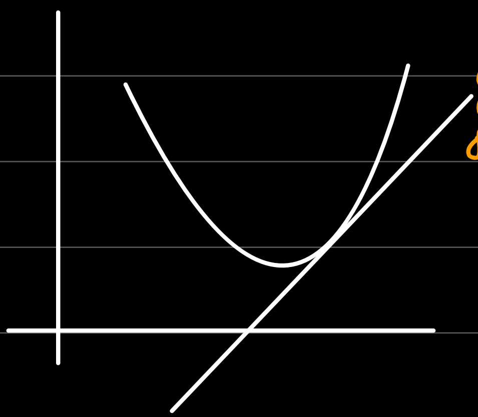
DIFFERENTIATION (8 - 10 marks) (P1)
(GRADIENT OF TANGENT/CURVE).

O-Levels:



gradient of Tangent.
$= \frac{dy}{dx}$

SYMBOLS:

$y \xrightarrow{\text{diff}} \frac{dy}{dx}$

$y \xrightarrow{\text{diff}} y'$

HOW TO DIFFERTIATE

BASE RULES:

$\boxed{1}$

$x \longrightarrow 1$

$2x \longrightarrow 2$

$10x \longrightarrow 10$

$-4x \longrightarrow -4$

$\boxed{2}$

alone constant $\longrightarrow 0$

$5 \longrightarrow 0$

$-3 \longrightarrow 0$

POWER RULE: $(\boxed{})^n \longrightarrow n(\boxed{})^{n-1} \times \boxed{}'$

Examples:

(i) $y = 2x + 5$ (ii) $y = 5 - 8x$

$\frac{dy}{dx} = 2 + 0$

$\frac{dy}{dx} = 2$

$\frac{dy}{dx} = 0 - 8$

$\frac{dy}{dx} = -8$

$$\left(\boxed{}\right)^{n} \longrightarrow n\left(\boxed{}\right)^{n-1} \times \square'$$

(iii) $y = x^7$

$y = \left(\boxed{x}\right)^{7}$

$\frac{dy}{dx} = 7\left(x\right)^6 \times 1$

$\frac{dy}{dx} = 7x^6$

(iv) $y = 3x^7$

$y = 3\left(\boxed{x}\right)^{7}$

$\frac{dy}{dx} = 3(7)(x)^6 \times 1$

$\frac{dy}{dx} = 21x^6$

(v) $y = (3x)^7$

$y = \left(\boxed{3x}\right)^{7}$

$\frac{dy}{dx} = 7(3x)^6 \times (3)$

$\frac{dy}{dx} = 21(3x)^6$

(vi) $y = (2x+3)^5$

$y = \left(\boxed{2x+3}\right)^{5}$

$2x + 3 \to 2 + 0$

$\frac{dy}{dx} = 5(2x+3)^4 \times (2+0)$

$\frac{dy}{dx} = 10(2x+3)^4$

(vii) $y = \sqrt{3x-5}$

$y = \left(\boxed{3x-5}\right)^{\frac{1}{2}}$

$3x - 5 \to 3 - 0$

$\frac{dy}{dx} = \frac{1}{2}(3x-5)^{-\frac{1}{2}} \times (3-0)$

$\frac{dy}{dx} = \frac{3}{2}(3x-5)^{-\frac{1}{2}}$

We cannot differentiate in DENOMINATOR.

(i) $y = \frac{2}{x} + \frac{5}{x^2}$

$y = 2\boxed{x}^{-1} + 5\boxed{x}^{-2}$

$\frac{dy}{dx} = 2(-1)x^{-2} \times 1 + 5(-2)x^{-3} \times 1$

$\frac{dy}{dx} = -2x^{-2} - 10x^{-3}$

$$\boxed{\frac{dy}{dx} = \frac{-2}{x^2} - \frac{10}{x^3}}$$

(ii) $y = \frac{3}{(2x-7)^2}$

$y = 3(\boxed{2x-7})^{-2}$

$\frac{dy}{dx} = 3(-2)(2x-7)^{-3} \times (2-0)$

$\frac{dy}{dx} = -12(2x-7)^{-3}$

$$\boxed{\frac{dy}{dx} = \frac{-12}{(2x-7)^3}}$$

EQUATION OF TANGENT & NORMAL

Q $y = x^2 + 4x - 3$

(i) Find $\frac{dy}{dx} = 2x^1(1) + 4 - 0 = 2x + 4$

(ii) Find gradient of tangent (curve) at $x = 2$

$\frac{dy}{dx} = 2x + 4$ $\quad x = 2$

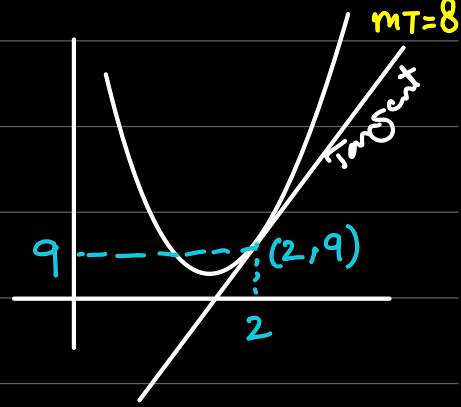
$m_T = 2(2) + 4 = 8$

(iii) Find equation of tangent at $x=2$.

First find value of y

$x=2$, $y = x^2 + 4x - 3$

$y = (2)^2 + 4(2) - 3 = 9$



Tangent: $m_T = 8$, $(2,9)$

$y - 9 = 8(x-2)$

$y - 9 = 8x - 16$

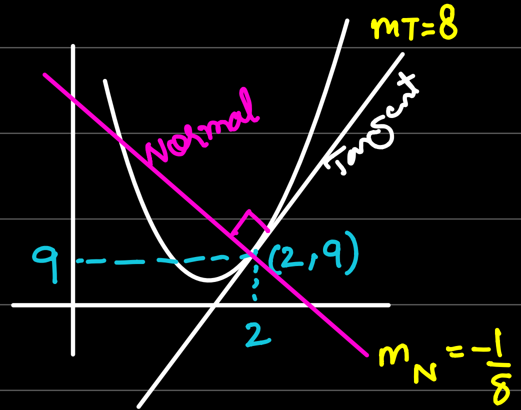
$y = 8x - 7$

(iv) Find equation of Normal at $x=2$.

$m_T = 8$

$\downarrow$

$m_N = -\frac{1}{8}$, $(2,9)$

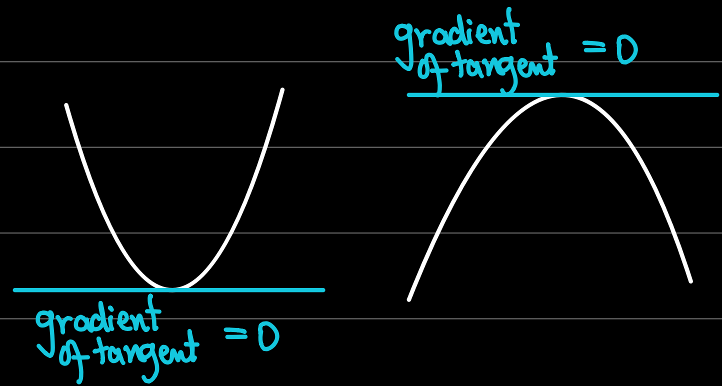


$y - 9 = -\frac{1}{8}(x-2)$

$8y - 72 = -x + 2$

$\boxed{8y = -x + 74}$

1) STATIONARY POINT
TURNING POINT
MAXIMUM POINT
MINIMUM POINT
CRITICAL POINT
VERTEX



$$\boxed{\frac{dy}{dx} = 0}$$

2) Nature of turning point.

$$y \xrightarrow{\text{DIFF}} \frac{dy}{dx} \xrightarrow{\text{DIFF}} \frac{d^2y}{dx^2}$$

+ve min

−ve max

STATIONARY POINT $\boxed{\frac{dy}{dx} = 0}$

NATURE OF STATIONARY POINT.

7 The equation of a curve is $y = (2x-3)^3 - 6x$.

(i) Express $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ in terms of x. [3]

(ii) Find the x-coordinates of the two stationary points and determine the nature of each stationary point. [5]

$\frac{dy}{dx} = 0$

(i) $y = (\boxed{2x-3})^3 - 6x$

$\frac{dy}{dx} = 3(2x-3)^2(2) - 6$

$$\boxed{\frac{dy}{dx} = 6(2x-3)^2 - 6}$$

$\frac{dy}{dx} = 6(\boxed{2x-3})^2 - 6$

$\frac{d^2y}{dx^2} = 6(2)(2x-3)^1 \times (2) - 0$

$\frac{d^2y}{dx^2} = 24(2x-3)$

(ii) $\frac{dy}{dx} = 0$

$6(2x-3)^2 - 6 = 0$

$6(2x-3)^2 = 6$

$\sqrt{(2x-3)^2} = \pm\sqrt{1}$

$2x-3 = \pm 1$

$x = \frac{3 \pm 1}{2}$

$x = 2 \quad , \quad x = 1$

For nature use $\frac{d^2y}{dx^2}$.

$x = 2, \ \frac{d^2y}{dx^2} = 24(2x-3) = 24(2(2)-3)$

$\frac{d^2y}{dx^2} = 24$ (+ve)(Min)

$x = 1, \ \frac{d^2y}{dx^2} = 24(2x-3) = 24(2(1)-3)$

$\frac{d^2y}{dx^2} = -24$ (−ve)(Max)

11 The equation of a curve is $y = \frac{12}{x^2+3}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [2]

(ii) Find the equation of the normal to the curve at the point $P\,(1,\ 3)$. [3]

$$y = 12\left(\boxed{x^2+3}\right)^{-1}$$

$$\square = x^2+3$$

$$\square' = 2x^1(1)+0$$

$$= 2x$$

$$\frac{dy}{dx} = 12(-1)(x^2+3)^{-2} \times (2x)$$

$$\frac{dy}{dx} = \frac{-24x}{(x^2+3)^2}$$

(ii) $$\frac{dy}{dx} = \frac{-24x}{(x^2+3)^2}$$

$P(1, 3)$

$x=1,\ y=3$

$$x=1,\ \frac{dy}{dx} = m_T = \frac{-24(1)}{(1^2+3)^2} = \frac{-24}{16} = -\frac{3}{2}$$

$$m_N = \frac{2}{3},\quad P(1,3)$$

$$y-3 = \frac{2}{3}(x-1)$$

$$3y-9 = 2x-2$$

$$\boxed{3y = 2x+7}$$

RATE OF CHANGE

$$\text{Rate of } \square = \frac{d\square}{dt}$$

$$\text{Rate of Change of } x = \frac{dx}{dt}$$

$$\text{Rate of change of } y = \frac{dy}{dt}$$

$$\text{Rate of change of Area} = \frac{dA}{dt}$$

Rate of increase is positive
Rate of decrease is negative.

LAYOUT

Rate

Two main variables + connecting equation.

x y $y = 2x^2+5x+1$

$\frac{dx}{dt}$ $\frac{dy}{dt}$ $\frac{dy}{dx}$

$$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$$

(Chain Rule)

Q: $y = 2x^2 - 6x + 3$

(i) Find $\frac{dy}{dx} = 4x - 6$

(ii) x coordinate is increasing at a rate of 3 units per second. Find the rate of increase of y at P(1, −1)

Two main variables + Connecting Equation

x $\quad$ y $\quad$ $y = 2x^2 - 6x + 3$

$\frac{dx}{dt}$ $\quad$ $\frac{dy}{dt}$ $\quad$ $\frac{dy}{dx}$

$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = ?$

$x = 1, y = -1$

$$\frac{dy}{dt} = \frac{dx}{dt} \times \frac{dy}{dx}$$

$$? = 3 \times (4x - 6)$$

$$\frac{dy}{dt} = 3 \times (4(1) - 6)$$

$$\boxed{\frac{dy}{dt} = -6}$$

Q. A circular pond is expanding in such a way that rate of increase of its radius is 5 m/s. Find the rate of increase of area when $r = 3$.

TWO MAIN VARIABLES + CONNECTING EQUATION

r $\quad$ A $\quad$ $A = \pi r^2$

$\downarrow$ $\quad$ $\downarrow$ $\quad$ $\downarrow$

$\frac{dr}{dt}$ $\quad$ $\frac{dA}{dt}$ $\quad$ $\frac{dA}{dr}$

$A = \pi r^2$

$A = \pi \boxed{r}^2$

$\frac{dA}{dr} = \pi (2) r^{'}(1)$

$\frac{dA}{dr} = 2\pi r$

$\frac{dr}{dt} = 5$

$r = 3$

$$\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi r \times 5$$

$$\frac{dA}{dt} = 2\pi(3) \times 5$$

$$\frac{dA}{dt} = 30\pi$$

PROOFS WALAY QUESTIONS

LAYOUT

AT start of question a substitution is given.

PROOF

$\downarrow$

START PROOF

$\downarrow$

you will get stuck in a few steps.

Plug in the Substitution from start

The proof works

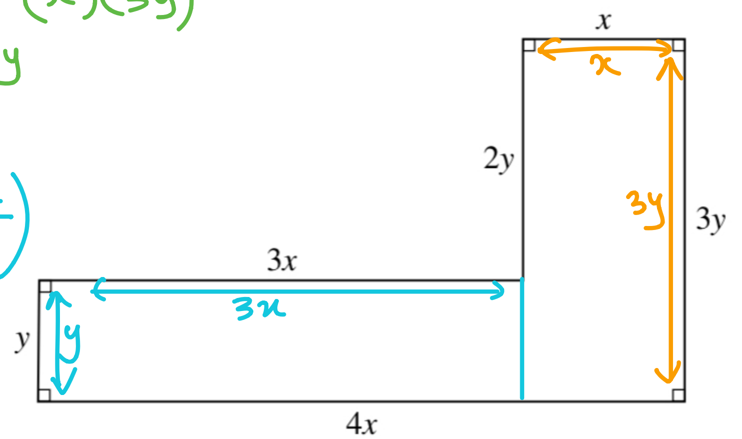
26 $A = (y)(3x) + (x)(3y)$

$A = 3xy + 3xy$

$A = 6xy$

$A = \cancel{6}x\left(\frac{48-8x}{\cancel{6}}\right)$

$\boxed{A = 48x - 8x^2}$



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

$y + 3x + 2y + x + 3y + 4x = 48$

$6y + 8x = 48$

(i) Find an expression for y in terms of x. [1]

$\boxed{y = \frac{48-8x}{6}}$

(ii) Given that the area of the garden is A m^2, show that $A = 48x - 8x^2$. [2]

(iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

diff $= 0$

$A = 48x - 8x^2$

$\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 48 - 16x$

$0 = 48 - 16x$

$x = 3$

$A = 48(3) - 8(3)^2$

$\boxed{A = 72}$

$\frac{dA}{dx} = 48 - 16x$

$\frac{d^2A}{dx^2} = -16$ (max)

ALWAYS ATTEMPT LAST PARTS BEFORE ATTEMPTING. PROOF

13 A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is $96\,\text{cm}^2$.

(i) Express h in terms of x and show that the volume, $V\,\text{cm}^3$, of the block is given by

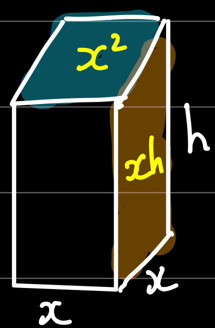
substitution

$$V = 24x - \tfrac{1}{2}x^3.$$ [3]

Given that x can vary,

(ii) find the stationary value of V, [3]

(iii) determine whether this stationary value is a maximum or a minimum. [2]



Total SA = 96

$2(x^2) + 4(xh) = 96$

$2x^2 + 4xh = 96$

$h = \dfrac{96 - 2x^2}{4x}$

$V = l \times w \times h$

$V = (x)(x)(h)$

$V = x^2 h$

$V = x^{\cancel{2}}\left[\dfrac{96 - 2x^2}{4\cancel{x}}\right]$

$V = \dfrac{96x - 2x^3}{4}$

$V = \dfrac{96x}{4} - \dfrac{2x^3}{4}$

$V = 24x - \dfrac{1}{2}x^3$

(ii) $V = 24x - \frac{1}{2}x^3$

$$\frac{dV}{dx} = 24 - \frac{1}{2}(3)x^2(1)$$

$$\boxed{\frac{dV}{dx} = 24 - \frac{3}{2}x^2}$$

$$0 = 24 - \frac{3}{2}x^2$$

$$\frac{3}{2}x^2 = 24$$

$$x^2 = 16$$

$$x = 4$$

$$V = 24(4) - \frac{1}{2}(4)^3$$

$$V = 64$$

(iii) $\frac{dV}{dx} = 24 - \frac{3}{2}x^2$

$$\frac{d^2V}{dx^2} = 0 - \frac{3}{\cancel{2}}(\cancel{2})x^1(1)$$

$$\frac{d^2V}{dx^2} = -3x$$

$$x = 4$$

$$\frac{d^2V}{dx^2} = -3(4)$$

$$= -12 \text{ (Max)}$$