FUNCTIONS (P1)

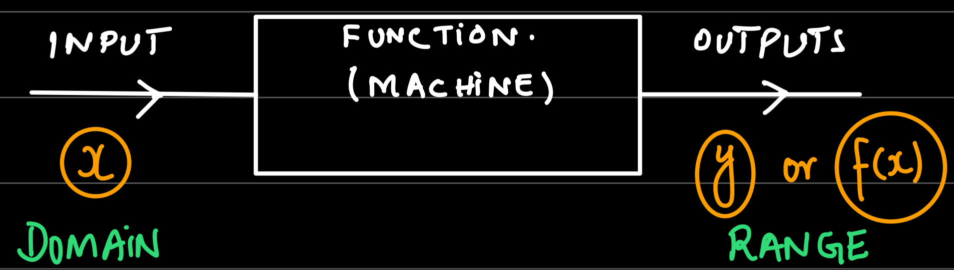
(10 MARKS)

FUNCTIONS ALONE	WITH QUADRATICS	WITH TRIG	WITH DIFFERENTIATION

VERY LONG BUT EASY CHAPTER.

MEMORIZE EVERY DETAIL!

FUNCTIONS ARE NUMBER MACHINES.



$f(x) = 2x + 3$ $\qquad\qquad$ $g(x) = x^2 - 5$

Name of the function $\qquad$ value of x

$f(1) = 2(1) + 3 = 5$

$f(4) = 2(4) + 3 = 11$

$g(3) = (3)^2 - 5 = 4$

$f(K) = 2K + 3$

$g(t-1) = (t-1)^2 - 5$

INVERSE OF A FUNCTION:

(2 Marks in O Levels) (3 MARKS IN A LEVELS)

THERE ARE ADDITIONAL STEPS. BE CAREFUL!

$$f(x) = 2x - 5$$

$$f(x) = y$$

$$y = 2x - 5$$

$$y + 5 = 2x$$

$$x = \frac{y+5}{2}$$

$$f^{-1}(y) = x$$

$$f^{-1}(y) = \frac{y+5}{2}$$

$$f^{-1}(x) = \frac{x+5}{2}$$

$$f(x) = y$$

$$x = f^{-1}(y)$$

$$f^{-1}(y) = x$$

Q: $f(x) = 2x + 8$ Find $f^{-1}(x)$

$$f(x) = y$$
$$y = 2x + 8$$
$$y - 8 = 2x$$
$$x = \frac{y-8}{2}$$

$$f^{-1}(y) = x$$
$$f^{-1}(y) = \frac{y-8}{2}$$

$$f^{-1}(x) = \frac{x-8}{2}$$

Q $g(x) = x^3 - 8$ Find $g^{-1}(x)$

$$g(x) = y$$
$$y = x^3 - 8$$
$$x^3 = y + 8$$
$$x = \sqrt[3]{y + 8}$$
$$g^{-1}(y) = x$$

$$g^{-1}(y) = \sqrt[3]{y + 8}$$

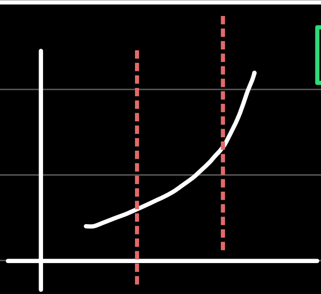
$$g^{-1}(x) = \sqrt[3]{x + 8}$$

TEST

VERTICAL LINE TEST	HORIZONTAL LINE TEST
CHECKS WHETER A GRAPH IS A FUNCTION OR NOT?	CHECKS IF THE INVERSE OF A FUNCTION EXISTS OR NOT?

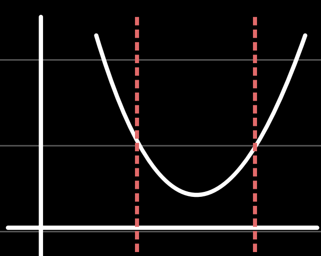
VERTICAL LINE TEST

CHECKS WHETER A GRAPH IS A FUNCTION OR NOT?



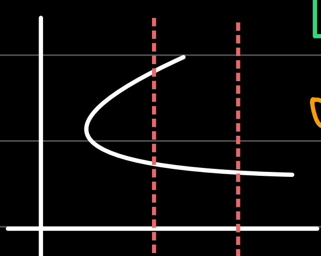
PASS FUNCTION

ONE-ONE FUNCTION



PASS FUNCTION

MANY-ONE FUNCION

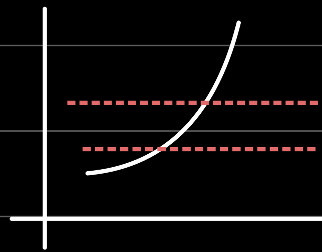


FAIL NOT A FUNCTION

ONE-MANY.

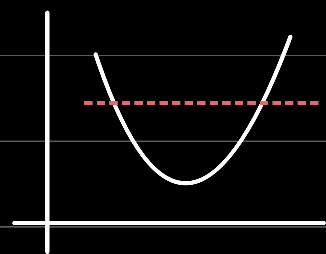
HORIZONTAL LINE TEST

CHECKS IF THE INVERSE OF A FUNCTION EXISTS OR NOT?



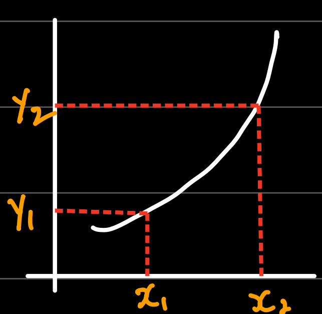
PASS INVERSE EXISTS.

ONE-ONE FUNCTION



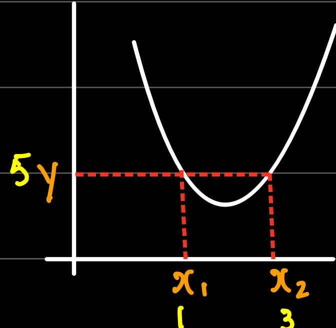
FAIL INVERSE DOES NOT EXIST.

MANY-ONE FUNCION



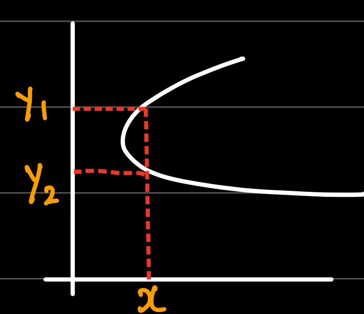
ONE-ONE FUNCTION

For one input you will get one output



MANY-ONE FUNCTION

Many inputs will give you one output.



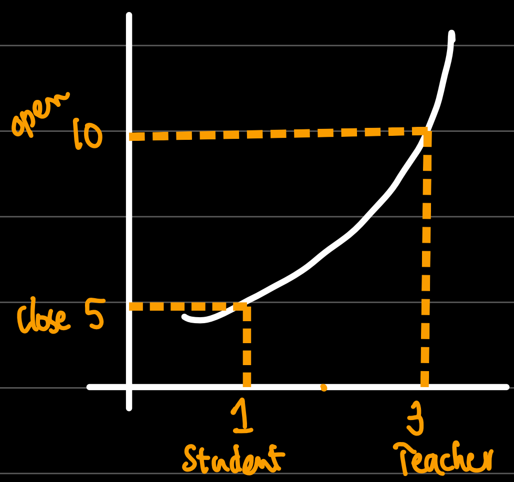
ONE - MANY

one input can give many outputs.

ACCESS POINTS

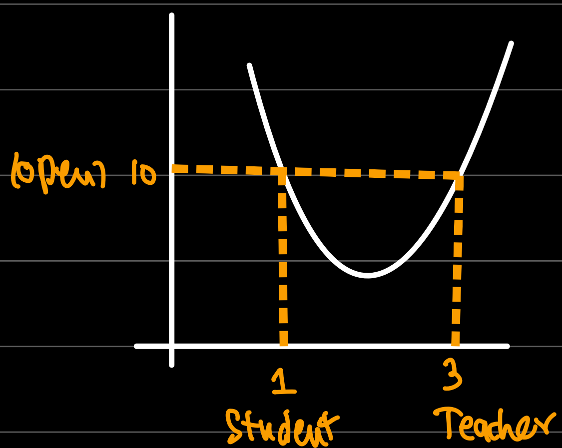
INPUT = 1 (STUDENT)
= 3 (TEACHER)

OUTPUT = 5 (CLOSE)
= 10 (OPEN)



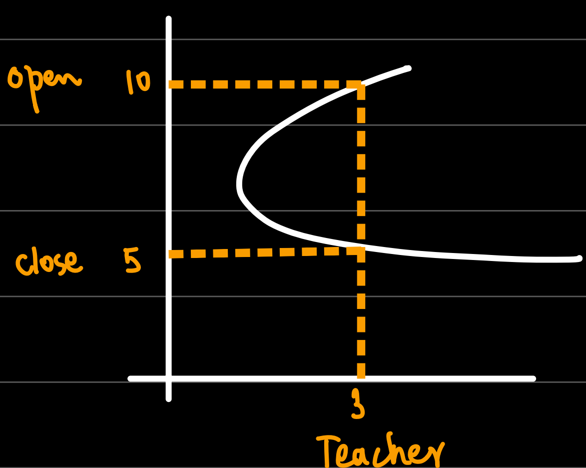
This can be used for STAFF ROOM or EXAM CENTRE.

ONE - ONE FUNCTION



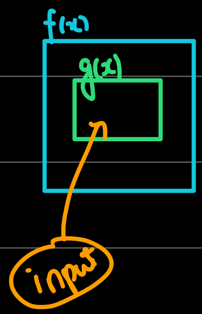
This can be used in classroom, Main gate, library, cafe.

MANY - ONE FUNCTION.



This is not a useful machine.

ONE - MANY is not useful machine.



COMPOSITE FUNCTIONS

(FUNCTION WITHIN A FUNCTION)

Inception
Dream within a dream.

$f(x) = 2x + 5$ $\qquad$ $g(x) = x^2 + 3$

$fg(x) = 2(x^2 + 3) + 5 = 2x^2 + 11$

Concept: $fg(x)$

$f(g(x)) = 2g(x) + 5$

$= 2(x^2+3) + 5$

$gf(x) = (2x + 5)^2 + 3$

$ff(x) = 2(2x + 5) + 5$

$gg(x) = (x^2 + 3)^2 + 3$

Q: $f(x) = 2x + 3$

Given that $ff(t) = 21$, find t.

$ff(x) = 2(2x+3) + 3$

$ff(x) = 4x + 9$

$ff(t) = 4t + 9$

$21 = 4t + 9$

$12 = 4t$

$t = 3$

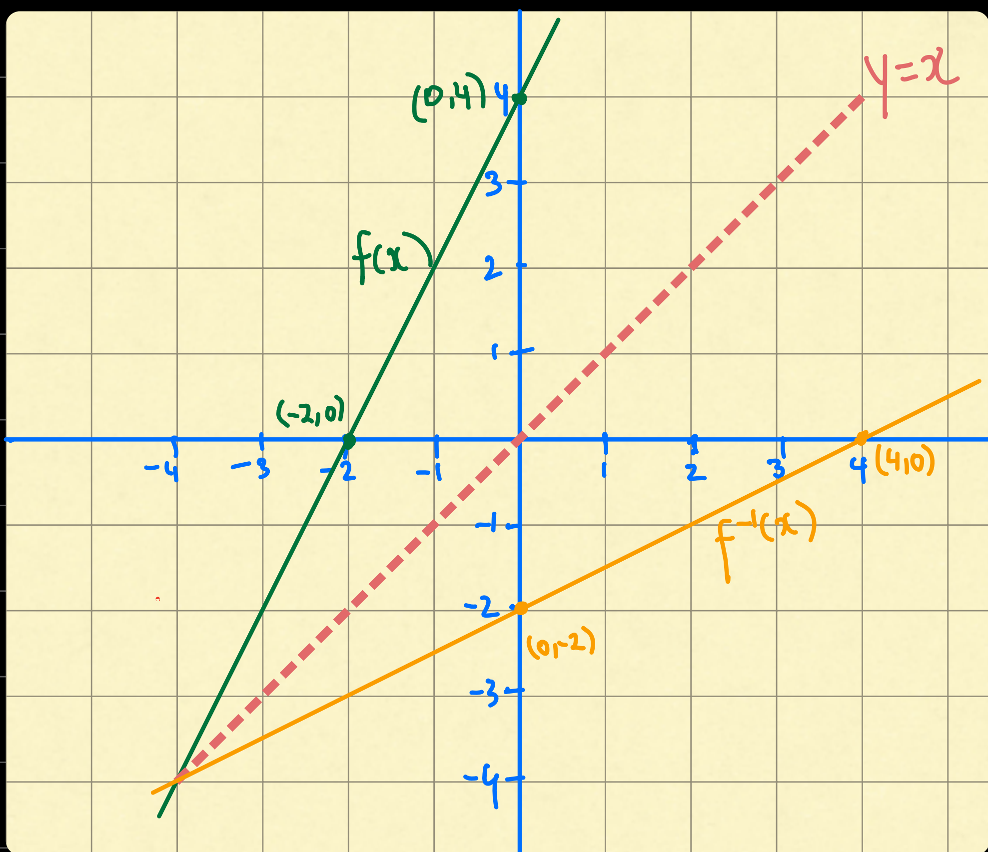
SKETCH OF $f(x)$ and $f^{-1}(x)$ ON SAME DIAGRAM.

Q: $f(x) = 2x + 4$

Sketch $f(x)$ and $f^{-1}(x)$ on same diagram making clear the relationship between them.

(3 marks).

IMP: THIS MUST BE DRAWN TO-SCALE OF 1:1 on x and y axis. (same scale on both axis)



$f(x) = 2x + 4$

$y = 2x + 4$

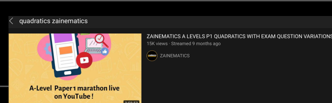
x-int: $y = 0$ $0 = 2x + 4$, $x = -2$

y-int: $x = 0$ $y = 2(0) + 4$, $y = 4$

$f^{-1}(x)$ is reflection of $f(x)$ in line $y = x$.

Tomm: Domain & Range

Quadratics (must do before class)



Q: $f(x) = 2x^2 - 8x + 14$

(a) Express $f(x)$ in form $a(x-b)^2 + c$

$2[x^2 - 4x + (2)^2 - (2)^2 + 7]$

$2[(x-2)^2 - 4 + 7]$

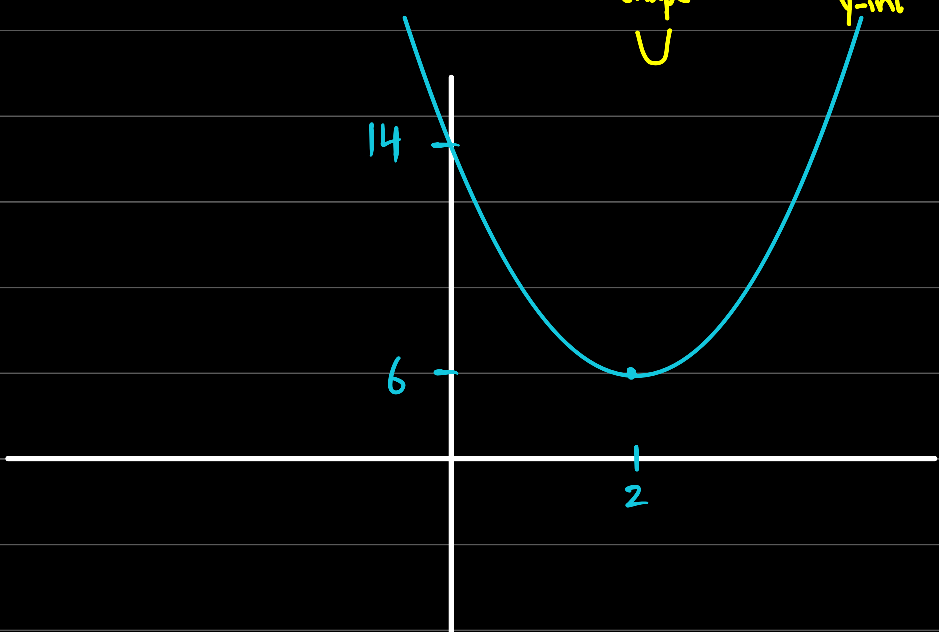
$2[(x-2)^2 + 3]$

$2(x-2)^2 + 6$

(b) Sketch $f(x)$.

$y = \underbrace{2x^2}_{\text{Shape } \cup} - 8x + \underbrace{14}_{y\text{-int}} = 2\underbrace{(x-2)}_{\substack{x-2=0 \\ x=2}}^2 + \underbrace{6}_{y=6}$

TP $(2, 6)$



(c) State with a reason whether $f^{-1}(x)$ exists or not?

No, because it is not one-one function.

$\boxed{\text{DOMAIN}}$ VALUES OF X FOR WHICH A FUNCTION DOES NOT GO CRAZY.

$\boxed{\text{TYPE 1}}$

$f(x) = \sqrt{x}$

Domain: $x \geq 0$
$x \in R$

$f(x) = \sqrt{x-2}$

Domain $x \geq 2$
$x \in R$

$\boxed{\text{GENERAL RULE}}$

$f(x) = \sqrt{\boxed{}}$

$x \in R$
x belongs to Real numbers
Any no.

Domain: $\boxed{} \geq 0$, $x \in R$
Solve this

$f(x) = \sqrt{2x+3}$

Domain: $2x + 3 \geq 0$

$2x \geq -3$

$x \geq \frac{-3}{2}$, $x \in R$

$\boxed{\text{TYPE 2}}$

$f(x) = \frac{3}{x}$

Domain: $x \neq 0$
$x \in R$

$f(x) = \frac{3}{x-2}$

Domain: $x \neq 2$
$x \in R$

GENERAL RULE.

$f(x) = \frac{\text{Anything}}{\boxed{}}$

Domain: $\boxed{} \neq 0$, $x \in R$
Solve

$$f(x) = \frac{2x+5}{3x-1}$$

Domain: $3x - 1 \neq 0$

$3x \neq 1$

$\boxed{x \neq \frac{1}{3},\ x \in R}$

RANGE: VALUES OF Y THAT GRAPH OF $f(x)$ COVERS.

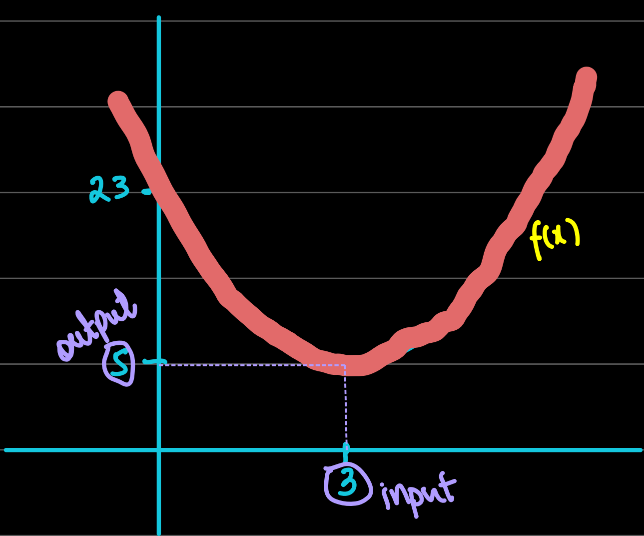
GOLDEN RULE: NEVER TELL RANGE OF A FUNCTION WITHOUT LOOKING AT ITS GRAPH.

Just like we can not tell the flavor of milkshake (output → range) by looking at blender (function).

FIND RANGE OF FOLLOWING FUNCTIONS:

(a) $f(x) = 2(x-3)^2 + 5 \quad \underline{x \in R}$

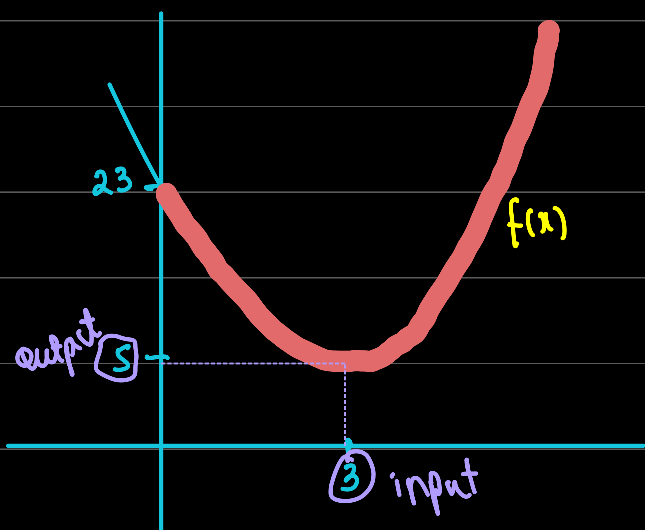
You are allowed all values of x.



Range: $y \geqslant 5$

$f(x) \geqslant 5$

(b) $f(x) = 2(x-3)^2 + 5 \quad x > 0$

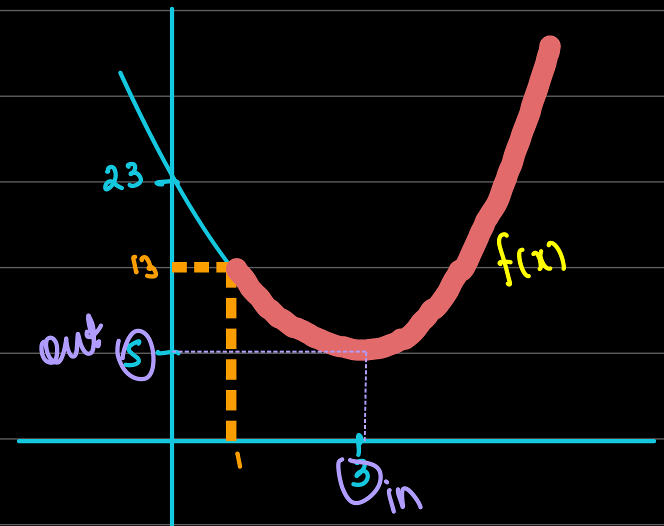


Range: $y \geqslant 5$

$f(x) \geqslant 5$

(c) $f(x) = 2(x-3)^2 + 5 \quad x \geqslant 1$

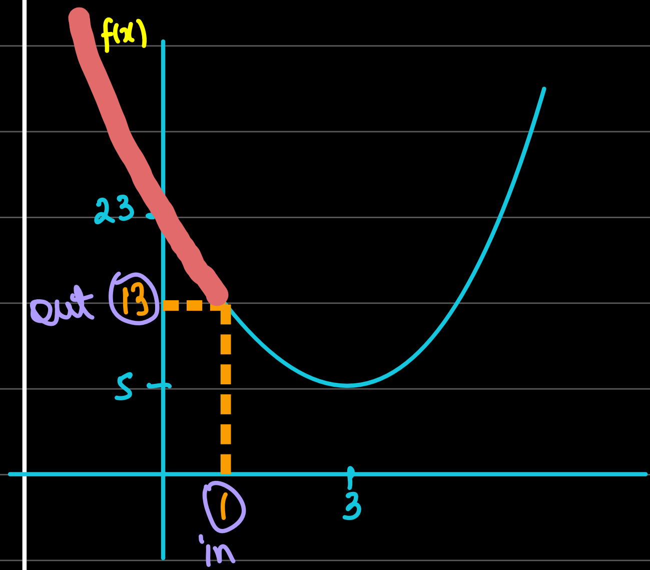
$x = 1 = 2(1-3)^2 + 5 = 13$



Range: $y \geqslant 5$

$f(x) \geqslant 5$

(d) $f(x) = 2(x-3)^2 + 5 \quad x < 1$

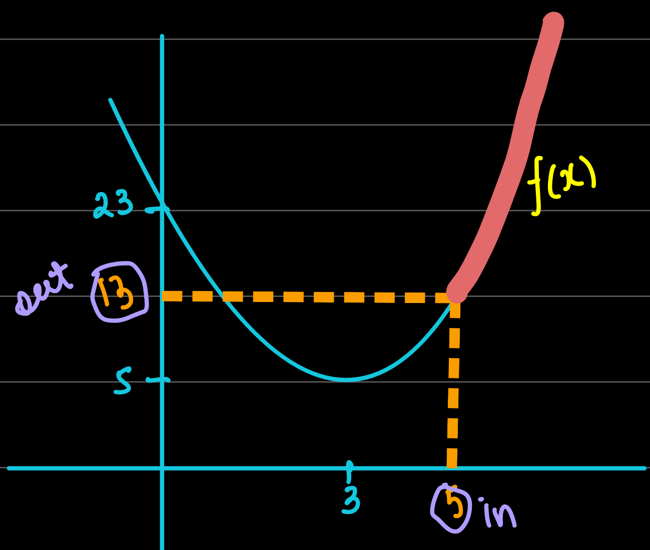


Range: $y > 13$

$f(x) > 13$

(e) $f(x) = 2(x-3)^2 + 5 \quad x \geqslant 5$

$x = 5, \ 2(5-3)^2 + 5 = 13$



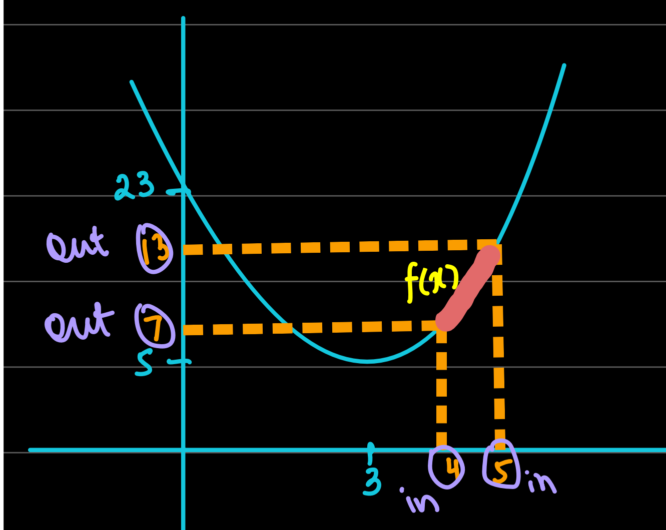
Range: $f(x) \geqslant 13$

$y \geqslant 13$

(f)

$f(x) = 2(x-3)^2 + 5 \quad 4 < x \leq 5$

$x = 4, \ 2(4-3)^2 + 5 = 7$

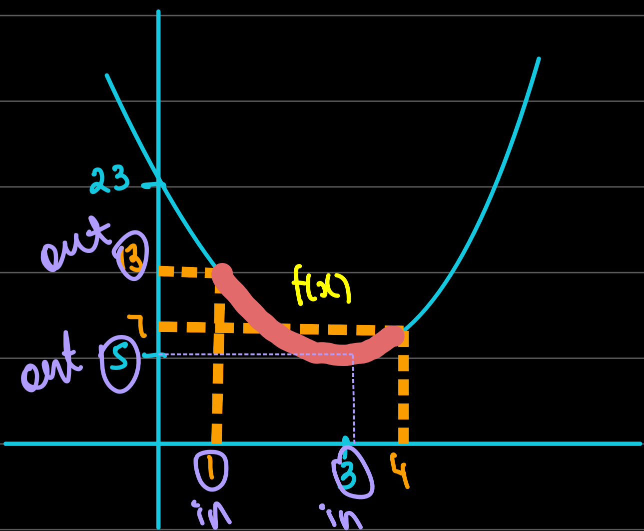


Range: $7 < y \leq 13$

$7 < f(x) \leq 13$

(g)

$f(x) = 2(x-3)^2 + 5 \qquad 1 < x < 4$

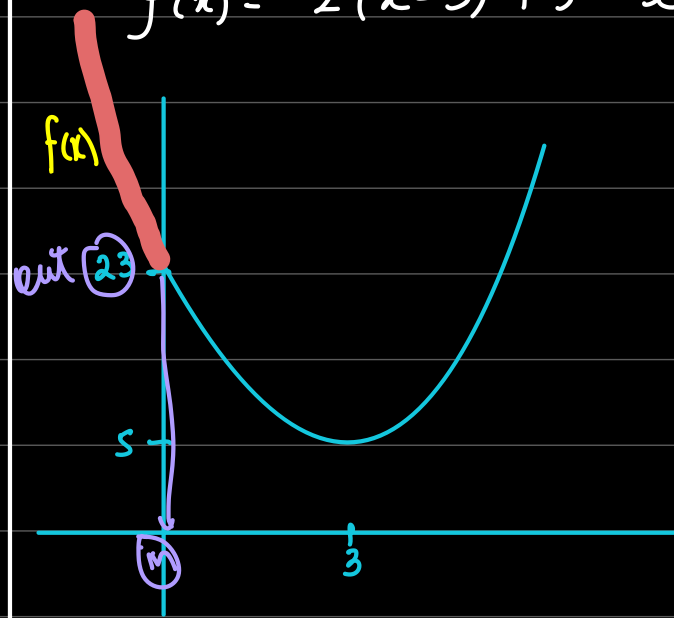


Range: $5 \leq y < 13$

$5 \leq f(x) < 13$

(h)

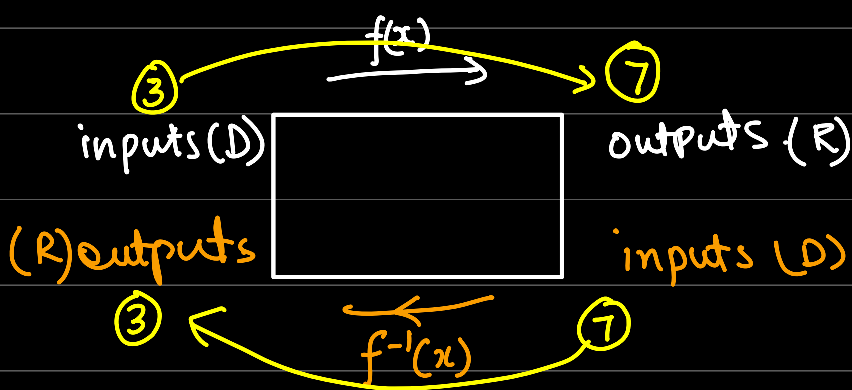
$f(x) = 2(x-3)^2 + 5 \qquad x < 0$



Range: $y > 23$

$f(x) > 23$

DOMAIN OF $f(x)$ = RANGE OF $f^{-1}(x)$
RANGE OF $f(x)$ = DOMAIN OF $f^{-1}(x)$



$f^{-1}(x)$ = inverse

$f'(x)$ = diff of $f(x)$

THIS QUESTION CONTAINS THREE V. IMP OUTCOMES TO MEMORIZE.

Q: $f(x) = 2x^2 - 12x + 10 \qquad x \in R$

(i) Express $f(x)$ in form $a(x-b)^2 + c$

$2[x^2 - 6x + (3)^2 - (3)^2 + 5]$

$2[(x-3)^2 - 9 + 5]$

$2[(x-3)^2 - 4]$

$2(x-3)^2 - 8$

(b) Sketch $f(x)$.

$y = \underbrace{2}_{\text{Shape}}x^2 - 12x + \underbrace{10}_{\text{y-int}} = 2(\underbrace{x-3}_{x-3=0}) \underbrace{^2 - 8}_{}$

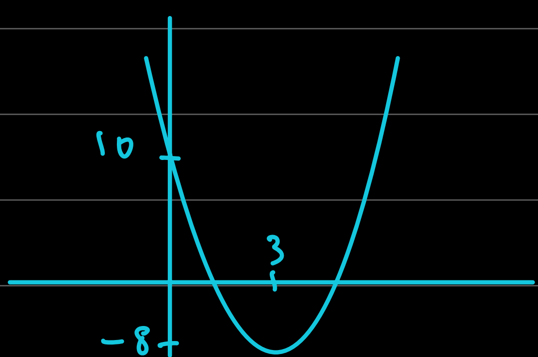
Shape ∪

y-int

$x - 3 = 0$

$x = 3 \quad y = -8$

$(3, -8)$



(iii) State with a reason whether or not $f^{-1}(x)$ exists?

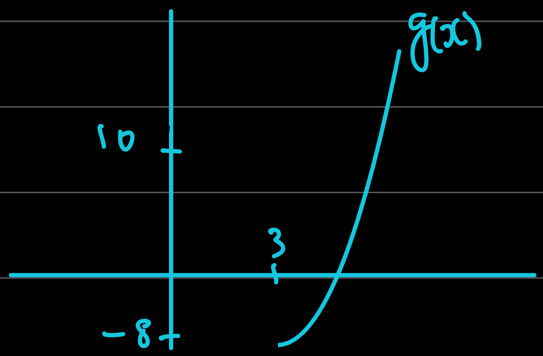
NO, the inverse doesnot exist because f is not a one-one function.

(iv) $g(x) = 2x^2 - 12x + 10 \qquad x \geqslant A$

$x \geqslant 3$

State smallest value of A for which inverse of $g(x)$ exists. (1 mark)

$\boxed{A=3}$



$x \geq 3$

CRAM:

A = x-coordinate of turning point.

Domain of $g(x)$ $\quad x \geq 3$

Range of $g(x)$ $\quad g(x) \geq -8$

VARIATIONS OF HOW Q IS PHRASED:

1) greatest value of A for which inverse exists
2) Smallest value of A for which inverse exists
3) greatest value of A for which $g(x)$ is one-one function
4) Smallest value of A for which $g(x)$ is one-one function

inverse exist = one-one function.

(v) For the value of A in last part, find $g^{-1}(x)$. Also state domain and range of $g^{-1}(x)$. (5 marks).

$g(x) = 2x^2 - 12x + 10$

$g(x) = y$

$y = 2x^2 - 12x + 10$

$y = 2(x-3)^2 - 8$

$y + 8 = 2(x-3)^2$

$(x-3)^2 = \dfrac{y+8}{2}$

You cannot make x-subject from standard form. Use completed square form in part (i).

$$x - 3 = \pm\sqrt{\frac{y+8}{2}}$$

$$x = 3 \pm \sqrt{\frac{y+8}{2}}$$

$$g^{-1}(y) = x$$

$$g^{-1}(y) = 3 \pm \sqrt{\frac{y+8}{2}}$$

$$g^{-1}(x) = 3 \pm \sqrt{\frac{x+8}{2}}$$

Domain of $g(x)$ $\Rightarrow$	Range of $g^{-1}(x)$
$x \geqslant 3$	$g^{-1}(x) \geqslant 3$

$g^{-1}(x)$ will give all outputs bigger than 3.

$$g^{-1}(x) = 3 + \sqrt{\frac{x+8}{2}}$$

Range of $g(x)$ $\Rightarrow$	Domain of $g^{-1}(x)$
$g(x) \geqslant -8$	$x \geqslant -8$

IMPORTANT TIP

Domains are always in terms of x

eg $x > \square$

$x < \square$

Ranges are written with y or Name of function.

$f(x) > \square$ preferred

$g^{-1}(x) > \boxed{}$.

46 The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(iii) Find the range of f. [1]

The function $g : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geqslant A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A, find an expression for $g^{-1}(x)$ in terms of x. [3]

10 The function f is def ned by $f : x \mapsto 2x^2 - 8x + 11$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants. [3]

(ii) State the range of f. [1]

(iii) Explain why f does not have an inverse. [1]

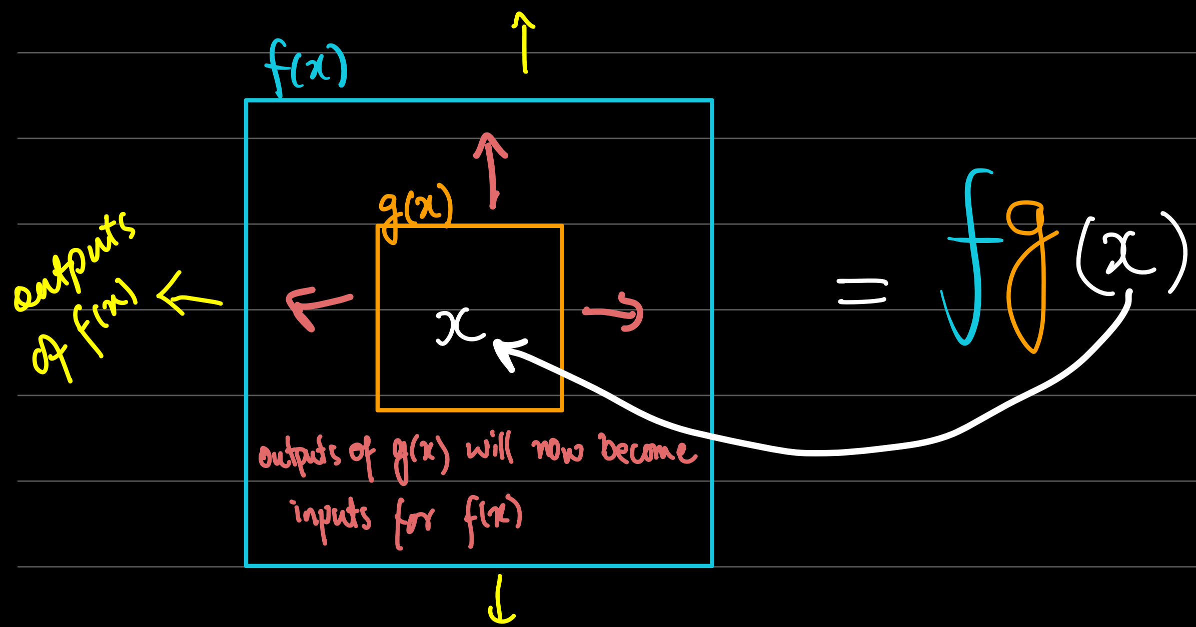
The function g is define by $g : x \mapsto 2x^2 - 8x + 11$ for $x \leqslant A$, where A is a constant.

(iv) State the largest value of A for which g has an inverse. [1]

(v) When A has this value, obtain an expression, in terms of x, for $g^{-1}(x)$ and state the range of g^{-1}. [4]

DOMAIN AND RANGE OF COMPOSITE FUNCTION.

(Machine within a machine)



Process:

1) Inputs are given to inner function.
Inner function processes inputs and give outputs.

2) outputs of inner function (Range) are inputs for outer function (Domain)

$\boxed{f(x) = x^2 + 2x + 3 \quad x \in R}$ $\quad$ $\boxed{g(x) = x + 3 \quad 0 < x < 2}$

Find Domain of $fg(x) = ?$

Domain of $fg(x) \Rightarrow \quad 0 < x < 2$

Note 1: Domain of a composite function is usually domain of inner most function.

Outputs (Range) of inner function $\longrightarrow$ inputs (Domain) of outer function

$\downarrow$

Range of $fg(x)$ composite function.

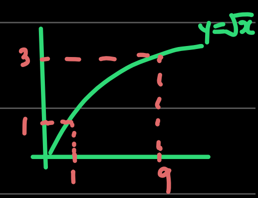
Q. $\boxed{f(x) = 3x + 5 \quad x \in R}$ $\boxed{g(x) = \sqrt{x} \quad 1 \leq x < 9}$

$fg(x)$ is a composite function.

(i) Domain of $fg(x)$: $1 \leq x < 9$

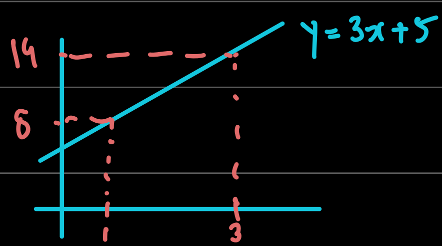
(ii) Range of $fg(x)$: outputs from $g(x)$ (Range of $g(x)$)

$1 \leq g(x) < 3$



These will now be used as inputs for outer function.

$f(x) = 3x + 5$ $\qquad$ $1 \leq x < 3$



Range of $f(x) \Rightarrow \quad 8 \leq f(x) < 14$

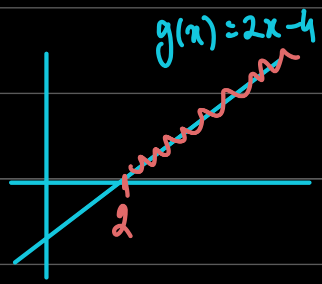
Range of $fg(x) \Rightarrow \quad 8 \leq fg(x) < 14$

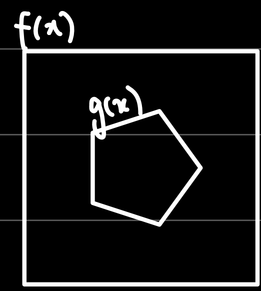
Q: $\boxed{f(x) = \sqrt{x} \quad x \geq 0}$ $\qquad \boxed{g(x) = 2x - 4 \quad x \geq a}$

(i) Find smallest value of a for which composite function $fg(x)$ will exist.

$\boxed{a = 2}$

Outputs of $g(x)$ $\longrightarrow$ inputs of $f(x)$ (must be ≥ 0)





EQUATION OF CIRCLE (V·Easy)

• understand that the equation $(x-a)^2+(y-b)^2=r^2$ represents the circle with centre (a, b) and radius r	Including use of the expanded form $x^2+y^2+2gx+2fy+c=0$.
• use algebraic methods to solve problems involving lines and circles	Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry. Implicit differentiation is not included.

Q EQUATION OF A CIRCLE IS

$$x^2+8x+y^2-12y-12=0$$

Find the centre and radius of circle.

- understand and use the transformations of the graph of $y = f(x)$ given by $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$ and simple combinations of these.

Including use of the terms 'translation', 'reflection' and 'stretch' in describing transformations. Questions may involve algebraic or trigonometric functions, or other graphs with given features.

TRANSFORMATION OF FUNCTIONS

This section tells how graphs behave if we introduce some changes to equations.

ORIGINAL $= f(x)$

TRANSFORMATION	EFFECT ON GRAPH.
$f(x) + a$	Graph shifts a boxes up.
$f(x) - a$	Graph shifts a boxes down.
$f(x) + 5$ ↑	Graph Shifts 5 boxes up
$f(x) - 3$ ↓	Graph shifts 3 boxes down.
$f(x+a)$	Graph shifts a boxes Left
$f(x-a)$	Graph shifts a boxes right
$f(x+2)$ ←	Graph shift 2 boxes left
$f(x-4)$ →	Graph shifts 4 boxes right.
$-f(x)$	Reflection in x-axis.
$f(-x)$	Reflection in y-axis.
$f^{-1}(x)$	Reflection in line $y=x$

Translation $\begin{pmatrix} x \\ y \end{pmatrix}$

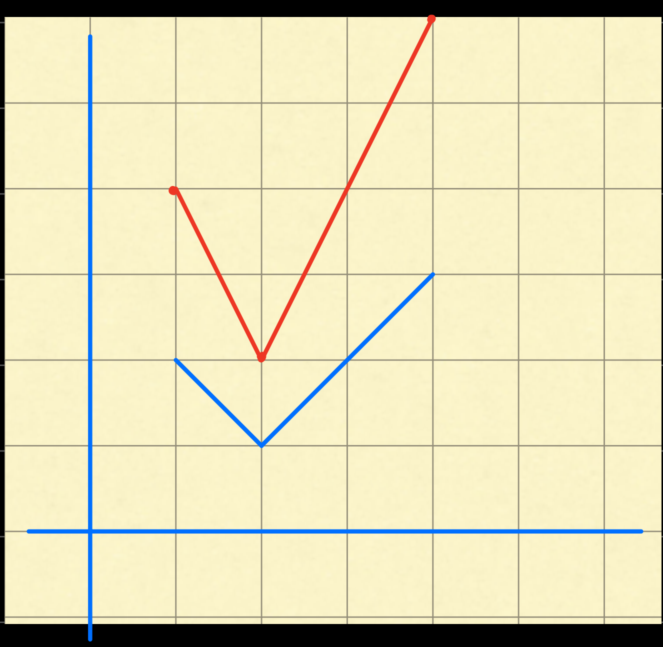
Reflection

$af(x)$ VERTICA STRETCH	STRETCH parallel to y axis with factor a
$2f(x)$	Stretch with factor of 2 on y-axis.
	$a>1$, STRETCH VERTICALLY.
	$a<1$, SQUEEZE VERTICALLY
$f(ax)$ HORIZONTAL STRETCH	STRETCH parallel to x axis with factor $\frac{1}{a}$
$f(2x)$	Stretch parallel to x-axis, factor $=\frac{1}{2}$
	$a>1$, SQUEEZE HORIZONTALY
	$a<1$, STRETCH HORIZONTALLY.

Translation vector $= \begin{pmatrix} x \\ y \end{pmatrix}$

$x \rightarrow$ +ve Right, -ve Left

$y \rightarrow$ +ve up, -ve down.

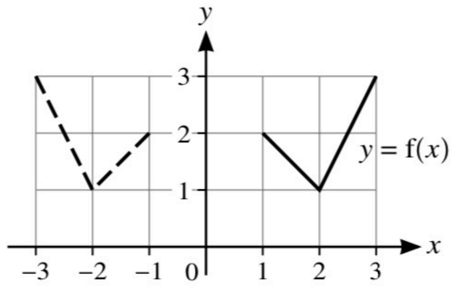


$2f(x)$

$2(y)$

3 In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = \mathrm{f}(x)$. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

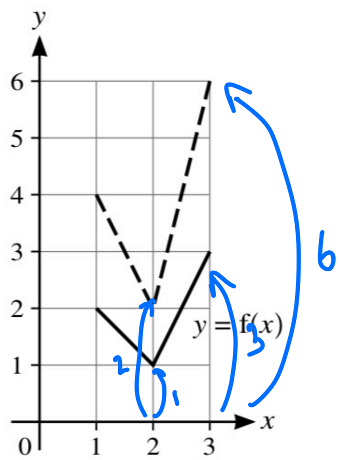
(a)



State, in terms of f, the equation of the graph shown with broken lines. [1]

$f(-x)$ ....

(b)

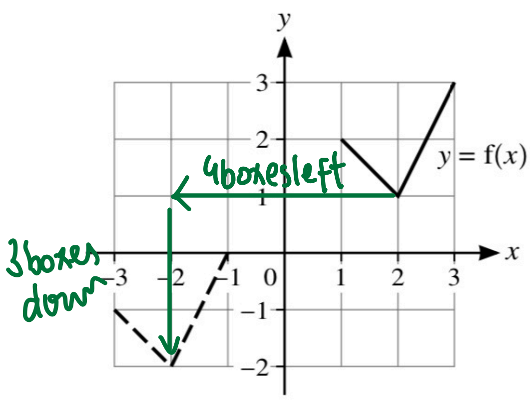


State, in terms of f, the equation of the graph shown with broken lines. [1]

$2f(x)$ ....

(c)

Vector $= \begin{pmatrix} -4 \\ -3 \end{pmatrix}$



State, in terms of f, the equation of the graph shown with broken lines. [2]

$f(x+4)-3$ ....

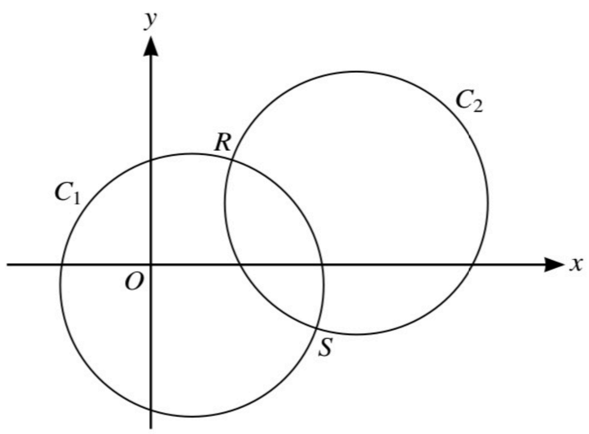
1 **(a)** Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

12 A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]



The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

2 The graph of $y = f(x)$ is transformed to the graph of $y = 1 + f\left(\frac{1}{2}x\right)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

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(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

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11 The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

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The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

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The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

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The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

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