1 **(a)** Find the number of different arrangements of the 8 letters in the word DECEIVED in which all three Es are together and the two Ds are together. [2]

No. of D's = 2

" " E's = 3

C, I, V = 1

Using Scope Method

5! {All D's as 1 block also E's as 1 block}

No arrangement within Scope, bcz of identical items

$= \boxed{120}$

(b) Find the number of different arrangements of the 8 letters in the word DECEIVED in which the three Es are not all together. [4]

If 3 E's are together $= \dfrac{6!}{2!} = 360$

If not ALL E's together $= \dfrac{8!}{3!\,2!} - 360$

$= 3360 - 360$

$= \boxed{3000}$

2 **(a)** Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the three Es are together and the two Ls are together. [2]

$6! = \boxed{720}$

(b) Find the number of different arrangements that can be made from the 9 letters of the word JEWELLERY in which the two Ls are not next to each other. [4]

If 2 Ls are next to each other

$= \frac{8!}{3!} = 6720$

when two Ls are not next to each other

$= \frac{9!}{3!\,2!} - 6720 = 30240 - 6720$

$= \boxed{23520}$

3 **(a)** How many different arrangements are there of the 8 letters in the word RELEASED? [1]

Es = 3

R, L, A, S, D = 1

$\frac{8!}{3!} = \boxed{6720}$

(b) How many different arrangements are there of the 8 letters in the word RELEASED in which the letters LED appear together in that order? [3]

_ [L E D] _ _ _ _

$\frac{6!}{2!} = \frac{720}{2} = \boxed{360}$

(c) An arrangement of the 8 letters in the word RELEASED is chosen at random.

Find the probability that the letters A and D are not together. [4]

Using Arrow method

$$\frac{6! \times {}^7P_2}{3!} = \boxed{5040}$$

$$P(\text{A + D Separated}) = \frac{5040}{8!/3!} = \frac{5040}{6720}$$

$$= \frac{3}{4} = \boxed{0.75}$$

6 **(a)** Find the total number of different arrangements of the 11 letters in the word CATERPILLAR. [2]

$\frac{11!}{2!2!2!} = \boxed{4989600}$

(b) Find the total number of different arrangements of the 11 letters in the word CATERPILLAR in which there is an R at the beginning and an R at the end, and the two As are not together. [4]

R _ _ A A _ _ _ _ _ R

If As are next to each other, & Rs at the ends

$\frac{8!}{2!} = 20160$

If As are not together

$\frac{9!}{2!2!} - 20160 = 90720 - 20160$

$= \boxed{70560}$

(c) Find the total number of different selections of 6 letters from the 11 letters of the word CATERPILLAR that contain both Rs and at least one A and at least one L. [4]

C, T, I, P, E

R R A L ~~X~~ ~~X~~ = $^5C_2$ = 10

R R A A L ~~X~~ = $^5C_1$ = 5

R R A L L ~~X~~ = $^5C_1$ = 5

R R A A L L = $^5C_0$ = 1

21

$\boxed{\text{21 selections}}$

3 **(a)** Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

$$\frac{8!}{2!\,3!} = \boxed{3360}$$

(b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

O _ _ _ _ _ _ N

$$\frac{6!}{2!\,2!} = \boxed{180}$$

(c) Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]

OOO CC _ _ _ = 5! 120

Be careful, this is a case of conditional probability

$P(3Os \mid 2Cs) = \frac{120}{P(2Cs)} = \frac{120}{7!/3!} = \frac{120}{840}$

$= \frac{1}{7} = 0.1428$

$\boxed{= \frac{1}{7} = 0.143}$

7 **(a)** Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which no consonant is next to another consonant. (The letters D, M, N and R are consonants and the letters A, E and O are **not** consonants.) [3]

C V C V C V C V C

$\frac{5! \times 4!}{2! \times 2!} = \boxed{720}$

(b) Find the number of different arrangements of the 9 letters in the word ANDROMEDA in which there is an A at each end and the Ds are **not** together. [3]

A _ $\boxed{D \quad D}$ _ _ _ _ A

If Ds are together $= 6! = 720$

If Ds are not together $= \frac{7!}{2!} - 6!$

$= 2520 - 720$

$= \boxed{1800}$

Four letters are selected at random from the 9 letters in the word ANDROMEDA.

(c) Find the probability that this selection contains at least one D and exactly one A. [4]

1D, 1A — — $= \frac{{}^2C_1 \times {}^2C_1 \times {}^5C_2}{{}^9C_4} = \frac{20}{63}$

2D, 1A — $= \frac{{}^2C_2 \times {}^2C_1 \times {}^5C_1}{{}^9C_4}$

$= \frac{5}{126}$

P(at least 1 D & exactly 1A) $= \frac{20}{63} + \frac{5}{126}$

$= \boxed{\frac{5}{14} = 0.3571}$

7 (a) Find the number of different possible arrangements of the 9 letters in the word CELESTIAL. [1]

$\frac{9!}{2!\,2!} = \boxed{90720}$

(b) Find the number of different arrangements of the 9 letters in the word CELESTIAL in which the first letter is C, the fifth letter is T and the last letter is E. [2]

C _ _ _ T _ _ _ E

$\frac{6!}{2!} = \frac{720}{2!} = \boxed{360}$

(c) Find the probability that a randomly chosen arrangement of the 9 letters in the word CELESTIAL does not have the two Es together. [4]

_ $\boxed{E\ E}$ _ _ _ _ _ _

$\frac{8!}{2!} = 20160$

Total possible arrangement $= \frac{9!}{2!\,2!} = 90720$

P(Es are together) $= \frac{20160}{90720} = \frac{2}{9}$

P(Es are not together) $= 1 - \frac{2}{9} = \boxed{7/9}$

C, S, T, I, A

5 letters are selected at random from the 9 letters in the word CELESTIAL.

(d) Find the number of different selections if the 5 letters include at least one E and at most one L. [3]

E L _ _ _ = $^5C_3$ = 10

E _ _ _ _ $^5C_4$ = 5

E E L _ _ $^5C_2$ = 10

E E _ _ _ $^5C_3$ = 10

35

$\boxed{\text{35 Selections}}$

5 The 8 letters in the word RESERVED are arranged in a random order.

(a) Find the probability that the arrangement has V as the first letter and E as the last letter. [3]

V _ _ _ _ _ _ E

$= \frac{6!}{2!2!} = 180$

Total possible arrangement $= \frac{8!}{3!2!} = 3360$

Prob (V......E) $= \frac{180}{3360} = \boxed{\frac{3}{56}} = 0.053$

(b) Find the probability that the arrangement has both Rs together given that all three Es are together. [4]

_ R R _ E E E _

2Rs & 3Es are together: $5! = 120$

3Es together: $\frac{6!}{2!} = 360$

$P(RR \cap EEE / EEE) = \frac{120}{360} = \boxed{\frac{1}{3}}$

6 **(a)** Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that there is an E at the beginning and an E at the end. [2]

E _ _ _ _ _ _ _ _ E

$$\frac{8!}{3!} = \boxed{6720}$$

(b) Find the number of different ways in which the 10 letters of the word SUMMERTIME can be arranged so that the Es are not together. [4]

Es are separated $\frac{10!}{3!\,2!} - \frac{9!}{3!} = 302400 - 60480$

$= \boxed{241920}$

(c) Four letters are selected from the 10 letters of the word SUMMERTIME. Find the number of different selections if the four letters include at least one M and exactly one E. [3]

M E _ _ $= {}^5C_2 = 10$

M M E _ $= {}^5C_1 = 5$

M M M E $= {}^5C_0 = 1$

$\boxed{16 \text{ Selections}}$

6 **(a)** Find the total number of different arrangements of the 8 letters in the word TOMORROW. [2]

$$\frac{8!}{2!\,3!} = \boxed{3360}$$

(b) Find the total number of different arrangements of the 8 letters in the word TOMORROW that have an R at the beginning and an R at the end, and in which the three Os are not all together. [3]

R _ $\boxed{O\ \ O\ \ O}$ _ _ R

If Os are together = 4!

If not ALL Os together $= \frac{6!}{3!} - 4!$

$= 120 - 24$

$= \boxed{96 \text{ permutations}}$

Four letters are selected at random from the 8 letters of the word TOMORROW.

(c) Find the probability that the selection contains at least one O and at least one R. [5]

O R X X = $\frac{{}^3C_1 \times {}^2C_1 \times {}^3C_2}{{}^8C_4} = \frac{18}{70}$

O O R X = $\frac{{}^3C_2 \times {}^2C_1 \times {}^3C_1}{{}^8C_4} = \frac{18}{70}$

O O R R = $\frac{{}^3C_2 \times {}^2C_2 \times {}^3C_0}{{}^8C_4} = \frac{3}{70}$

O O O R = $\frac{{}^3C_3 \times {}^2C_1 \times {}^3C_0}{{}^8C_4} = \frac{2}{70}$

R R O X = $\frac{{}^3C_1 \times {}^2C_2 \times {}^3C_1}{{}^8C_4} = \frac{9}{70}$

P(at least one O & at least one R) $= \frac{18+18+3+2+9}{70}$

$= \frac{50}{70} = \boxed{\frac{5}{7}}$

7 **(a)** Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that all 3 Es are together. [2]

_ _ $\boxed{E \quad E \quad E}$ _ _ _ _ _

$\frac{8!}{2!} = \boxed{20160}$

(b) Find the number of different ways in which the 10 letters of the word SHOPKEEPER can be arranged so that the Ps are not next to each other. [4]

Total permutations with no restrictions $= \frac{10!}{2!\,3!}$

$= 302400$

If Ps are together $=$ _ _ _ _ $\boxed{PP}$ _ _ _ _

$= \frac{9!}{3!} = 60480$

No. of permutations, if Ps are not next to each other

$= 302400 - 60480$

$= \boxed{241920}$

(c) Find the probability that a randomly chosen arrangement of the 10 letters of the word SHOPKEEPER has an E at the beginning and an E at the end. [2]

E _ _ _ _ _ _ _ _ E

$P(\text{Es at the beginning \& Es at the end}) = \dfrac{8!/2!}{10!/2!\,3!}$

$= \dfrac{40320}{604800}$

$= \boxed{\dfrac{1}{15} = 0.0667}$

Four letters are selected from the 10 letters of the word SHOPKEEPER.

(d) Find the number of different selections if the four letters include exactly one P. [3]

P E E E $= {}^5C_0 = 1$

P E E X $= {}^5C_1 = 5$

P E X X $= {}^5C_2 = 10$

P X X X $= {}^5C_3 = 10$

26

$\boxed{\text{26 selections}}$

4 **(a)** In how many different ways can the 9 letters of the word TELESCOPE be arranged? [2]

$$\frac{9!}{3!} = \boxed{60480}$$

(b) In how many different ways can the 9 letters of the word TELESCOPE be arranged so that there are exactly two letters between the T and the C? [4]

$$6 \times \frac{7!}{3!} \times 2 = \boxed{10080}$$

6 **(a)** How many different arrangements are there of the 11 letters in the word REQUIREMENT? [2]

$$\frac{11!}{3!\,2!} = \boxed{3326400}$$

(b) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which the two Rs are together and the three Es are together? [1]

$\boxed{R\ R}$ $\boxed{E\ E\ E}$

$$8! = \boxed{40320}$$

(c) How many different arrangements are there of the 11 letters in the word REQUIREMENT in which there are exactly three letters between the two Rs? [3]

x x $\boxed{R\ x\ x\ x\ R}$ x x x x

$$7 \times \frac{9!}{3!} = \boxed{423360}$$

Five of the 11 letters in the word REQUIREMENT are selected.

(d) How many possible selections contain at least two Es and at least one R? [4]

E E R X X $= {}^6C_2 = 15$

E E R R X $= {}^6C_1 = 6$

E E E R X $= {}^6C_1 = 6$

E E E R R $= {}^6C_0 = 1$

$\underline{\underline{28}}$

$\boxed{\text{28 Selections}}$

7 (a) Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

If Cs are next to each other $= \frac{9!}{4!} = 15120$

If Cs are NOT together $= \frac{10!}{2!4!} - \frac{9!}{4!}$

$= 75600 - 15120$

$= \boxed{60480}$

(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

A _ C _ _ _ C _ _ A

IF 2 As and 1 other between 2 Cs $= \frac{^4C_1 \times 3! \times 4!}{2!} = 288$

If 1A & 2 others between 2 Cs $= {}^4C_2 \times 3! \times 4! = 864$

If No As $= \frac{^4C_3 \times 3! \times 4!}{2!} = 288$

$\boxed{1440}$

OR $4 \times \frac{6!}{2!} = \boxed{1440}$

Five letters are selected from the 10 letters in the word CASABLANCA.

(c) Find the number of different selections in which the five letters include at least two As and at most one C. [3]

A A C ~~X~~ ~~X~~ $= {}^4C_2 = 6$

A A ~~X~~ ~~X~~ ~~X~~ $= {}^4C_3 = 4$

A A A C ~~X~~ $= {}^4C_1 = 4$

A A A ~~X~~ ~~X~~ $= {}^4C_2 = 6$

A A A A C $= {}^4C_0 = 1$

A A A A ~~X~~ $= {}^4C_1 = 4$

$\underline{\underline{25}}$

$\boxed{25 \text{ selections}}$

7 **(a)** Find the number of different arrangements of the 9 letters in the word DELIVERED in which the three Es are together and the two Ds are **not** next to each other. [4]

D D D D D D

$\boxed{E\ E\ E}$

$5! \times {}^6C_2 = 120 \times 15$

$= \boxed{1800}$

(b) Find the probability that a randomly chosen arrangement of the 9 letters in the word DELIVERED has exactly 4 letters between the two Ds. [5]

$\boxed{D \_ \_ \_ \_ D}$

No. of arrangements with 4 letters b/w 2 Ds

$4 \times \frac{7!}{3!} = \underline{3360}$

Total No. of arrangements

$\frac{9!}{2!\,3!} = \underline{\underline{30240}}$

P(4 letters between 2 Ds) = $\frac{3360}{30240}$

$= \boxed{\frac{1}{9}}$

Plz do a good practice for similar question involving PROBABILITY

Five letters are selected from the 9 letters in the word DELIVERED.

(c) Find the number of different selections if the 5 letters include at least one D and at least one E. [3]

D E X X X $= {}^4C_3 = 4$

D E E X X $= {}^4C_2 = 6$

D E E E X $= {}^4C_1 = 4$

D D E X X $= {}^4C_2 = 6$

D D E E X $= {}^4C_1 = 4$

D D E E E $= {}^4C_0 = 1$

$= \boxed{25 \text{ selections}}$

6 **(a)** Find the number of different arrangements of the 9 letters in the word ACTIVATED. [2]

As = 2

Ts = 2

C, I, V, E, D = 1 each

$$\frac{9!}{2!\,2!} = \boxed{90720}$$

(b) Find the number of different arrangements of the 9 letters in the word ACTIVATED in which there are at least 5 letters between the two As. [3]

_ A _ _ _ _ _ A _

$$3 \times \frac{7!}{2!} = 7560$$

A _ _ _ _ _ _ A _

$$2 \times \frac{7!}{2!} = 5040$$

A _ _ _ _ _ _ _ A

$$\frac{7!}{2!} = 2520$$

$7560 + 5040 + 2520$

$= \boxed{15120}$

Five letters are selected at random from the 9 letters in the word ACTIVATED.

(c) Find the probability that the selection does **not** contain more Ts than As. [5]

As = 2

Ts = 2

$$\frac{{}^2C_2 \times {}^2C_0 \times {}^5C_3 + {}^2C_2 \times {}^2C_1 \times {}^5C_2 + {}^2C_2 \times {}^2C_2 \times {}^5C_1 + {}^2C_1 \times {}^2C_1 \times {}^5C_3 + {}^2C_1 \times {}^2C_0 \times {}^5C_4 + {}^5C_5}{{}^9C_5}$$

$$\frac{10 + 20 + 5 + 40 + 10 + 1}{126}$$

$$= \frac{86}{126} = \boxed{\frac{43}{63} = 0.683}$$

7 (a) Find the number of different arrangements of the 9 letters in the word ALLIGATOR in which the two As are together and the two Ls are together. [2]

A A L L

$\boxed{7! = 5040}$ permutations

Challenging Quest

(b) The 9 letters in the word ALLIGATOR are arranged in a random order.

Find the probability that the two Ls are together and there are exactly 6 letters between the two As. [5]

A L L A

$$\frac{5C_1 \times 5! \times 2}{\frac{9!}{2!\,2!}} = \frac{1200}{90720}$$

$$P(2L \text{ and } A\ldots\ldots A) = \boxed{\frac{5}{378} = 0.0132}$$

(c) Find the number of different selections of 5 letters from the 9 letters in the word ALLIGATOR which contain at least one A and at most one L. [3]

As = 2
Ls = 2

A L X X X = $^5C_3$ = 10

A X X X X = $^5C_4$ = 5

A A L X X = $^5C_2$ = 10

A A X X X = $^5C_3$ = 10

35

$\boxed{35 \text{ Selections}}$

6 **(a)** Find the number of different arrangements of the 9 letters in the word CROCODILE. [1]

$C_s = 2$

$O_s = 2$

$\frac{9!}{2!\,2!} = \boxed{90720}$

(b) Find the number of different arrangements of the 9 letters in the word CROCODILE in which there is a C at each end and the two Os are not together. [3]

C _ $\boxed{\text{O O}}$ _ _ _ _ C

If Os are together = $6! = 720$

If Os are separated = $\frac{7!}{2!} - 6!$

$= \boxed{1800}$

(c) Four letters are selected from the 9 letters in the word CROCODILE.

Find the number of selections in which the number of Cs is not the same as the number of Os. [3]

Cs = 2

Os = 2

C C O X = $^5C_1$ = 5

C C X X = $^5C_2$ = 10

C O O X = $^5C_1$ = 5

C X X X = $^5C_3$ = 10

O X X X = $^5C_3$ = 10

O O X X = $^5C_2$ = 10

$\boxed{\text{50 selection}}$

(d) Find the number of ways in which the 9 letters in the word CROCODILE can be divided into three groups, each containing three letters, if the two Cs must be in different groups. [3]

① C O O   C X X   X X X | $^5C_2 \times {}^3C_3 = 10$

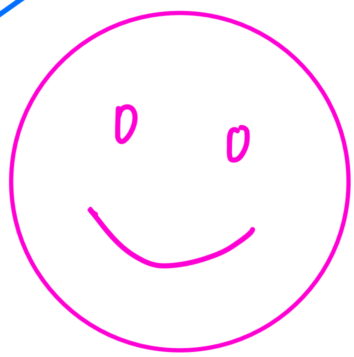
② C O X   C O X   X X X | $\frac{^5C_1 \times {}^4C_1}{2!} = 10$

③ C X X   C X X   O O X | $\frac{^5C_2 \times {}^3C_2}{2!} = 15$

④ C O X   C X X   O X X | $^5C_1 \times {}^4C_2 \times {}^2C_2 = 30$

$10 + 10 + 15 + 30 = \boxed{65}$

ENJOY YOUR LEARNING



From the Desk of
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(Nixor, Cedar, Highbrow)