QUADRATICS (P1)

SUPER IMPORTANT FOR (FUNCTIONS P1)(11 marks)

1 STANDARD FORM

$$y = \underbrace{a}_{\text{SHAPE}}x^2 + bx + \underbrace{c}_{\text{y-intercept}}$$

2 VERTEX FORM / COMPLETED SQUARE FORM

$$y = \underbrace{a}_{\text{SHAPE}}(\underbrace{x-b}_{x-b=0})^2 + \underbrace{c}$$

TURNING POINT : $\boxed{x = b \quad y = c}$

3 ROOT FORM

$$y = \underbrace{a}_{\text{SHAPE}}(\underbrace{x-b}_{x-b=0})(\underbrace{x-c}_{x-c=0})$$

x-intercepts. $\boxed{x = b \qquad x = c}$

STANDARD FORM	$\longrightarrow$	COMPLETED SQUARE FORM

Q: Express $2x^2 - 16x + 44$ in form $a(x-b)^2 + c$.

$$2x^2 - 16x + 44$$

HALF

$$2\left[x^2 - 8x + (4)^2 - (4)^2 + 22\right]$$

$$2\left[(x-4)^2 - 16 + 22\right]$$

$$2\left[(x-4)^2 + 6\right]$$

$$2(x-4)^2 + 12$$

$$a(x-b)^2 + c.$$

Q: Express $2x^2 + 16x + 25$ in form $a(x-b)^2 + c$

HALF

$$2\left[x^2 + 8x + (4)^2 - (4)^2 + \frac{25}{2}\right]$$

$$2\left[(x+4)^2 - 16 + \frac{25}{2}\right]$$

$$2(x+4)^2 - 32 + 25$$

$$2(x+4)^2 - 7$$

$$a(x-b)^2 + c$$

ADVANCED

Q: Express $4x^2 - 24x + 48$ in form $(2x-a)^2 + b$

State values of a and b. (4 marks)

STEP 1: CONVERT TO COMPLETED SQUARE FORM.

$4[x^2 - 6x + (3)^2 - (3)^2 + 12]$

$4[(x-3)^2 - 9 + 12]$

$4[(x-3)^2 + 3]$

$4(x-3)^2 + 12$

$4(x-3)^2 + 12$

$2^2(x-3)^2 + 12$

Now take power common

$[2(x-3)]^2 + 12$

$(2x-6)^2 + 12$

$(2x-a)^2 + b$

STANDARD FORM $\longrightarrow$ ROOT FORM

$y = ax^2 + bx + c$ $\quad$ $y = a(x-b)(x-c)$

FACTORIZATION.

$y = 7x^2 - 63$

$y = 7[x^2 - 9]$

$y = 7[(x)^2 - (3)^2]$

$y = 7(x+3)(x-3)$

$y = a(x-b)(x-c)$

$y = x^2 - 6x - 16$

$y = x^2 - 8x + 2x - 16$

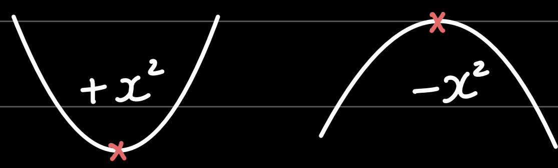
$y = x(x-8) + 2(x-8)$

$y = (x+2)(x-8)$

$y = a(x-b)(x-c)$

SKETCH OF A QUADRATIC CURVE

1 SHAPE



$a>0$ $\quad$ $a<0$

we can tell about shape from all the forms.

2 TURNING POINT / VERTEX / STATIONARY POINT.

This is found from completed square form.

$$y = \underset{\substack{\downarrow \\ \text{SHAPE} \\ \oplus}}{3}\,(\underbrace{x-2}_{x-2=0})^2 - 7$$

$\boxed{x = 2 \quad y = -7}$

Turning point $(2,-7)$

3 y-intercept $(x=0)$

This is found on STANDARD FORM.

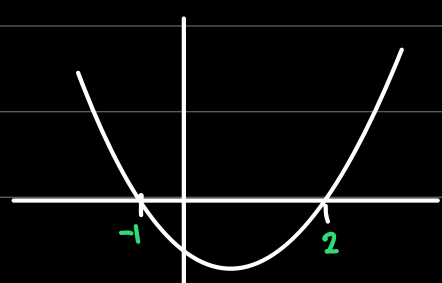
$$y = 2x^2 + 12x \underset{\substack{\downarrow \\ \text{y-int}}}{+5}$$

4 x-intercepts (ROOTS) only find these if a question particularly asks.

We will find these on ROOT FORM.

$$y = \underset{\substack{\downarrow \\ \text{SHAPE} \\ \oplus}}{3}(\underbrace{x-2}_{x-2=0})(\underbrace{x+1}_{x+1=0})$$

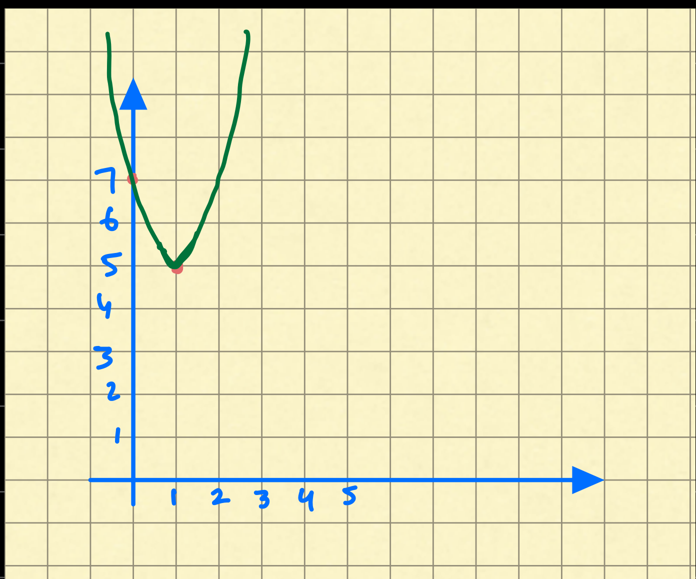
$\boxed{x = 2 \quad x = -1}$



x-intercepts.

Shape ∪   y-int

Q: SKETCH GRAPH OF $y = 2x^2 - 4x + 7$ (3marks)



$$y = 2x^2 - 4x + 7$$

$$= 2\left[x^2 - 2x + 1^2 - 1^2 + \frac{7}{2}\right]$$

$$2\left[(x-1)^2 - 1 + \frac{7}{2}\right]$$

$$2(x-1)^2 - 2 + 7$$

$$2(x-1)^2 + 5$$

$x - 1 = 0$,

Turning point ⟶ $\boxed{x = 1 \quad y = 5}$

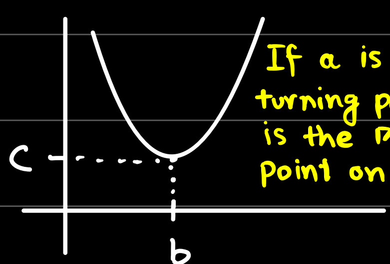
Maximum / Minimum Value

$$y = a(x-b)^2 + c$$

Min value if a is +ve

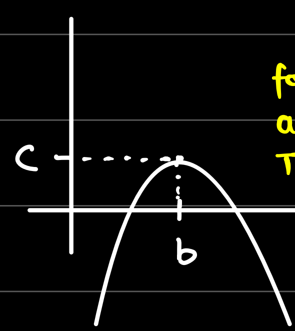
Max value if a is -ve.

If a is positive



If a is +ve, turning point is the MIN point on curve.

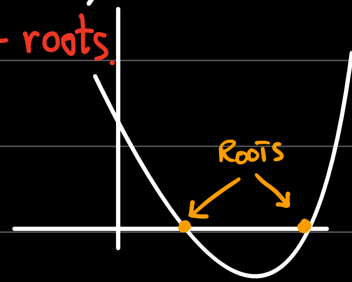
If a is negative.



for a sad face a is -ve. TP is MAX point on curve.

$\boxed{\text{ROOTS}}$ x-intercepts $(y=0)$

max. power of an equation tells total no. of roots.



eg: $y = 2x^2 + 3x + 11$

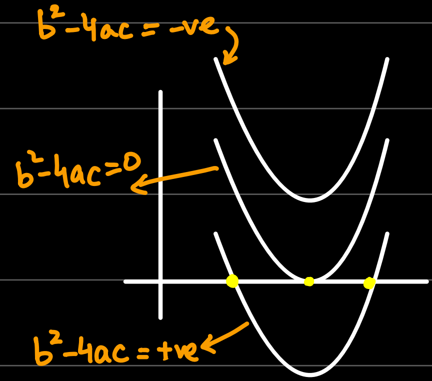
For roots (x-axis) put $y=0$

$0 = 2x^2 + 3x + 11$

$2x^2 + 3x + 11 = 0$

$x = \dfrac{-b \pm \sqrt{\boxed{b^2 - 4ac}}}{2a}$

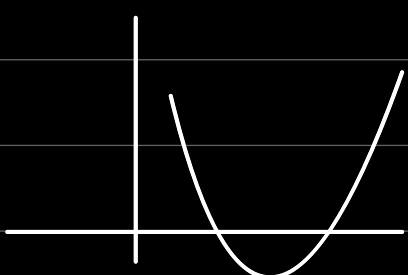
→ +ve 2 Ans
→ zero 1 Ans
→ -ve No Ans

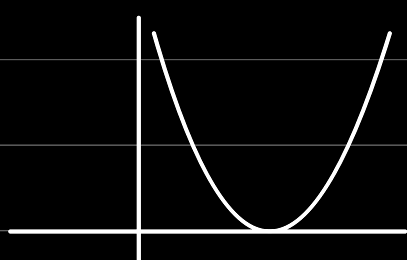


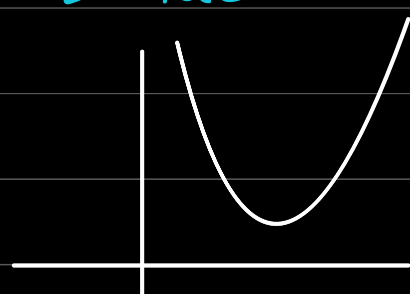
$\boxed{\text{ROOT}}$

$\boxed{b^2-4ac}$ → Discriminant $= b^2-4ac$

(2 Ans)	(1 Ans)	(No Ans)
$b^2 - 4ac = +ve$	$b^2-4ac = 0$	$b^2-4ac = -ve$







$\boxed{b^2-4ac>0}$	$\boxed{b^2-4ac=0}$	$\boxed{b^2-4ac<0}$
TWO DISTINCT REAL ROOTS	TWO EQUAL (REPEATED) REAL ROOTS.	No Real Roots.

$x^2 - 4x + 4 = 0$

$x^2 - 2x - 2x + 4 = 0$

This still has 2 roots. They are

$x(x-2)-2(x-2)=0$

$(x-2)(x-2)=0$

$x=2$

roots. They are not real. They are imaginary (complex)

If the question says "Real Roots" and does not mention "DISTINCT" ($b^2-4ac>0$) or "EQUAL" ($b^2-4ac=0$),

$$\boxed{b^2-4ac \geq 0}$$

Q. Find the set of values (inequality) of K for which $2x^2-6x+K-8=0$ has two distinct real roots. ($b^2-4ac>0$)

$2x^2-6x+K-8=0$

$b^2-4ac>0$

$(-6)^2-4(2)(K-8)>0$

$36-8K+64>0$

$100-8K>0$

$-8K>-100$

$K<\frac{-100}{-8}$

$$\boxed{K<12.5}$$

IMPORTANT

EXAMINER → VS

$\sqrt{2x+1} = \sqrt{2}\,x+1$

$\sqrt{(2x)}+1 = \sqrt{2x}+1$

$\sqrt{(2x+1)} = \sqrt{2x+1}$

Some questions ask for answers in exact form. In these questions you must **not** use your calculator to evaluate answers and you must show the steps in your working. Exact answers may include fractions or square roots and you should simplify them as far as possible.

NO CALCULATOR

Q: Find the exact values of p for which $2x^2+3px+4=0$ has two equal roots

$b^2-4ac=0$

$$2x^2+3px+4=0$$

$$b^2-4ac=0$$

$$(3p)^2-4(2)(4)=0$$

$$9p^2-32=0$$

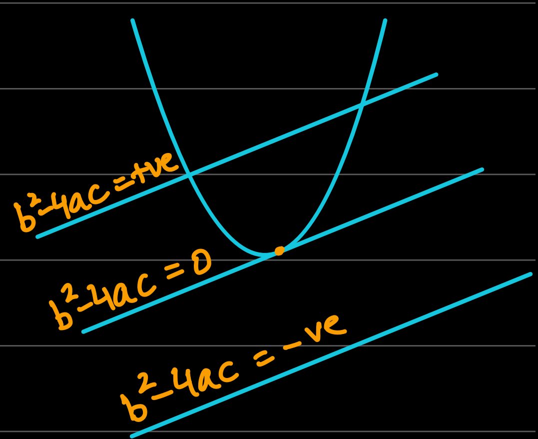
$$p^2=\frac{32}{9}$$

$$p=\pm\sqrt{\frac{32}{9}}$$

POINTS OF INTERSECTION OF A LINE AND A QUADRATIC CURVE

$$y=2x^2+6x-3$$

$$y=5x+1$$



STEP 1: EQUATE AND BRING TO STANDARD FORM.

$$2x^2+6x-3=5x+1$$

$$2x^2+6x-5x-3-1=0$$

$$2x^2+x-4=0$$

$$x=\frac{-b\pm\sqrt{\boxed{b^2-4ac}}}{2a}$$

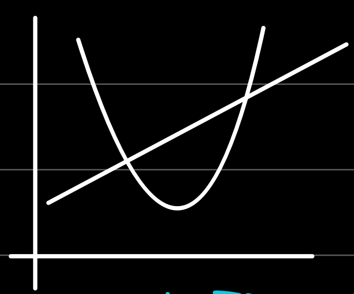
- +ve 2Ans
- Zero 1Ans
- −ve No Ans.

A line + A Quadratic Curve

STEP1: EQUATE BOTH AND BRING TO STANDARD FORM

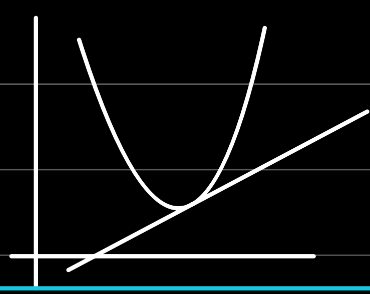
STEP2 : ANALYSE $\boxed{b^2-4ac}$

$b^2-4ac>0$



TWO POINTS OF INTERSECTION

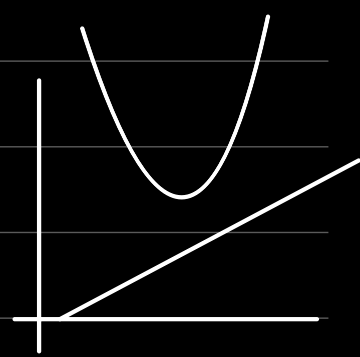
$b^2-4ac=0$



LINE IS TANGENT TO CURVE

LINE TOUCHES CURVE

$b^2-4ac<0$



Line does not intersect the curve.

IF QUESTION SAYS THAT

LINE MEETS THE CURVE
AND DOES NOT SPECIFY HOW MANY TIMES

$b^2-4ac>0$ and $b^2-4ac=0$

$$b^2-4ac\geq 0$$

Q. A line $y = 2x + c$ meets the curve $y = 2x^2 - 6x + 3c - 2$ at two distinct points. Find set of values of c.

$b^2 - 4ac > 0$

$$y = 2x^2 - 6x + 3c - 2 \qquad y = 2x + c$$

$$2x^2 - 6x + 3c - 2 = 2x + c$$

$$2x^2 - 8x + 2c - 2 = 0$$

$$b^2 - 4ac > 0$$

$$(-8)^2 - 4(2)(2c-2) > 0$$

$$64 - 16c + 16 > 0$$

$$80 - 16c > 0$$

$$-16c > -80$$

$$c < \frac{-80}{-16}$$

$$\boxed{c < 5}$$

FOR $b^2 - 4ac > 0$ and $b^2 - 4ac < 0$ we are dealing with inequalities.

INEQUALITIES (P1, P3, M1)

LINEAR (O-Levels)

1) $2x - 3 > 7$

$2x > 10$

$x > 5$

2) $-2x < 8$

$x > \frac{8}{-2}$

$x > -4$

3) $\frac{x}{-3} < 6$

$x > (6)(-3)$

$x > -18$

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1) $x^2 - 16 < 0$

$x^2 < 16$

You are not allowed to take square root on a inequality sign.

2) $(x-3)(x+1) < 0$

you are not allow this

either $x - 3 < 0$ or $x + 1 < 0$

for inequalities.

(x^2 term) QUADRATIC INEQUALITIES

(4Mark)(V.IMP)(P1, P3, M1)

STEPS: 1. Factorize

2. Sketch using $\boxed{\text{SHAPE}}$ & $\boxed{\text{x-intercepts}}$

3. Colour the correct region

□ >0 above x-axis

□ <0 below x-axis.

4. Write down an inequality for x values of coloured region.

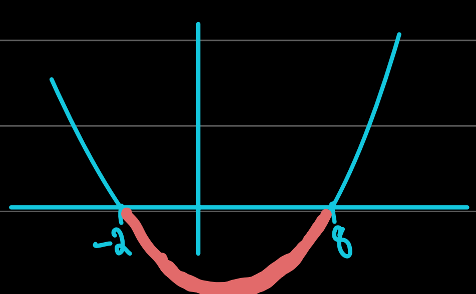
$\boxed{1}$ $x^2 - 6x - 16 < 0$

$x^2 - 8x + 2x - 16 < 0$

$x(x-8) + 2(x-8) < 0$

$(x-8)(x+2) < 0$ ↓ below x-axis

$x - 8 = 0$, $x = 8$ $\qquad$ $x + 2 = 0$, $x = -2$



STEPS: ✓1. Factorize

✓2. Sketch using $\boxed{\text{SHAPE}}$ & $\boxed{\text{x-intercepts}}$

✓3. Colour the correct region

✓4. Write down an inequality for x values of coloured region.

$$\boxed{-2 < x < 8}$$

STEPS: ✓1. Factorize
✓2. Sketch using $\boxed{\text{SHAPE}}$ & $\boxed{\text{x-intercepts}}$
✓3. Colour the correct region
✓4. Write down an inequality for x values of coloured region.

$\boxed{2}$

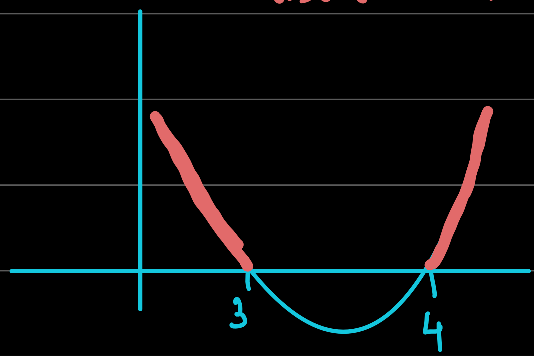
$$x^2 - 7x + 12 > 0$$

$$x^2 - 3x - 4x + 12 > 0$$

$$x(x-3) - 4(x-3) > 0$$

$$(x-3)(x-4) > 0$$

↓
above x-axis



$\boxed{x < 3}$ or $\boxed{x > 4}$

$\boxed{3}$

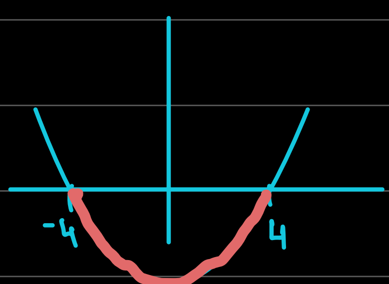
$$x^2 < 16$$

$$x^2 - 16 < 0$$

$$(x)^2 - (4)^2 < 0$$

$$(x+4)(x-4) < 0$$

STEPS: ✓1. Factorize
2. Sketch using $\boxed{\text{SHAPE}}$ & $\boxed{\text{x-intercepts}}$
3. Colour the correct region
4. Write down an inequality for x values of coloured region.



$$\boxed{-4 < x < 4}$$

$-x^2$ (sadface)

$\boxed{4}$ $(3-x)(x+2) > 0$

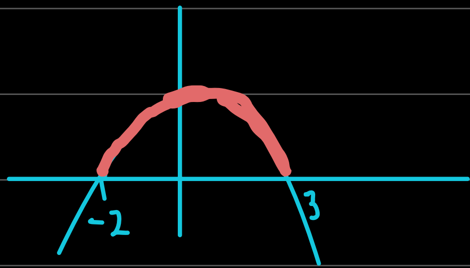
THIS IS ALREADY FACTORIZED.

STEPS: 1. Factorize

2. Sketch using $\boxed{\text{SHAPE}}$ & $\boxed{\text{x-intercepts}}$

3. Colour the correct region

4. Write down an inequality for x values of coloured region.



$\boxed{-2 < x < 3}$

Quad

Forms + Sketch.

Root

Line and Curve.

Quad Ineq..

7 The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

(i) In the case where $k = 8$, find the coordinates of the points of intersection of the line and the curve. [4]

(ii) Find the value of k for which the line is a tangent to the curve. [3]

(i) $y^2 + 2x = 13$ $\qquad$ $2y + x = 8$

$y = \sqrt{13-2x}$ $\qquad$ $y = \dfrac{8-x}{2}$

$$\left(\sqrt{13-2x}\right)^2 = \left(\frac{8-x}{2}\right)^2$$

$$13 - 2x = \frac{64 - 16x + x^2}{}$$

$$52 - 8x = 64 - 16x + x^2$$
$$0 = x^2 - 8x + 12$$
$$0 = x^2 - 6x - 2x + 12$$
$$0 = x(x-6) - 2(x-6)$$
$$0 = (x-6)(x-2)$$

$x - 6 = 0$, $x - 2 = 0$

$x = 6$ $\qquad$ $x = 2$

$y = \dfrac{8-6}{2} = 1$ $\qquad$ $y = \dfrac{8-2}{2} = 3$

$(6, 1)$ $\qquad$ $(2, 3)$

(iii)

Step1: EQUATE AND BRING TO STANDARD FORM.

$y^2 + 2x = 13$ $\qquad$ $2y + x = K$

$y = \sqrt{13 - 2x}$ $\qquad$ $y = \dfrac{K - x}{2}$

$$\left(\sqrt{13-2x}\right)^2 = \left(\frac{K-x}{2}\right)^2$$
$$13 - 2x = \frac{K^2 - 2Kx + x^2}{4}$$
$$52 - 8x = K^2 - 2Kx + x^2$$
$$0 = x^2 + 8x - 2Kx + K^2 - 52$$
$$0 = x^2 + (8 - 2K)x + K^2 - 52$$

LINE IS TANGENT TO CURVE

$$b^2 - 4ac = 0$$
$$(8 - 2K)^2 - 4(1)(K^2 - 52) = 0$$

$64 - 32k + \cancel{4k^2} - \cancel{4k^2} + 208 = 0$

$272 = 32k$

$k = \frac{17}{2}$

10 The equation of a line is $2y + x = k$, where k is a constant, and the equation of a curve is $xy = 6$.

(i) In the case where $k = 8$, the line intersects the curve at the points A and B. Find the equation of the perpendicular bisector of the line AB. [6]

$b^2 - 4ac > 0$

(ii) Find the set of values of k for which the line $2y + x = k$ intersects the curve $xy = 6$ at two distinct points. [3]

inequality.

$2y + x = k$

$y = \frac{k - x}{2}$

$yx = 6$

$y = \frac{6}{x}$

$\frac{k - x}{2} = \frac{6}{x}$

$kx - x^2 = 12$

$$0 = x^2 - kx + 12$$

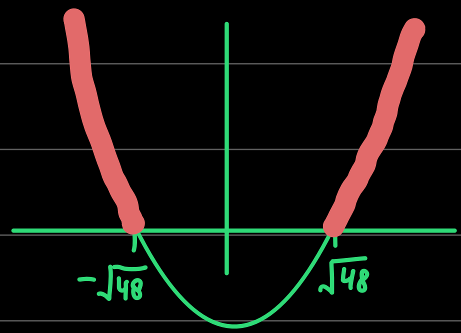
$$b^2 - 4ac > 0$$

$$(-k)^2 - 4(1)(12) > 0$$

$$k^2 - 48 > 0$$

$$(k)^2 - (\sqrt{48})^2 > 0$$

$$(k+\sqrt{48})(k-\sqrt{48}) > 0$$



$$k < -\sqrt{48} \quad \text{or} \quad k > \sqrt{48}$$

5 Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]

3 **(i)** Express $2x^2 - 4x + 1$ in the form $a(x+b)^2 + c$ and hence state the coordinates of the minimum point, A, on the curve $y = 2x^2 - 4x + 1$. [4]

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at points P and Q. It is given that the coordinates of P are $(3, 7)$.

(ii) Find the coordinates of Q. [3]

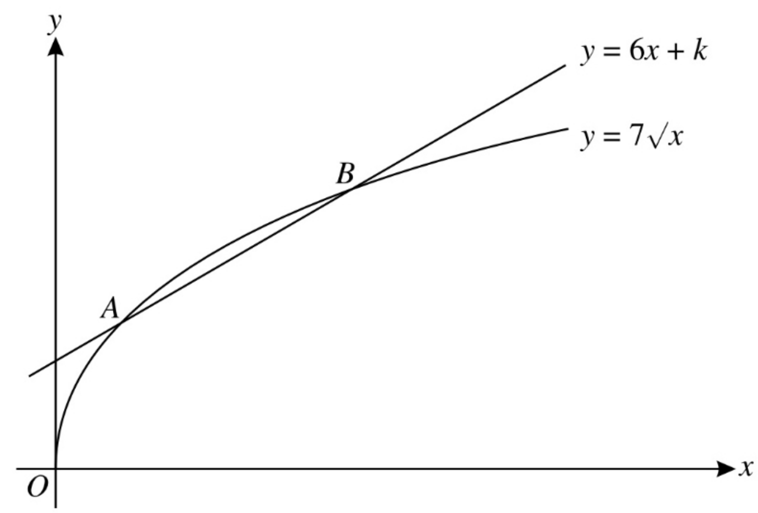
(iii) Find the equation of the line joining Q to the mid-point of AP. [3]

4 The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5.

(i) Find the values of p and q. [2]

(ii) Using these values of p and q, find the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots. [3]

9



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant. The curve and the line intersect at the points A and B.

(i) For the case where $k = 2$, find the x-coordinates of A and B. [4]

(ii) Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

10 The equation of a line is $2y + x = k$, where k is a constant, and the equation of a curve is $xy = 6$.

(i) In the case where $k = 8$, the line intersects the curve at the points A and B. Find the equation of the perpendicular bisector of the line AB. [6]

(ii) Find the set of values of k for which the line $2y + x = k$ intersects the curve $xy = 6$ at two distinct points. [3]