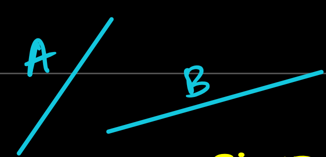
COORDINATE GEOMETRY
(7 + 4 Marks)

$A(x_1, y_1)$ $B(x_2, y_2)$

Midpoint $= \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$

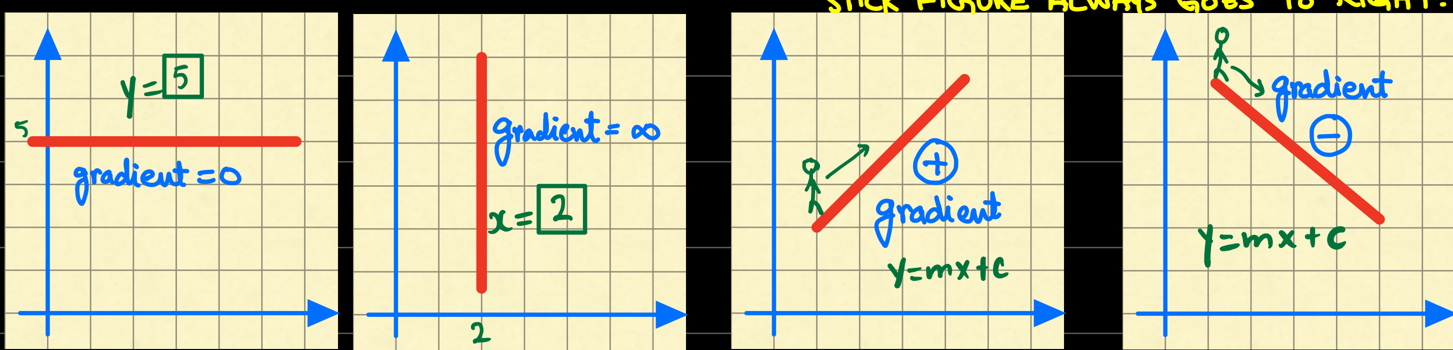
Distance : $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$



GRADIENTS (STEEPNESS)

STICK FIGURE ALWAYS GOES TO RIGHT.

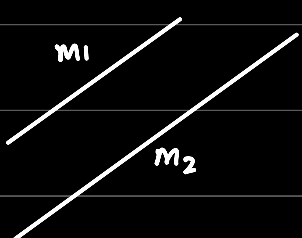


grad: +ve constant

grad: −ve constant.

PARALLEL LINES

Parallel lines have same gradient.

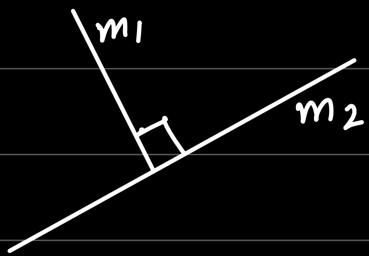


$m_1 = m_2$

$\boxed{90}$ → Right angle
→ Perpendicular
→ Normal

PERPENDICULAR LINES

↓ 90°



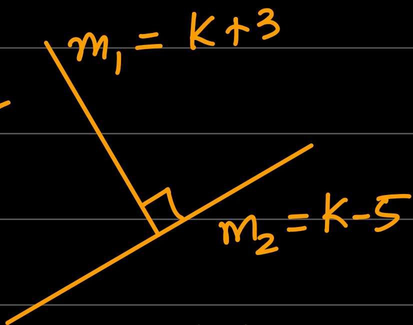
$$\boxed{m_1 \times m_2 = -1}$$

This is very important to remember

Negative Reciprocal.

$m_1 = \frac{3}{4} \longrightarrow m_2 = -\frac{4}{3}$

eg



Find value of K.

$m_1 \times m_2 = -1$

$(K+3)(K-5) = -1$

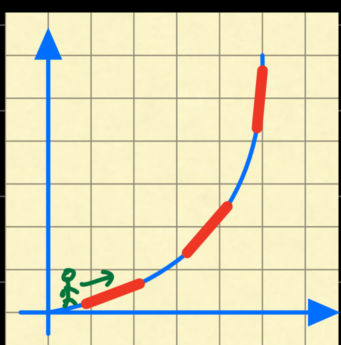
$K^2 + 3K - 5K - 15 = -1$

$K^2 - 2K - 14 = 0$

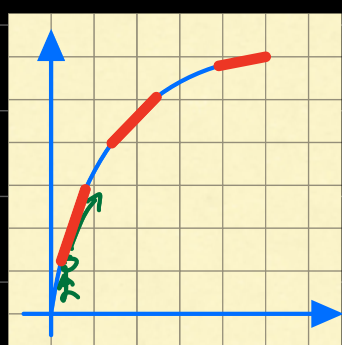
Solve Quadric eq. for value of K.

TO COMMENT ON GRADIENT
PICK ONE FROM EACH BOX.

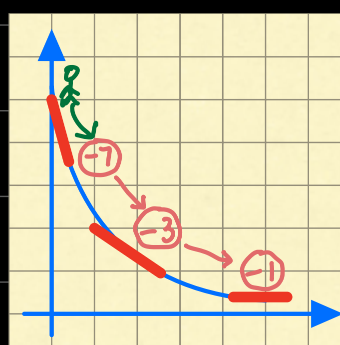
POSITIVE / NEGATIVE	(straight lines) CONSTANT, INCREASING, DECREASING.



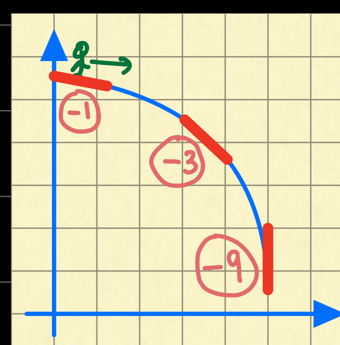
gradient: +ve
increasing



grad: +ve
decreasing



grad = -ve
decreasing

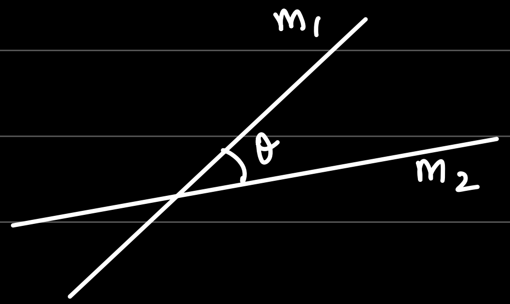


grad = -ve
increasing.

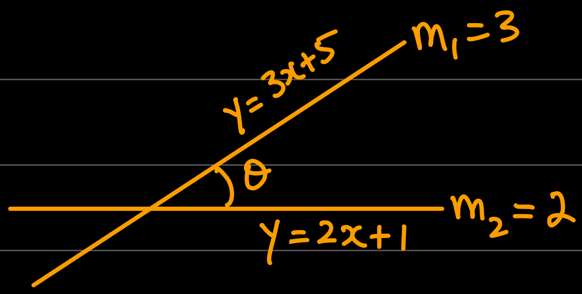
EQUATION OF A LINE

Slope-intercept form	Point-Slope form (MOST IMPORTANT)	Two Point form (optional)
$y = mx + c$	$y - y_1 = m(x - x_1)$	$\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$
Given : 1) Slope (m) 2) y-int (c)	Given : 1) Slope (m) 2) Point on line (x_1, y_1)	Given : 1) Two points (x_1, y_1) & (x_2, y_2)
Q. Find equation of line with slope 3 and cuts y axis at 2. $m = 3$, $c = 2$ $y = mx + c$ $\boxed{y = 3x + 2}$	Q: Find equation of line with slope 3 and passes through point $(\underset{x_1}{1}, \underset{y_1}{5})$ $m = 3$ $y - y_1 = m(x - x_1)$ $y - 5 = 3(x - 1)$ $y = 3x - 3 + 5$ $\boxed{y = 3x + 2}$	Q. Find equation of line passing through $(\underset{x_1}{1}, \underset{y_1}{5})$ and $(\underset{x_2}{4}, \underset{y_2}{11})$ $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$ $\dfrac{y - 5}{11 - 5} = \dfrac{x - 1}{4 - 1}$ $\dfrac{y - 5}{\cancel{6}\ 2} = \dfrac{x - 1}{\cancel{3}\ 1}$ $y - 5 = 2x - 2$ $y = 2x - 2 + 5$ $y = 2x + 3$

ACUTE ANGLE BETWEEN TWO LINES



$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$



$$\tan\theta = \frac{2-3}{1+(2)(3)}$$

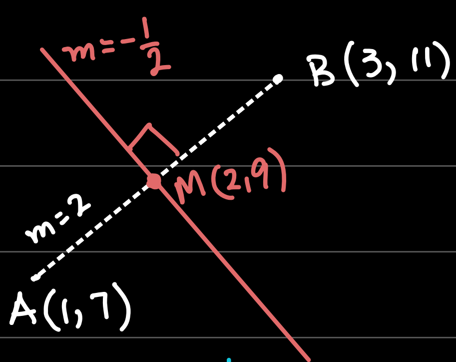
$$\tan\theta = \frac{-1}{7}$$

NOTE: ALWAY IGNORE -ve sign while taking $\sin^{-1}/\cos^{-1}/\tan^{-1}$. (we will discuss details in TRIG)

$$\theta = \tan^{-1}\left(\frac{1}{7}\right) = 8.1301$$

EQUATION OF A $\underbrace{\text{PERPENDICULAR}}_{90^\circ}$ $\underbrace{\overbrace{\text{BI}}^{2}\overbrace{\text{SECTOR}}^{\text{Divide}}}_{\text{cuts in two equal halves.}}$

(4 Marks)

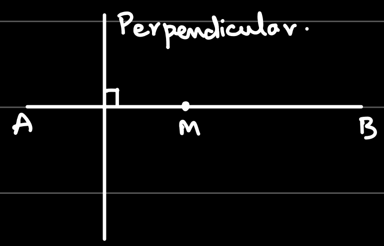


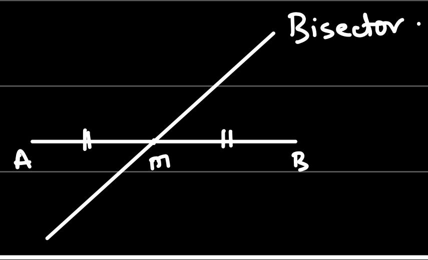
Q. Find equation of perpendicular bisector of A(1,7) and B(3,11)

Midpoint (M)	Gradients:	EQUATION:
$M\left(\frac{1+3}{2}, \frac{7+11}{2}\right)$	$m_{AB} = \frac{11-7}{3-1} = \frac{4}{2} = 2$	$m = \frac{-1}{2}$, $(\underset{x_1}{2}, \underset{y_1}{9})$
$M(2,9)$	$m_{\perp} = \frac{-1}{2}$	$y - y_1 = m(x - x_1)$ $y - 9 = \frac{-1}{2}(x - 2)$

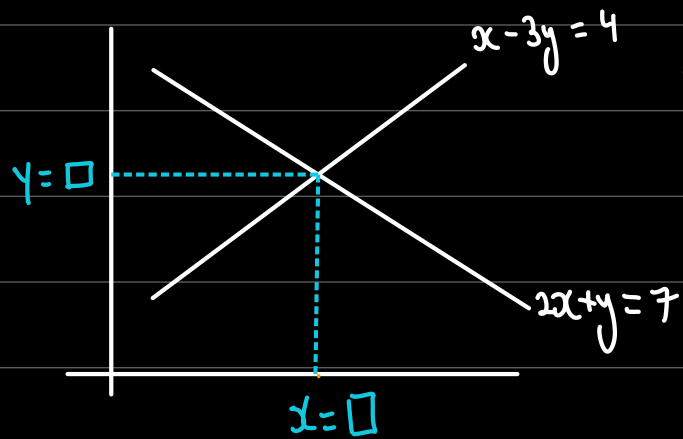
$$2y - 18 = -x + 2$$

$$\boxed{2y + x = 20}$$





POINTS OF INTERSECTION OF GRAPHS



$x = \square$, $y = \square$ $\begin{cases} 2x + y = 7 \\ x - 3y = 4 \end{cases}$

SIMULTANEOUS SOLVING:

1) ELIMINATION (only works for lines).

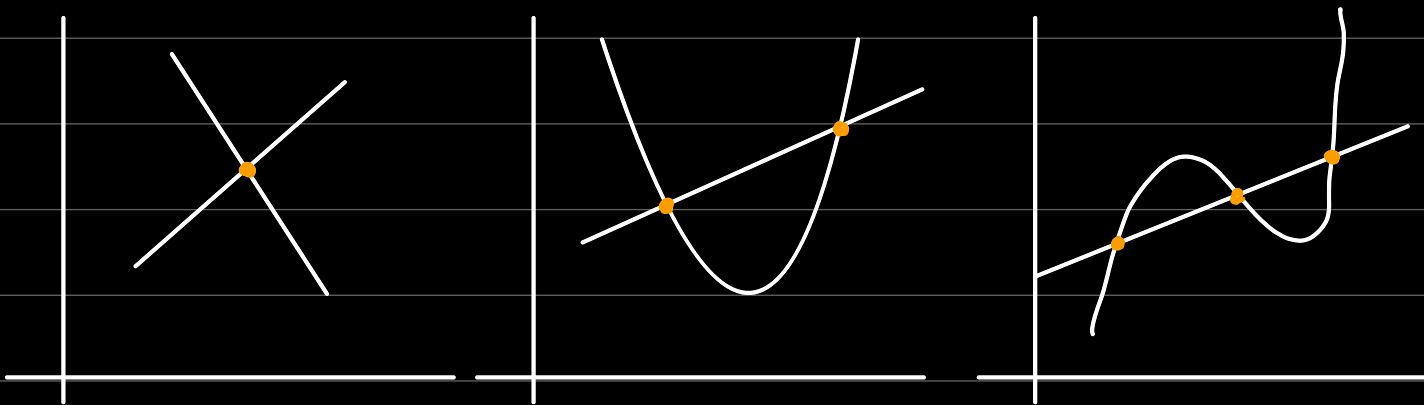
2) SUBSTITUTION (This is one we will use all time)

3) SIR ZAIN'S METHOD:

Make y subject and put both

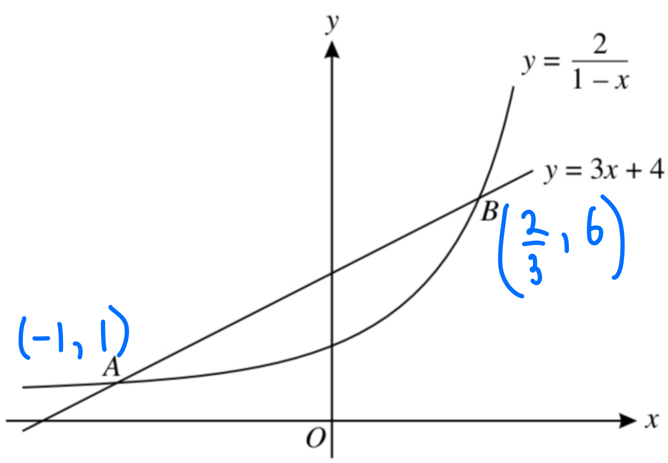
equations equal to each other.

HOW MANY POINTS OF INTERSECTION ???



It depends upon shape of graph how many points of intersection are there.

16



The diagram shows part of the curve $y = \frac{2}{1-x}$ and the line $y = 3x + 4$. The curve and the line meet at points A and B. (Points of intersection)

(i) Find the coordinates of A and B. (Simultaneously solve) [4]

(ii) Find the length of the line AB and the coordinates of the mid-point of AB. [3]

$y = \frac{2}{1-x}$ $\qquad$ $y = 3x+4$

$\frac{2}{1-x} = 3x + 4$

$$2 = (1-x)(3x+4)$$
$$2 = 3x + 4 - 3x^2 - 4x$$
$$3x^2 + 4x - 3x + 2 - 4 = 0$$
$$3x^2 + x - 2 = 0$$
$$3x^2 + 3x - 2x - 2 = 0$$
$$3x(x+1) - 2(x+1) = 0$$
$$(3x-2)(x+1) = 0$$

Note: (Preferrable)
P1 → factorization
P3 → Formula.

$3x - 2 = 0$ $\qquad$ $x + 1 = 0$

$x = \frac{2}{3}$ $\qquad$ $x = -1$

$y = 3\left(\frac{2}{3}\right) + 4$ $\qquad$ $y = 3(-1) + 4$

$y = 6$ $\qquad$ $y = 1$

$\left(\frac{2}{3}, 6\right)$ $\qquad$ $(-1, 1)$

(ii) Distance $= \sqrt{\left(-1-\frac{2}{3}\right)^2 + (1-6)^2} = \boxed{}$

Midpoint $= \left(\frac{-1+\frac{2}{3}}{2}, \frac{6+1}{2}\right) = (\quad , \quad)$

EQUATION OF CIRCLE

$$\underbrace{(x-a)}_{\substack{x-a=0\\ x=a}}{}^2 + \underbrace{(y-b)}_{\substack{y-b=0\\ y=b}}{}^2 = r^2 \quad (\text{radius})^2$$

Centre (a, b)

Q: $(x+2)^2 + (y-5)^2 = 25$

$x+2=0 \qquad y-5=0 \qquad r^2 = 25$

$x=-2 \qquad y=5$

$C(-2, 5) \qquad r=5$

Q: Equation of a circle is $x^2+4x+y^2-6y-12=0$. Find centre and radius of this circle.

$$x^2+4x+y^2-6y-12=0$$

$$\underbrace{x^2+4x+(2)^2}\ \underbrace{+y^2-6y+(3)^2} = 12+(2)^2+(3)^2$$

$$\underbrace{(x+2)}{}^2 + \underbrace{(y-3)}{}^2 = \underbrace{25}$$

$x+2=0 \qquad y-3=0 \qquad r^2 = 25$

$x=-2 \qquad y=3 \qquad r = \sqrt{25}$

Centre $(-2, 3)$ $\qquad$ radius $= 5$

Circles
- Coordinate (equations)
- Trig (Circular Measure)

Q: Equation of a circle is $x^2 - 8x + y^2 + 3y - 5 = 0$
Find centre and radius of this circle.

$$\underbrace{x^2 - 8x + (4)^2} + \underbrace{y^2 + 3y + \left(\frac{3}{2}\right)^2} = 5 + (4)^2 + \left(\frac{3}{2}\right)^2$$

$$(x-4)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{93}{4}$$

$x - 4 = 0$ | $y + \frac{3}{2} = 0$

$x = 4$ | $y = -1.5$

$C(4, -1.5)$

$r^2 = \frac{93}{4}$

$r = \sqrt{\frac{93}{4}} = 4.82182$

8

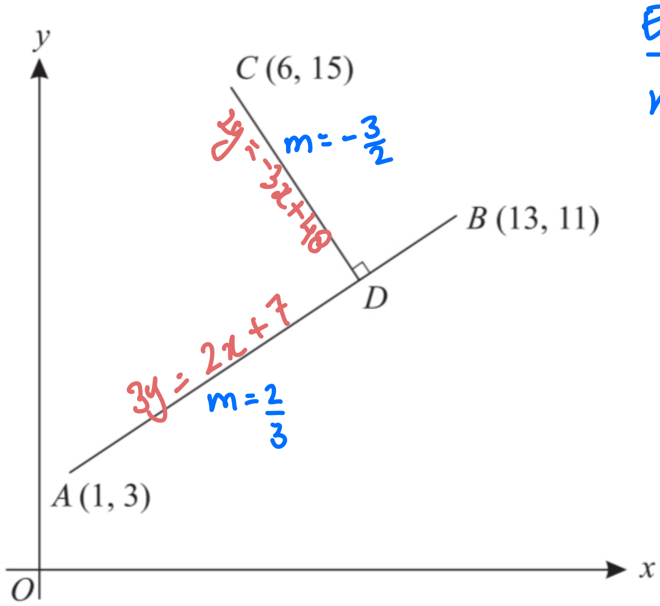
$m_{AB} = \frac{11-3}{13-1} = \frac{8}{12} = \frac{2}{3}$

$m_{CD} = -\frac{3}{2}$ $\quad C(6, 15)$ $\quad x_1 \; y_1$

$y - 15 = -\frac{3}{2}(x - 6)$

$2y - 30 = -3x + 18$

$\boxed{2y = -3x + 48}$



EQUATION OF AB

$m = \frac{2}{3}$, $A(1, 3)$

$y - 3 = \frac{2}{3}(x - 1)$

$3y - 9 = 2x - 2$

$\boxed{3y = 2x + 7}$

The three points $A\,(1,\ 3)$, $B\,(13,\ 11)$ and $C\,(6,\ 15)$ are shown in the diagram. The perpendicular from C to AB meets AB at the point D. Find

(i) the equation of CD, [3]

(ii) the coordinates of D. [4]

$y = \frac{-3x + 48}{2}$, $\quad y = \frac{2x + 7}{3}$

$\frac{-3x + 48}{2} = \frac{2x + 7}{3}$

$-9x + 144 = 4x + 14$

$144 - 14 = 4x + 9x$

$130 = 13x$

$x = 10$

$y = \frac{2(10) + 7}{3} = 9$

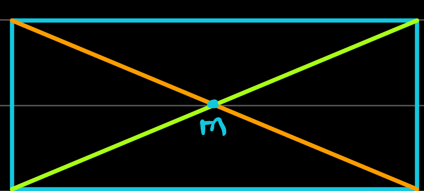
$D\,(10, 9)$

V.V. IMP

SPECIAL SHAPES

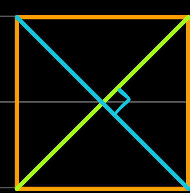
DIAGONALS MEET AT MIDPOINT

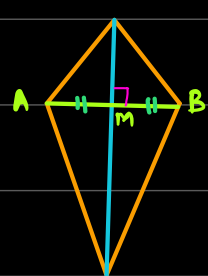
1- SQUARE
2- RHOMBUS
3- RECTANGLE
4- PARALLELOGRAM



DIAGONALS CUT AT 90°

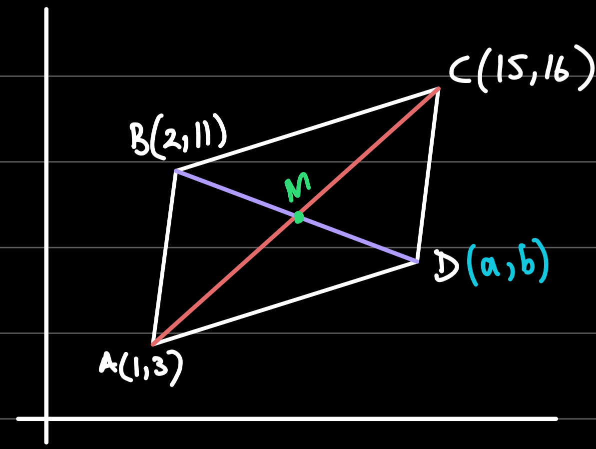
1- SQUARE
2- RHOMBUS
3- KITE





In a kite the smaller diagonal cuts at midpoint.

Q:



Given that ABCD is parallelogram, find coordinates of D (2 mark).

Diagonals cut at midpoint

midpoint of AC = midpoint of BD

$$\left(\frac{1+15}{2}, \frac{3+16}{2}\right) = \left(\frac{2+a}{2}, \frac{11+b}{2}\right)$$

$\frac{16}{\cancel{2}} = \frac{2+a}{\cancel{2}}$ $\qquad$ $\frac{19}{\cancel{2}} = \frac{11+b}{\cancel{2}}$ $\qquad$ D(14, 8)

$a = 14$ $\qquad$ $b = 8$

9 (i) $m_{AB} = \frac{14-8}{2-(-2)} = \frac{6}{4} = \frac{3}{2}$

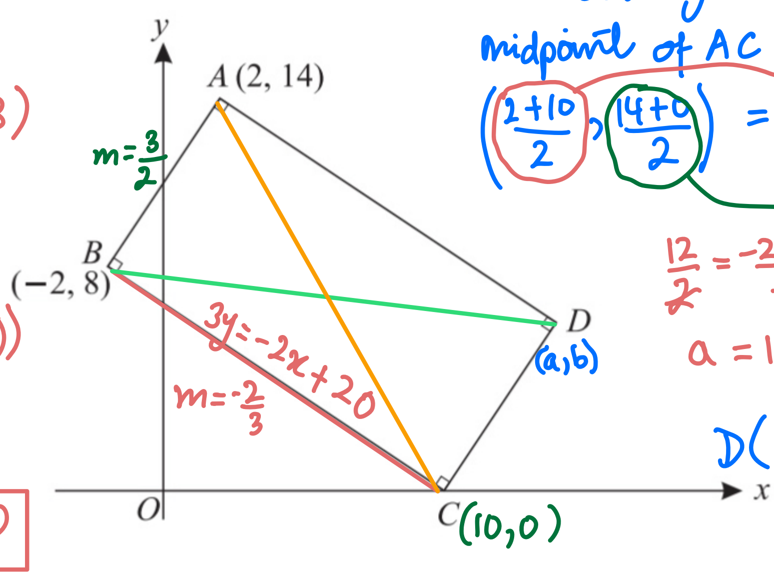
$m_{BC} = -\frac{2}{3}$, B(−2, 8)

$y - y_1 = m(x - x_1)$

$y - 8 = -\frac{2}{3}(x - (-2))$

$3y - 24 = -2x - 4$

$\boxed{3y = -2x + 20}$



Rectangle - Special shape

midpoint of AC = Midpoint of BD

$$\left(\frac{2+10}{2}, \frac{14+0}{2}\right) = \left(\frac{-2+a}{2}, \frac{8+b}{2}\right)$$

$\frac{12}{\cancel{2}} = \frac{-2+a}{\cancel{2}}$ $\qquad$ $\frac{14}{\cancel{2}} = \frac{8+b}{\cancel{2}}$

$a = 14$ $\qquad$ $b = 6$

D(14, 6)

The diagram shows a rectangle *ABCD*. The point *A* is (2, 14), *B* is (−2, 8) and *C* lies on the *x*-axis. Find

(i) the equation of *BC*, [4]

(ii) the coordinates of *C* and *D*. [3]

(ii) C (x-axis) $y = 0$

$3y = -2x + 20$

$3(0) = -2x + 20$

$x = 10$

1

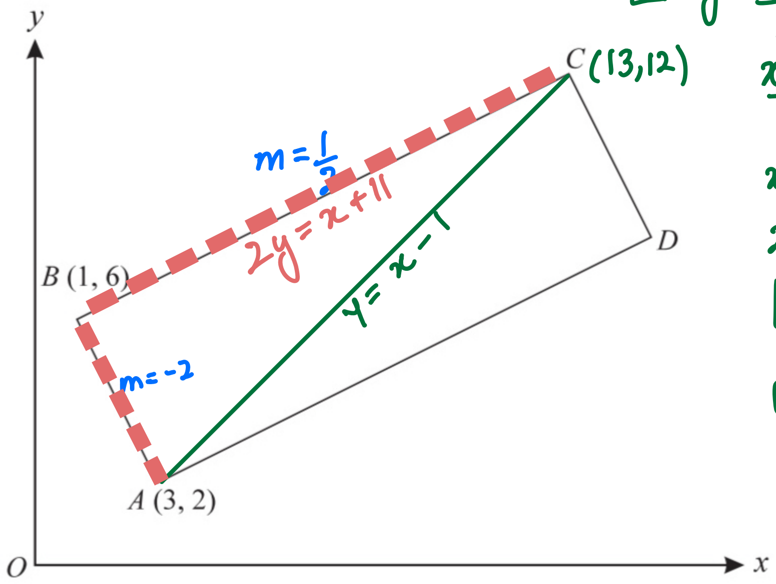
(i) $m_{AB} = \frac{6-2}{1-3} = \frac{4}{-2} = -2$

$m_{BC} = \frac{1}{2}$ $\quad B(1,6)$

$y - 6 = \frac{1}{2}(x-1)$

$2y - 12 = x - 1$

$\boxed{2y = x + 11}$



C $\quad y = \frac{x+11}{2}$, $y = x - 1$

$\frac{x+11}{2} = x - 1$

$x + 11 = 2x - 2$

$2 + 11 = 2x - x$

$\boxed{x = 13}$

$y = x - 1$
$= 13 - 1$

$\boxed{y = 12}$

$C(13, 12)$

The diagram shows a rectangle $ABCD$, where A is (3, 2) and B is (1, 6).

(i) Find the equation of BC. [4]

Given that the equation of AC is $y = x - 1$, find

(ii) the coordinates of C, (point of intersection of AC and BC) [2]

(iii) the perimeter of the rectangle $ABCD$. [3]

$A(3,2) \qquad B(1,6) \qquad C(13,12)$

$AB = \sqrt{(1-3)^2 + (6-2)^2} = \sqrt{4 + 16} = \sqrt{20}$

$BC = \sqrt{(13-1)^2 + (12-6)^2} = \sqrt{144 + 36} = \sqrt{180}$

$P = 2(L + W)$

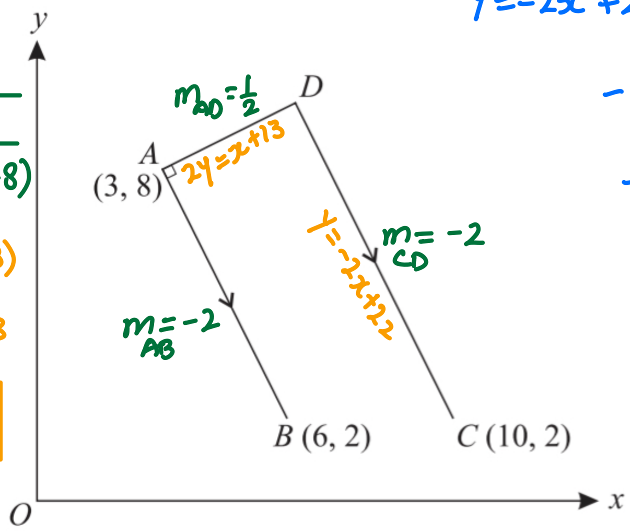
$= 2(\sqrt{20} + \sqrt{180})$

$P = 35.77$

10

$m_{AB} = \frac{2-8}{6-3} = \frac{-6}{3} = -2$

CD	AD
$m = -2$, $C(10,2)$	$m_{AD} = \frac{1}{2}$, $A(3,8)$
$y - 2 = -2(x - 10)$	$y - 8 = \frac{1}{2}(x - 3)$
$y - 2 = -2x + 20$	$2y - 16 = x - 3$
$\boxed{y = -2x + 22}$	$\boxed{2y = x + 13}$



D = point of intersection of AD and CD

$y = -2x + 22$, $y = \frac{x+13}{2}$

$-2x + 22 = \frac{x+13}{2}$

$-4x + 44 = x + 13$

$44 - 13 = x + 4x$

$5x = 31$

$\boxed{x = 6.2}$

$y = \frac{6.2 + 13}{2}$

$\boxed{y = 9.6}$

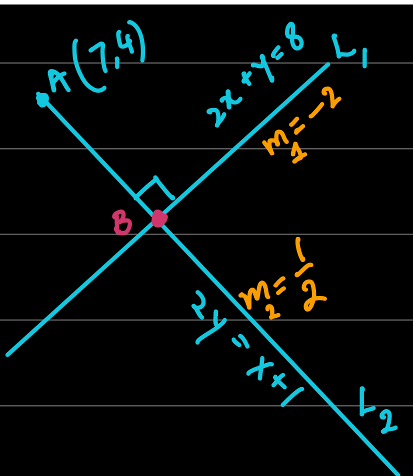
$D(6.2, 9.6)$

The three points A (3, 8), B (6, 2) and C (10, 2) are shown in the diagram. The point D is such that the line DA is perpendicular to AB and DC is parallel to AB. Calculate the coordinates of D. [7]

2 The line L_1 has equation $2x + y = 8$. The line L_2 passes through the point $A\ (7,\ 4)$ and is perpendicular to L_1.

(i) Find the equation of L_2. [4]

(ii) Given that the lines L_1 and L_2 intersect at the point B, f nd the length of AB. [4]



$2x + y = 8$

$y = -2x + 8$

$m_1 = -2$

$m_2 = \frac{1}{2}$ $\quad A\ (7, 4)$

$y - 4 = \frac{1}{2}(x - 7)$

$2y - 8 = x - 7$

$2y = x + 1$

(ii) B is point of intersection

$2x + y = 8$ $\qquad$ $2y = x + 1$

$y = -2x + 8$ $\qquad$ $y = \frac{x+1}{2}$

$-2x + 8 = \frac{x+1}{2}$

$$-4x + 16 = x + 1$$

$$16 - 1 = x + 4x$$

$$15 = 5x$$

$$x = 3$$

$$y = -2(3) + 8 = 2$$

$$B(3, 2)$$

$A(7, 4)$ $\quad B(3, 2)$

$$AB = \sqrt{(3-7)^2 + (2-4)^2}$$

$$= \sqrt{20}$$

$$= 4.472$$

5

M = Midpoint of AC

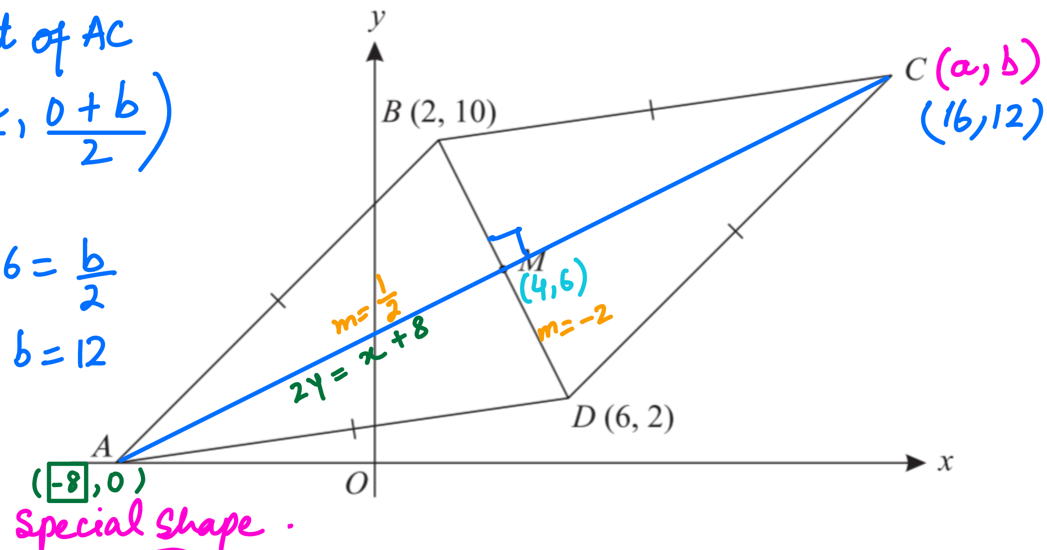
$$(4, 6) = \left(\frac{-8+a}{2}, \frac{0+b}{2}\right)$$

$$4 = \frac{-8+a}{2} \qquad 6 = \frac{b}{2}$$

$$8 = -8 + a \qquad b = 12$$

$$a = 16$$

$$C(16, 12)$$



The diagram shows a rhombus $ABCD$. The points B and D have coordinates $(2, 10)$ and $(6, 2)$ respectively, and A lies on the x-axis. The mid-point of BD is M. Find, by calculation, the coordinates of each of M, A and C. [6]

$$M = \left(\frac{2+6}{2}, \frac{10+2}{2}\right) = (4, 6)$$

$$m_{BD} = \frac{2-10}{6-2} = \frac{-8}{4} = -2$$

$$m_{AC} = \frac{1}{2} \qquad M(4, 6)$$

$$y - 6 = \frac{1}{2}(x-4)$$

$$2y - 12 = x - 4$$

$\boxed{2y = x + 8}$ EQUATION OF AC

For A : put $y = 0$

$$2(0) = x + 8$$

$$0 = x + 8$$

$$x = -8 \qquad A(-8, 0)$$

WORK SUBMISSIONS

→ Every Monday.

→ on Google ClassRoom.

CAMSCANNER

P1 Coordinate Geometry.

11

$2y + x = 16$

$y = -\frac{1}{2}x + \frac{16}{2}$

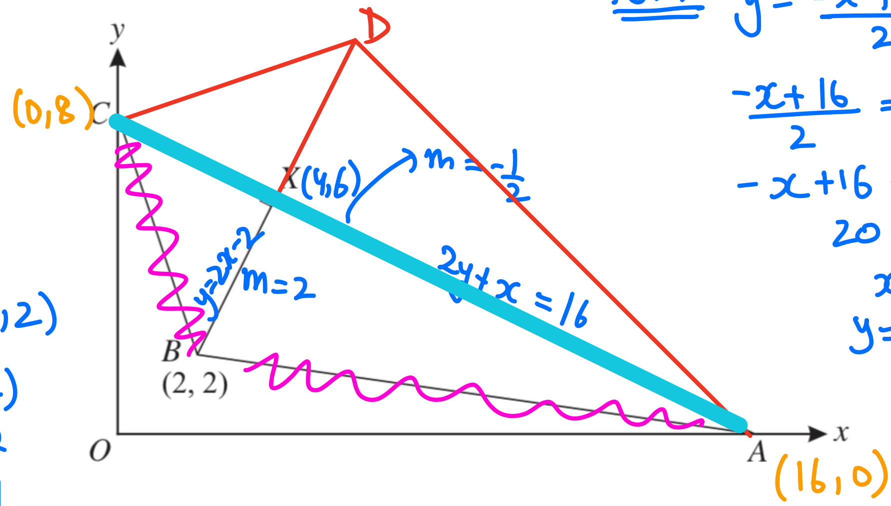
$m_{CA} = -\frac{1}{2}$

$m_{BX} = 2$, $B(2,2)$

$y - 2 = 2(x - 2)$

$y = 2x - 4 + 2$

$\boxed{y = 2x - 2}$



For X: $y = \frac{-x+16}{2}$, $y = 2x - 2$

$\frac{-x+16}{2} = 2x - 2$

$-x + 16 = 4x - 4$

$20 = 5x$

$x = 4$

$y = 2(4) - 2 = 6$

In the diagram, the points A and C lie on the x- and y-axes respectively and the equation of AC is $2y + x = 16$. The point B has coordinates $(2, 2)$. The perpendicular from B to AC meets AC at the point X.

(i) Find the coordinates of X. [4]

The point D is such that the quadrilateral $ABCD$ has AC as a line of symmetry.

(ii) Find the coordinates of D. [2]

(iii) Find, correct to 1 decimal place, the perimeter of $ABCD$. [3]

(ii) $X(4,6)$ = Midpoint of $B(2,2)$ and $D(a, b)$

$(4, 6) = \left(\frac{2+a}{2}, \frac{2+b}{2}\right)$

$4 = \frac{2+a}{2}$, $6 = \frac{2+b}{2}$

$a = 6$ $\qquad$ $b = 10$ $\qquad$ $\boxed{D(6,10)}$

(iii) $2y + x = 16$

$\boxed{C}$ y-axis $x = 0$

$2y + 0 = 16$

$y = 8$

$C(0,8)$

$\boxed{A}$ x-axis $y = 0$

$2(0) + x = 16$

$x = 16$

$A(16, 0)$

14

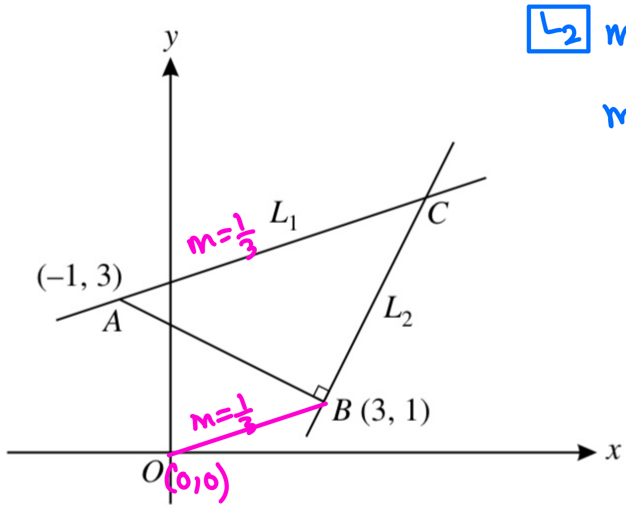
$m_{OB} = \frac{1-0}{3-0} = \frac{1}{3}$

$\boxed{L_1}$ $m = \frac{1}{3}$, $A(-1,3)$

$y - 3 = \frac{1}{3}(x-(-1))$

$3y - 9 = x + 1$

$\boxed{3y = x + 10}$



$\boxed{L_2}$ $m_{AB} = \frac{1-3}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$

$m_{BC} = 2$, $B(3,1)$

$y - 1 = 2(x-3)$

$y = 2x - 6 + 1$

$\boxed{y = 2x - 5}$

In the diagram, A is the point $(-1, 3)$ and B is the point $(3, 1)$. The line L_1 passes through A and is parallel to OB. The line L_2 passes through B and is perpendicular to AB. The lines L_1 and L_2 meet at C. Find the coordinates of C. [6]

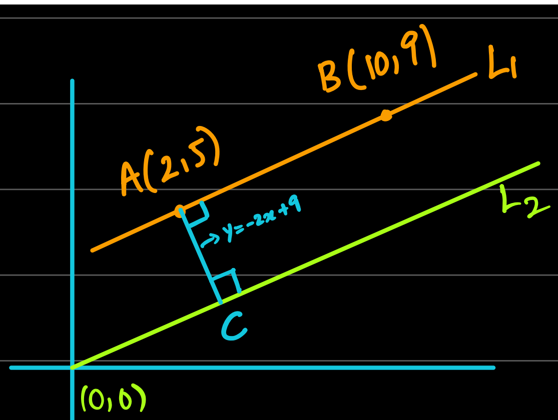
For C Simultaneously Solve

$3y = x + 10$, $y = 2x - 5$

18 The line L_1 passes through the points $A(2, 5)$ and $B(10, 9)$. The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2. Find

(i) the coordinates of C, [5]

(ii) the distance AC. [2]



$m_{AB} = \frac{9-5}{10-2} = \frac{4}{8} = \frac{1}{2}$

$m_{AC} = -2$, $A(2,5)$

$y - 5 = -2(x-2)$

$y - 5 = -2x + 4$

$\boxed{y = -2x + 9}$

$\boxed{L_2}$ $m = \frac{1}{2}$, $(0,0)$

$y - 0 = \frac{1}{2}(x - 0)$

$\boxed{y = \frac{1}{2}x}$

c is point of intersection of AC and L_2.

$-2x + 9 = \frac{1}{2}x$

$-4x + 18 = x$

$5x = 18$

$x = 3.6$

$y =$

19 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, meets the x-axis at P and the y-axis at Q. Given that $PQ = \sqrt{(45)}$ and that the gradient of the line PQ is $-\frac{1}{2}$, find the values of a and b. [5]

P(x-axis) (y=0)

$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{x}{a} + \frac{0}{b} = 1$

$\frac{x}{a} = 1$

$x = a$

$P(a, 0)$

Q(y-axis)(x=0)

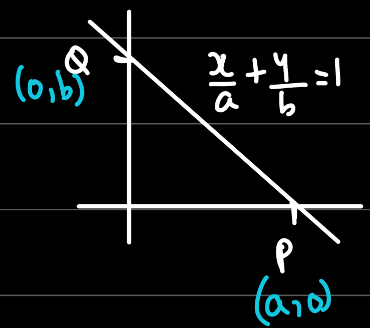
$\frac{x}{a} + \frac{y}{b} = 1$

$\frac{0}{a} + \frac{y}{b} = 1$

$\frac{y}{b} = 1$

$y = b$

$Q(0, b)$



$PQ = \sqrt{(a-0)^2 + (0-b)^2}$

$\sqrt{45} = \sqrt{a^2 + b^2}$

$\boxed{45 = a^2 + b^2}$

$m_{PQ} = \frac{b - 0}{0 - a}$

$-\frac{1}{} = \frac{b}{}$

$45 = (2b)^2 + b^2$ $\qquad$ $2 \qquad -a$

$45 = 4b^2 + b^2$ $\qquad$ $\boxed{a = 2b}$

$45 = 5b^2$

$b^2 = 9$

$b = \sqrt{9}$

$b = 3$ $\qquad$ $a = 2b = 2(3) = 6$

30 The coordinates of points A and B are $(a, 2)$ and $(3, b)$ respectively, where a and b are constants. The distance AB is $\sqrt{(125)}$ units and the gradient of the line AB is 2. Find the possible values of a and of b. [6]

$A(a, 2)$ $\qquad$ $B(3, b)$

$AB = \sqrt{125}$ $\qquad$ grad $= 2$

$\sqrt{(3-a)^2 + (b-2)^2} = \sqrt{125}$ $\qquad$ $\frac{b-2}{3-a} = 2$

$(3-a)^2 + (b-2)^2 = 125$ $\qquad$ $b - 2 = 6 - 2a$

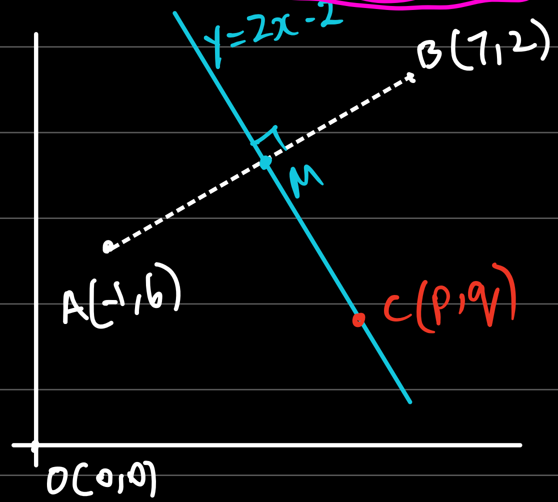
$(3-a)^2 + (8-2a-2)^2 = 125$ $\qquad$ $b = 8 - 2a$

27 The point A has coordinates $(-1, 6)$ and the point B has coordinates $(7, 2)$.

(i) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$. [4]

$y = 2x - 2$

(ii) A point C on the perpendicular bisector has coordinates (p, q). The distance OC is 2 units, where O is the origin. Write down two equations involving p and q and hence find the coordinates of the possible positions of C. [5]



$OC = \sqrt{(p-0)^2 + (q-0)^2}$

$2 = \sqrt{p^2 + q^2}$

$\boxed{p^2 + q^2 = 4}$

$y = 2x - 2 \longrightarrow C(p, q)$

$\boxed{q = 2p - 2}$

$p^2 + (2p-2)^2 = 4$

$p^2 + 4p^2 - 8p + 4 = 4$

$5p^2 - 8p = 0$

$p(5p - 8) = 0$

$p = 0 \quad , \quad p = \frac{8}{5}$

$q = 2(0) - 2$

$q = -2$

$q = 2\left(\frac{8}{5}\right) - 2$

$q = 1.2$

$C(0, -2) \qquad C\left(\frac{8}{5}, 1.2\right)$