1 Two fair dice are thrown. Let the random variable X be the smaller of the two scores if the scores are different, or the score on one of the dice if the scores are the same.

(i) Copy and complete the following table to show the probability distribution of X. [3]

x	1	2	3	4	5	6	
$P(X = x)$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$	
$x \cdot p(x)$	$\frac{11}{36}$	$\frac{18}{36}$	$\frac{21}{36}$	$\frac{20}{36}$	$\frac{15}{36}$	$\frac{6}{36}$	$E(X) = \frac{91}{36}$

(ii) Find $E(X)$.

$X=1$	$X=2$	$X=3$	$X=4$	$X=5$	$X=6$
(1,1)	(2,2)	(3,3)	(4,4)	(5,5)	(6,6)
(1,2) (2,1)	(2,3) (3,2)	(3,4)(4,3)	(4,5) (5,4)	(5,6)(6,5)	
(1,3) (3,1)	(2,4) (4,2)	(3,5)(5,3)	(4,6) (6,4)		
(1,4) (4,1)	(2,5) (5,2)	(3,6)(6,3)			
(1,5) (5,1)	(2,6) (6,2)				
(1,6) (6,1)					
11	9	7	5	3	1

9 Every day Eduardo tries to phone his friend. Every time he phones there is a 50% chance that his friend will answer. If his friend answers, Eduardo does not phone again on that day. If his friend does not answer, Eduardo tries again in a few minutes' time. If his friend has not answered after 4 attempts, Eduardo does not try again on that day.

(i) Draw a tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unanswered phone calls made by Eduardo on a day. Copy and complete the table showing the probability distribution of X. [4]

x	0	1	2	3	4	
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$	
$x \cdot P(x)$	0	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{16}$	$\frac{4}{16}$	$E(X) = \frac{15}{16}$

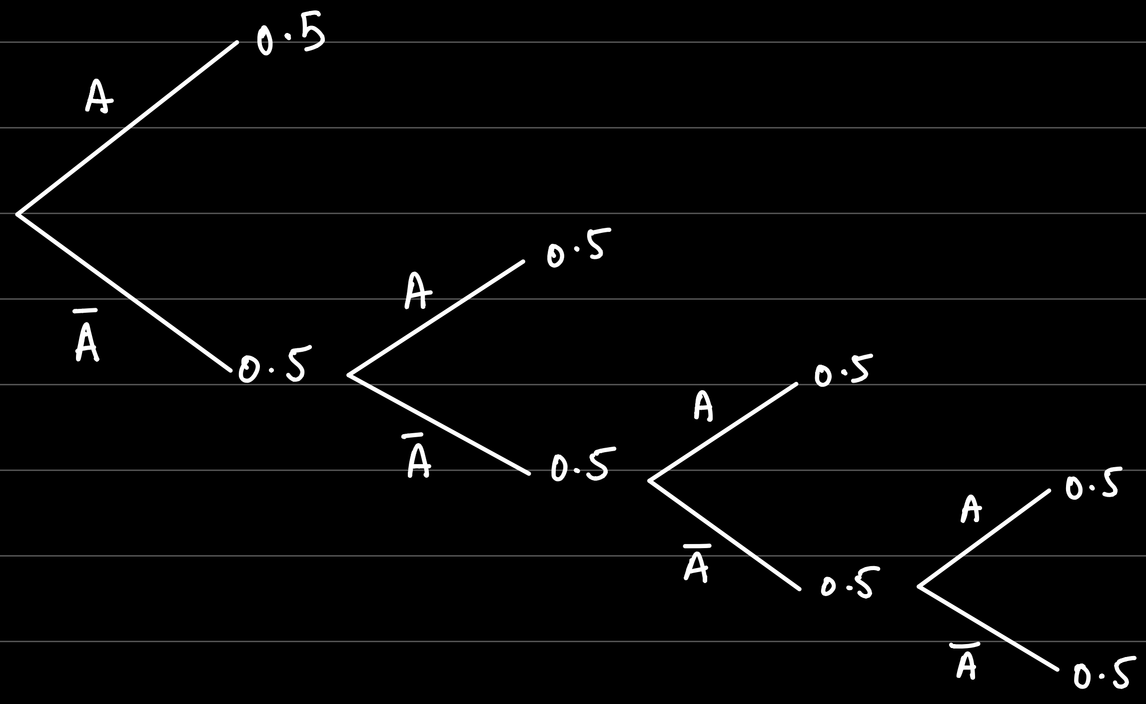
(iii) Calculate the expected number of unanswered phone calls on a day. [2]

$E(X)$

$X = 0$ means No unanswered phone call $\rightarrow$ A

$X = 1$ means 1 unanswered call $\rightarrow$ $\bar{A}\,A$

$X = 4$ means 4 unanswered calls $\rightarrow$ $\bar{A}\,\bar{A}\,\bar{A}\,\bar{A}$



4 In a competition, people pay \$1 to throw a ball at a target. If they hit the target on the first throw they receive \$5. If they hit it on the second or third throw they receive \$3, and if they hit it on the fourth or fifth throw they receive \$1. People stop throwing after the first hit, or after 5 throws if no hit is made. Mario has a constant probability of $\frac{1}{5}$ of hitting the target on any throw, independently of the results of other throws.

$H = \frac{1}{5}$, $M = \frac{4}{5}$

(i) Mario misses with his first and second throws and hits the target with his third throw. State how much profit he has made. [1]

(ii) Show that the probability that Mario's profit is \$0 is 0.184, correct to 3 significant figures. [2]

(iii) Draw up a probability distribution table for Mario's profit. [3]

(iv) Calculate his expected profit. [2] $E(X)$.

$\boxed{n, p, q \text{ not constant}}$ NOT BINOMIAL.

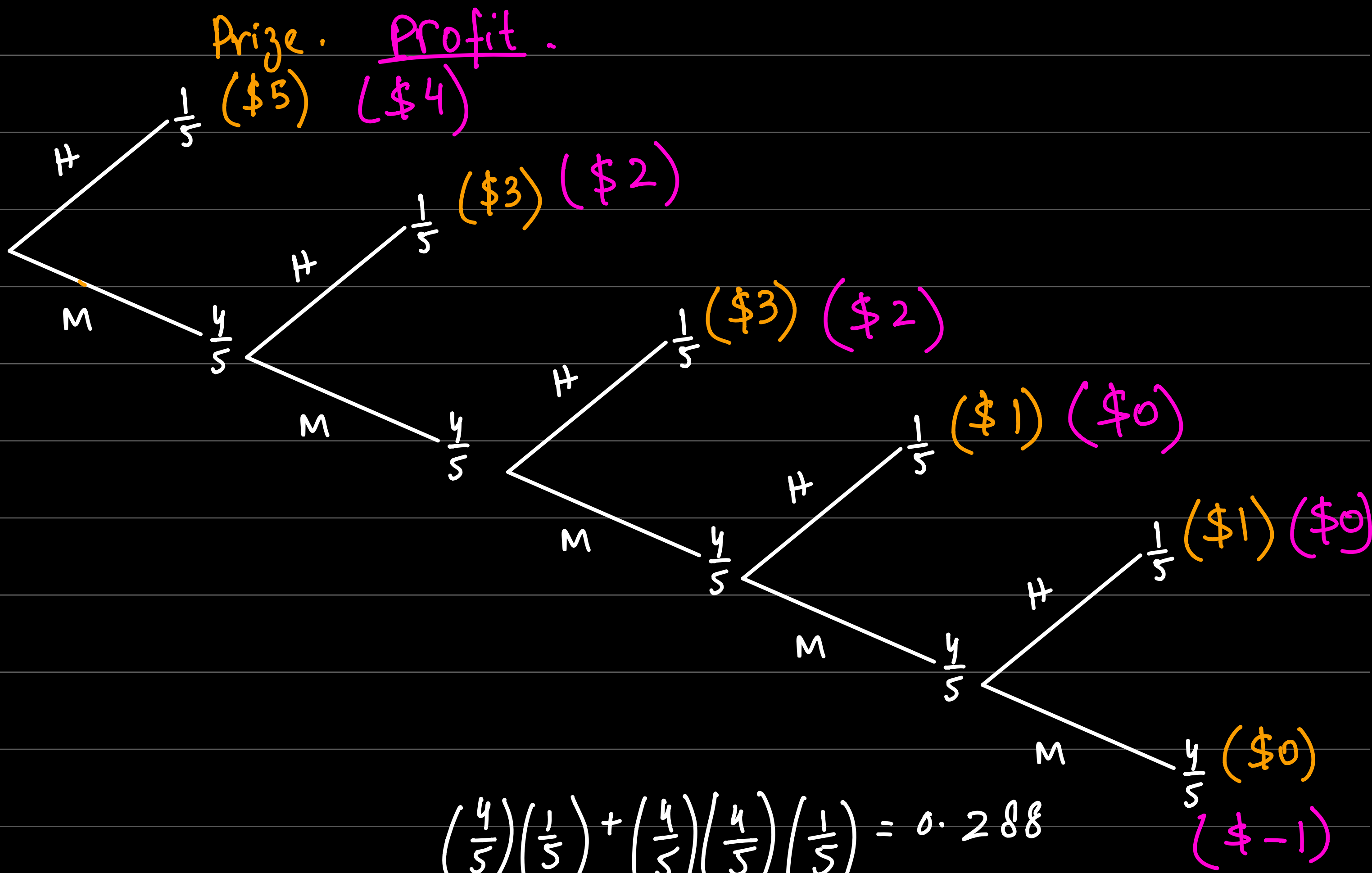
(i) Profit = Prize − 1
= 3 − 1
= \$2

(ii) Profit = Prize − 1
0 = Prize − 1
$\boxed{\text{prize} = 1}$ Hits on fourth or fifth.

FOURTH OR FIFTH

MMMH OR MMMMH

$$\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right) = 0.184$$



$$\left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{4}{5}\right)\left(\frac{1}{5}\right) = 0.288$$

X = profit	-1	0	2	4	
$P(x)$	0.328	0.184	0.288	0.2	
$x \cdot P(x)$	-0.328	0	0.576	0.8	$E(X) = 1.048$ Mean (Average)

21 Judy and Steve play a game using five cards numbered 3, 4, 5, 8, 9. Judy chooses a card at random, looks at the number on it and replaces the card. Then Steve chooses a card at random, looks at the number on it and replaces the card. If their two numbers are equal the score is 0. Otherwise, the smaller number is subtracted from the larger number to give the score.

(i) Show that the probability that the score is 6 is 0.08. [1]

(ii) Draw up a probability distribution table for the score. [2]

(iii) Calculate the mean score. Expected value. [1]

If the score is 0 they play again. If the score is 4 or more Judy wins. Otherwise Steve wins. They continue playing until one of the players wins.

(iv) Find the probability that Judy wins with the second choice of cards. [3]

(v) Find an expression for the probability that Judy wins with the nth choice of cards. [2]

TOTAL = 25 outcomes.

$\boxed{\text{Score} = 6}$ (3,9) (9,3) $= \frac{2}{25} = 0.08$

S J S J

$\boxed{\text{SCORE} = 1}$ (3,4) (4,3) (4,5) (5,4) (8,9) (9,8)

$P(X=1) = \frac{6}{25}$

S J

$\boxed{X=0}$ (3,3) (4,4) (5,5) (8,8) (9,9) $= \frac{5}{25}$

$\boxed{X=2}$ (3,5) (5,3) $= \frac{2}{25} = 0.08$

$\boxed{X=3}$ (5,8) (8,5) $= \frac{2}{25} = 0.08$

$\boxed{X=4}$ (4,8) (8,4) (5,9) (9,5) $= \frac{4}{25} = 0.16$

$\boxed{X=5}$ (9,4) (4,9) (3,8) (8,3) $= \frac{4}{25} = 0.16$

DRAW

	DRAW Again	STEVE WINS.			JUDY WINS.		
X	0	1	2	3	4	5	6
P(X)	0.2	0.24	0.08	0.08	0.16	0.16	0.08
x·P(x)	0	0.24	0.16	0.24	0.64	0.8	0.48

E(X) = mean
2.56

Draw = 0.2
Steve = 0.4
Judy = 0.4

P(Judy wins second game) = DJ = (0.2)(0.4) = 0.08
third game = DDJ = D^2J
FOURTH Game = DDDJ = D^3J

n^{th} game = $D^{n-1}J$
$= \boxed{(0.2)^{n-1}(0.4)}$

31 The discrete random variable X has the following probability distribution.

x	-3	0	2	4
$\mathrm{P}(X = x)$	p	q	r	0.4

Given that $\mathrm{E}(X) = 2.3$ and $\mathrm{Var}(X) = 3.01$, find the values of p, q and r. [6]

x^2	9	0	4	16	
x	-3	0	2	4	
P(X)	p	q	r	0.4	$\Sigma p(x)=1$
$x\cdot P(x)$	$-3p$	0	$2r$	1.6	E(X) = 2.3

$\mathrm{Var} = E(x^2) - [E(x)]^2$
$3.01 = E(x^2) - (2.3)^2$
$E(x^2) = 8.3$

$x^2 \cdot P(x)$	$9p$	0	$4r$	6.4	$E(x^2) = 8.3$

$p+q+r+0.4=1$, $-3p+2r+1.6=2.3$, $9p+4r+6.4=8.3$

$\boxed{p+q+r=0.6}$ $\boxed{2r-3p=0.7}$ $\boxed{4r+9p=1.9}$

$q=\boxed{}$

$p=\boxed{}$, $r=\boxed{}$

15 A small farm has 5 ducks and 2 geese. Four of these birds are to be chosen at random. The random variable X represents the number of geese chosen.

(i) Draw up the probability distribution of X. [3]

(ii) Show that $E(X) = \frac{8}{7}$ and calculate $Var(X)$. [3]

(iii) When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]

X = Number of geese.

X	0	1	2
$P(x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$

$\boxed{X=0}$ No geese DDDD $= \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} = \frac{1}{7}$

(5D
2G)

$\boxed{X=1}$

$$\left.\begin{aligned} GDDD &= \frac{2}{7} \times \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \\ DGDD &= \frac{5}{7} \times \frac{2}{6} \times \frac{4}{5} \times \frac{3}{4} \\ DDGD &= \frac{5}{7} \times \frac{4}{6} \times \frac{2}{5} \times \frac{3}{4} \\ DDDG &= \frac{5}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{2}{4} \end{aligned}\right\} = 4 \times \left(\frac{2}{7} \times \frac{5}{6} \times \frac{4}{5} \times \frac{3}{4} \right)$$

$$= \frac{4}{7}$$

$\boxed{X=2}$

$$\left.\begin{matrix} DDGG & GGDD \\ DGDG & GDGD \\ GDDG & DGGD \end{matrix}\right\} = 6 \times \left(\frac{5}{7} \times \frac{4}{6} \times \frac{2}{5} \times \frac{1}{4} \right)$$

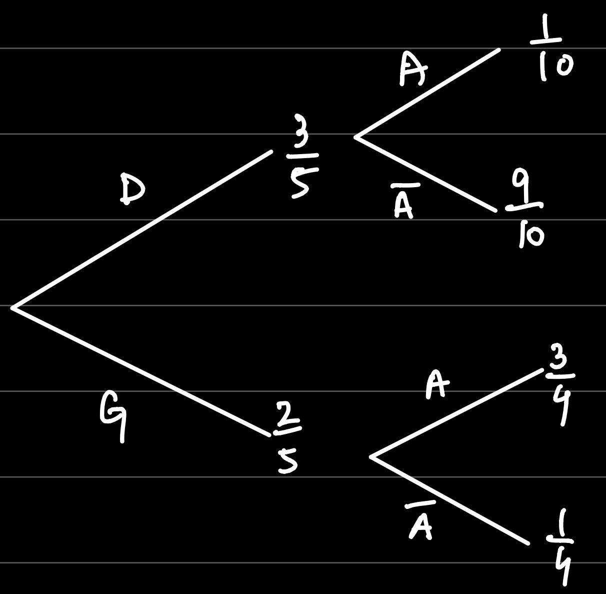
$$= \frac{2}{7}$$

x^2	0	1	4	
X	0	1	2	
$P(x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$	
$x \cdot p(x)$	0	$\frac{4}{7}$	$\frac{4}{7}$	$E(x) = \frac{8}{7}$
$x^2\, p(x)$	0	$\frac{4}{7}$	$\frac{8}{7}$	$E(x^2) = \frac{12}{7}$

Variance $= E(x^2) - [E(x)]^2$

$$= \frac{12}{7} - \left(\frac{8}{7}\right)^2 = \frac{20}{49}$$

(iii) When the farmer's dog is let loose, it chases either the ducks with probability $\frac{3}{5}$ or the geese with probability $\frac{2}{5}$. If the dog chases the ducks there is a probability of $\frac{1}{10}$ that they will attack the dog. If the dog chases the geese there is a probability of $\frac{3}{4}$ that they will attack the dog. Given that the dog is not attacked, find the probability that it was chasing the geese. [4]



$$P(G \mid \bar{A}) = \frac{P(G \text{ and } \bar{A})}{P(\bar{A})}$$

$$= \frac{\frac{2}{5} \times \frac{1}{4}}{\left(\frac{3}{5} \times \frac{9}{10}\right) + \left(\frac{2}{5} \times \frac{1}{4}\right)}$$

$$= \frac{5}{32}$$