39 Coin A is weighted so that the probability of throwing a head is $\frac{2}{3}$. Coin B is weighted so that the probability of throwing a head is $\frac{1}{4}$. Coin A is thrown twice and coin B is thrown once.

(i) Show that the probability of obtaining exactly 1 head and 2 tails is $\frac{13}{36}$. [3]

(ii) Draw up the probability distribution table for the number of heads obtained. [4]

(iii) Find the expectation of the number of heads obtained. [2]

Coin A: $H_A = \frac{2}{3}$ Coin B: $H_B = \frac{1}{4}$

$T_A = \frac{1}{3}$ $T_B = \frac{3}{4}$

A	A	B	
$H = \frac{2}{3}$	$T\ \frac{1}{3}$	$T\ \frac{3}{4}$	$= \left(\frac{2}{3} \times \frac{1}{3} \times \frac{3}{4}\right)$
$T = \frac{1}{3}$	$H\ \frac{2}{3}$	$T\ \frac{3}{4}$	$= \left(\frac{1}{3} \times \frac{2}{3} \times \frac{3}{4}\right)$
$T = \frac{1}{3}$	$T\ \frac{1}{3}$	$H\ \frac{1}{4}$	$= \left(\frac{1}{3} \times \frac{1}{3} \times \frac{1}{4}\right)$

$= \frac{13}{36}$

X	0	1	2	3
P(x)	$\frac{3}{36}$	$\frac{13}{36}$		

$T_A\ T_A\ T_B$

$\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)\left(\frac{3}{4}\right)$

$H_A\ H_A\ T_B = \left(\frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}\right)$

$H_A\ T_A\ H_B =$

$T_A\ H_A\ H_B$

34 The 12 houses on one side of a street are numbered with even numbers starting at 2 and going up to 24. A free newspaper is delivered on Monday to 3 different houses chosen at random from these 12. Find the probability that at least 2 of these newspapers are delivered to houses with numbers greater than 14. [4]

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24

P(at least two) = P(two houses greater than 14) + P(three houses greater than 14)

P(TWO HOUSES GREATER THAN 14)

H_1	H_2	H_3
$G_{14} = \frac{5}{12}$	$G_{14} = \frac{4}{11}$	$S_{14} = \frac{7}{10}$
$G_{14} = \frac{5}{12}$	$S_{14} = \frac{7}{11}$	$G_{14} = \frac{4}{10}$
$S_{14} = \frac{7}{12}$	$G_{14} = \frac{5}{11}$	$G_{14} = \frac{4}{10}$

$$= 3\left(\frac{5}{12} \times \frac{4}{11} \times \frac{7}{10}\right)$$

P(THREE HOUSES GREATER THAN 14)

H_1	H_2	H_3
$G_{14} = \frac{5}{12}$	$G_{14} = \frac{4}{11}$	$G_{14} = \frac{3}{10}$

$$= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$$

$$= 3\left(\frac{5}{12} \times \frac{4}{11} \times \frac{7}{10}\right) + \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$$

$$= \frac{4}{11}$$

37 Dayo chooses two digits at random, without replacement, from the 9-digit number 113 333 555.

(i) Find the probability that the two digits chosen are equal. [3]

(ii) Find the probability that one digit is a 5 and one digit is not a 5. [3]

(iii) Find the probability that the first digit Dayo chose was a 5, given that the second digit he chose is not a 5. [4]

(iv) The random variable X is the number of 5s that Dayo chooses. Draw up a table to show the probability distribution of X. [3]

1 1 3 3 3 3 5 5 5

$P(\text{Both equal}) = P(1,1) + P(3,3) + P(5,5)$

$= \left(\frac{2}{9} \times \frac{1}{8}\right) + \left(\frac{4}{9} \times \frac{3}{8}\right) + \left(\frac{3}{9} \times \frac{2}{8}\right)$

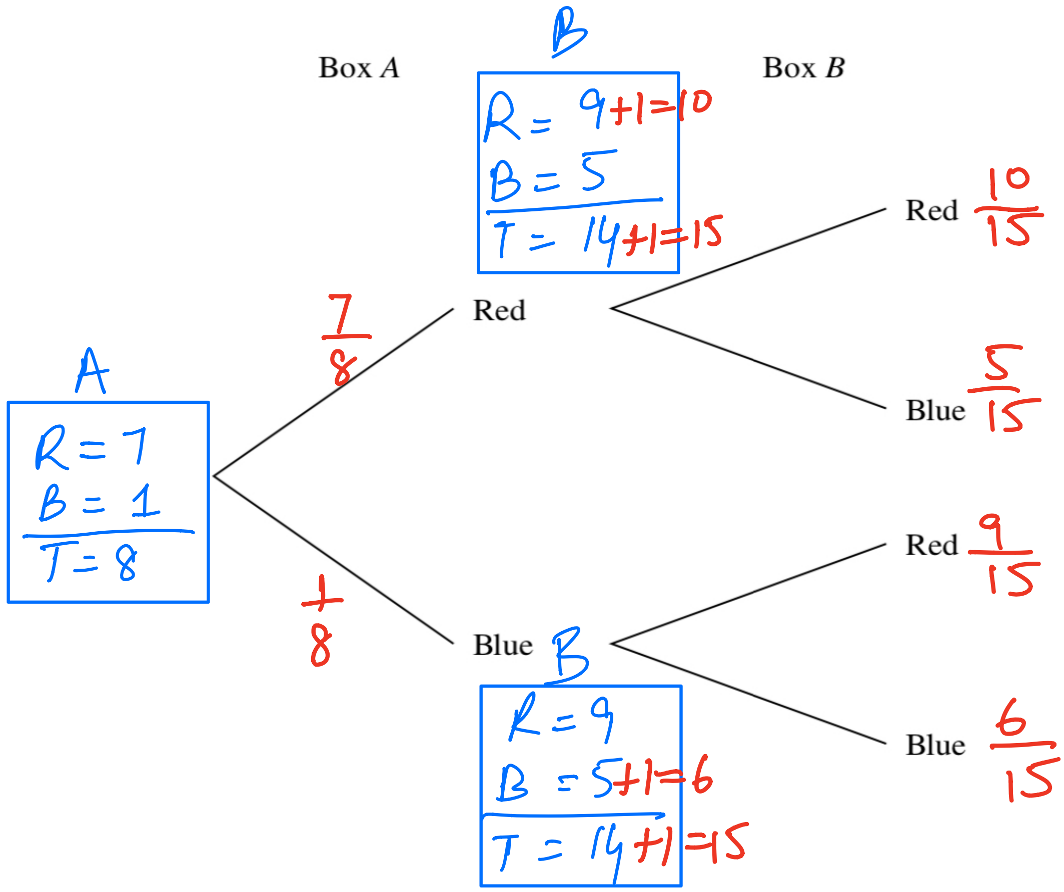
$= \frac{5}{18}$

$P(\text{one } 5 \text{ and one } \bar{5}) = P(5,\bar{5}) + P(\bar{5},5)$

$\left(\frac{3}{9} \times \frac{6}{8}\right) + \left(\frac{6}{9} \times \frac{3}{8}\right)$

6 Box A contains 7 red balls and 1 blue ball. Box B contains 9 red balls and 5 blue balls. A ball is chosen at random from box A and placed in box B. A ball is then chosen at random from box B. The tree diagram below shows the possibilities for the colours of the balls chosen.

(a) Complete the tree diagram to show the probabilities. [3]



15 A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

(i) Draw up the probability distribution of X. [3]

X	0	1	2
$P(X)$			

DDDD

GDDD
DGDD
DDGD
DDDG

$\frac{4!}{3!}$ = 4 ways

$4\left(\frac{5}{7}\times\frac{4}{6}\times\frac{3}{5}\times\frac{2}{4}\right)$

DDGG

$\frac{4!}{2!\,2!}$ = 6 ways

DDGG GGDD
DGGD GDDG
DGDG GDGD

35 James has a fair coin and a fair tetrahedral die with four faces numbered 1, 2, 3, 4. He tosses the coin once and the die twice. The random variable X is defined as follows.

- If the coin shows a **head** then X is the **sum** of the scores on the two throws of the die.
- If the coin shows a **tail** then X is the score on the **first throw** of the die only.

(i) Explain why $X = 1$ can only be obtained by throwing a tail, and show that $P(X = 1) = \frac{1}{8}$. [2]

(ii) Show that $P(X = 3) = \frac{3}{16}$. [4]

(iii) Copy and complete the probability distribution table for X. [3]

x	1	2	3	4	5	6	7	8
$P(X = x)$	$\frac{1}{8}$		$\frac{3}{16}$		$\frac{1}{8}$		$\frac{1}{16}$	$\frac{1}{32}$

Event Q is 'James throws a tail'. Event R is 'the value of X is 7'.

(iv) Determine whether events Q and R are exclusive. Justify your answer. [2]

(i) If head, X = Sum of two scores.
No two scores on a dice can give sum of 1.

(iii) $P(X=3)$

$H \to (1,2)$ $T \to 3$

$H \to (2,1)$

$\left(\frac{1}{2}\times\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{2}\times\frac{1}{6}\times\frac{1}{6}\right)+\left(\frac{1}{2}\times\frac{1}{6}\right)=\frac{3}{16}$

Q32 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21

Prob $= \frac{3}{15} = \frac{1}{5} = 0.2$

$\boxed{1\,2\,3\,4}$

R = Sum of 3 scores is 9

(3, 3, 3)
(4, 4, 1)
(1, 4, 4)
(4, 1, 4)
(3, 2, 4)
(2, 3, 4)
(4, 3, 2)
(3, 4, 2)

R and S

S = Product of 3 scores is 16

(4, 4, 1)
(1, 4, 4)
(4, 1, 4)
(4, 2, 2)
(2, 4, 2)
(2, 2, 4)

(4, 2, 3)

(2, 4, 3)

$P(R) = \frac{10}{64}$

$P(S) = \frac{6}{64}$

$P(R \& S) = \frac{3}{64}$

27 The random variable X has the probability distribution shown in the table.

x	2	4	6
$P(X = x)$	0.5	0.4	0.1

Two independent values of X are chosen at random. The random variable Y takes the value 0 if the two values of X are the same. Otherwise the value of Y is the larger value of X minus the smaller value of X.

(i) Draw up the probability distribution table for Y. [4]

(ii) Find the expected value of Y. [1]

$\left.\begin{array}{l}(2,2)\\(4,4)\\(6,6)\end{array}\right\}\ Y=0$

(2, 4) $Y = 2$

(4, 2) $Y = 2$

(4, 6) $Y = 2$

(6, 4) $Y = 2$

(2, 6) $Y = 4$

(6, 2) $Y = 4$

17 Sanket plays a game using a biased die which is twice as likely to land on an even number as on an odd number. The probabilities for the three even numbers are all equal and the probabilities for the three odd numbers are all equal.

$x = \frac{1}{9}$

$P(\text{odd}) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{3}{9}$

$x + 2x + x + 2x + x + 2x = 1$

1	2	3	4	5	6
$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{9}$	$\frac{2}{9}$

(i) Find the probability of throwing an odd number with this die. [2]

Sanket throws the die once and calculates his score by the following method.

- If the number thrown is 3 or less he multiplies the number thrown by 3 and adds 1.
- If the number thrown is more than 3 he multiplies the number thrown by 2 and subtracts 4.

The random variable X is Sanket's score.

(ii) Show that $P(X = 8) = \frac{2}{9}$. [2]

The table shows the probability distribution of X.

x	4	6	7	8	10
$P(X = x)$	$\frac{3}{9}$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

(iii) Given that $E(X) = \frac{58}{9}$, find $\text{Var}(X)$. [2]

Sanket throws the die twice.

(iv) Find the probability that the total of the scores on the two throws is 16. [2]

(v) Given that the total of the scores on the two throws is 16, find the probability that the score on the first throw was 6. [3]

Dice : 1, 2, 3

$X = 3\square + 1$

$3(1) + 1 = 4$

$3(2) + 1 = 7$

$3(3) + 1 = 10$

Dice : 4, 5, 6

$X = 2\square - 4$

$2(4) - 4 = 4$

$2(5) - 4 = 6$

$\boxed{2(6) - 4 = 8}$

$X = 8$ when dice has 6. $P(6) = \frac{2}{9}$

18 A factory makes a large number of ropes with lengths either 3 m or 5 m. There are four times as many ropes of length 3 m as there are ropes of length 5 m.

(i) One rope is chosen at random. Find the expectation and variance of its length. [4]

(ii) Two ropes are chosen at random. Find the probability that they have different lengths. [2]

(iii) Three ropes are chosen at random. Find the probability that their total length is 11 m. [3]

X	3	5
$P(X)$	$\frac{4}{5}$	$\frac{1}{5}$

3m : 5m
4 : 1

22 Ronnie obtained data about the gross domestic product (GDP) and the birth rate for 170 countries. He classified each GDP and each birth rate as either 'low', 'medium' or 'high'. The table shows the number of countries in each category.

		Birth rate			
		Low	Medium	High	
GDP	Low	3	5	45	53
	Medium	20	42	12	74
	High	35	8	0	43
		58	55	57	$\boxed{170}$

One of these countries is chosen at random.

(i) Find the probability that the country chosen has a medium GDP. $\frac{74}{170}$ [1]

(ii) Find the probability that the country chosen has a low birth rate, given that it does not have a medium GDP. $\frac{35+3}{53+43} = \frac{38}{96}$ [2]

(iii) State with a reason whether or not the events 'the country chosen has a high GDP' and 'the country chosen has a high birth rate' are exclusive. Yes. Since there is no country which has both high GDP and Birthrate. [2]

One country is chosen at random from those countries which have a medium GDP and then a different country is chosen at random from those which have a medium birth rate.

(iv) Find the probability that both countries chosen have a medium GDP and a medium birth rate. [3]

$\frac{42}{74} \times \frac{41}{54}$

14 **(i)** Find the number of different ways that a set of 10 different mugs can be shared between Lucy and Monica if each receives an odd number of mugs. [3]

(ii) Another set consists of 6 plastic mugs each of a different design and 3 china mugs each of a different design. Find in how many ways these 9 mugs can be arranged in a row if the china mugs are all separated from each other. [3]

(iii) Another set consists of 3 identical red mugs, 4 identical blue mugs and 7 identical yellow mugs. These 14 mugs are placed in a row. Find how many different arrangements of the colours are possible if the red mugs are kept together. [3]

(i)

L	M		
1	9	$= {}^{10}C_1 \times {}^{9}C_9$	$=$
3	7	$= {}^{10}C_3 \times {}^{7}C_7$	$=$
5	5	$= {}^{10}C_5 \times {}^{5}C_1$	$=$
7	3	$= {}^{10}C_7 \times {}^{3}C_3$	$=$
9	1	$= {}^{10}C_9 \times {}^{1}C_1$	$=$

* _ * _ * _ * _ * _ * _ *

$6! \times {}^{7}P_3 =$

(iii) $\boxed{RRR}$ B B B B Y Y Y Y Y Y Y

$$\frac{3! \times 12!}{3!\,4!\,7!}$$

38 **(i)** Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the fi st letter is R. [2]

(ii) Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if the 3 letters G are together, both letters A are together and both letters E are together. [2]

(iii) The letters G, R and T are consonants and the letters A and E are vowels. Find the number of different ways that the 9 letters of the word AGGREGATE can be arranged in a line if consonants and vowels occur alternately. [3]

AA, EE, T, R

(iv) Find the number of different selections of 4 letters of the word A~~GG~~RE~~G~~ATE which contain exactly 2 Gs or exactly 3 Gs. [3]

TWO G's

G G _ _

1 × 1

$\boxed{\begin{array}{l} GGAA \\ GGEE \\ GGAE \\ GGAR \\ GGAT \\ GGER \\ GGET \\ GGRT \end{array}}$

THREE G's

G G G _

$\boxed{\begin{array}{l} GGGR \\ GGGA \\ GGGE \\ GGGT \end{array}}$

= 12 ways.

18 A committee of 6 people, which must contain at least 4 men and at least 1 woman, is to be chosen from 10 men and 9 women.

(i) Find the number of possible committees that can be chosen. [3]

(ii) Find the probability that one particular man, Albert, and one particular woman, Tracey, are both on the committee. [2]

(iii) Find the number of possible committees that include either Albert or Tracey but not both. [3]

(iv) The committee that is chosen consists of 4 men and 2 women. They queue up randomly in a line for refreshments. Find the probability that the women are not next to each other in the queue. [3]