$$(a+b)^n = a^n + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + {}^nC_3\, a^{n-3} b^3 \ldots$$

Examples: Expand upto first three terms.

$$(2x+5)^6 = (2x)^6 + {}^6C_1 (2x)^{6-1} (+5)^1 + {}^6C_2 (2x)^{6-2} (5)^2$$

$\boxed{6\ \boxed{nCr}\ 1 \times 2^\wedge 5 \times 5 = 960}$ $\boxed{6\ \boxed{nCr}\ 2 \times 2^\wedge 4 \times 5^2 = 6000}$

$$64x^6 + 960x^5 + 6000x^4$$

$$(2-x)^6 = (2)^6 + {}^6C_1 (2)^5 (-x)^1 + {}^6C_2 (2)^4 (-x)^2$$

$$= 64 - 192x + 240x^2$$

$\boxed{\text{TYPE 1}}$

Q: (a) Expand $(2-x)^6$ upto first three terms.

$$(2)^6 + {}^6C_1 (2)^5 (-x)^1 + {}^6C_2 (2)^4 (-x)^2$$

$$(2-x)^6 = 64 - 192x + 240x^2$$

(b) Find coficient of x^2 in expansion of $(2x+3)(2-x)^6$

$$(2x+3)(2-x)^6 \longrightarrow x^2$$

$$2x \times -192x = -384x^2$$
$$+3 \times 240x^2 = 720x^2$$
$$336x^2 \quad \text{(Final Ans)}$$

(c) Find cofficient of x^2 in expansion of $(7+3x-2x^2)(2-x)^6$

$$(2-x)^6 = 64 - 192x + 240x^2$$

$$(7+3x-2x^2)(2-x)^6 \longrightarrow x^2$$

$$7 \times 240x^2 = 1680x^2$$
$$+3x \times -192x = -576x^2$$
$$-2x^2 \times 64 = -128x^2$$
$$976x^2$$

ADVANCED: (REVERSE) (FINAL ANS is GIVEN IN QUESTION)

(d) The cofficient of x^2 is 48 in expansion of $(2ax+7)(2-x)^6$. find a.

$$(2-x)^6 = 64 - 192x + 240x^2$$

$$(2ax+7)(2-x)^6 \longrightarrow x^2$$

$$2ax \times -192x = -384a\,x^2$$
$$+7 \times 240x^2 = 1680\,x^2$$
$$\overline{48x^2}$$

$$-384a + 1680 = 48$$

$$a = \frac{48-1680}{-384} = \boxed{\frac{17}{4}}$$

(e) The cofficient of x is 30 in expansion $(2x-a)(2-x)^6$. Find a

$$(2-x)^6 = 64 - 192x + 240x^2$$

$$(2x-a)(2-x)^6 \longrightarrow x$$

$$2x \times 64 = 128\,x$$
$$-a \times -192x = 192a\,x$$
$$\overline{30\,x}$$

$$128 + 192a = 30$$

$$a = \frac{30-128}{192} = \boxed{\frac{-49}{96}}$$

<u>Reverse working</u>

final Answer = $0x$

↑

(f) There is $\boxed{\text{no term in } x}$ in expansion of $(kx+3)(2-x)^6$, find k.

$$(2-x)^6 = 64 - 192x + 240x^2$$

$$kx \times 64 = 64k\,x$$

$$+3 \times -192x = -576\,x$$

$$0\,x$$

$$64k - 576 = 0$$

$$k = \frac{576}{64} = 9.$$

VOCAB

No term in x ⟶ Final Ans $= 0x$

No term in x^2 ⟶ Final Ans $= 0x^2$

TYPE 2 A PARTICULAR TERM

QUESTION WILL REQUIRE ONE PARTICULAR TERM.

In this case you should never go for expansion.

ONE PARTICULAR TERM

$$T_{r+1} = {}^{n}C_{r}\, a^{n-r} b^{r}$$

TERM NO	POWER OF X
Q: Find the 5^{th} term in	Q. Find the term of x^2 in

expansion of $(2x-3)^8$.

$T_{r+1} = {}^nC_r\, a^{n-r} b^r$

$T_{r+1} = {}^8C_r\,(2x)^{8-r}(-3)^r$

you need T_5, $r=4$

$T_{4+1} = {}^8C_4\,(2x)^{8-4}(-3)^4$

$T_5 = (70)(16x^4)(+81)$

$T_5 = 90720x^4$

Too easy. Never tested in CAIE.

expansion of $\left(2x+\frac{3}{x}\right)^8$.

$T_{r+1} = {}^nC_r\, a^{n-r} b^r$

$T_{r+1} = {}^8C_r\,(2x)^{8-r}\left(\frac{3}{x}\right)^r$

$= {}^8C_r \cdot 2^{8-r} \cdot x^{8-r} \cdot \frac{3^r}{x^r}$

STEP1: Expand all powers and isolate x-terms

$\frac{x^{8-r}}{x^r} = x^2$

STEP2: EQUATE these isolated x terms to required term and find r.

$x^{8-r-r} = x^2$

$8-2r = 2$

$2r = 6$

$\boxed{r=3}$

Step3: Now put this value of r in first line

$T_{3+1} = {}^8C_3\,(2x)^{8-3}\left(\frac{3}{x}\right)^3$

$(56)(2x)^5\left(\frac{27}{x^3}\right)$

$= 56\,(32x^{\cancel{5}\,2})\left(\frac{27}{\cancel{x^3}}\right)$

$= 48384x^2$

VOCAB: TERM INDEPENDENT OF X

1) APPLY FORMULA FOR PARTICULAR TERM.

2) Find the term of x^0.

Q: Find the term independent of x in $\left(2x+\frac{3}{x}\right)^8$

$\downarrow x^0$

$$T_{r+1} = {}^8C_r (2x)^{8-r}\left(\frac{3}{x}\right)^r$$

$${}^8C_r \cdot 2^{8-r} \cdot x^{8-r} \cdot \frac{3^r}{x^r}$$

$$\frac{x^{8-r}}{x^r} = x^0$$

$$x^{8-r-r} = x^0$$

$$8-2r = 0$$

$$\boxed{r=4}$$

$$T_{4+1} = {}^8C_4 (2x)^{8-4}\left(\frac{3}{x}\right)^4$$

$$= (70)(2x)^4\left(\frac{3}{x}\right)^4$$

$$= 70\left(16\cancel{x^4}\right)\left(\frac{81}{\cancel{x^4}}\right)$$

$$= \boxed{90720}$$

15 Find the term independent of x in the expansion of $\left(x-\frac{1}{x^2}\right)^9$. [3]

$$T_{r+1} = {}^9C_r (x)^{9-r}\left(\frac{-1}{x^2}\right)^r$$

$$= {}^9C_r \cdot x^{9-r} \cdot \frac{(-1)^r}{x^{2r}}$$

$$\frac{x^{9-r}}{x^{2r}} = x^0$$

$$x^{9-r-2r} = x^0$$

$$9-3r = 0$$

$$T_{3+1} = {}^9C_3 (x)^{9-3}\left(\frac{-1}{x^2}\right)^3$$

$$=(84)x^6\,\frac{(-1)}{x^6}$$

$$= -84$$

$\boxed{r=3}$

1 Time practice of worksheets = NO PRACTICE AT ALL.

10 **(i)** Find the first 3 terms in the expansion of $\left(2x - \frac{3}{x}\right)^5$ in descending powers of x. [3]

(ii) Hence find the coefficient of x in the expansion of $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5$. [2]

(i) $\left(2x - \frac{3}{x}\right)^5 = (2x)^5 + {}^5C_1(2x)^4\left(\frac{-3}{x}\right)^1 + {}^5C_2(2x)^3\left(\frac{-3}{x}\right)^2$

$= 32x^5 + (5)(16x^4)\left(-\frac{3}{x}\right) + (10)(8x^3)\left(\frac{9}{x^2}\right)$

$= 32x^5 - 240x^3 + 720x$

(ii) $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5 \longrightarrow x$

$1 \times 720x = 720x$

$\frac{2}{x^2} \times -240x^3 = -480x$

$240x$

4 The first three terms in the expansion of $(2 + ax)^n$, in ascending powers of x, are $32 - 40x + bx^2$. Find the values of the constants n, a and b. [5]

$$(2+ax)^n = (2)^n + {}^nC_1(2)^{n-1}(ax)^1 + {}^nC_2(2)^{n-2}(ax)^2$$
$$\qquad\qquad 32 \qquad -40x \qquad + bx^2$$

$2^n = 32$	${}^nC_1\, 2^{n-1}(ax) = -40x$	${}^nC_2(2)^{n-2}(ax)^2 = bx^2$
$2^n = 2^5$	${}^5C_1\,(2)^{5-1}(ax) = -40x$	${}^5C_2(2)^{5-2}(-\frac{1}{2}x)^2 = bx^2$
$n = 5$	$(5)(16)(ax) = -40x$	$(10)(8)(\frac{1}{4}x^2) = bx^2$
	$80ax = -40x$	$20x^2 = bx^2$
	$a = \frac{-1}{2}$	$b = 20.$

20 **(i)** Find the first 3 terms in the expansion of $(2 - y)^5$ in ascending powers of y. [2]

(ii) Use the result in part **(i)** to find the $\boxed{\text{coefficient of } x^2}$ in the expansion of $\left(2 - (2x - x^2)\right)^5$. [3]

(i) $(2-y)^5 = (2)^5 + {}^5C_1(2)^{5-1}(-y)^1 + {}^5C_2(2)^{5-2}(-y)^2$

$(2-y)^5 = 32 - 80y + 80y^2$

$\downarrow$

$y = 2x - x^2$

$\downarrow$

$(2-(2x-x^2))^5 = 32 - 80(2x - x^2) + 80(2x - x^2)^2$

$= 32 - 160x + 80x^2 + 80[4x^2 - 2(2x)(x^2) + (x^2)^2]$

$80x^2 + 320x^2$

$400x^2$

18 The coefficient of x^3 in the expansion of $(a+x)^5 + (1-2x)^6$, where a is positive, is 90. Find the value of a. [5]

$(a+x)^5 + (1-2x)^6 \longrightarrow 90x^3$

$\boxed{\text{you will need } x^3 \text{ term from both brackets.}}$

$(a+x)^5 \rightarrow {}^5C_r (a)^{5-r} (x)^r$

$x^r = x^3$

$r = 3$

${}^5C_3 (a)^2 (x^3)$

$10a^2x^3$

$(1-2x)^6 = {}^6C_r (1)^{6-r} (-2x)^r$

$= {}^6C_r \cdot 1^{6-r} \cdot (-2)^r \cdot x^r$

$x^r = x^3$

$r = 3$

${}^6C_3 (1)^{6-3} (-2x)^3$

$(20)(1)(-8x^3)$

$-160x^3$

$(a+x)^5 + (1-2x)^6 \longrightarrow 90x^3$

$10a^2x^3 + (-160x^3) = 90x^3$

$10a^2 - 160 = 90$

$10a^2 = 250$

$a^2 = 25$

$a = 5$