscle is negligible. [1]

[Total: 8]

- 4 (a) (i) State what is meant by the *specific acoustic impedance* of a medium. $Z = \rho c$

Product of density of a medium and speed of sound wave in it

[2]

- (ii) The intensity reflection coefficient α is given by the expression

$$\alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

Explain the meanings of the symbols in this expression.

α : $= \frac{I_R}{I_0}$: Ratio of reflected intensity to incident intensity.

Z_2 and Z_1 : Specific acoustic impedances of medium 2 and medium 1 respectively.

[2]

- (b) A parallel beam of ultrasound has intensity I_0 as it enters a muscle. The beam passes through a thickness of 3.4 cm of muscle before being incident on the boundary with a bone, as shown in Fig. 4.1.

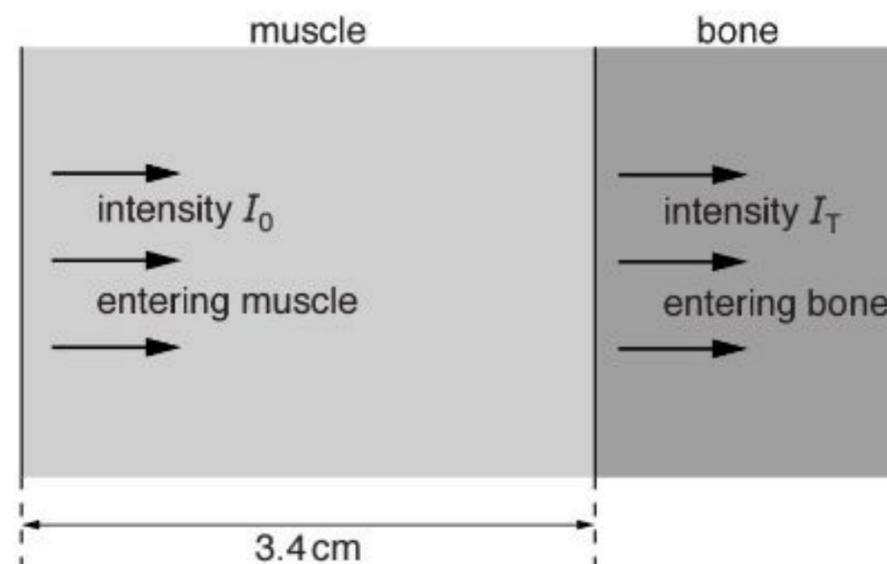


Fig. 4.1

The intensity of the ultrasound beam as it passes into the bone is I_T .

Some data for muscle and bone are given in Fig. 4.2.

	linear absorption coefficient/ m^{-1} μ	specific acoustic impedance $/\text{kg m}^{-2} \text{s}^{-1}$ Z
muscle	23	1.7×10^6
bone	130	6.3×10^6

Fig. 4.2

Calculate the ratio $\frac{I_T}{I_0}$.

$$\frac{I_{\text{Transmitted}}}{I_{\text{incident}}} = (1 - \alpha) e^{-\mu x}, \quad \alpha = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}$$

$$\alpha = \frac{[(6.3 - 1.7) \times 10^6]^2}{[(6.3 + 1.7) \times 10^6]^2} = 0.33$$

$$\begin{aligned} \frac{I_T}{I_0} &= (1 - \alpha) e^{-\mu_m x_m} \\ &= (1 - 0.33) e^{-(23)(3.4 \times 10^{-2})} \end{aligned}$$

ratio = 0.31 [5]

= 0.31

[Total: 9]

24.2 Production and use of X-rays

Candidates should be able to:

- 1 explain that X-rays are produced by electron bombardment of a metal target and calculate the minimum wavelength of X-rays produced from the accelerating p.d.
- 2 understand the use of X-rays in imaging internal body structures, including an understanding of the term contrast in X-ray imaging
- 3 recall and use $I = I_0 e^{-\mu x}$ for the attenuation of X-rays in matter
- 4 understand that computed tomography (CT) scanning produces a 3D image of an internal structure by first combining multiple X-ray images taken in the same section from different angles to obtain a 2D image of the section, then repeating this process along an axis and combining 2D images of multiple sections

X-rays:- e.m. wave having frequency of the order of 10^{18} Hz

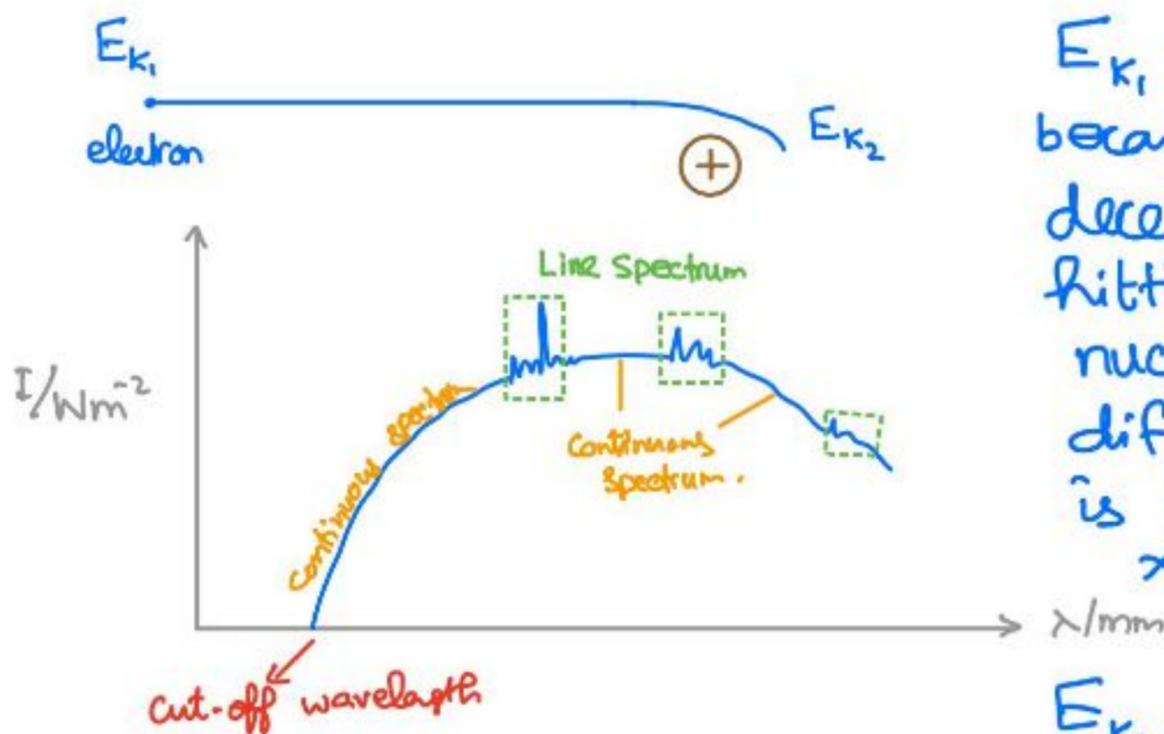
Production Principle:-

X-rays are produced when fast moving electrons hit a metallic target. {decelerate}

X-ray spectrum:-

→ Continuous spectrum
→ Line spectrum

Continuous spectrum:



$E_{k_1} > E_{k_2}$
because electron decelerate on hitting the target nucleus and this different of K.E is radiated as X-ray photon

$$E_{k_1} - E_{k_2} = \frac{hc}{\lambda}$$

Cut-off wavelength:

$$\Delta E = \frac{hc}{\lambda}$$

$\lambda = \text{minimum}$ if $\Delta E = \text{maximum}$

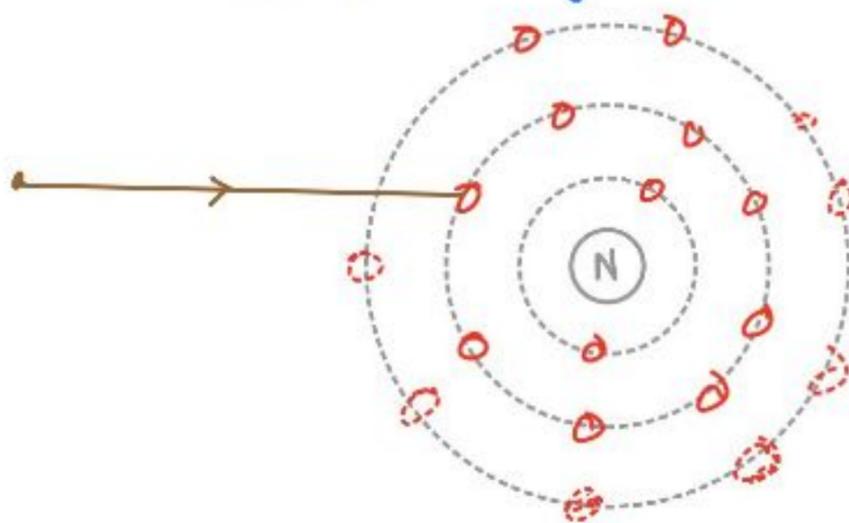
$$\Delta E = E_{K_1} - E_{K_2} = \text{maximum}$$

$\Delta E = \text{maximum}$ if there is no subtraction operation.

So this is possible if an electron comes so close to +ve nucleus of a target metal, so that this electron is captured and its final K.E. is zero.



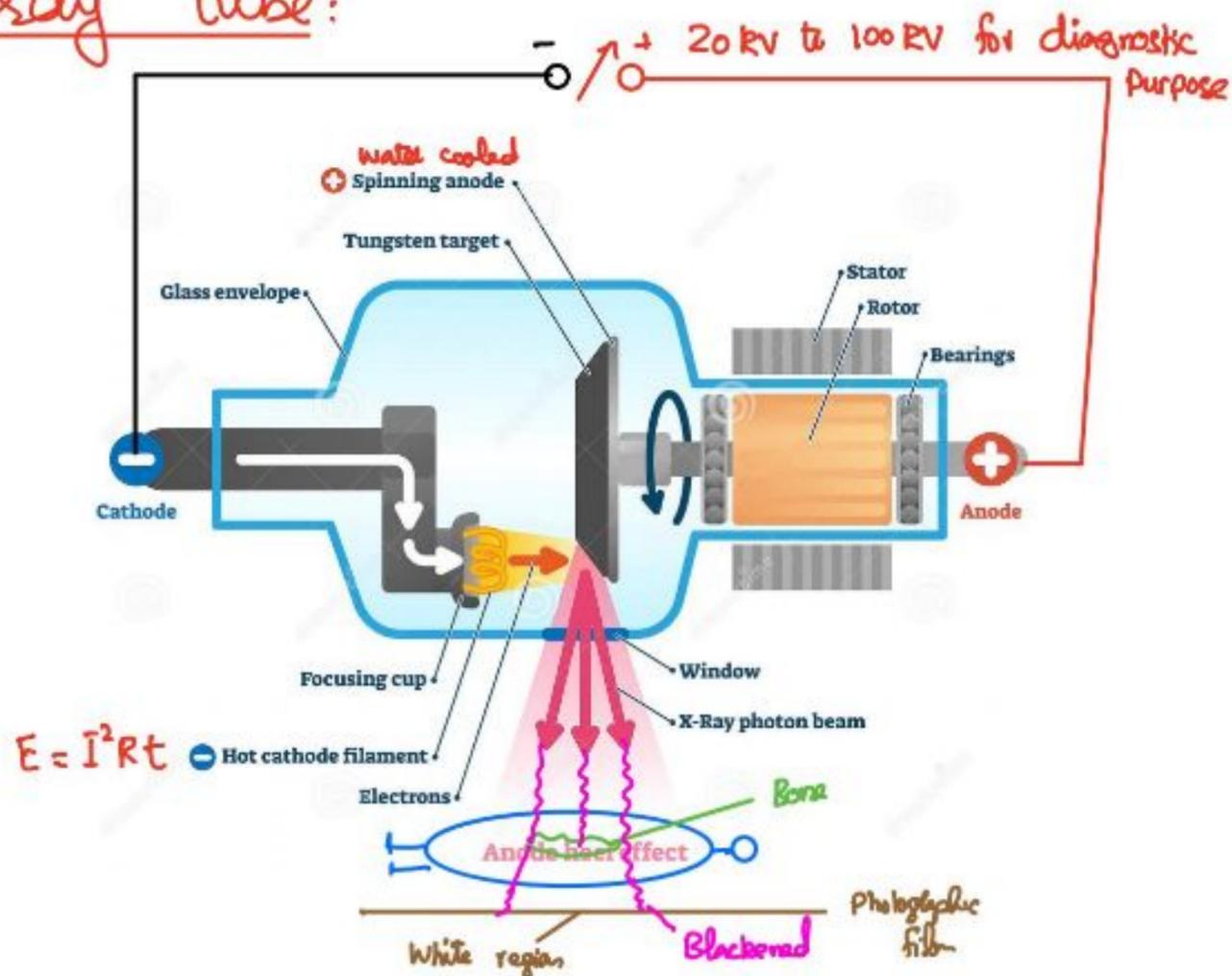
Line/Characteristic spectrum: These x-rays are emitted due to de-excitation of excited atoms of target metal.



$$E_n - E_p = \frac{hc}{\lambda}$$

$$Q = mc \Delta Q$$
$$\Delta Q = \frac{Q}{m c}$$

X-ray tube:



Intensity of X-ray beam:-

$$I = \frac{\text{Total Energy}}{\text{time} \times \text{Area}} \Rightarrow I = \frac{n E}{t A}$$

$$e = R \phi = \text{const}$$

(rate of arrival of electrons at the metal target) ↑
 i.e (tube current) ↑. This tube current is controlled by current provided to heater/filament cathode.

Hard and Soft X-rays:-

Hard X-rays: Greater penetration ability. This is controlled by accelerating voltage provided

b/w cathode filament and Anode metallic target
i.e

(P.d) \uparrow , (E_k) \uparrow , More penetrating x-rays
with higher photon energy
are produced.

$$\begin{aligned} E_k &= E_p \\ \uparrow E_k &= \uparrow V_e \end{aligned}$$

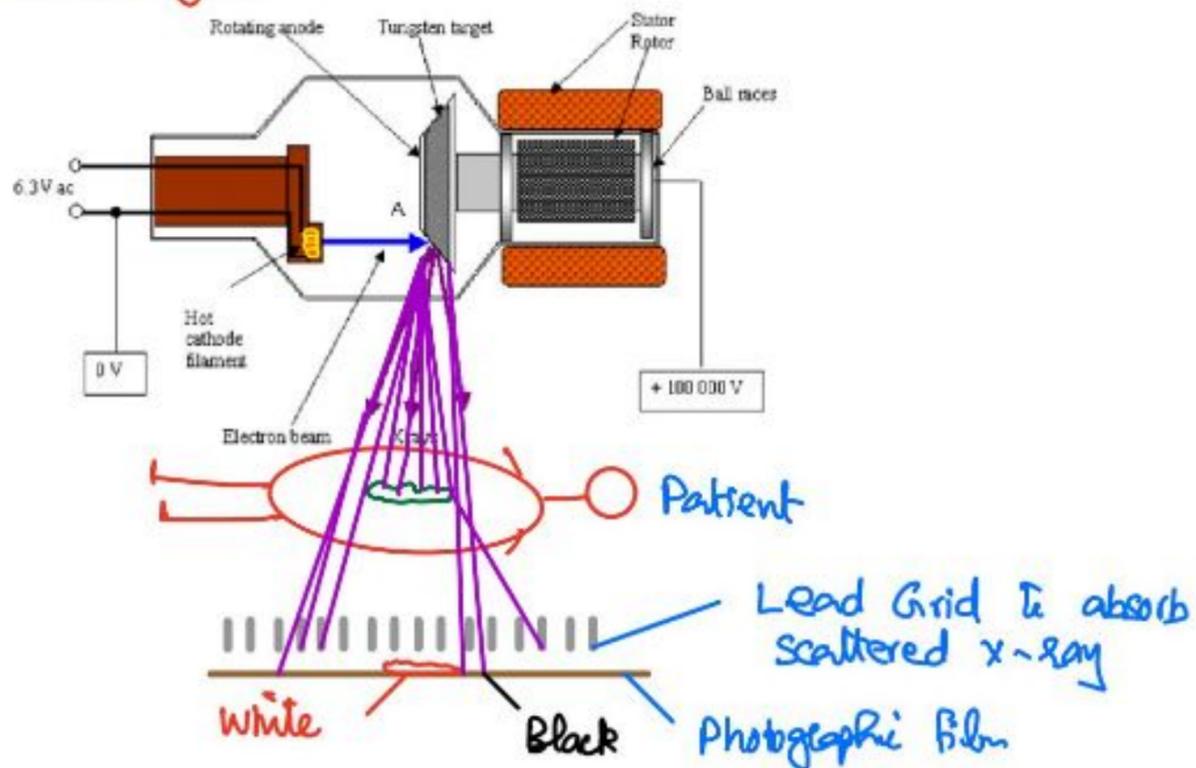
Soft x-rays: Weak penetration ability and even
can not pass through flesh and do not
contribute in image formation.

$$E = \frac{hc}{\lambda}$$

\downarrow \uparrow

So soft x-rays must be absorbed at the
x-ray window of tube.

X-ray image.



X-rays are blocked by bones and can pass through flesh. Area of photographic film exposed to X-rays is blackened and the region where X-rays can not hit is white. So a black and white image is obtained at film.

Contrast of X-ray image:-

It is the difference b/w black and white part on the photographic film.

(Contrast) \uparrow if

- (i) no change in white part of image.
- (ii) but dark region becomes jet black.

(a) Use the contrast medium:- Barium Sulphate for stomach. Barium Sulphate sticks to the walls of stomach and absorbs X-rays to provide a white image on photographic film. Similarly contrast medium is injected in blood to scan blood vessels.

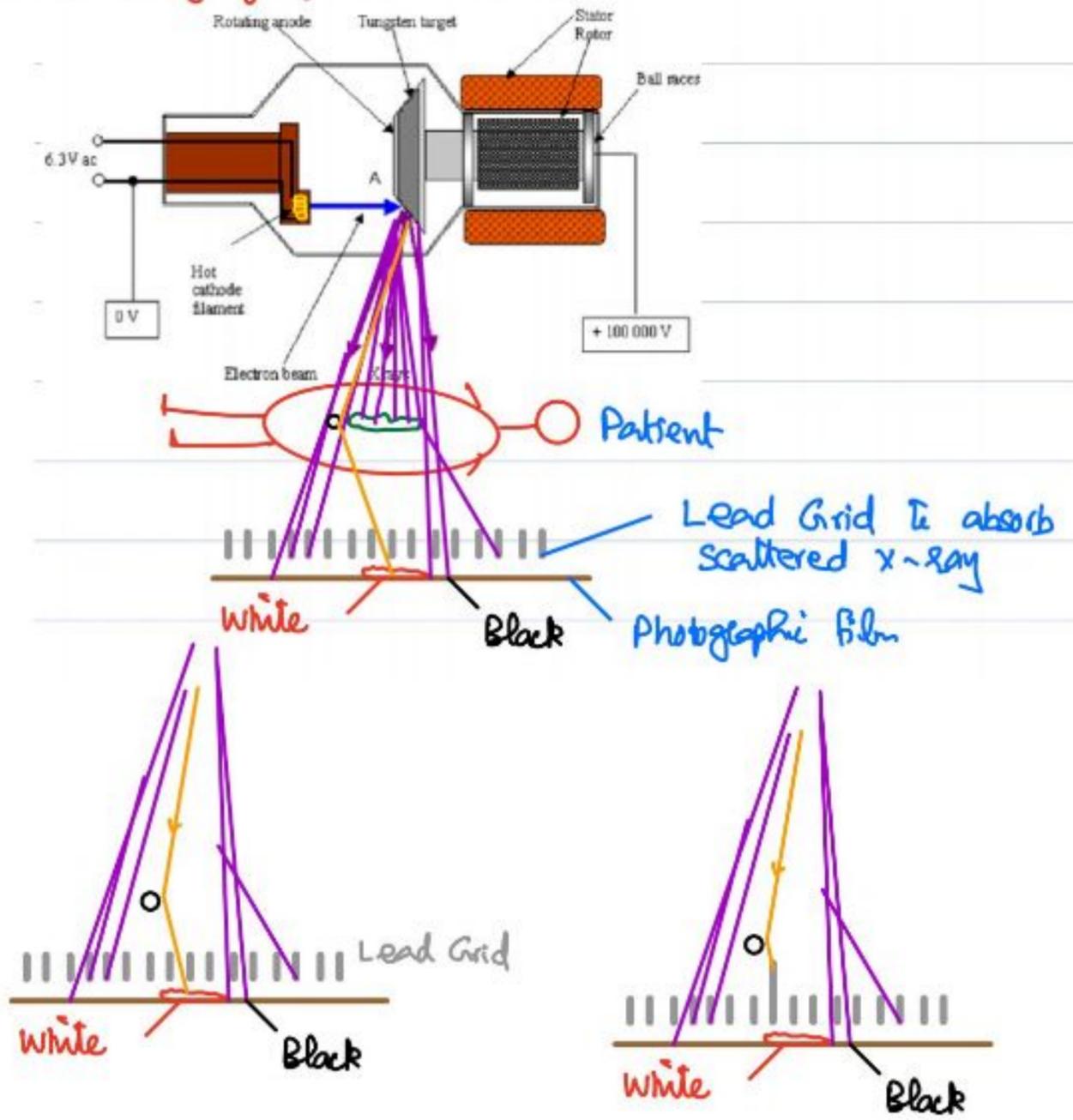


(b) Exposure time of X-rays to patient increases

Bone keeps on absorbing X-rays but flesh allows them to pass through. So greater no. of X-rays hit the photographic film to make it jet black.

(c) Paste a fluorescent material on the photographic film: Fluorescent material is blacked when exposed to X-rays.

(c) Scattering of x-rays from the patient's body is reduced or scattered x-rays are absorbed by Lead grid b/w patient and the photographic film



- 11 (a) Electrons are accelerated through a potential difference of 15 kV. The electrons collide with a metal target and a spectrum of X-rays is produced.

- (i) Explain why a continuous spectrum of energies of X-ray photons is produced.

X-rays are produced when electrons are decelerated on hitting the metallic target.

$$E_{k_1} - E_{k_2} = \frac{hc}{\lambda} \Rightarrow \Delta E_e \propto \frac{1}{\lambda}$$

Different wavelengths show the range of decelerations produced on hitting target. [3]

- (ii) Calculate the wavelength of the highest energy X-ray photon produced.

$$\begin{aligned}
 E_{k_1} - E_{k_2} &= \frac{hc}{\lambda} \\
 ve - 0 &= \frac{hc}{\lambda} \\
 (15 \times 10^3)(1.60 \times 10^{-19}) - 0 &= \frac{(6.63 \times 10^{-34})(3.0 \times 10^8)}{\lambda} \\
 \lambda &= \frac{8.29 \times 10^{-13}}{\lambda} \text{ m}
 \end{aligned}$$

$E_k = E_p$
 $E_k = ve$

wavelength = m [3]

- (b) A beam of X-rays has an initial intensity I_0 . The beam is directed into some body tissue. After passing through a thickness x of tissue the intensity is I . The graph in Fig. 11.1 shows the variation with x of $\ln(I/I_0)$.

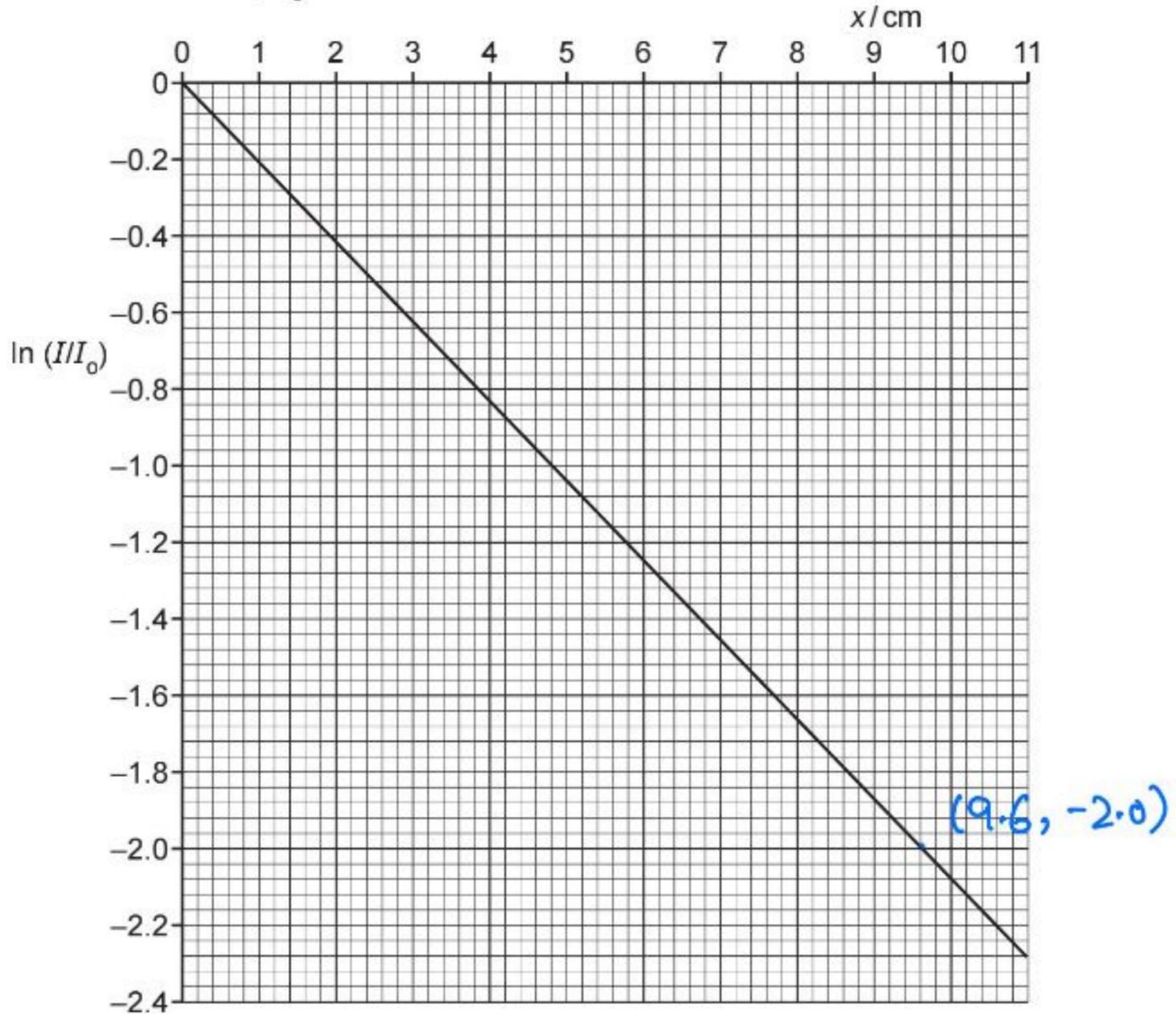


Fig. 11.1

- (i) Determine the linear attenuation (absorption) coefficient μ for this beam of X-rays in the tissue.

$$I = I_0 e^{-\mu x}$$

$$\frac{I}{I_0} = e^{-\mu x}$$

$$\ln\left(\frac{I}{I_0}\right) = -\mu x \quad \ln e$$

$$\mu = -\frac{\ln\left(\frac{I}{I_0}\right)}{x} \Rightarrow \mu = -(\text{Gradient of graph})$$

$$\mu = -\left(\frac{-2.0 - 0}{9.6 - 0}\right) = 0.21$$

$\mu = \dots\dots\dots 0.21 \dots\dots\dots \text{cm}^{-1}$ [2]

- (ii) Determine the thickness of tissue that the X-ray beam must pass through so that the intensity of the beam is reduced to 5.0% of its initial value.

$$I = I_0 e^{-\mu x}$$

$$\frac{5}{100} I_0 = I_0 e^{-(0.21)x}$$

$$0.05 = e^{-(0.21)x}$$

$$\ln(0.05) = (-0.21)x$$

$$x = 14.3$$

thickness = $\dots\dots\dots 14.3 \dots\dots\dots \text{cm}$ [2]

[Total: 10]

12 High-energy electrons collide with a metal target, producing X-ray photons.

The variation with wavelength of the intensity of the X-ray beam is illustrated in Fig. 12.1.

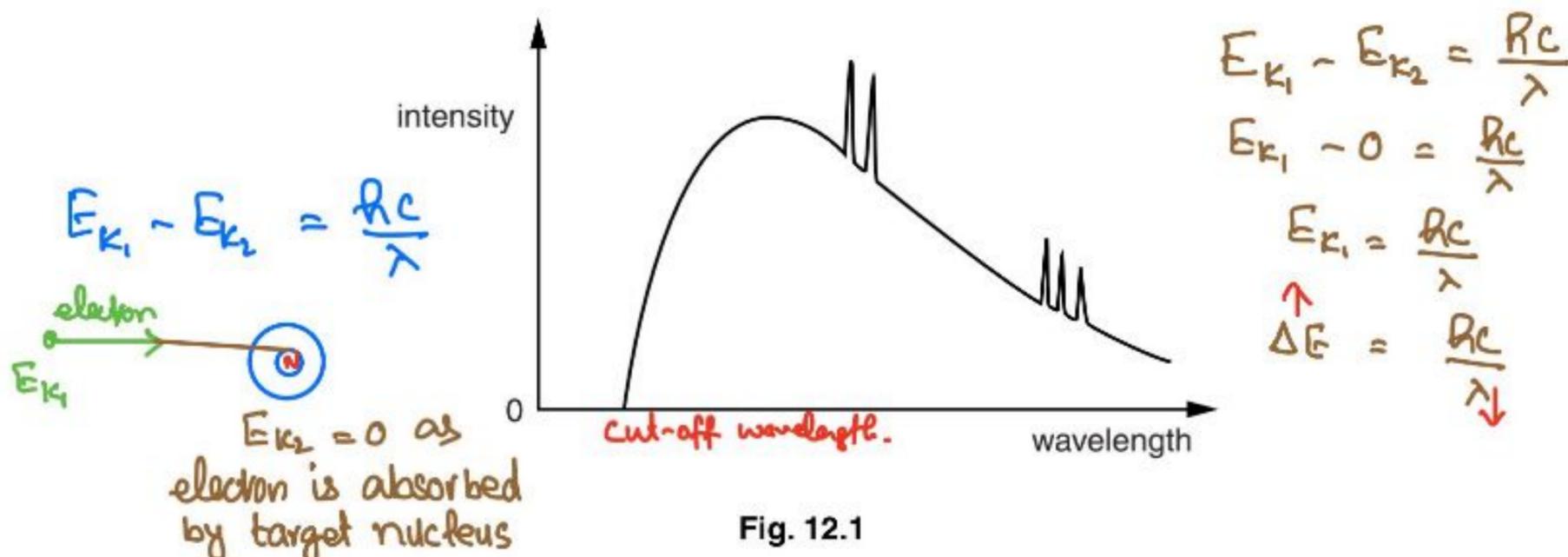


Fig. 12.1

(a) Explain why there is

(i) a continuous distribution of wavelengths,

X-rays are produced when electrons are decelerated on hitting the metallic target.

$$E_{K_1} - E_{K_2} = \frac{hc}{\lambda} \Rightarrow \Delta E \propto \frac{1}{\lambda}$$

Different wavelengths show the range of decelerations produced on hitting target. [3]

(ii) a sharp cut-off at short wavelength,

Now electron is absorbed by the nucleus of target metal and is stopped resulting an increase of difference of K.E of electron to emit X-rays of shortest wavelength, [2]

(iii) a series of peaks superimposed on the continuous distribution of wavelengths.

This is caused due to de-excitation of atoms of target nucleus from different energy level. [1]

(b) In the X-ray imaging of body structures, longer wavelength photons are frequently filtered out of the X-ray beam.

$$E = \frac{hc}{\lambda}$$

(i) State how this filtering is achieved.

Aluminium filters are used to block soft X-rays [1]

(ii) Suggest the reason for this filtering.

Longer wavelength x-rays have lower energy
and can not pass through patient and [1]
do not contribute in image formation, [Total: 8]
but the dose of x-rays to patient is
increased.

$$Ve = \Delta E_k = \frac{hc}{\lambda}$$

- 11 (a) State and explain how, in an X-ray tube, the hardness of the X-ray beam is controlled.

It is controlled by accelerating voltage defined b/w Cathode and +vely charge metallic target. The Hard x-rays have shorter wavelength and depends upon the range of different of K.E of electrons. [3]

- (b) A parallel beam of X-rays has intensity I_0 and is incident on a medium having a linear absorption (attenuation) coefficient μ .

- (i) State an equation for the variation of the intensity I with the thickness x of the medium.

$$I = I_0 e^{-\mu x} \quad [1]$$

- (ii) Data for the linear absorption (attenuation) coefficient μ for an X-ray beam in blood and in muscle is shown in Fig. 11.1.

	μ/cm^{-1}
blood	0.23
muscle	0.22

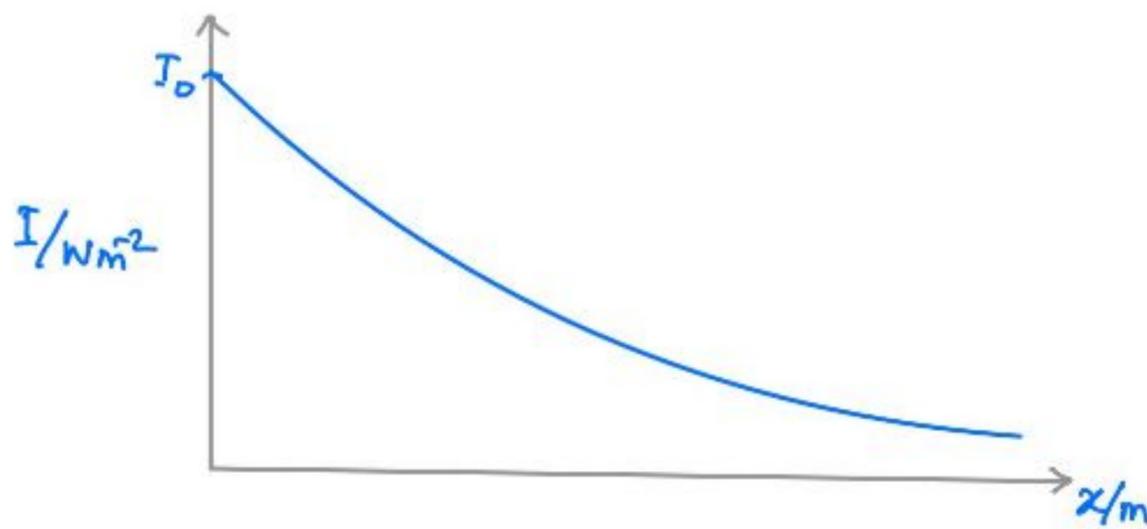
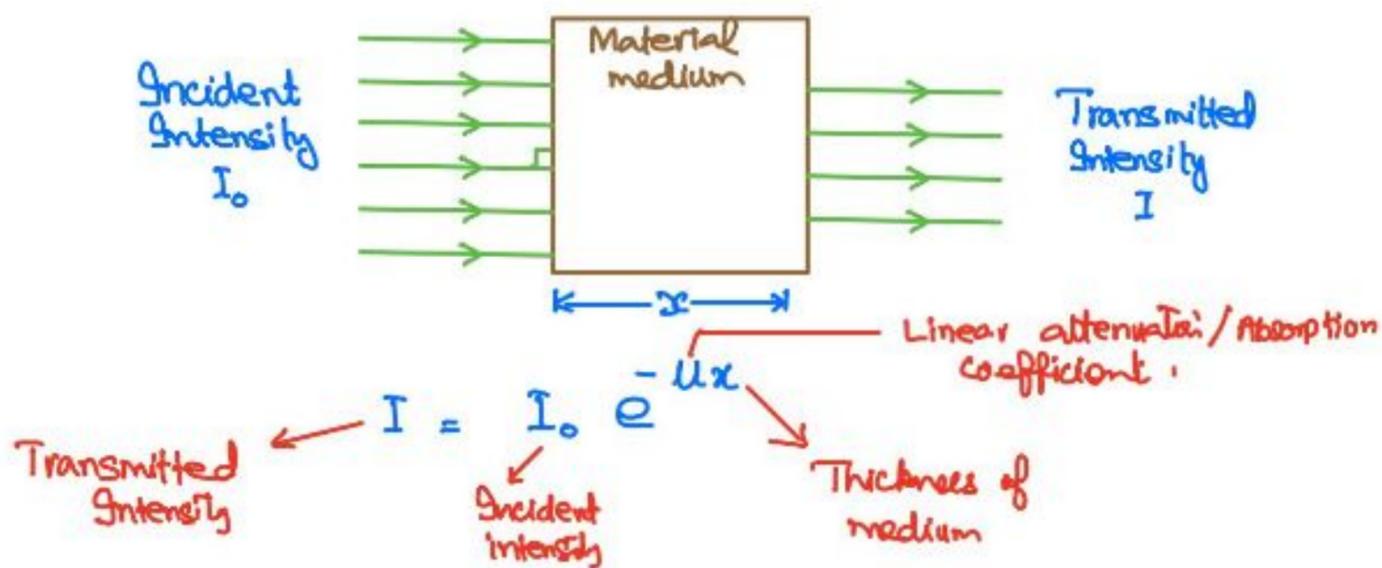
Fig. 11.1

Suggest why, if this X-ray beam is used to image blood vessels in muscle, contrast on the image would be poor.

Readability depends upon the contrast b/w blood vessels and muscle. Since the difference in the absorption coefficient is very small, so there is a very little difference b/w black and white part on photographic film. [2]

Linear absorption coefficient / Linear attenuation coefficient:

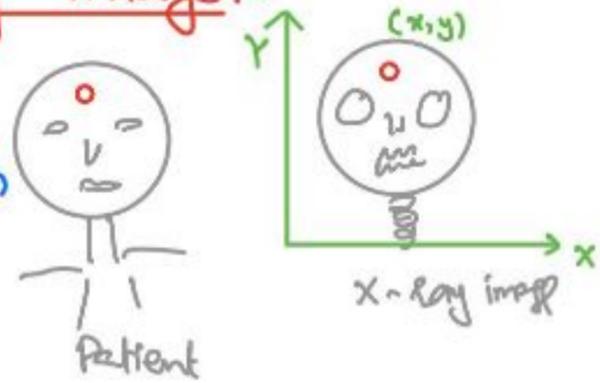
Decrease in amplitude/energy/intensity of a wave due to loss of energy in passing through a medium.



Condition: The incident wave must be perpendicular to the surface of material medium i.e angle of incidence is zero.

Disadvantage of an X-ray image:-

Flat 2D image which does not provide any impression of depth.



Remedy: Computed Tomography scanning (CT scanning)

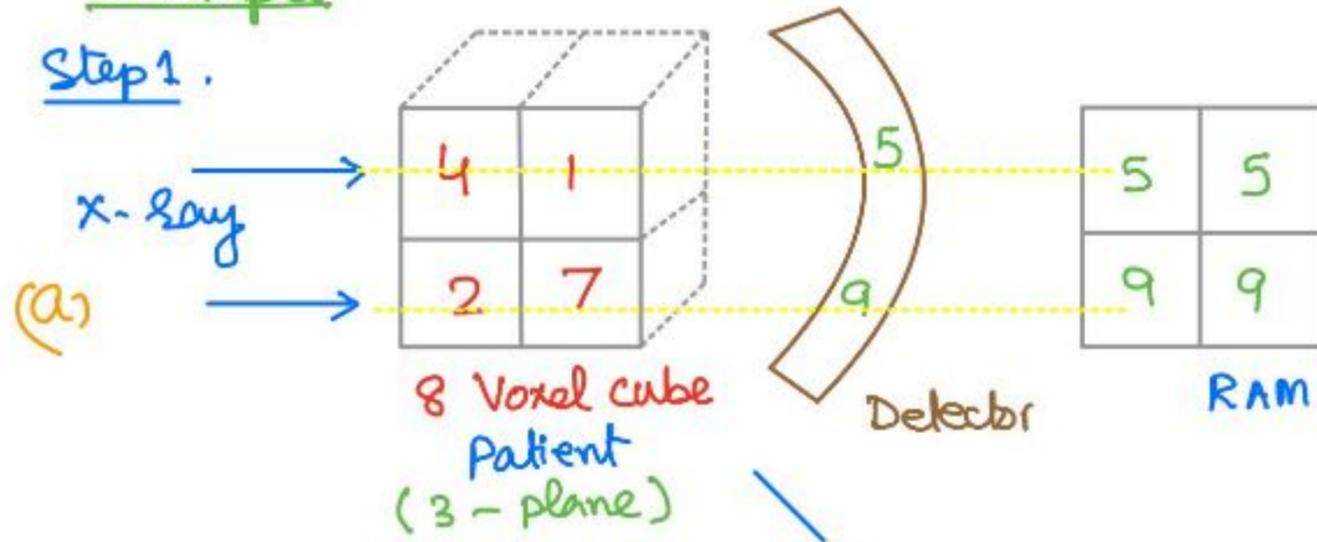
Computed tomography scanning:

Principle:

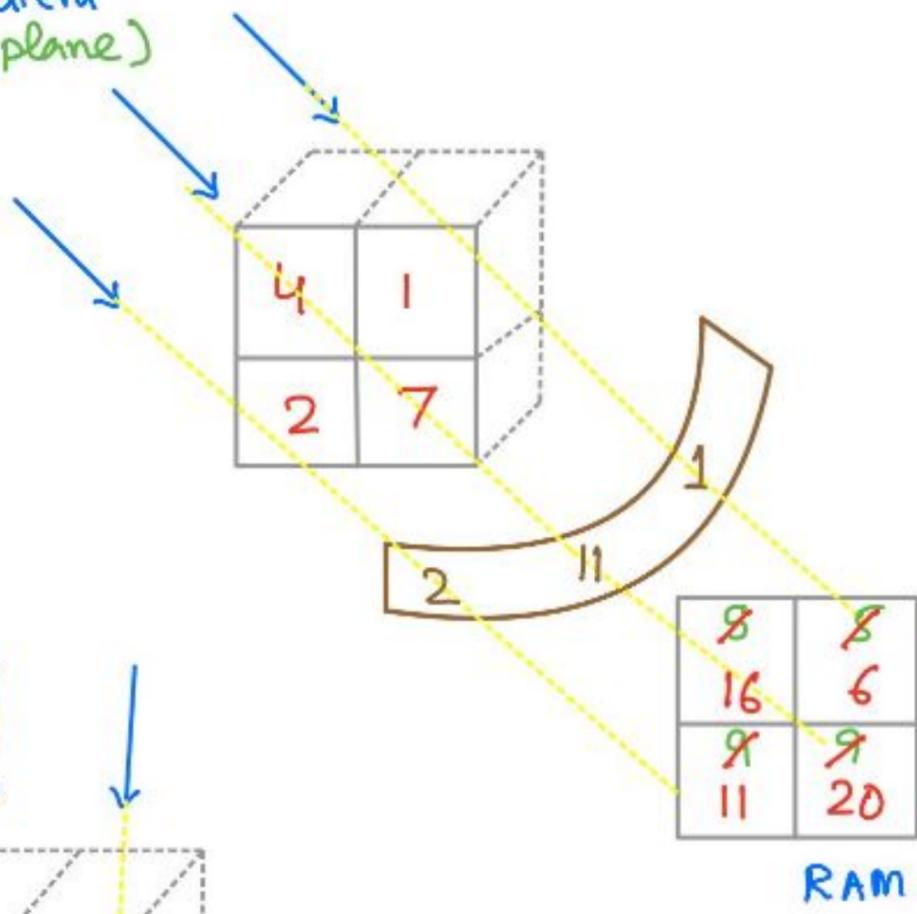
- 1- Multiple X-rays images of a slice and in the plane of slice from different angles.
- 2- These images are combined using a software algorithm to form a 2D-image.
- 3- Repeat step 1 and 2 for other plane of the same slice.
- 4- Use a software programme which combine all these images from different planes to form a 3D image. This 3D image can be magnified, rotated and can be viewed from any angle. The computer hardware used for this purpose is very fast which can execute billions of instructions in the least possible time.

Example:

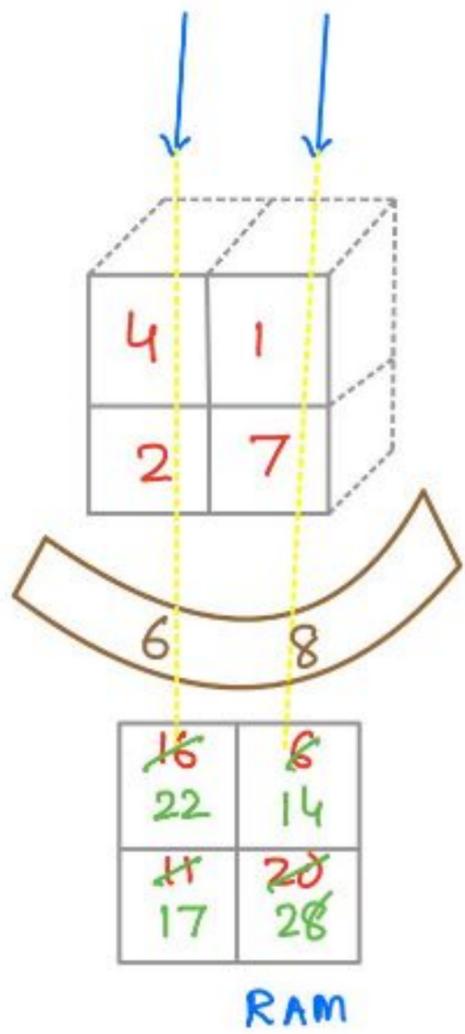
Step 1.



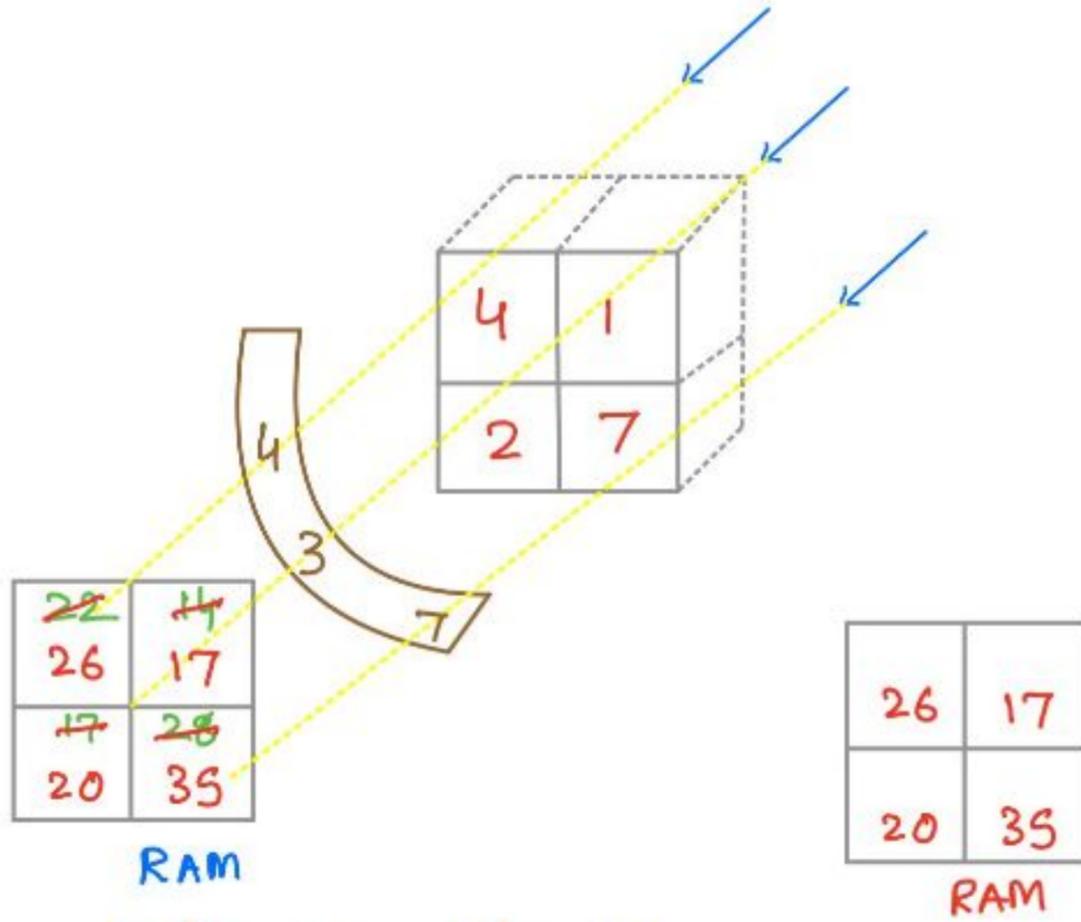
(b)



(c)



(d)



Step 2: Software algorithm

(i) Subtract background intensity from each voxel intensity. Background intensity is the value detected by detector.

$26-14$	$17-14$
12	3
$20-14$	$35-14$
6	21

RAM

(ii) Divide each voxel value/intensity by 3

$\frac{12}{3}$	$\frac{3}{3}$
$\frac{6}{3}$	$\frac{21}{3}$

RAM

=

4	1
2	7

RAM

12 (a) Outline briefly the principles of CT scanning.

- * Multiple x-rays images of a slice and in the plane of slice from different angles.
- * These image are combined to get a 2D image.
- * Repeat same process explained earlier with other planes of same slice to get their respective 2D images.
- * All these 2D images of a slice are combined using a software program to get a 3D image. This 3D image can be rotated and viewed from any angle. [5]

- (b) In a model for CT scanning, a section is divided into four voxels. The pixel numbers P , Q , R and S of the voxels are shown in Fig. 12.1.

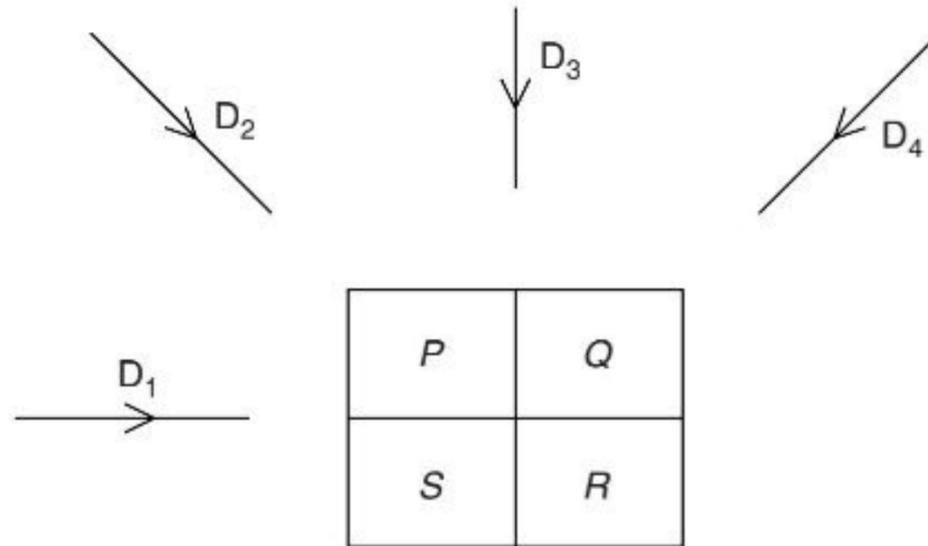


Fig. 12.1

The section is viewed from the four directions D_1 , D_2 , D_3 and D_4 . The detector readings for each direction are noted.

The detector readings are summed as shown in Fig. 12.2.

49	61
73	55

Fig. 12.2

$49-34$ $=15$	$61-34$ $=27$
$73-34$ $=39$	$55-34$ $=21$
15 $\frac{3}{3}$ $=5$	27 $\frac{3}{3}$ $=9$
39 $\frac{3}{3}$ $=13$	21 $\frac{3}{3}$ $=7$

The background reading is 34.

Determine the pixel numbers P , Q , R and S as shown in Fig. 12.3.

P $=5$	Q $=9$
S $=13$	R $=7$

Fig. 12.3

$P =$ 5

$Q =$ 9

$S =$ 13

$R =$ 7

[4]

POSITRON EMISSION TOMOGRAPHY:-

24.3 PET scanning

Candidates should be able to:

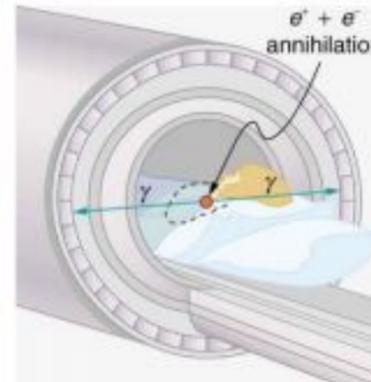
- 1 understand that a tracer is a substance containing radioactive nuclei that can be introduced into the body and is then absorbed by the tissue being studied
- 2 recall that a tracer that decays by β^+ decay is used in positron emission tomography (PET scanning)
- 3 understand that annihilation occurs when a particle interacts with its antiparticle and that mass-energy and momentum are conserved in the process
- 4 explain that, in PET scanning, positrons emitted by the decay of the tracer annihilate when they interact with electrons in the tissue, producing a pair of gamma-ray photons travelling in opposite directions
- 5 calculate the energy of the gamma-ray photons emitted during the annihilation of an electron-positron pair
 $E = hf = \frac{hc}{\lambda}$, $p = \frac{E}{c}$
- 6 understand that the gamma-ray photons from an annihilation event travel outside the body and can be detected, and an image of the tracer concentration in the tissue can be created by processing the arrival times of the gamma-ray photons

24.3 - PET Scanning

Positron emission tomography (PET) scans can be used to form both 3D images and cross-sections of the body through the following process:

1. The patient is injected with a **positron-emitting (β^+ decaying) radionuclide attached to a substance** used by the region of the body under investigation.
2. The patient is left for around an hour to **allow the radionuclide to move** to the region of interest.
3. The radionuclide will then be absorbed and broken down, **releasing positrons** which will collide with electrons present in the body, causing them to become **annihilated**. The minimum energy of each photon emitted is equal to the rest energy of the electron/positron. The energy of the electron and positron is assumed to be shared equally between the gamma ray photons. The photon energy is $hf = E_0$.
4. This releases two high-energy gamma rays, moving in opposite directions, which are recorded by detectors. These signals are sent to a computer for processing, and an **image of the radioactivity in that region can be formed**.

The image formed depends on the **metabolic activity** of the cells in the region, this is because **cells with a high metabolism will break down more of the radionuclide**, causing more annihilation and therefore more gamma radiation to be emitted and detected.



The **advantages** of PET scanners:

- The **metabolic activity** of a region can be measured.
- **Tumours can be detected** and information about **if they are spreading/malignant can be found**.
- **Brain activity** can be easily investigated because gamma rays produced inside the brain can easily pass through the skull.

The **disadvantages** of PET scanners:

- **Ionising radiation** is used, which could cause **damage to the patient's cells**.
- Scans take a **long time** and require patients to stay **very still** inside the scanner, which may be **uncomfortable** and may cause some patients to feel claustrophobic.
- They are **very large** and **expensive**, meaning that a **patient may need to travel a long distance** to get to a hospital which has a PET scanner.

30.8 Positron Emission Tomography

Positron Emission Tomography or PET scanning is another tool in the diagnostic toolbox of modern medicine. It has a range of uses: **investigating**, **diagnosing** and **monitoring treatment of cancers**, **heart disease**, **gastrointestinal disorders** and **brain function**.

The principle operation of PET is different from CT and ultrasound scanning. CT and ultrasound look at the patient from the outside, whereas PET looks at the patient from the inside. A small amount of tracer, sometimes referred to as a **radiotracer**, is injected into a vein, travels round the body and is absorbed by organs and tissues. It is the radiation from this that is used to produce the image.

Radiotracers

There are several different radiotracers used in PET, an example being a glucose based molecule, onto which a radioactive-nuclide, fluorine-18, is attached. This substance is known as fluorodeoxyglucose. The fluorine-18 nuclide decays by emitting a β^+ -particle, a positron. The advantage of using a glucose-based tracer is that it is taken up at different rates by different tissues or organs. Cancer cells are more metabolically active than surrounding healthy cells, consequently they absorb glucose at a higher rate and thus emit radiation at a greater rate. This will then appear on the screen as a bright area, allowing doctors to identify diseases and also determine the progress and effectiveness of any treatment used for the disease. PET scans are not only used for the detection of cancers but are a diagnostic tool in investigating blood flow, heart disease and brain injuries, and they are also being used to investigate Alzheimer's disease and other forms of dementia.

PET scans are unique in that they are able to pinpoint molecular activity within the patient's body, rather than looking at the body from outside. Consequently, they can identify disease in its earliest stages, meaning that there is a greater chance of successful treatment. They can also be used to track a patient's immediate and ongoing response to treatments.

What happens in positron emission?

PET scanners require a radioactive isotope that decays by β^+ emission, the emission of a positron, the antiparticle of the electron, which you met in [Chapter 15](#). Most β^+ emitters are not naturally occurring isotopes and are made by firing protons at target nuclei.

The positron moves through the patient's tissue and within a very short distance (significantly less than a millimetre) it will encounter an electron. The pair will annihilate and their mass becomes pure energy in the form of two γ -rays that move apart in opposite directions. The concept of mass-energy is discussed in detail in [Chapter 29](#).

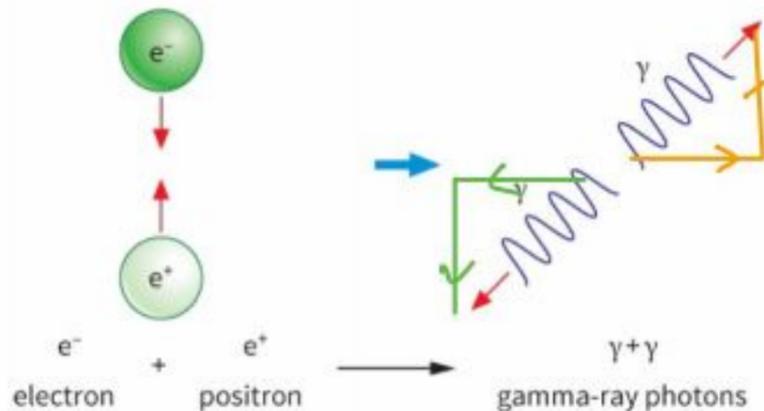


Figure 30.20: Energy is released in the annihilation of a positron and an electron.

In the annihilation process, as in all collisions, both mass-energy and momentum are conserved. The initial kinetic energy of the positron is small - negligible compared to their rest mass-energy - hence, the γ -ray photons have a specific energy and a specific frequency that are determined, solely, by the mass-energy of the positron-electron pair.

The energy of a photon is given by:

$$E = hf$$

where h is Planck's constant, and f is the frequency of the photon.

The momentum of a photon is given by:

$$P = \frac{E}{c}$$

where c is the speed of electromagnetic radiation in a vacuum.

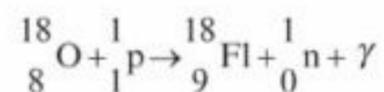
KEY EQUATION

$$\begin{aligned} \text{energy of a photon} &= E = hf \\ \text{momentum of a photon} &= p = \frac{E}{c} \end{aligned}$$

The production of suitable radioisotopes

You will remember from your work on magnetic fields that a charged particle entering a magnetic field at right angles to the field will travel in a circular path. The cyclotron works on this principle, however, the particles are continuously accelerated by an alternating electric field as they go round the circle, thus they travel in a spiral path before they are released and collide with the target nuclei. Figure 30.21 shows an early cyclotron. The principle of the cyclotron is shown in Figure 30.22.

In the production of fluorine-18, oxygen-18 nuclei are bombarded with protons and the following reaction takes place:



The isotope, fluorine-18, has a half-life of just under two hours. This means the patient is not subjected to radiation for a long period of time. However, it also means that the radiotracer needs to be made up freshly, probably on site, to be most effective.

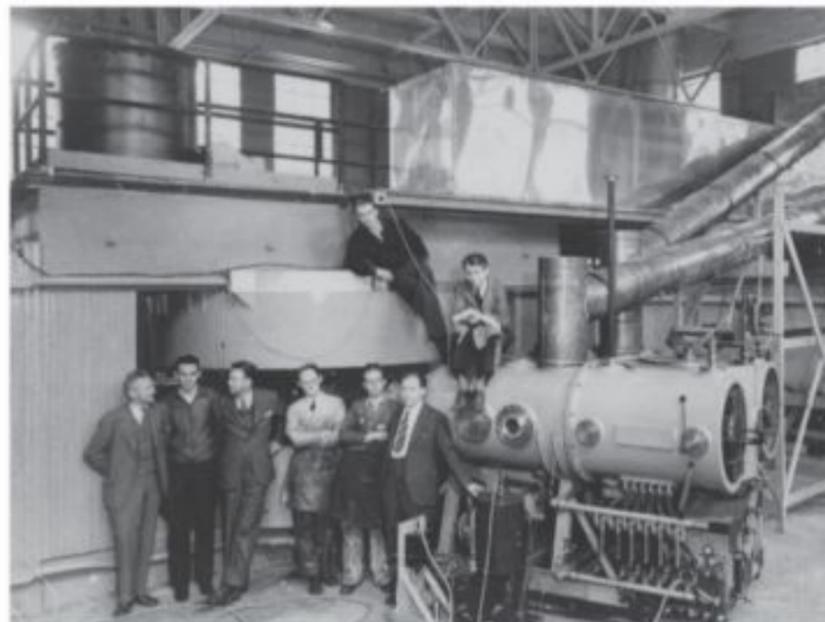


Figure 30.21: The cyclotron at the Lawrence Radiation Laboratory, Berkeley, soon after completion in 1939.

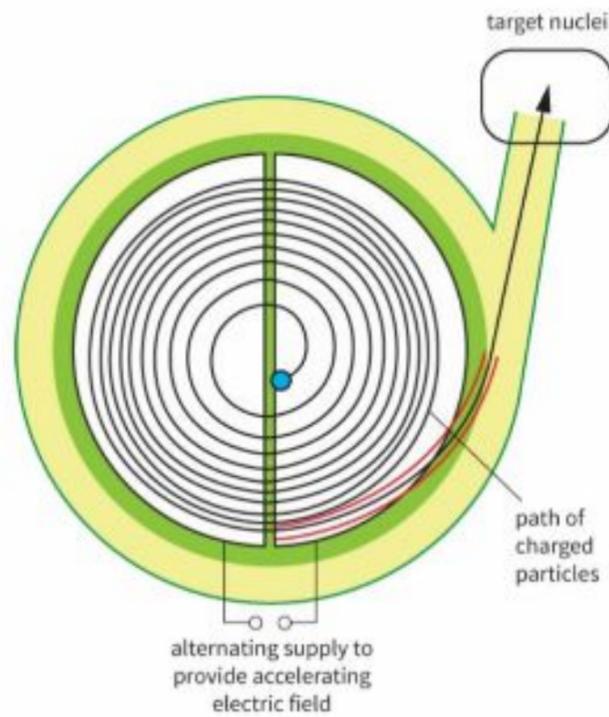


Figure 30.22: The principle of the cyclotron. Note, the magnetic field is into the plane of the page.

Questions

- 18 Suggest the reason why, in PET scanning, it is important that the positron meets an electron within a very short distance from its point of emission.
- 19 Explain why the γ -rays produced in positron-electron annihilation must travel at 180° to each other.
- 20 Fluorine-18 decays by β^+ emission. Write a nuclear equation to show this decay.
- 21 a Calculate the energy released when a positron and an electron annihilate.
(Mass of an electron = mass of a positron = 9.1×10^{-31} kg.)
- b Calculate the frequency of the γ -rays emitted.
- c Calculate the momentum of the one of the γ -rays emitted.

$$E = hf \Rightarrow mc^2 = hf \Rightarrow (9.1 \times 10^{-31}) (3.0 \times 10^8)^2 = (6.63 \times 10^{-34}) f$$

$$p = \frac{E}{c} = \frac{mc^2}{c} = \frac{mc}{2}$$

Detecting the γ -rays

The patient being scanned is placed on a bed with a series of rings of detectors, in a donut type shape. The patient on the bed is moved through the detectors, so that a series of images of 'slices' through the patient are made in similar manner to those made by a CT scan. Indeed, PET scans are often combined with CT scans so that more information is gathered.



Figure 30.23: A patient being prepared for a PET/CT scan by a radiologist. Note the donut shaped ring,

which contains the detectors and through which the patient will be moved.

The detectors of the γ -ray photons consist of two parts: a crystal that scintillates and a photomultiplier. When a high energy γ -ray photon is incident on the crystal, an electron is excited into a very high energy state. As the electron travels through the crystal, it loses energy and excites more electrons; these electrons then decay back to their original state, emitting visible light photons. The photons produced by the scintillator are then converted into an electrical signal by the photomultiplier tube - these signals are then fed to a computer that can plot back where the photon pair was originally produced.

Reconstruction of the image

Figure 30.24 shows a simplified view of the detectors in a PET scanners. They form a series of rings around the patient. The γ -ray photons, formed by an electron-positron annihilation, travel from a point very near to the event. They travel in a straight line and in opposite directions and strike the detectors as shown. A line (known as the **line of response**) can be drawn, joining the two detectors. Using the time lapse between the two photons arriving at the detectors, the position on the line of response can be established. In practice, there are many annihilations and sophisticated computers analyse the data and convert it into an image. The numbers of photons arriving from a particular point determine the concentration of the tracer at that point. Where there are many arriving per unit time, it means that there is a high concentration of tracer and this will appear as a bright point on the image.

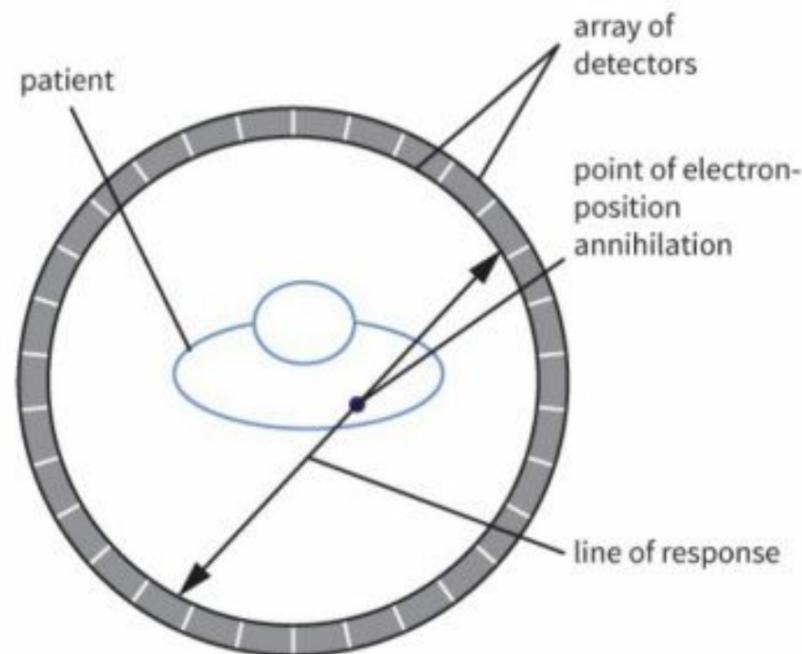


Figure 30.24: The arrangement of detectors in a PET scanner.

REFLECTION

It is about 120 years since X-rays were discovered. Modern medicine has many methods for looking inside the bodies of people who are unwell or have suffered injuries. Use the internet to find as many different methods as you can. Try and draw a timeline to show when these methods were developed.

What did you learn about yourself as you worked on this activity? Did you find it a useful way of learning?

Summary of PET:

WHAT IS A POSITRON?

- BELONG TO THE FAMILY OF LEPTONS
- FUNDAMENTAL PARTICLES
- ANTIMATTER PARTICLE OF ELECTRON

RADIOTRACERS

- IT IS A SUBSTANCE THAT CONSISTS OF :
 1. A RADIO ACTIVE MATERIAL [F-18]
 2. A NATURAL CHEMICAL SUBSTANCE [GLUCOSE]
 3. INJECTED INTRAVENOUSLY

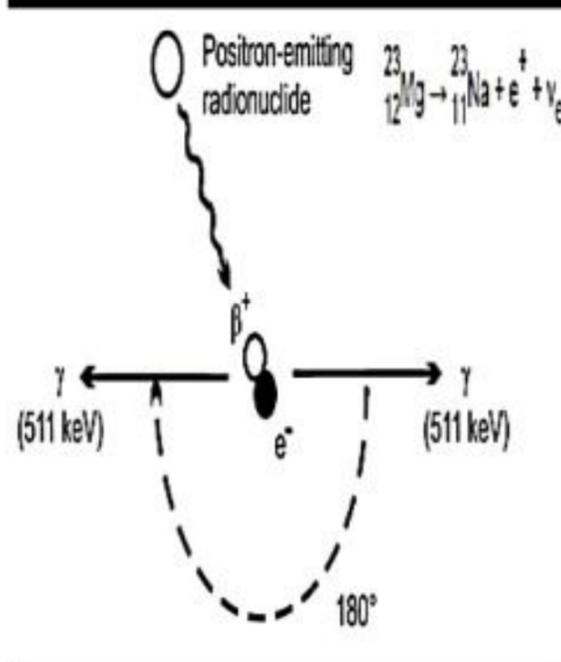
WHY USE GLUCOSE?

- IT IS A NATURAL ORGANIC COMPOUND THAT IS READILY USED BY METABOLICALLY ACTIVE CELLS AND TISSUES OF BODY
- RATE OF UPTAKE BY TUMORS IS MUCH FASTER AS THEY ARE CONTINUOUSLY DIVIDING
- HENCE THEY EMIT RADIATIONS AT A FASTER RATE

MECHANISM OF ACTION

- ONCE ABSORBED BY TARGET CELLS [CANCER CELLS] THE ISOTOPE DECAYS BY EMITTING POSITRON [B+ particle]
- This cannot travel a long distance from the target cell, significantly less than a few millimeters
- They come across an electron which are present in the tissues of body

ANNIHILATION



- IT OCCURS WHEN A PARTICLE [ELECTRON] COMES ACROSS ITS ANTI-PARTICLE [POSITRON]
- THE PAIR ANNIHILATES AND IN THE PROCESS MASS ENERGY AND MOMENTUM IS CONSERVED
- THEY WILL BECOME PURE ENERGY IN THE FORM OF GAMMA RAY PHOTONS AND WILL MOVE IN OPPOSITE DIRECTION

How to calculate energy of photon

- $E=hf$
- E is the energy of photon
- h is the planks constant
- f is the frequency of photon

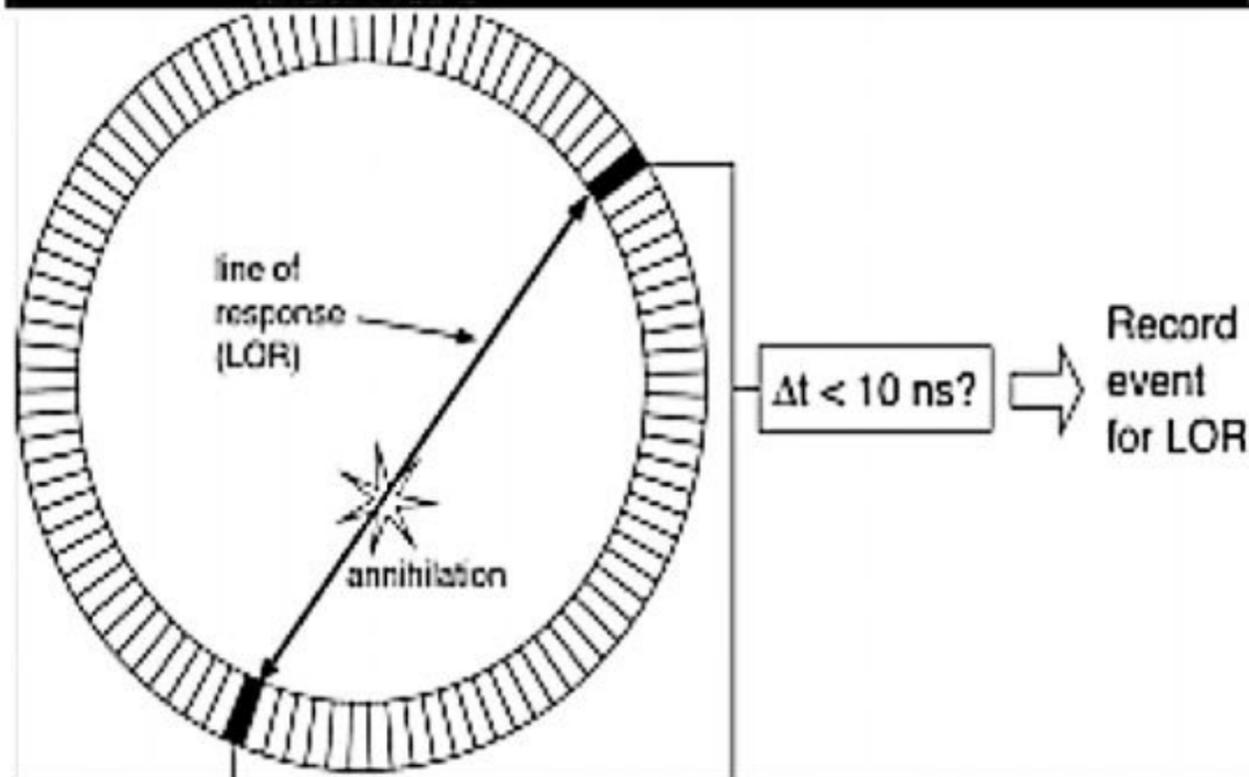
- Momentum is also conserved during annihilation and is given by
- $P=E/c$
- Where is the speed of electromagnetic radiation

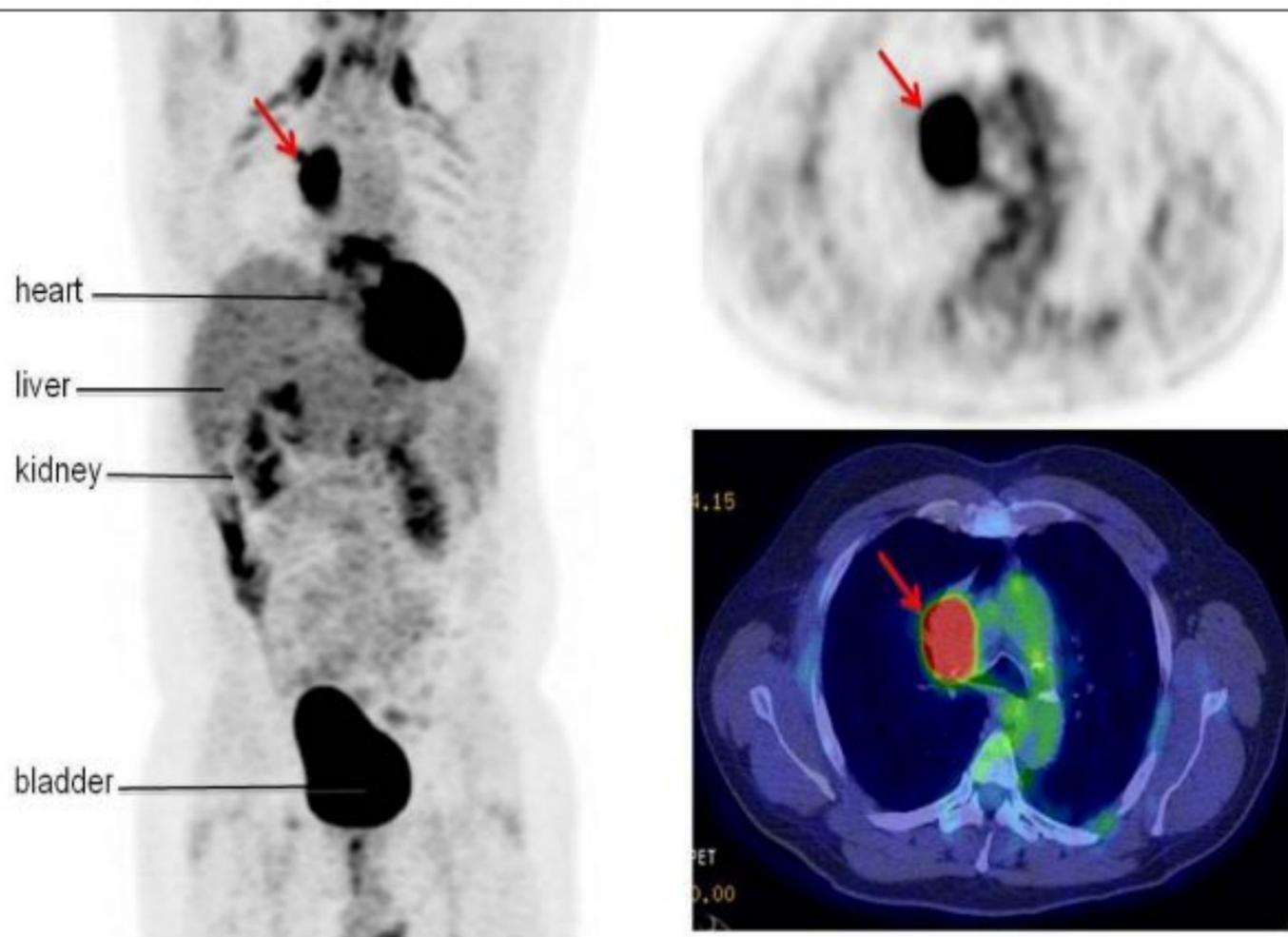
Producing an image

- The gamma ray photons travel outside the body
- A series of ring detectors, in a donut type shape, detect these radiations
- As gamma ray photons travel in a straight line and in opposite direction, a line of response can be drawn joining the two detectors
- Using time lapse between the arrival of two photons the position of the annihilation can be located hence pinpointing the site of tumour

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This 55-year-old smoking man developed a new cough.