

A LEVEL PHYSICS

DERIVATIONS

**40 of the most important
derivations for A Level Physics**

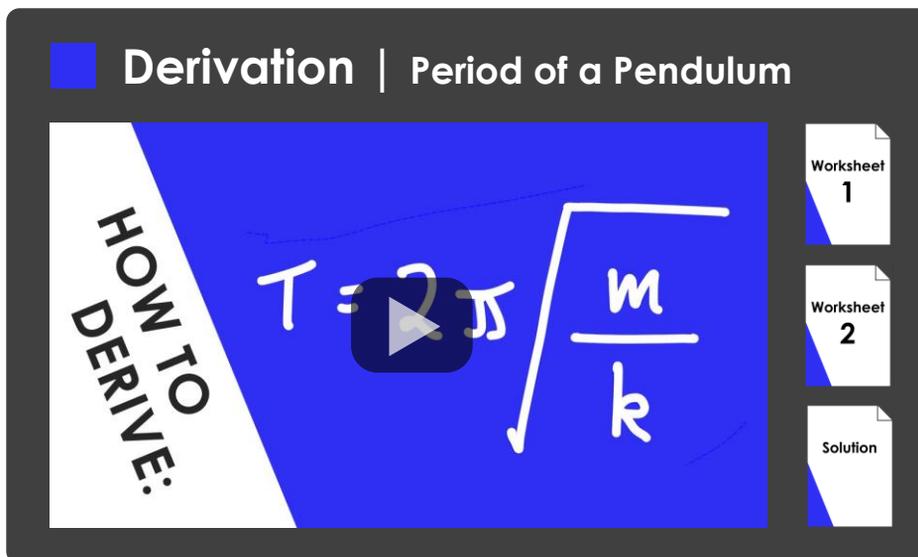
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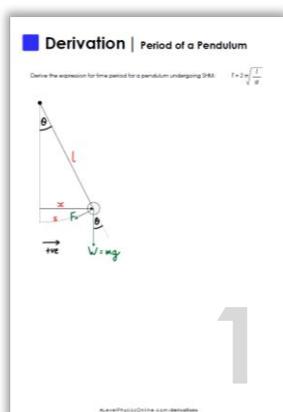
Derivations | Introduction

Thank you for downloading this guide, I really hope you find it useful!

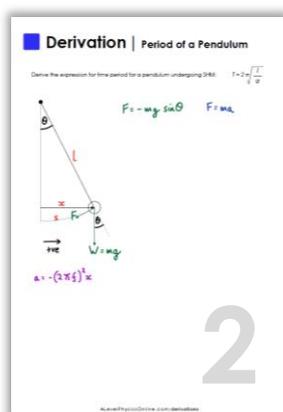
I have brief derivations for forty of the most important equations that you may come across during your A Level Physics course. Some of these are listed in the specification, many have come up in previous exam questions and the others just help you understand the underlying physics.



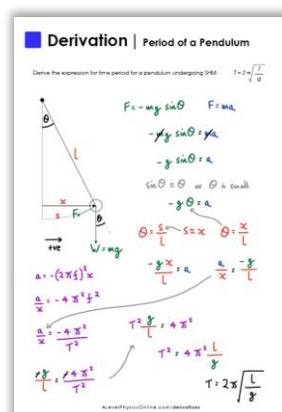
Don't forget that I also have short videos where I explain each of these derivations in a little more detail at ALEvelPhysicsOnline.com. There are also worksheets (with varying levels of support) you can use when you're revising these.



1



2



While you're on the website, don't forget that you can sign up for a **Premium Plan** (or your teachers can buy a **School Subscription**) to access hundreds more videos and resources.

There is also a workbook called the **Daily Workout** with short questions you can complete every single day – available to buy on Amazon.



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Derivation | Electrical Power

Derive the series of expressions linking power, current, potential difference and resistance:

$$P = I^2 R$$

$$P = V^2 / R$$

$$P = IV \quad \leftarrow \quad V = IR$$

$$P = IIR$$

$$P = I^2 R$$

$$P = IV \quad \leftarrow \quad V = IR \quad I = \frac{V}{R}$$

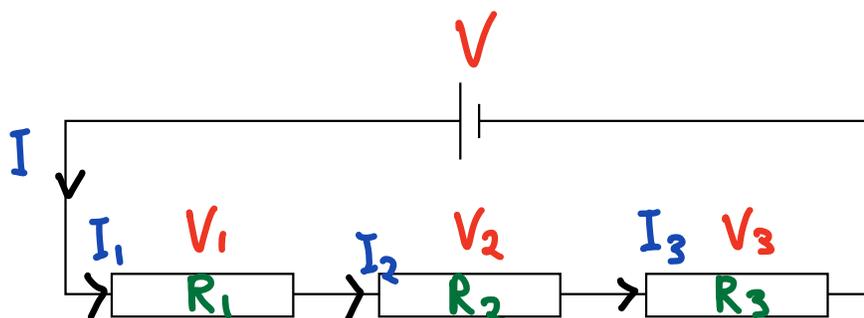
$$P = \frac{VV}{R}$$

$$P = \frac{V^2}{R}$$

Derivation | Resistors in Series

Derive the expression for calculating the total resistance of resistors in series:

$$R_T = R_1 + R_2 + \dots + R_n$$



$$V = V_1 + V_2 + V_3$$

$$V = I R_T \quad V_1 = I_1 R_1 \quad V_2 = I_2 R_2 \quad V_3 = I_3 R_3$$

$$I R_T = I_1 R_1 + I_2 R_2 + I_3 R_3$$

$$I = I_1 = I_2 = I_3$$

$$R_T = R_1 + R_2 + R_3$$

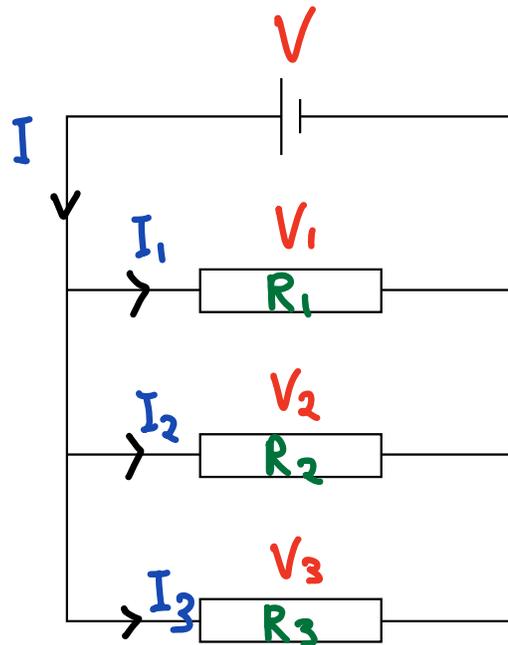
$$\therefore R_T = R_1 + R_2 + \dots + R_n$$



Derivation | Resistors in Parallel

Derive the expression for calculating the total resistance of resistors in parallel:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$



$$I = I_1 + I_2 + I_3$$
$$I = \frac{V}{R_T} \quad I_1 = \frac{V_1}{R_1} \quad I_2 = \frac{V_2}{R_2} \quad I_3 = \frac{V_3}{R_3}$$

$$\frac{V}{R_T} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

$$V = V_1 = V_2 = V_3$$

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

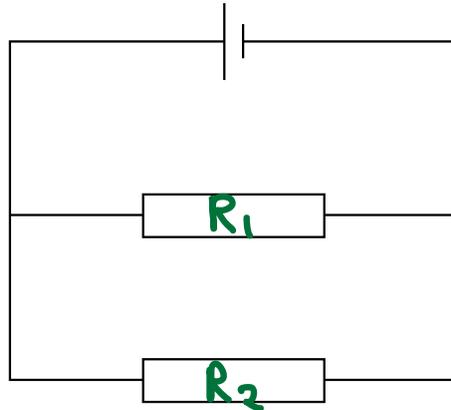
$$\therefore \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}$$



Derivation | Two Resistors in Parallel

Derive the expression for calculating the total resistance of resistors in parallel:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

$\times R_T$

$$1 = \frac{R_T}{R_1} + \frac{R_T}{R_2}$$

$\times R_1$

$$R_1 = R_T + \frac{R_T R_1}{R_2}$$

$\times R_2$

$$R_1 R_2 = R_T R_2 + R_T R_1$$

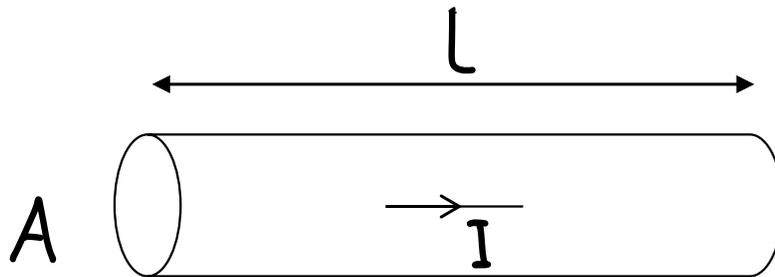
$$R_1 R_2 = R_T (R_1 + R_2)$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Derivation | Drift Velocity

Derive the expression for calculating the drift velocity of charge carriers:

$$I = Anev$$



n = no. of charge carriers per m^3

nV = total no. of charge carriers

nVe = total charge, $Q = nLAe$

$$V = LA$$

$$v = \frac{L}{t} \quad l = vt$$

$$Q = It \quad I = \frac{Q}{t} = \frac{nLAe}{t} = \frac{nvt \cancel{Ae}}{\cancel{t}}$$

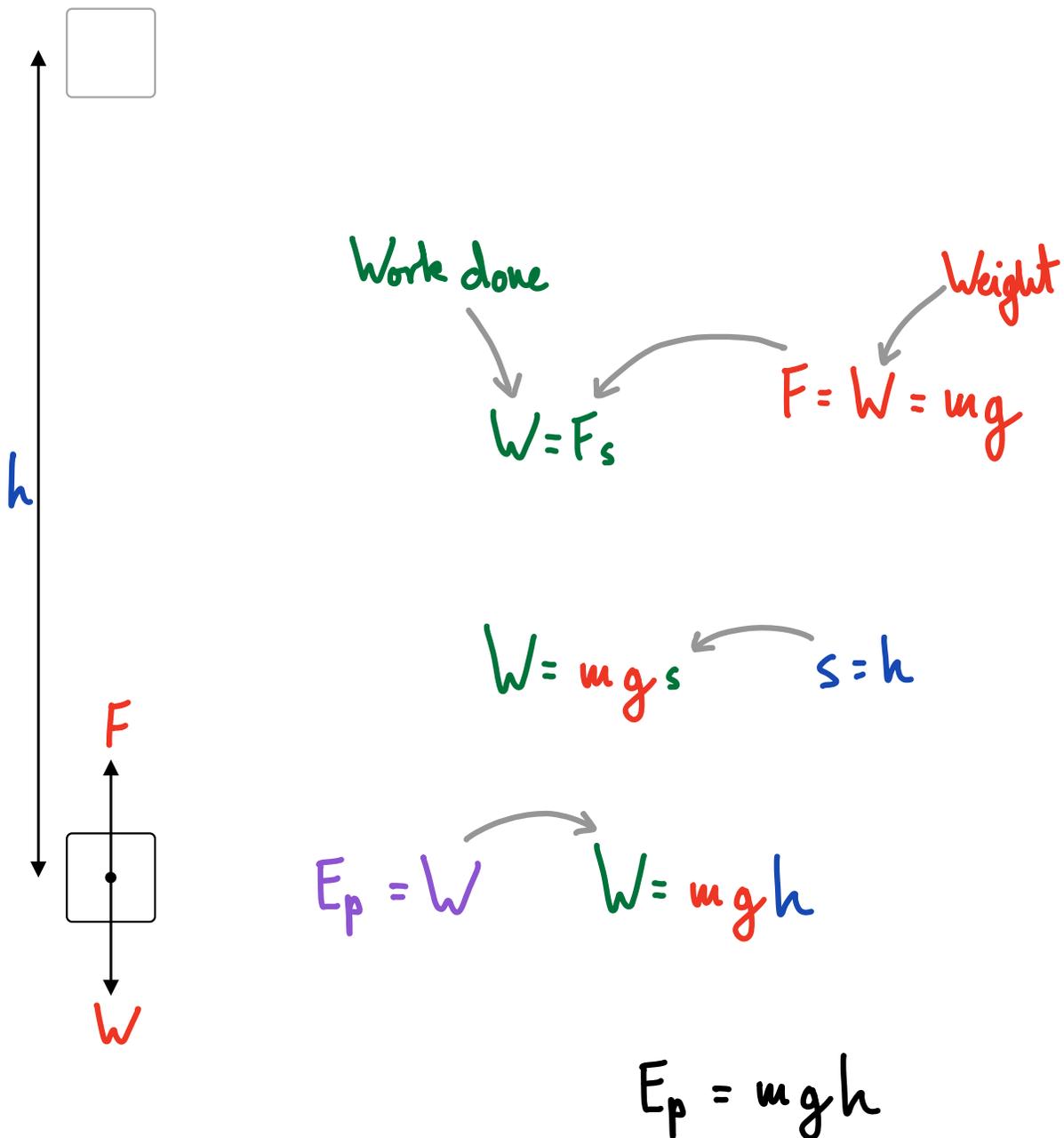
$$I = n v A e$$

$$I = A n e v$$

Derivation | Potential Energy

Derive the expression for change in gravitational potential energy in a uniform field:

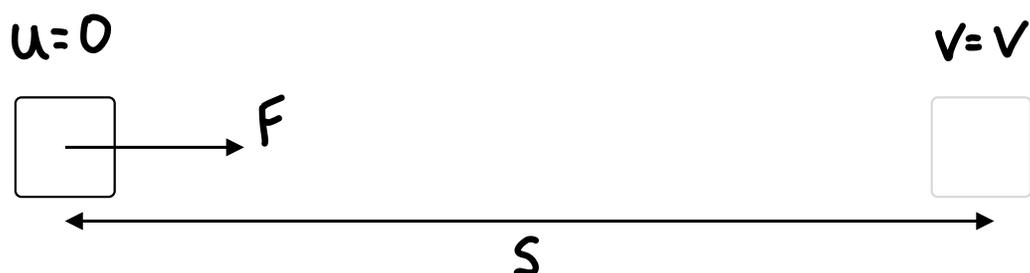
$$E_p = mgh$$



Derivation | Kinetic Energy

Derive the expression for kinetic energy:

$$E_k = \frac{1}{2}mv^2$$



$$W = Fs \quad F = ma \quad v^2 = \cancel{u^2} + 2as$$

$$v^2 = 2as$$

$$W = mas \quad as = \frac{v^2}{2}$$

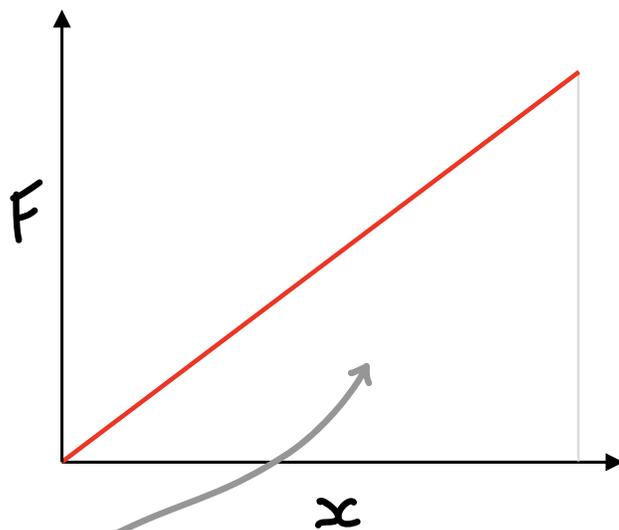
$$W = E_k \quad W = m \frac{v^2}{2}$$

$$E_k = \frac{1}{2}mv^2$$

Derivation | Elastic Potential Energy

Derive the expression for elastic potential energy stored in a spring:

$$E_e = \frac{1}{2} kx^2$$



Area = Energy stored = $\frac{1}{2} Fx$ $F = kx$

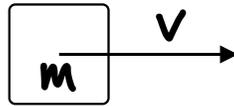
$$E = \frac{1}{2} kx \cdot x$$

$$E = \frac{1}{2} kx^2$$

Derivation | Energy and Momentum

Derive the expression for kinetic energy in terms of momentum and mass:

$$E_k = \frac{p^2}{2m}$$



$$E_k = \frac{1}{2} m v^2$$

$$p = mv$$

$$v = \frac{p}{m}$$

$$E_k = \frac{\cancel{m} p^2}{2 m^2}$$

$$v^2 = \frac{p^2}{m^2}$$

$$E_k = \frac{p^2}{2m}$$

Derivation | Power, Force and Velocity

Derive the expression for power in terms of force and velocity:

$$P = Fv$$

$$P = \frac{W}{t} \quad W = Fs$$


$$P = \frac{Fs}{t} \quad v = \frac{s}{t}$$


$$P = Fv$$



Derivation | 'suvat' Equations

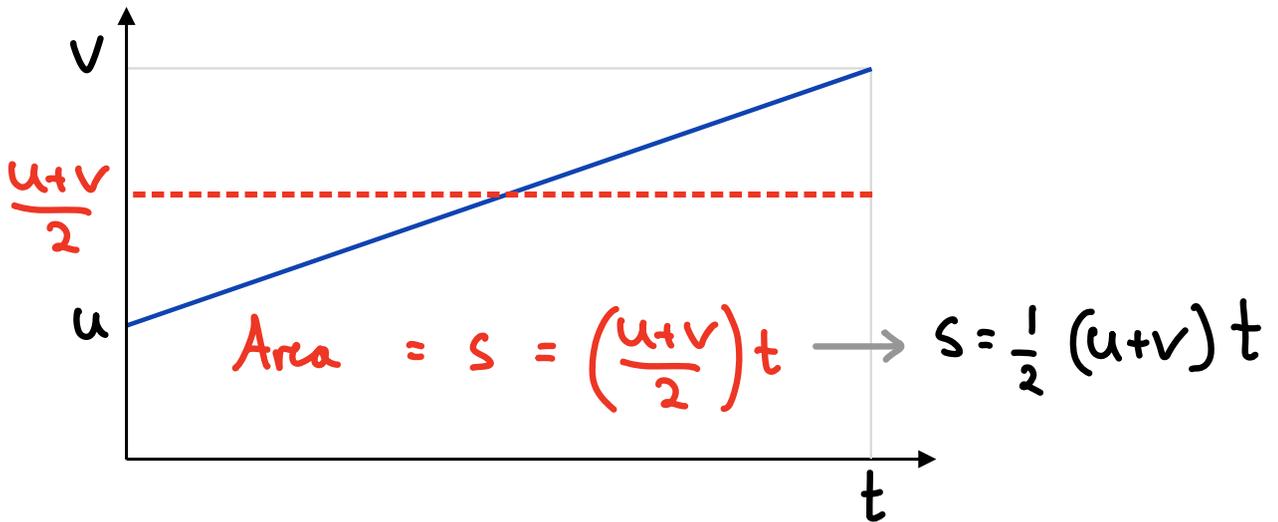
Derive the 'suvat' equations for an object undergoing uniform acceleration:

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$



$$\text{Gradient} = a = \frac{v-u}{t} \rightarrow at = v-u \rightarrow v = u + at$$

$$s = \left(\frac{u+v}{2}\right)t \quad v = u + at$$

$$s = \left(\frac{u+v}{2}\right)t \quad t = \left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$s = \left(\frac{2u}{2}\right)t + \left(\frac{at}{2}\right)t$$

$$s = \frac{(u+v)(v-u)}{2a}$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{\cancel{uv} + v^2 - \cancel{uv} - u^2}{2a} = \frac{v^2 - u^2}{2a}$$

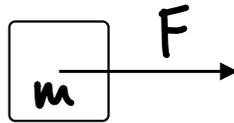
$$2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

Derivation | Newton's 2nd Law

Derive the expression to show how the resultant force is related to the rate of change of momentum:

$$F = \frac{\Delta p}{\Delta t}$$



$$F = m a \quad a = \frac{v - u}{t}$$

$$F = m \left(\frac{v - u}{t} \right)$$

$$F = \frac{m v - m u}{t} \quad p = m v$$

$$F = \frac{\Delta p}{\Delta t}$$

Derivation | Upthrust

Derive the expression for the upthrust on an object submerged in a fluid:

$$U = \rho V g$$

$\rho = \frac{F}{A}$ $F = \rho A$
 $\rho = \rho g h$
 $F = \rho g h A$
 $F_1 = \rho g h A$

$F_2 = \rho g (h+x) A$
 $F_2 = \rho g h A + \rho g x A$

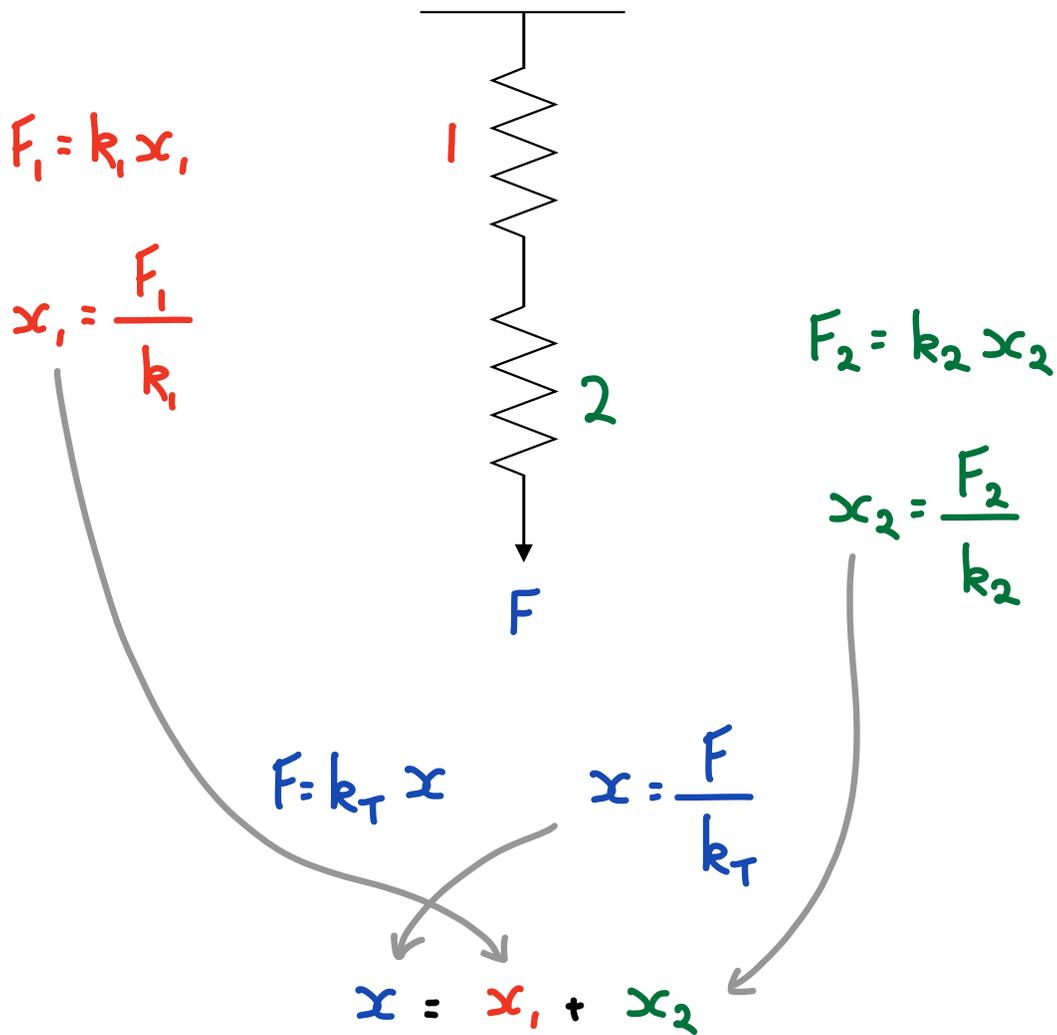
Weight = Upthrust

$W = F_2 - F_1$
 $W = U$ $U = \cancel{\rho g h A} + \rho g x A - \cancel{\rho g h A}$
 $U = \rho g x A$ $x A = V$
 $U = \rho g V$ $U = \rho V g$

Derivation | Springs in Series

Derive the expression for calculating the combined stiffness of springs in series:

$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} + \dots + \frac{1}{k_n}$$



$$\frac{F}{k_T} = \frac{F_1}{k_1} + \frac{F_2}{k_2}$$

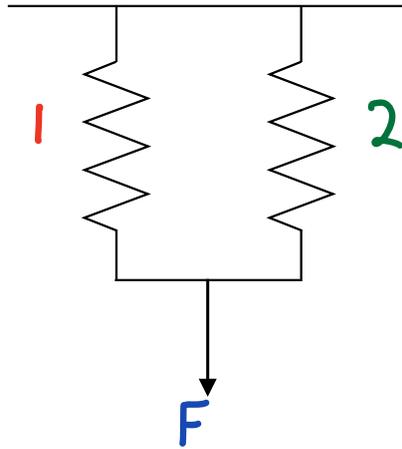
$$F = F_1 = F_2$$

$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2}$$

Derivation | Springs in Parallel

Derive the expression for calculating the combined stiffness of springs in parallel:

$$k_T = k_1 + k_2 + \dots + k_n$$



$$F_1 = k_1 x_1$$

$$F = k_T x$$

$$F_2 = k_2 x_2$$

$$F = F_1 + F_2$$

$$k_T x = k_1 x_1 + k_2 x_2$$

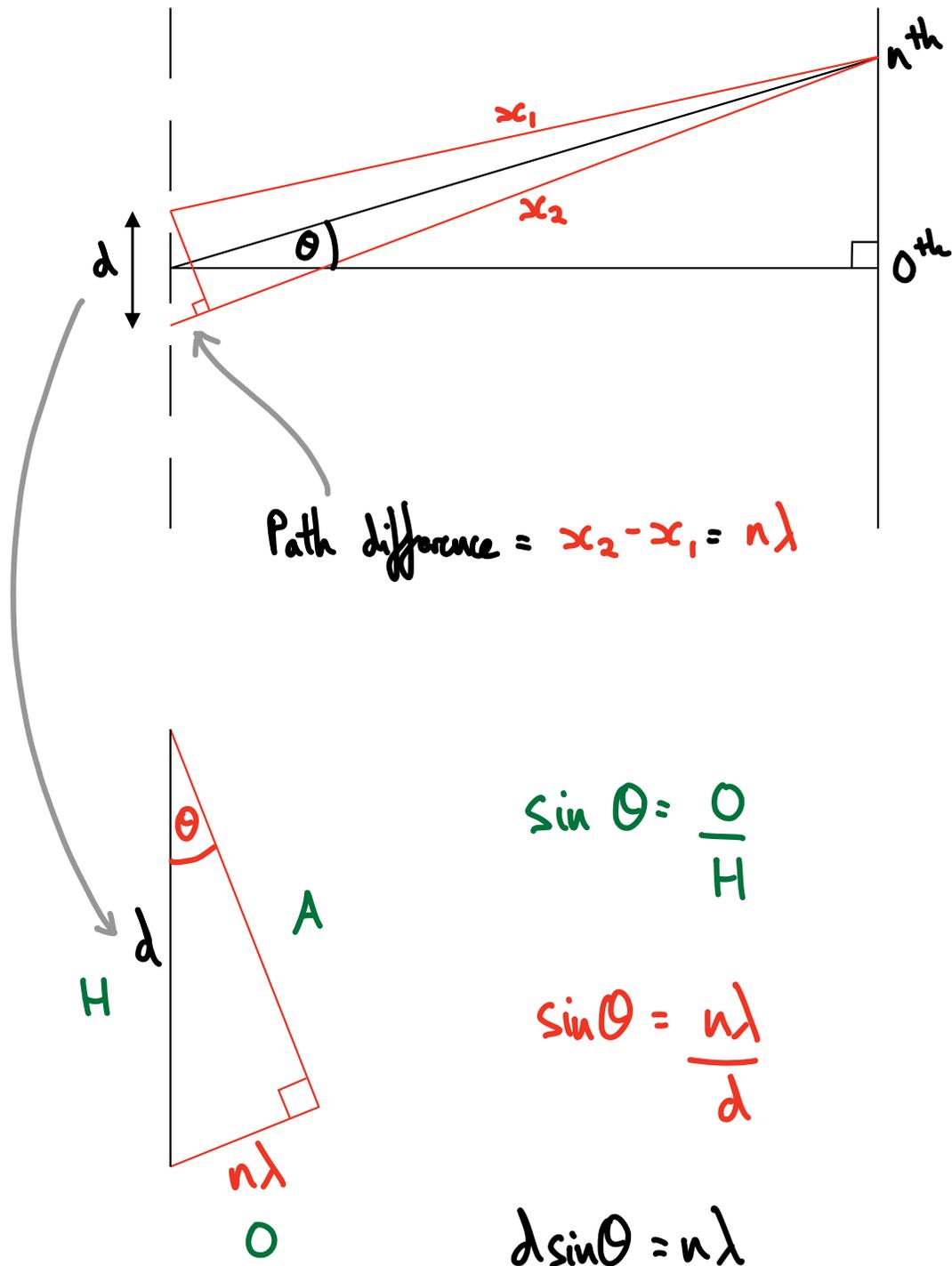
$$x = x_1 = x_2$$

$$k_T = k_1 + k_2$$

Derivation | Diffraction Grating

Derive the expression for a diffraction grating:

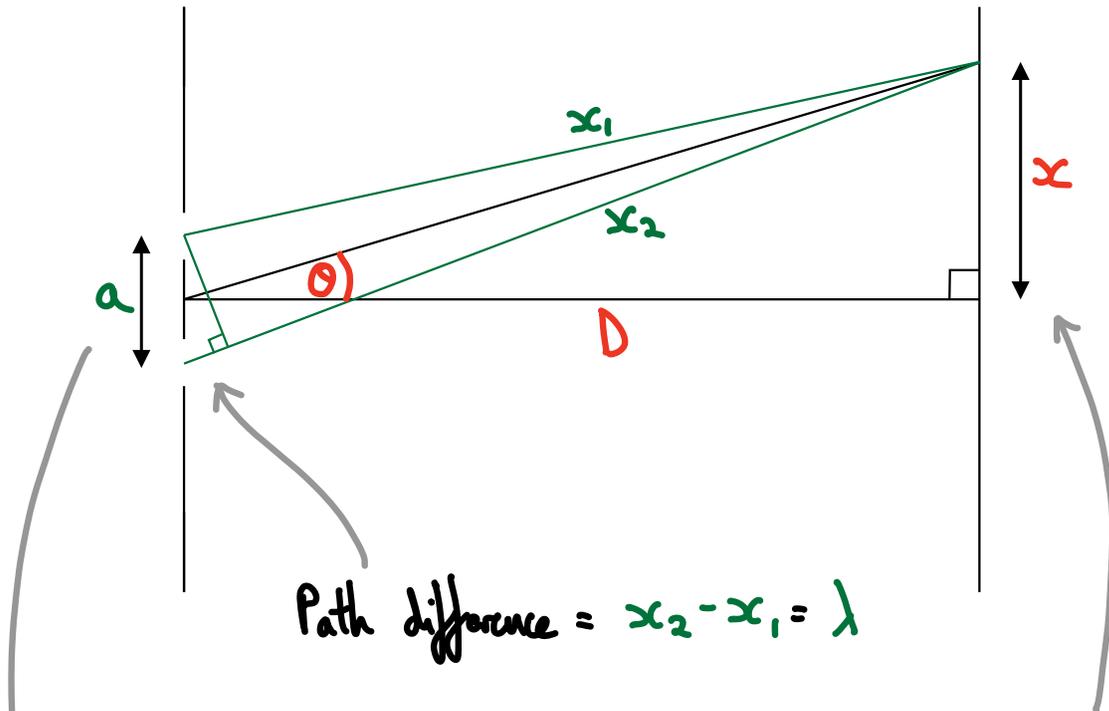
$$d \sin \theta = n \lambda$$



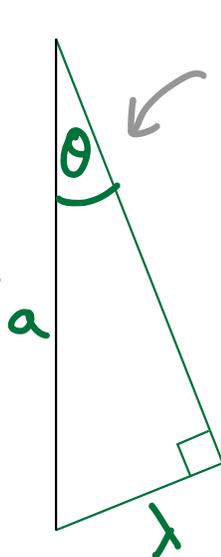
Derivation | Double Slit

Derive the expression for the fringe spacing from a double slit:

$$\lambda = \frac{ax}{D}$$



$$\text{Path difference} = x_2 - x_1 = \lambda$$



$$\sin \theta = \frac{O}{H}$$

$$\sin \theta = \frac{\lambda}{a}$$

$$\tan \theta = \frac{O}{A}$$

$$\tan \theta = \frac{x}{D}$$

$$\sin \theta \approx \tan \theta \quad \text{as } \theta \rightarrow 0$$

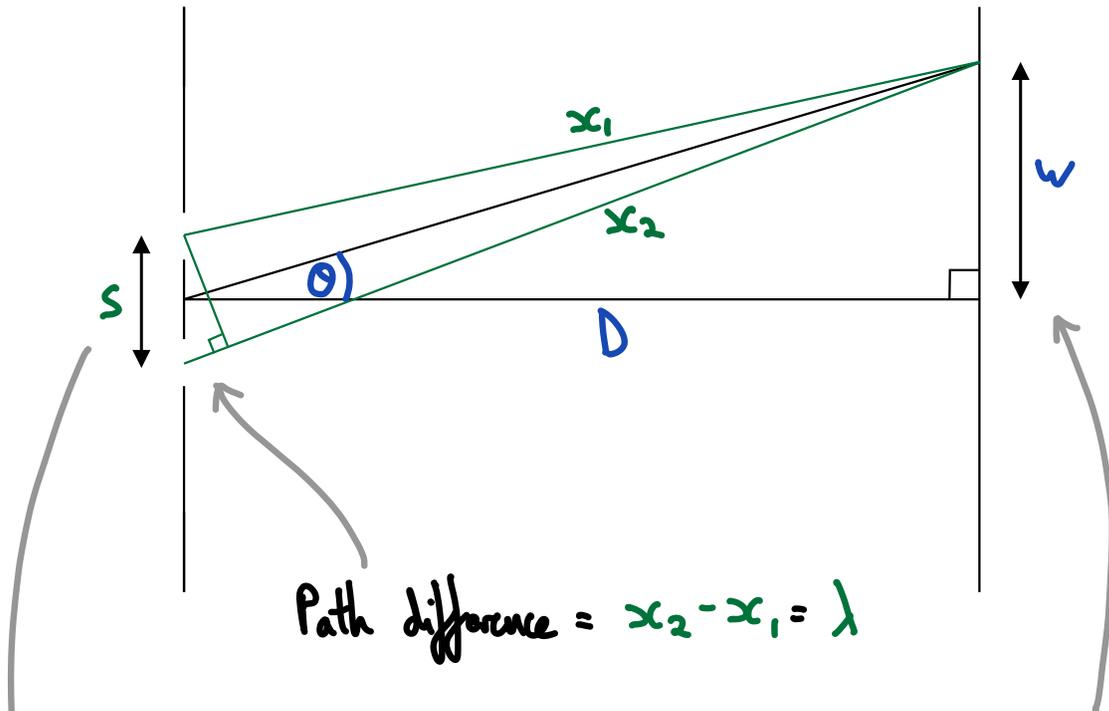
$$\frac{\lambda}{a} = \frac{x}{D}$$

$$\lambda = \frac{ax}{D}$$

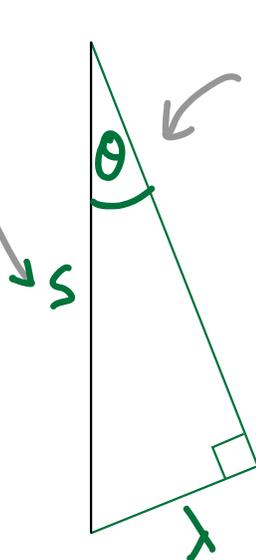
Derivation | Double Slit (AQA)

Derive the expression for the fringe spacing from a double slit:

$$w = \frac{\lambda D}{s}$$



$$\text{Path difference} = x_2 - x_1 = \lambda$$



$$\sin \theta = \frac{\text{O}}{\text{H}}$$

$$\sin \theta = \frac{\lambda}{s}$$

$$\tan \theta = \frac{\text{O}}{\text{A}}$$

$$\tan \theta = \frac{w}{D}$$

$$\sin \theta \approx \tan \theta \quad \text{as } \theta \rightarrow 0$$

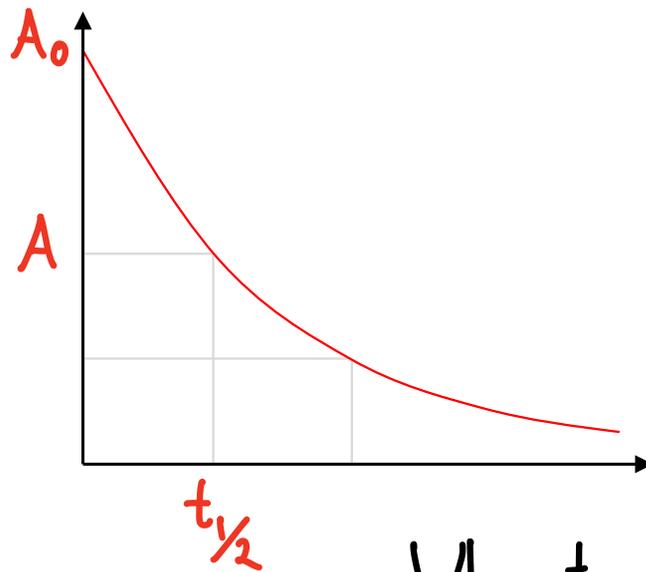
$$\frac{\lambda}{s} = \frac{w}{D}$$

$$w = \frac{\lambda D}{s}$$

Derivation | Half-life

Derive the expression for the half-life of a radioactive isotope in terms of its decay constant:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$



$$A = A_0 e^{-\lambda t}$$

When $t_{1/2}$, $\frac{A}{A_0} = \frac{1}{2}$

$$\frac{A}{A_0} = e^{-\lambda t}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln \frac{1}{2} = -\lambda t_{1/2}$$

$$-\ln \frac{1}{2} = \lambda t_{1/2}$$

$$-\ln \frac{1}{2} = \ln 2 \quad \therefore \quad \ln 2 = \lambda t_{1/2}$$

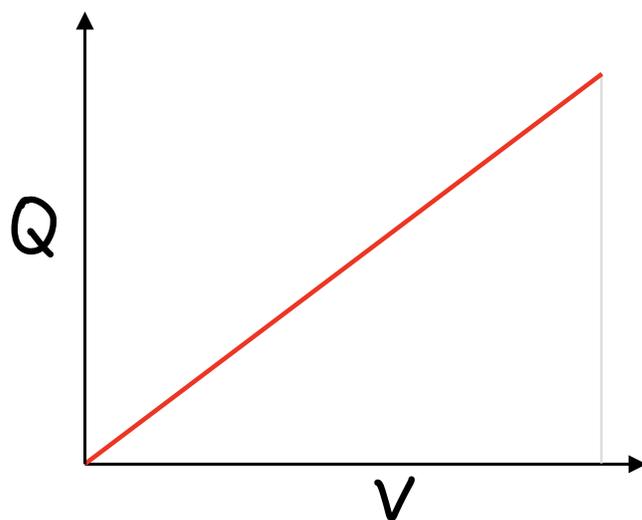
$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Derivation | Energy in a Capacitor

Derive the series of expressions for the energy stored by a capacitor to its capacitance, charge stored and potential difference:

$$E = \frac{1}{2} CV^2$$

$$E = \frac{1}{2} Q^2 / C$$



$$\text{Area} = E = \frac{1}{2} QV \quad Q = CV$$

$$E = \frac{1}{2} CVV$$

$$E = \frac{1}{2} CV^2$$

$$E = \frac{1}{2} QV \quad V = \frac{Q}{C}$$

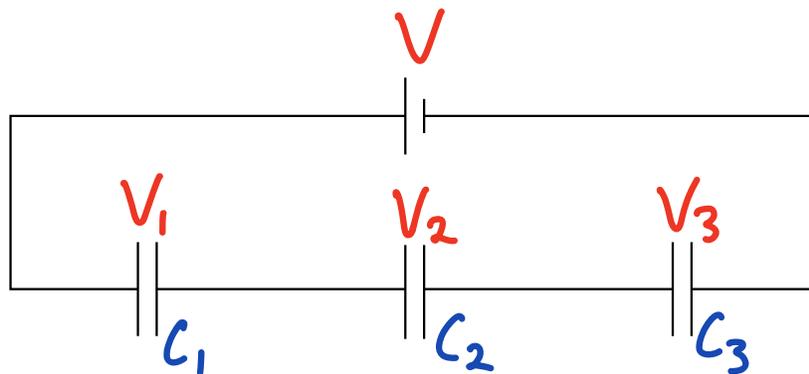
$$E = \frac{1}{2} Q \frac{Q}{C}$$

$$E = \frac{1}{2} Q^2 / C$$

Derivation | Capacitors in Series

Derive the expression for calculating the total capacitance of capacitors in series:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$



$$V = V_1 + V_2 + V_3$$

$$Q = CV \quad V = \frac{Q}{C}$$

$$\frac{Q}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

$$Q = Q_1 = Q_2 = Q_3$$

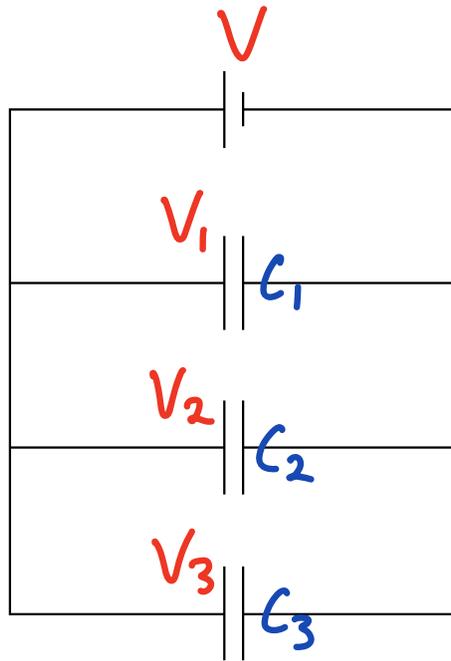
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\therefore \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_n}$$

Derivation | Capacitors in Parallel

Derive the expression for calculating the total capacitance of capacitors in parallel:

$$C_T = C_1 + C_2 + \dots + C_n$$



$$Q = Q_1 + Q_2 + Q_3$$

$$Q = CV$$

$$C_T V = C_1 V_1 + C_2 V_2 + C_3 V_3$$

$$V = V_1 = V_2 = V_3$$

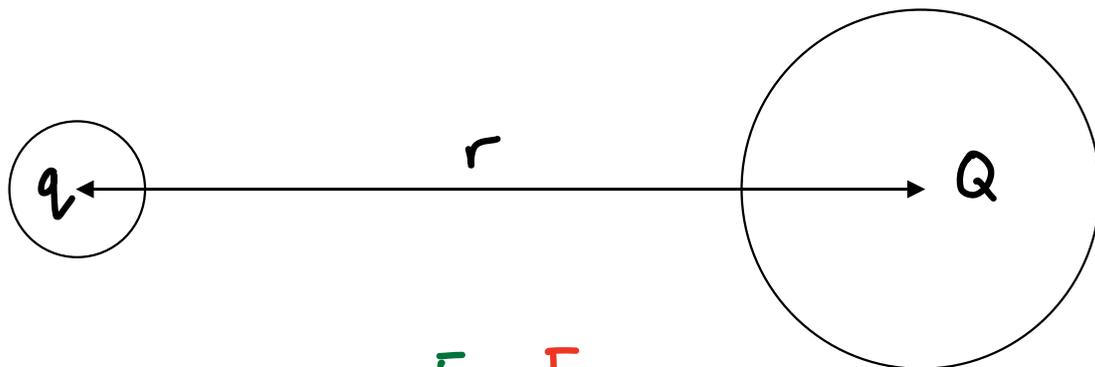
$$C_T = C_1 + C_2 + C_3$$

$$\therefore C_T = C_1 + C_2 + \dots + C_n$$

Derivation | Closest Approach

Derive the expression for the distance of closest approach for an alpha particle to a nucleus:

$$r = \frac{qQ}{2\pi\epsilon_0 mv^2}$$



$$E_k = \frac{1}{2}mv^2$$
$$E_k = E_p$$
$$E_p = \frac{qQ}{4\pi\epsilon_0 r}$$

$$\frac{1}{2}mv^2 = \frac{qQ}{4\pi\epsilon_0 r}$$

$$\frac{1}{2}mv^2 r = \frac{qQ}{4\pi\epsilon_0}$$

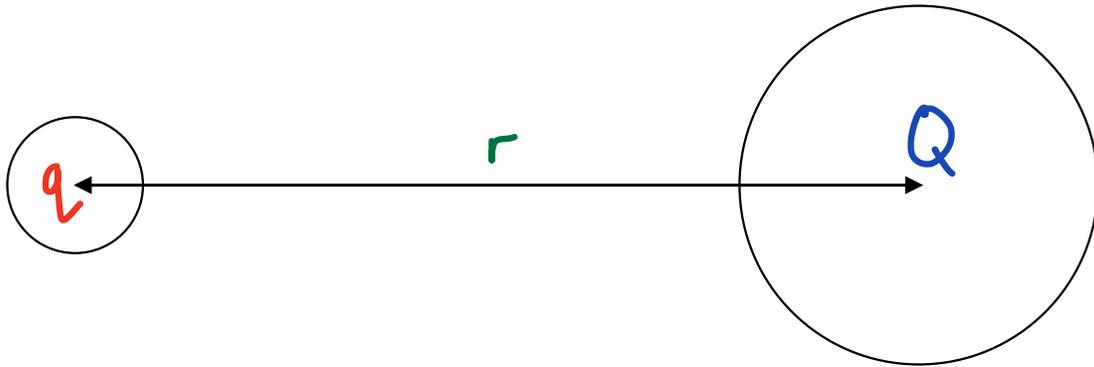
$$r = \frac{2qQ}{4\pi\epsilon_0 mv^2}$$

$$r = \frac{qQ}{2\pi\epsilon_0 mv^2}$$

Derivation | Coulomb's Law

Derive Coulomb's law between two charged objects:

$$F = \frac{qQ}{4\pi\epsilon_0 r^2}$$



$$F \propto q$$

$$F \propto \frac{1}{r^2}$$

$$F \propto Q$$

$$F \propto \frac{qQ}{r^2}$$

$$F = k \frac{qQ}{r^2}$$

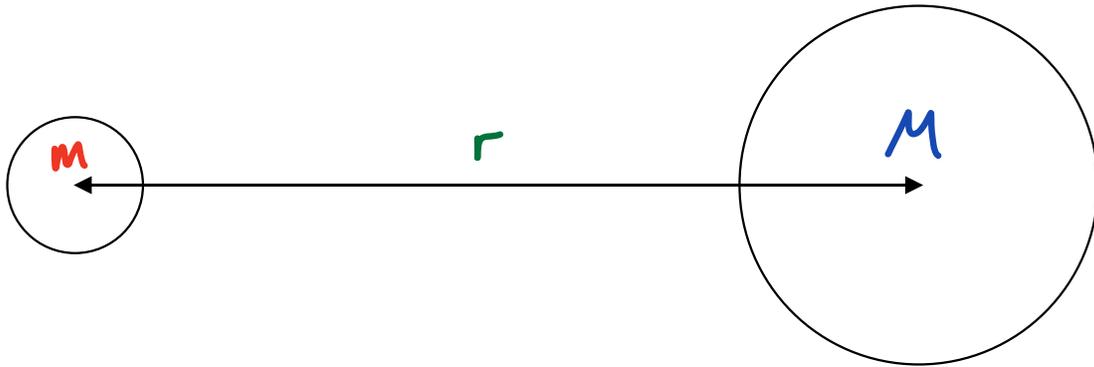
$$k = \frac{1}{4\pi\epsilon_0}$$

$$F = \frac{qQ}{4\pi\epsilon_0 r^2}$$

Derivation | Gravitational Attraction

Derive Newton's law of universal gravitation between two objects:

$$F = \frac{-GMm}{r^2}$$



$$F \propto m$$

$$F \propto \frac{1}{r^2}$$

$$F \propto M$$

$$F \propto \frac{mM}{r^2}$$

$$F = k \frac{mM}{r^2}$$

$$k = G = 6.67 \times 10^{-11} \text{ Nm}^2 \text{ kg}^{-2}$$

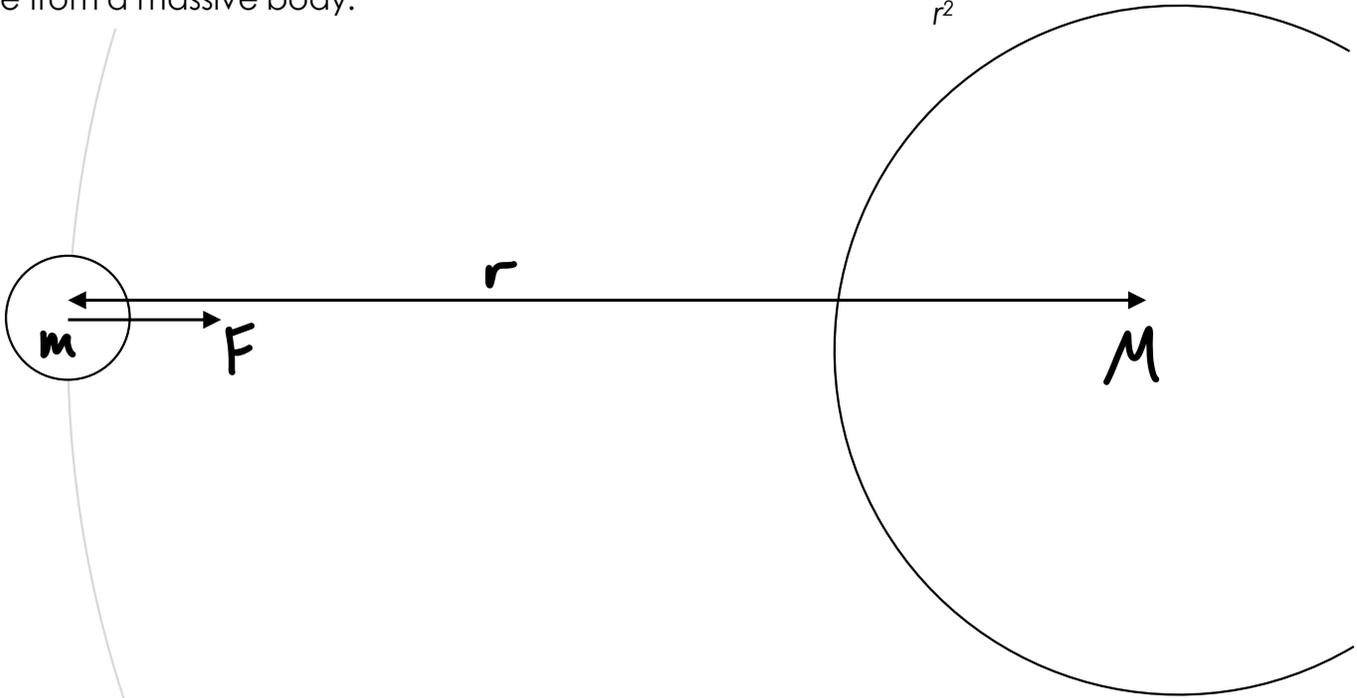
$$F = \frac{-GMm}{r^2}$$

Attractive force

Derivation | Gravitational Field Strength

Derive the expression for gravitational field strength at a distance from a massive body:

$$g = \frac{-GM}{r^2}$$



$$F = mg$$

$$F = \frac{-GmM}{r^2}$$

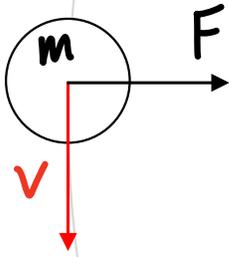
$$\cancel{m}g = \frac{-G\cancel{m}M}{r^2}$$

$$g = \frac{-GM}{r^2}$$

Derivation | Radial Path

Derive the expression for the radius of a path of a charged particle in a magnetic field:

$$r = \frac{p}{BQ}$$



$$F = \frac{mv^2}{r}$$

$$F = BQv$$

$$\frac{mv^2}{r} = BQv$$

$$\frac{mv}{r} = BQ$$

$$r = \frac{mv}{BQ}$$

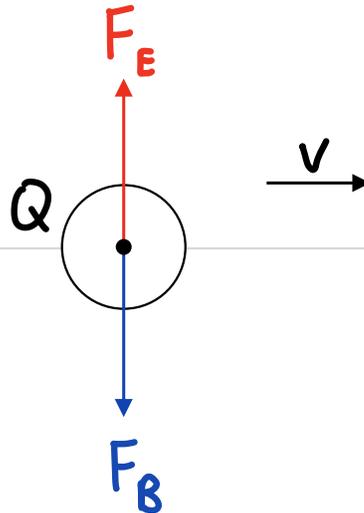
$$p = mv$$

$$r = \frac{p}{BQ}$$

Derivation | Velocity Selector

Derive the expression for the velocity of a particle travelling in a straight line in a magnetic and electric field:

$$v = \frac{E}{B}$$



$$F_B = BQv$$

$$F_E = EQ$$

$$F_B = F_E$$

$$\cancel{BQv} = \cancel{EQ}$$

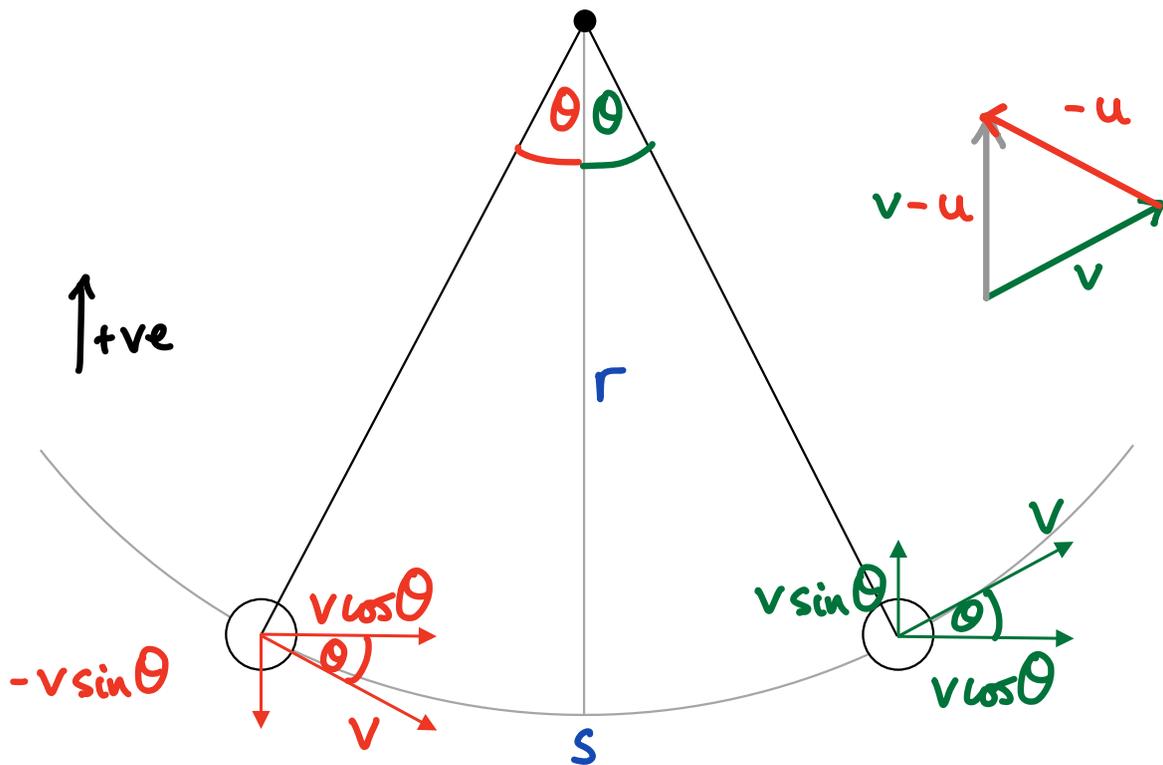
$$Bv = E$$

$$v = \frac{E}{B}$$

Derivation | Centripetal Acceleration

Derive the expression for centripetal acceleration:

$$a = \frac{v^2}{r}$$



$$a = \frac{v-u}{t}$$

$$a = \frac{v \sin \theta - (-) v \sin \theta}{t} = \frac{2v \sin \theta}{t}$$

$$t = \frac{s}{v} = \frac{2r\theta}{v}$$

$$a = \frac{2v \sin \theta}{2r\theta/v} = \frac{2v^2 \sin \theta}{2r\theta} = \frac{v^2 \sin \theta}{r\theta}$$

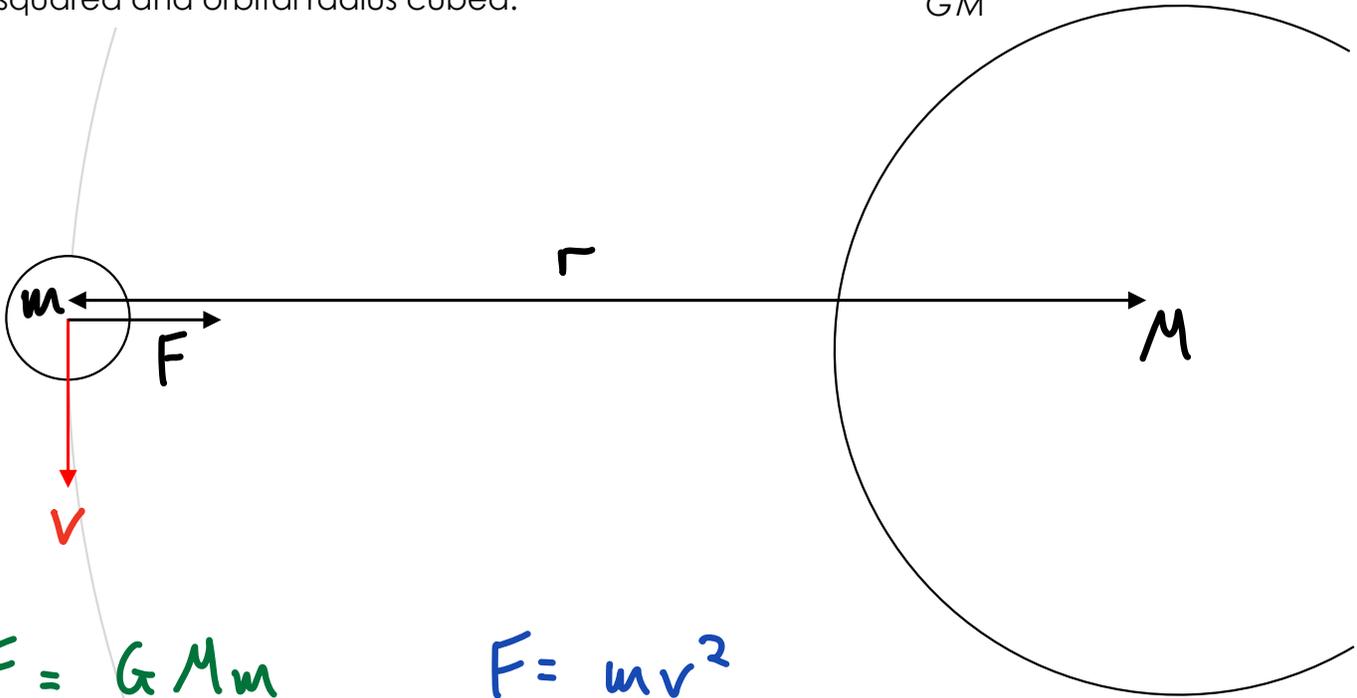
$$\frac{\sin \theta}{\theta} \approx 1 \quad \text{as } \theta \rightarrow 0$$

$$a = \frac{v^2}{r}$$

Derivation | Orbital Period and Radius

Derive the expression for the relationship between orbital period squared and orbital radius cubed:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$



$$F = \frac{GMm}{r^2}$$

$$F = \frac{mv^2}{r}$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = v^2$$

$$v^2 = \frac{4\pi^2 r^2}{T^2}$$

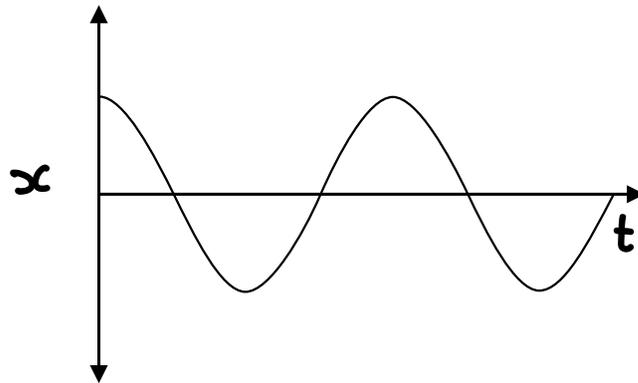
$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

Derivation | Simple Harmonic Motion

Show that for a system undergoing simple harmonic motion:

$$a \propto -x$$



$$x = A \cos(\omega t)$$

$$v = \frac{dx}{dt} = -A\omega \sin(\omega t)$$

$$a = \frac{dv}{dt} = -A\omega^2 \cos(\omega t)$$

$$a = -A\omega^2 \cos(\omega t)$$

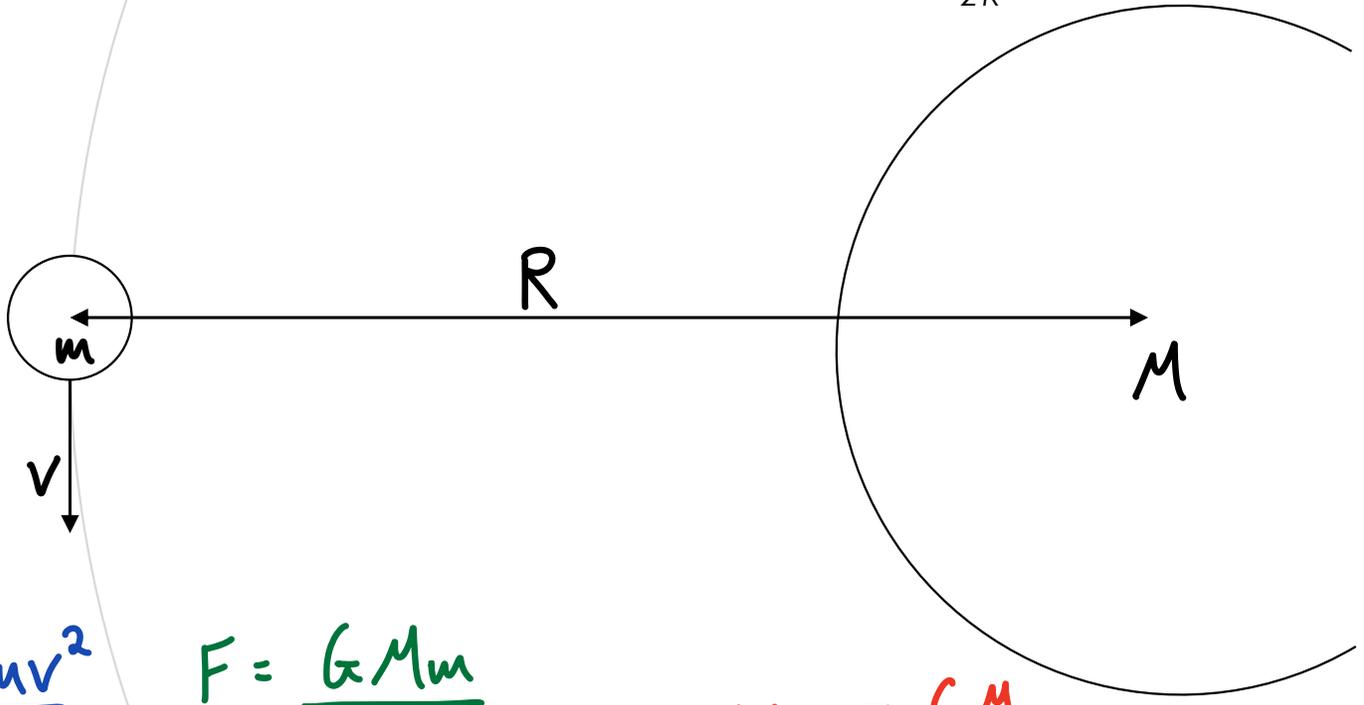
$$a = -\omega^2 x \quad (\omega = 2\pi f)$$

$$\therefore a \propto -x$$

Derivation | Total Energy of a Satellite

Derive the expression for the total energy of a satellite:

$$E_T = \frac{-GmM}{2R}$$



$$F = \frac{mv^2}{R}$$

$$F = \frac{GMm}{R^2}$$

$$V_g = \frac{-GM}{R}$$

$$E_p = \frac{-GmM}{R}$$

$$\frac{\cancel{mv^2}}{\cancel{R}} = \frac{GM\cancel{m}}{R^2}$$

$$v^2 = \frac{GM}{R}$$

$$E_T = E_K + E_p$$

$$E_K = \frac{1}{2}mv^2$$

$$E_T = \frac{GmM}{2R} + \frac{(-)GmM}{R}$$

$$E_K = \frac{1}{2}m \frac{GM}{R}$$

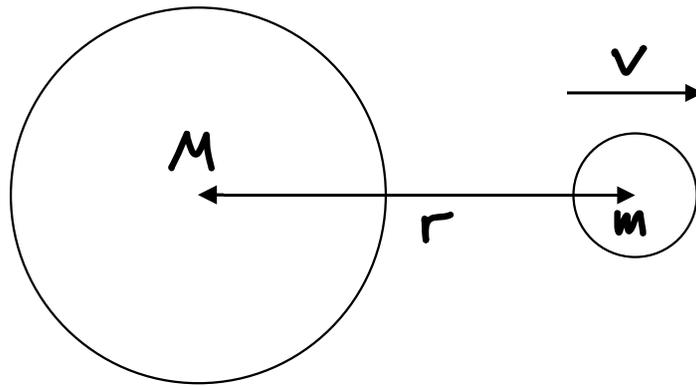
$$E_T = \frac{-GmM}{2R}$$

$$E_K = \frac{GmM}{2R}$$

Derivation | Escape Velocity

Derive the expression for the escape velocity from a planet:

$$v = \sqrt{\frac{2GM}{r}}$$



$$E_k = E_p$$
$$E_k = \frac{1}{2}mv^2$$
$$E_p = \frac{GmM}{r}$$

$$\frac{1}{2}mv^2 = \frac{GmM}{r}$$

$$\frac{v^2}{2} = \frac{GM}{r}$$

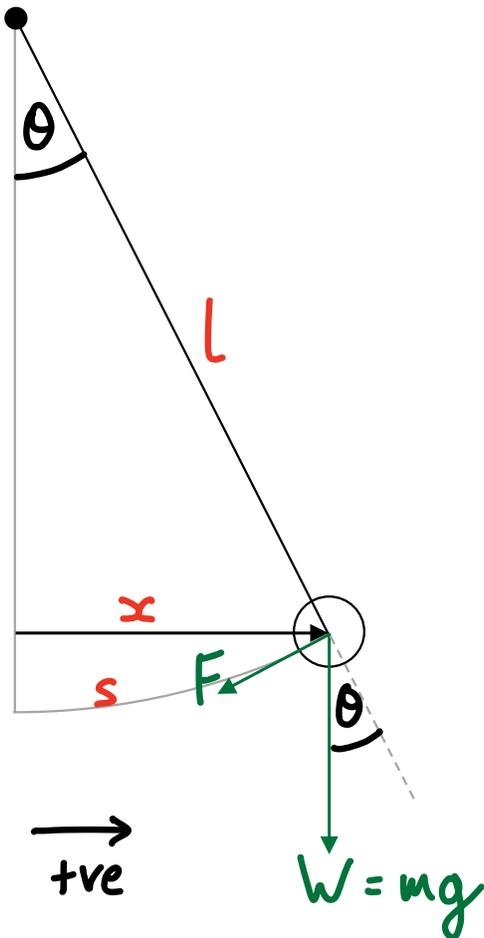
$$v^2 = \frac{2GM}{r}$$

$$v = \sqrt{\frac{2GM}{r}}$$

Derivation | Period of a Pendulum

Derive the expression for time period for a pendulum undergoing SHM:

$$T = 2\pi \sqrt{\frac{l}{g}}$$



$$F = -mg \sin\theta \quad F = ma$$

$$-mg \sin\theta = ma$$

$$-g \sin\theta = a$$

$\sin\theta \approx \theta$ as θ is small

$$-g\theta = a$$

$$\theta = \frac{s}{l} \quad s \approx x \quad \theta = \frac{x}{l}$$

$$-\frac{gx}{l} = a \quad \frac{a}{x} = -\frac{g}{l}$$

$$a = -(2\pi f)^2 x$$

$$\frac{a}{x} = -4\pi^2 f^2$$

$$\frac{a}{x} = \frac{-4\pi^2}{T^2}$$

$$T^2 \frac{g}{l} = 4\pi^2$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$\frac{+g}{l} = \frac{+4\pi^2}{T^2}$$

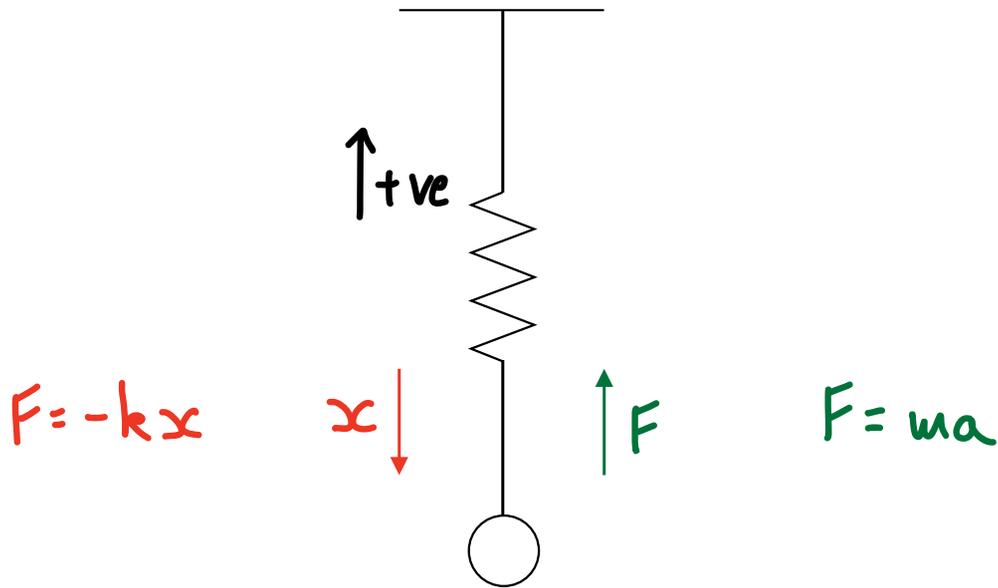
$$T = 2\pi \sqrt{\frac{l}{g}}$$



Derivation | Period of a Spring-Mass

Derive the expression for period for a spring-mass system undergoing SHM:

$$T = 2\pi \sqrt{\frac{m}{k}}$$



$$F = -kx$$

$$x \downarrow$$

$$F \uparrow$$

$$F = ma$$

$$-kx = ma$$

$$\frac{a}{x} = \frac{-k}{m}$$

$$a = -(2\pi f)^2 x$$

$$\frac{a}{x} = -4\pi^2 f^2$$

$$\frac{a}{x} = \frac{-4\pi^2}{T^2}$$

$$\frac{-k}{m} = \frac{-4\pi^2}{T^2}$$

$$T^2 \frac{k}{m} = 4\pi^2$$

$$T^2 = 4\pi^2 \frac{m}{k}$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Derivation | Ideal Gas Equation

Derive the ideal gas equation:

$$pV = nRT$$

$$pV = NkT$$

$$p \propto T$$

$$\frac{p}{T} = k_1$$

$$p \propto \frac{1}{V}$$

$$pV = k_2$$

$$V \propto T$$

$$\frac{V}{T} = k_3$$

$$\frac{pV}{T} = k$$

$$pV = kT$$

$$pV = nRT$$

$$pV = NkT$$

$$(nR = Nk)$$

Derivation | E_k and Temperature

Derive the expression relating the kinetic energy of a particle to the temperature of an ideal gas:

$$E_k = \frac{3}{2} kT$$

$$pV = \frac{1}{3} Nm\bar{c}^2 \qquad pV = NkT$$



$$\frac{1}{3} Nm\bar{c}^2 = NkT$$

$$\times 3 \qquad m\bar{c}^2 = 3kT$$

$$\div 2 \qquad \frac{1}{2} m\bar{c}^2 = \frac{3}{2} kT$$

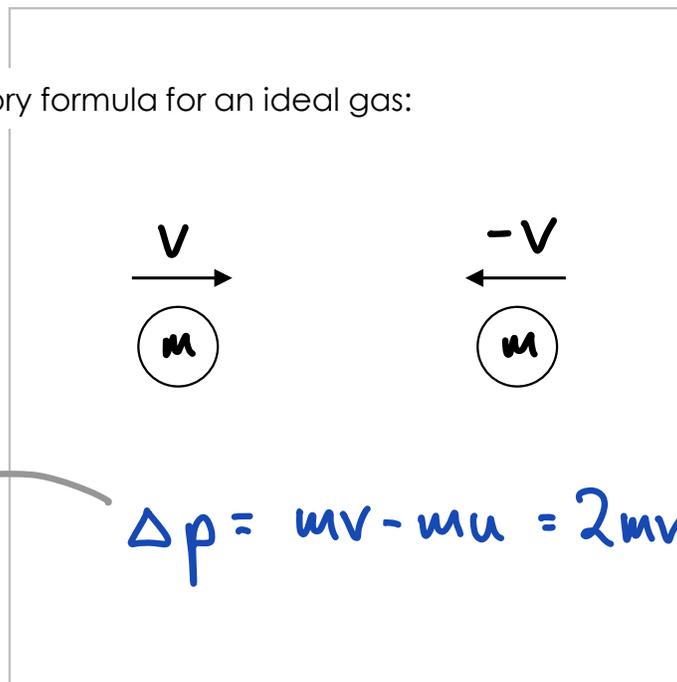
$$E_k = \frac{3}{2} kT$$

$$(E_k \propto T)$$

Derivation | Kinetic Theory

Derive the kinetic theory formula for an ideal gas:

$$pV = \frac{1}{3} Nm\bar{c}^2$$



Cube of side 'L'

$$A = L^2$$

$$V = L^3$$

$$F = \frac{\Delta p}{t}$$

$$\Delta p = mv - mu = 2mv$$

$$F = \frac{2mv}{t}$$

$$v = \frac{2L}{t} \quad t = \frac{2L}{v}$$

$$F = \frac{2mv}{2L/v}$$

In x direction, n molecules at \bar{c}

$$F = \frac{mv^2}{L}$$

$$F_T = \frac{nm\bar{c}^2}{L}$$

$$p = \frac{F_T}{A} = \frac{nm\bar{c}^2/L}{L^2} = \frac{nm\bar{c}^2}{L^3} = \frac{nm\bar{c}^2}{V}$$

$$N = 3n(x, y, z)$$

$$n = \frac{N}{3}$$

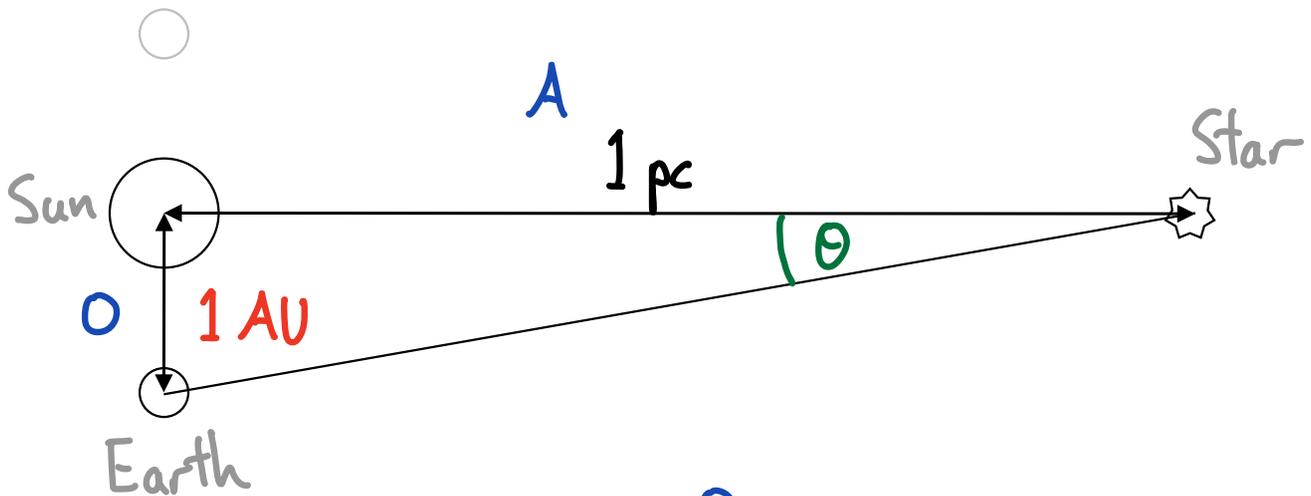
$$p = \frac{Nm\bar{c}^2}{3V}$$

$$pV = \frac{1}{3} Nm\bar{c}^2$$

Derivation | The Parsec

Derive the parsec and show that:

$$1 \text{ pc} = 3.1 \times 10^{16} \text{ m}$$



$$\tan \theta = \frac{O}{A}$$

$$A = \frac{O}{\tan \theta}$$

$$\text{If } O = 1 \text{ AU} = 1.5 \times 10^{11} \text{ m}$$

$$\text{and } \theta = 1 \text{ second of arc} = \frac{1^\circ}{60 \times 60}$$

$$A = \frac{1.5 \times 10^{11}}{\tan\left(\frac{1}{3600}\right)}$$

$$A = 3.1 \times 10^{16} \text{ m} = 1 \text{ pc}$$

Derivation | Specific Charge on an e⁻

Derive the expression for the specific charge on an electron:

$$\frac{e}{m} = \frac{2V}{B^2 r^2}$$

$$r = \frac{mv}{Be}$$

$$\frac{1}{2}mv^2 = eV$$

$$r^2 = \frac{m^2 v^2}{B^2 e^2}$$

$$v^2 = \frac{2eV}{m}$$

$$r^2 = \frac{m^2 \cancel{2eV}}{B^2 e^2 \cancel{m}}$$

$$r^2 = \frac{m 2V}{B^2 e}$$

$$e = \frac{m 2V}{B^2 r^2}$$

$$\frac{e}{m} = \frac{2V}{B^2 r^2}$$