

# LISTING TYPE

- (b) There are 7 Chinese, 6 European and 4 American students at an international conference. Four of the students are to be chosen to take part in a television broadcast. Find the number of different ways the students can be chosen if at least one Chinese and at least one European student are included. [5]

C(7)	E(6)	A(4)		
1	1	2	$= {}^7C_1 \times {}^6C_1 \times {}^4C_2$	$= 252$
1	2	1	$= {}^7C_1 \times {}^6C_2 \times {}^4C_1$	$= 420$
2	1	1	$= {}^7C_2 \times {}^6C_1 \times {}^4C_1$	$= 504$
1	3	0	$= {}^7C_1 \times {}^6C_3 \times {}^4C_0$	$= 140$
3	1	0	$= {}^7C_3 \times {}^6C_1 \times {}^4C_0$	$= 210$
2	2	0	$= {}^7C_2 \times {}^6C_2 \times {}^4C_0$	$= 315$
				<u>1841</u>

- 48 Rachel has 3 types of ornament. She has 6 different wooden animals, 4 different sea-shells and 3 different pottery ducks.

- (i) She lets her daughter Cherry choose 5 ornaments to play with. Cherry chooses at least 1 of each type of ornament. How many different selections can Cherry make? [5]

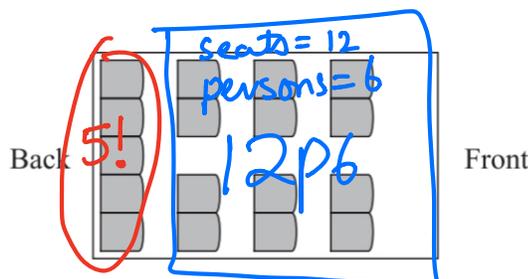
WA(6)	SS(4)	PD(3)		
1	1	3	$= {}^6C_1 \times {}^4C_1 \times {}^3C_3$	$= 24$
1	3	1	$= {}^6C_1 \times {}^4C_3 \times {}^3C_1$	$= 72$
3	1	1	$= {}^6C_3 \times {}^4C_1 \times {}^3C_1$	$= 240$
1	2	2	$= {}^6C_1 \times {}^4C_2 \times {}^3C_2$	$= 108$
2	1	2	$= {}^6C_2 \times {}^4C_1 \times {}^3C_2$	$= 180$
2	2	1	$= {}^6C_2 \times {}^4C_2 \times {}^3C_1$	$= 270$
				<u>894</u>

- (b) Sandra wishes to buy some applications (apps) for her smartphone but she only has enough money for 5 apps in total. There are 3 train apps, 6 social network apps and 14 games apps available. Sandra wants to have at least 1 of each type of app. Find the number of different possible selections of 5 apps that Sandra can choose. [5]

T (3)	S (6)	G (14)	
1	1	3	=
1	3	1	=
3	1	1	=
1	2	2	=
2	1	2	=
2	2	1	=

## SEATING PROBLEMS (SLOW DOWN AND UNDERSTAND PROBLEM)

6



The diagram shows the seating plan for passengers in a minibus, which has 17 seats arranged in 4 rows. The back row has 5 seats and the other 3 rows have 2 seats on each side. 11 passengers get on the minibus.

- (i) How many possible seating arrangements are there for the 11 passengers?  ${}^{17}P_{11} =$  [2]
- (ii) How many possible seating arrangements are there if 5 particular people sit in the back row?  $5! \times {}^{12}P_6 = 79833600.$  [3]

Of the 11 passengers, 5 are unmarried and the other 6 consist of 3 married couples.

- (iii) In how many ways can 5 of the 11 passengers on the bus be chosen if there must be 2 married couples and 1 other person, who may or may not be married? [3]





choose two couples out of 3 AND 1 OTHER PERSON

$$= {}^3C_2 \times {}^7C_1$$

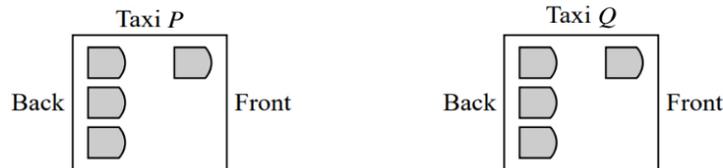
$$= 21$$

52 A group of 8 friends travels to the airport in two taxis,  $P$  and  $Q$ . Each taxi can take 4 passengers.

(i) The 8 friends divide themselves into two groups of 4, one group for taxi  $P$  and one group for taxi  $Q$ , with Jon and Sarah travelling in the same taxi. Find the number of different ways in which this can be done. [3]

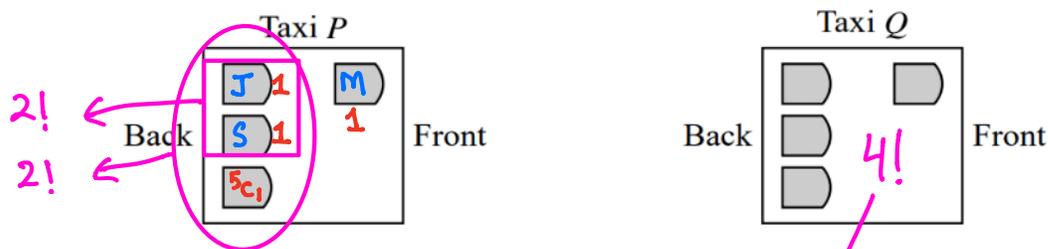
$$\begin{matrix} \boxed{P} & \boxed{Q} \\ \hline \text{J} & \text{S} & \text{---} & \text{---} \\ \hline (1 & 1 & {}^6C_2) & \times ({}^4C_4) \end{matrix} \quad \text{OR} \quad \begin{matrix} \boxed{P} & \boxed{Q} \\ \hline \text{---} & \text{---} & \text{---} & \text{---} \\ \hline ({}^6C_4) & \times (1 & 1 & \times {}^2C_2) \end{matrix}$$

$$= 30 \text{ ways.}$$



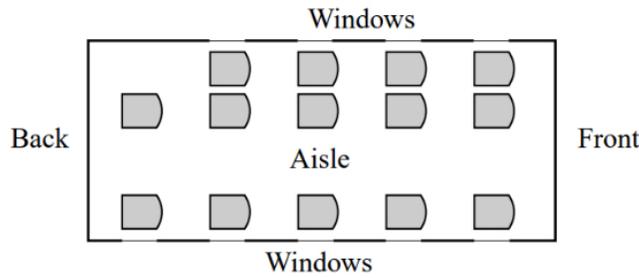
Each taxi can take 1 passenger in the front and 3 passengers in the back (see diagram). Mark sits in the front of taxi  $P$  and Jon and Sarah sit in the back of taxi  $P$  next to each other.

(ii) Find the number of different seating arrangements that are now possible for the 8 friends. [4]



$$(1 \times 1 \times {}^5C_1) \times 2! \times 2! \times 1 \times 4P_4 = 480$$

WORK ON IT TILL  
12:30



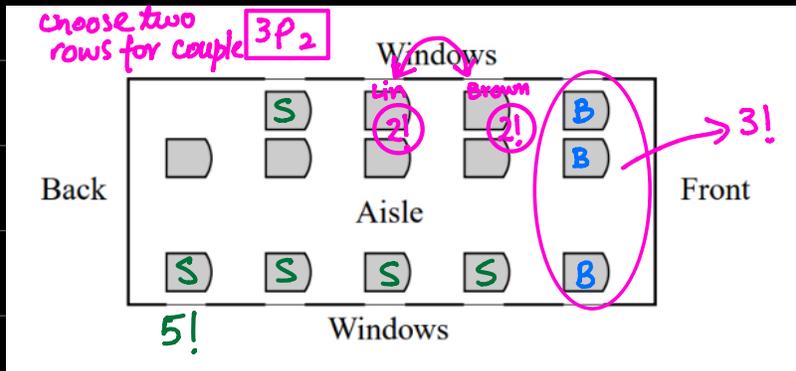
A small aeroplane has 14 seats for passengers. The seats are arranged in 4 rows of 3 seats and a back row of 2 seats (see diagram). 12 passengers board the aeroplane.

- (i) How many possible seating arrangements are there for the 12 passengers? Give your answer correct to 3 significant figures.  $14P_{12} = 4.36 \times 10^{10}$  [2]

These 12 passengers consist of 2 married couples (Mr and Mrs Lin and Mr and Mrs Brown), 5 students and 3 business people.

- (ii) The 3 business people sit in the front row. The 5 students each sit at a window seat. Mr and Mrs Lin sit in the same row on the same side of the aisle. Mr and Mrs Brown sit in another row on the same side of the aisle. How many possible seating arrangements are there? [4]
- (iii) If, instead, the 12 passengers are seated randomly, find the probability that Mrs Lin sits directly behind a student and Mrs Brown sits in the front row. [4]

(ii)



Business =  $3!$  = 6

Couples :  $({}^3C_2 \times 2!) \times 2! \times 2!$  = 24

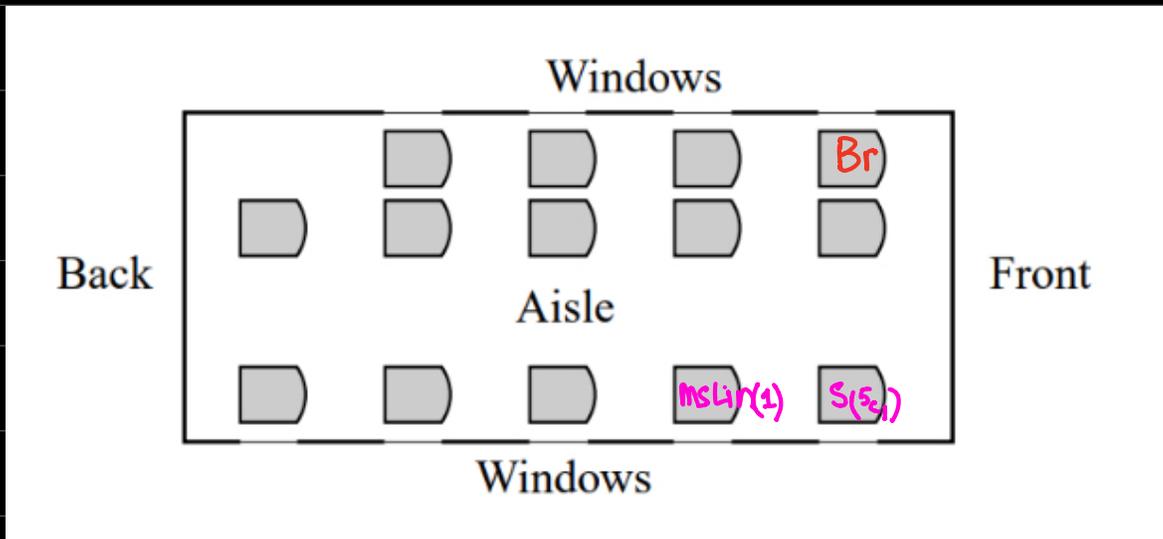
Students =  $5!$  = 120

BUSINESS AND COUPLES AND STUDENTS

= 6 x 24 x 120

= 17280

- (iii) If, instead, the 12 passengers are seated randomly, find the probability that Mrs Lin sits directly behind a student and Mrs Brown sits in the front row. [4]



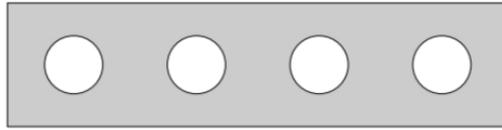
Mrs Brown = 3 ways

$$\text{Mrs Lin \& Student} = 1 \times {}^5C_1 \times 10$$

$$\text{Remaining people} = \begin{matrix} \text{spaces} = 11 \\ \text{persons} = 9 \end{matrix} \rightarrow {}^{11}P_9$$

$$\begin{aligned} \text{No of ways} &= 3 \times (1 \times {}^5C_1) \times 10 \times {}^{11}P_9 \\ &= 2.99 \times 10^9 \end{aligned}$$

$$\text{Probability} = \frac{\text{Fav}}{\text{Total}} = \frac{2.99 \times 10^9}{4.36 \times 10^{10}} = 0.0687$$



Pegs are to be placed in the four holes shown, one in each hole. The pegs come in different colours and pegs of the same colour are identical. Calculate how many different arrangements of coloured pegs in the four holes can be made using

(i) 6 pegs, all of different colours,  ${}^6P_4 = 360$  [1]

(ii) 4 pegs consisting of 2 blue pegs, 1 orange peg and 1 yellow peg. [1]

Beryl has 12 pegs consisting of 2 red, 2 blue, 2 green, 2 orange, 2 yellow and 2 black pegs. Calculate how many different arrangements of coloured pegs in the 4 holes Beryl can make using

(iii) 4 different colours, [1]

(iv) 3 different colours, [3]

(v) any of her 12 pegs. [3]

$$(ii) \quad B B O Y = \frac{4!}{2!} = 12$$

(iii) 2R, 2Blue, 2G, 2O, 2Y, 2Black.

choose 4 different colors AND Arrange

$${}^6C_4 \times 4! = 360$$

(iv) choose 3 colours AND Arrange

$${}^6C_3$$

x

$$\begin{matrix} (C_1) & (C_1) & (C_2) & (C_3) & \frac{4!}{2!} \end{matrix}$$

$$\begin{matrix} (C_1) & (C_2) & (C_2) & (C_3) & \frac{4!}{2!} \end{matrix}$$

$$\begin{matrix} (C_1) & (C_2) & (C_3) & (C_3) & \frac{4!}{2!} \end{matrix}$$

$${}^6C_3 \times \left( \frac{4!}{2!} + \frac{4!}{2!} + \frac{4!}{2!} \right) = 720.$$

choose 2 different colours AND ARRANGE

$${}^6C_2$$

$$\textcircled{C_1} \textcircled{C_1} \textcircled{C_2} \textcircled{C_2} \frac{4!}{2!2!}$$

$${}^6C_2 \times \frac{4!}{2!2!} = 90 \text{ ways.}$$

WITHOUT RESTRICTIONS : All 4 pegs of different colours OR 3 different colours OR Two different colours

$$= 360 + 720 + 90$$

$$= 1170.$$

11 A choir consists of 13 sopranos, 12 altos, 6 tenors and 7 basses. A group consisting of 10 sopranos, 9 altos, 4 tenors and 4 basses is to be chosen from the choir.

(i) In how many different ways can the group be chosen? [2]

(ii) In how many ways can the 10 chosen sopranos be arranged in a line if the 6 tallest stand next to each other? [3]

(iii) The 4 tenors and 4 basses in the group stand in a single line with all the tenors next to each other and all the basses next to each other. How many possible arrangements are there if three of the tenors refuse to stand next to any of the basses? [3]

(i) S(13)    A(12)    T(6)    B(7)

10            9            4            4

$$= {}_{10}C_1 \times {}_9C_1 \times {}_4C_1 \times {}_4C_1 = 33033000.$$

(ii)

$$\boxed{\text{---} \text{---} \text{---} \text{---} \text{---} \text{---}} \text{---} \text{---} \text{---} \text{---} = 6! \times 5! = 86400$$

5!

(iii)

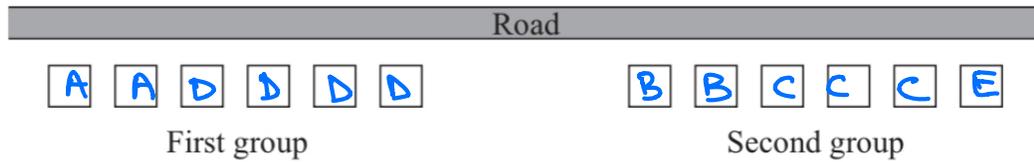
T	B
--- 3! ---	--- 4! ---
1	
2!	

$$= (3! \times 1) \times (4!) \times 2! = 288.$$

10 A builder is planning to build 12 houses along one side of a road. He will build 2 houses in style *A*, 2 houses in style *B*, 3 houses in style *C*, 4 houses in style *D* and 1 house in style *E*.

(i) Find the number of possible arrangements of these 12 houses. 
$$= \frac{12!}{2! 2! 3! 4!} = 831600$$
 [2]

(ii)



The 12 houses will be in two groups of 6 (see diagram). Find the number of possible arrangements if all the houses in styles *A* and *D* are in the first group and all the houses in styles *B*, *C* and *E* are in the second group. [3]

(iii) Four of the 12 houses will be selected for a survey. Exactly one house must be in style *B* and exactly one house in style *C*. Find the number of ways in which these four houses can be selected. [2]

(ii) 
$$\frac{6!}{2! 4!} \times \frac{6!}{2! 3!} = 900$$

(iii) Assume all houses are not identical.



13 (a) (i) Find how many different four-digit numbers can be made using only the digits 1, 3, 5 and 6 with no digit being repeated. [1]

spaces = 4 } Factorial = 4! = 24  
objects = 4 }

32 (ii) How many different numbers between 20 000 and 30 000 can be formed using 5 different digits from the digits 1, ~~4~~, 6, 7, 8? [2]

spaces = 5 } permutation  
objects = 6 }



$$1 \times {}^5P_4 = 120 \text{ ways.}$$

42 A committee of 6 people is to be chosen from 5 men and 8 women. In how many ways can this be done

(i) if there are more women than men on the committee, [4]

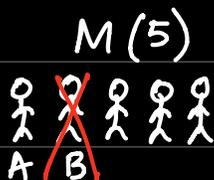
(ii) if the committee consists of 3 men and 3 women but two particular men refuse to be on the committee together? [3]

One particular committee consists of 5 women and 1 man.

(iii) In how many different ways can the committee members be arranged in a line if the man is not at either end? [3]

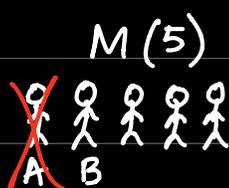
(i)	M(5)	W(8)	
	2	4	$= {}^5C_2 \times {}^8C_4 = 700$
	1	5	$= {}^5C_1 \times {}^8C_5 = 280$
	0	6	$= {}^5C_0 \times {}^8C_6 = 28$
			<u>1008</u>

(ii) A goes:



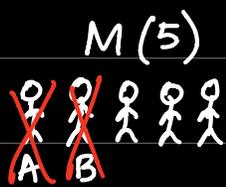
$\boxed{A}$	$\boxed{\quad}$	$\boxed{\quad}$	$\times$	${}^8C_3$	$=$	168
1	${}^3C_2$					

B goes:



$\boxed{B}$	$\boxed{\quad}$	$\boxed{\quad}$	$\times$	${}^8C_3$	$=$	168
1	${}^3C_2$					

None of A & B go:



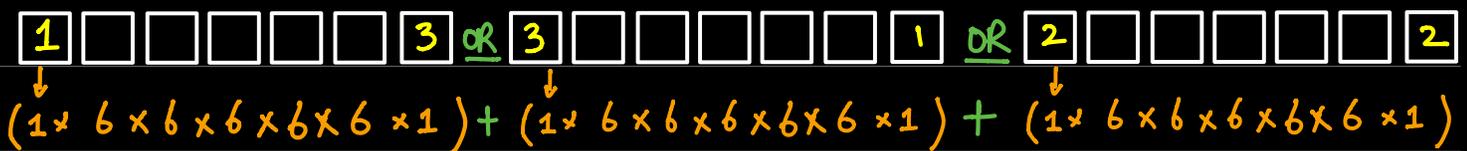
w(8)

$${}^3C_3 + {}^8C_3 = 56$$

$$168 + 168 + 56 = 392$$

- 45 (a) Seven fair dice each with faces marked 1, 2, 3, 4, 5, 6 are thrown and placed in a line. Find the number of possible arrangements where the sum of the numbers at each end of the line add up to 4. [3]
- (b) Find the number of ways in which 9 different computer games can be shared out between Wainah, Jingyi and Hebe so that each person receives an odd number of computer games. [6]

(i)



$$= 3 \times (1 \times 6^5 \times 1)$$

$$= 23328$$

(b)	W	J	H	
	1	1	7	$\left. \begin{array}{l} \longrightarrow \\ \frac{3!}{2!} = 3 \end{array} \right\}$
	1	7	1	
	7	1	1	
	3	5	1	$\left. \begin{array}{l} \longrightarrow \\ 3! = 6 \end{array} \right\}$
	3	1	5	
	5	1	3	
	5	3	1	
	1	3	5	
	1	5	3	

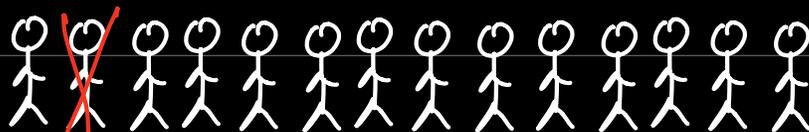
${}^9C_1 \times {}^8C_1 \times {}^7C_7 = 72$
${}^9C_1 \times {}^8C_7 \times {}^1C_1 = 72$
${}^9C_7 \times {}^2C_1 \times {}^1C_1 = 72$
${}^9C_3 \times {}^6C_5 \times {}^1C_1 = 504$
$= 504$
$= 504$
$= 504$
$= 504$
$= 504$

3 3 3

$${}^9C_3 + {}^6C_3 + {}^3C_3 = 1680$$

$$4920$$

47. (b) Find the number of ways of selecting a group of 9 people from 14 if two particular people cannot both be in the group together. [3]



A B

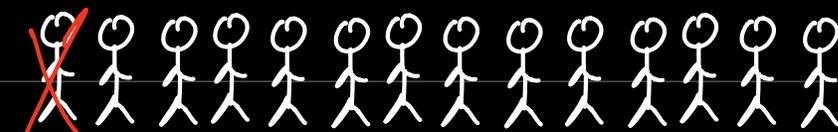
A goes



1

$${}^{12}C_8$$

$$= 495$$



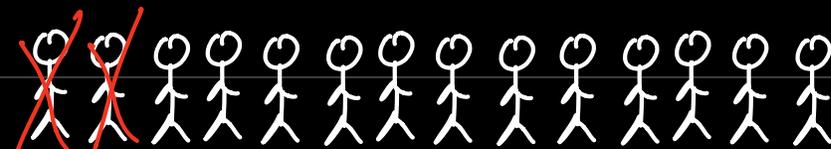
A B

B goes



$$1 \times {}^{12}C_8$$

$$= 495$$



A B

None go.



$${}^{12}C_9$$

$$= 220$$

Q25

1210

- (b) Find the number of different ways in which the 9 letters of the word GREENGAGE can be arranged if exactly two of the Gs are next to each other. [3]

$$\text{Total ways} = \text{All three Gs together} + \text{Exactly Two Gs together} + \text{All three Gs separate.}$$

$$10080 = 840 + \text{Exactly Two Gs together} + 4200$$

$$\text{Exactly Two Gs together} = 5040$$

$$\text{Total ways: } \frac{9!}{3! 3!} = 10080$$

G E

$$\text{All Three G's together: } \frac{3!}{7!} = \frac{3! \times 7!}{3! 3!} = 840$$

All G's Separate.



$$\frac{6! \times {}^7P_3}{3! 3!} = 4200$$

16 Three identical cans of cola, 2 identical cans of green tea and 2 identical cans of orange juice are arranged in a row. Calculate the number of arrangements if

(i) the first and last cans in the row are the same type of drink, [3]

(ii) the 3 cans of cola are all next to each other and the 2 cans of green tea are not next to each other. [5]

C C C G G O O

(ii)

$$\begin{array}{c}
 * \boxed{\text{--- } 3! \text{ ---}} * \text{---} * \text{---} * \\
 \hline
 \frac{3! \quad 4P_2 \times 3!}{3! \quad 2! \quad 2!} = \boxed{18 \text{ way}}
 \end{array}$$

colas together, no restriction on rest:  $\frac{3! \times 5!}{3! \cdot 2! \cdot 2!} = 30$

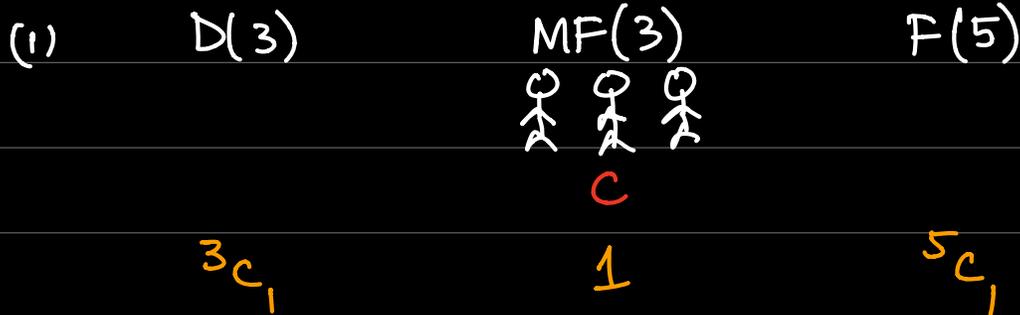
colas together, greeteas together:  $\frac{3! \cdot 2! \times 4!}{3! \cdot 2! \cdot 2!} = 12$

$30 - 12 = 18 \text{ ways.}$

3 ( a ) A football team consists of 3 players who play in a defence position, 3 players who play in a midfi ld position and 5 players who play in a forward position. Three players are chosen to collect a gold medal for the team. Find in how many ways this can be done

(i) if the captain, who is a midfi ld player, must be included, together with one defence and one forward player, [2]

(ii) if exactly one forward player must be included, together with any two others. [2]



Q32

(c) Helen has some black tiles, some white tiles and some grey tiles. She places a single row of 8 tiles above her washbasin. Each tile she places is equally likely to be black, white or grey. Find the probability that there are no tiles of the same colour next to each other. [3]



$$B = \frac{1}{3}$$

$$G = \frac{1}{3}$$

$$W = \frac{1}{3}$$

$$1 \times \left(\frac{2}{3}\right)^7 = \square$$

24 Twelve coins are tossed and placed in a line. Each coin can show either a head or a tail.

(i) Find the number of different arrangements of heads and tails which can be obtained. [2]

(ii) Find the number of different arrangements which contain 7 heads and 5 tails. [1]

(i)  $\boxed{2} \boxed{2} = 2^{12}$



$$\frac{12!}{7! 5!} = 792$$

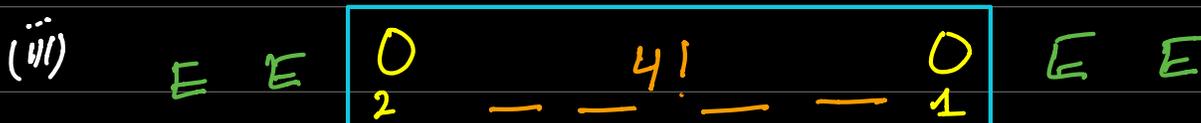
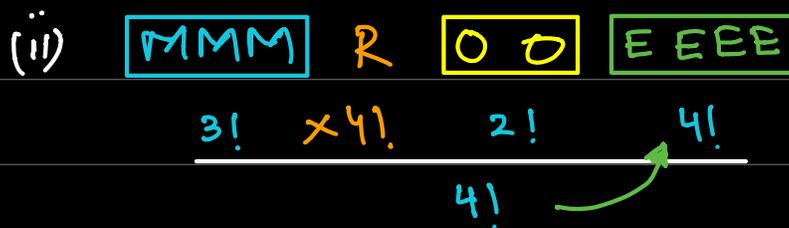
36 A shop has 7 different mountain bicycles, 5 different racing bicycles and 8 different ordinary bicycles on display. A cycling club selects 6 of these 20 bicycles to buy.

(i) How many different selections can be made if there must be no more than 3 mountain bicycles and no more than 2 of each of the other types of bicycle? [4]

The cycling club buys 3 mountain bicycles, 1 racing bicycle and 2 ordinary bicycles and parks them in a cycle rack, which has a row of 10 empty spaces.

✓ (ii) How many different arrangements are there in the cycle rack if the mountain bicycles are all together with no spaces between them, the ordinary bicycles are both together with no spaces between them and the spaces are all together? [3]

✓ (iii) How many different arrangements are there in the cycle rack if the ordinary bicycles are at each end of the bicycles and there are no spaces between any of the bicycles? [3]



5!

$$= \frac{2! \times 5! \times 4!}{4!} = \underline{\hspace{2cm}}$$

13)

(b) Six cards numbered 1, 2, 3, 4, 5, 6 are arranged randomly in a line. Find the probability that the cards numbered 4 and 5 are **not** next to each other. [3]

4 and 5 Not next to each other

TOTAL - (4 & 5 Together)

$$= 6! - (2! \times 5!) \\ = 480$$

$$P(4 \text{ and } 5 \text{ not together}) = \frac{\text{4 \& 5 NOT Together}}{\text{Total.}}$$

$$= \frac{480}{6!} = \frac{480}{720} = \frac{2}{3}$$

15 Nine cards, each of a different colour, are to be arranged in a line.

(i) How many different arrangements of the 9 cards are possible?  $= 9! = 362880$  [1]

The 9 cards include a pink card and a green card.

(ii) How many different arrangements do not have the pink card next to the green card? [3]

Total - Both together

$$9! - (2! \times 8!) = 282240$$

choose using  ${}^n C_r$

arrange.

Consider all possible choices of 3 cards from the 9 cards with the 3 cards being arranged in a line.

(iii) How many different arrangements in total of 3 cards are possible? [2]

(iv) How many of the arrangements of 3 cards in part (iii) contain the pink card? [2]

(v) How many of the arrangements of 3 cards in part (iii) do not have the pink card next to the green card? [2]

(iii) choose 3 cards out of 9 AND ARRANGE

$${}^9 P_3 = {}^9 C_3 \times 3! = 504.$$

(iv) choose 3 cards out of 9 AND ARRANGE

$$\begin{array}{ccc} \boxed{P} & \boxed{\phantom{P}} & \boxed{\phantom{P}} \\ 1 & \underbrace{\phantom{P}} & \phantom{P} \\ & 8C_2 & \end{array} \times 3! = (1 \times {}^8 C_2) \times 3! = 168$$

(v) FIND WAYS OF PINK & GREEN TOGETHER.

choose 3 cards out of 9 AND ARRANGE

$$\begin{array}{ccc} \boxed{P} & \boxed{G} & \boxed{\phantom{P}} \\ 1 & 1 & 7C_1 \\ & & \phantom{P} \end{array} \times \boxed{\begin{array}{ccc} \boxed{P} & \boxed{G} & \boxed{\phantom{P}} \\ & & \phantom{P} \end{array}} = \boxed{28}$$

$2! \times 2!$

$$\begin{aligned} \text{PINK \& GREEN NOT NEXT TO EACH OTHER} &= 504 - 28 \\ &= 476. \end{aligned}$$

30 (a) In a sweet shop 5 identical packets of toffees, 4 identical packets of fruit gums and 9 identical packets of chocolates are arranged in a line on a shelf. Find the number of different arrangements of the packets that are possible if the packets of chocolates are kept together. [2]

(b) Jessica buys 8 different packets of biscuits. She then chooses 4 of these packets.

(i) How many different choices are possible if the order in which Jessica chooses the 4 packets is taken into account?  $8P_4 = 8C_4 \times 4! = 1680$  [2]

The 8 packets include 1 packet of chocolate biscuits and 1 packet of custard creams.

(ii) How many different choices are possible if the order in which Jessica chooses the 4 packets is taken into account and the packet of chocolate biscuits and the packet of custard creams are both chosen? [3]

(a) T T T T T F F F F C C C C C C C C C

$10! \qquad 9!$

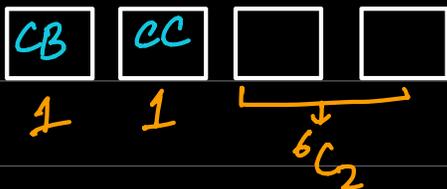
$$= \frac{10! \times 9!}{5! 4! 9!} = 1260$$

$T \quad F \quad C$



CHOOSE

ARRANGE



$4!$

$$= (1 \times 1 \times 6C_2) \times 4! = 360$$

4 A staff car park at a school has 13 parking spaces in a row. There are 9 cars to be parked.

(i) How many different arrangements are there for parking the 9 cars and leaving 4 empty spaces? [2]

(ii) How many different arrangements are there if the 4 empty spaces are next to each other? [3]

(iii) If the parking is random, find the probability that there will **not** be 4 empty spaces next to each other. [2]

$$(i) \left. \begin{array}{l} \text{spaces} = 13 \\ \text{objects} = 9 \end{array} \right\} \text{Permutation} = {}_{13}P_9 = 259,459,200$$



$$\frac{10! \times 4!}{4!} = 3628800$$

(iii) Not all spaces next to each other

$$= 259,459,200 - 3628800 =$$

$$P(\text{not all together}) = \frac{\text{Total.}}{\text{Total.}}$$

18 A committee of 6 people, which must contain at least 4 men and at least 1 woman, is to be chosen from 10 men and 9 women.

- (i) Find the number of possible committees that can be chosen. [3]
- (ii) Find the probability that one particular man, Albert, and one particular woman, Tracey, are both on the committee. [2]
- (iii) Find the number of possible committees that include either Albert or Tracey but not both. [3]
- (iv) The committee that is chosen consists of 4 men and 2 women. They queue up randomly in a line for refreshments. Find the probability that the women are not next to each other in the queue. [3]

M(10)

W(9)

4

2

$$= {}^{10}C_4 \times {}^9C_2 =$$

5

1

$$= {}^{10}C_5 \times {}^9C_1 =$$

9828

~~M(10)~~

~~W(9)~~

3

1

$$= ({}^9C_3 \times {}^8C_1)$$

4

0

$$= ({}^9C_4 \times {}^8C_0)$$

798.

