TRIGONOMETRY

(15 - 20 Marks)

ANGLE

UNITS

- Degrees
- RADIANS

$180^\circ = \pi$ rad

$360^\circ = 2\pi$

$90^\circ = \frac{\pi}{2}$

$45^\circ = \frac{\pi}{4}$

$30^\circ = \frac{\pi}{6}$

$60^\circ = \frac{\pi}{3}$

$180^\circ \longrightarrow \pi$ rad

Convert 110° to radians

$180^\circ \rightarrow \pi$ rad

Convert 4.5 rad to degrees.

$180^\circ \rightarrow \pi$

$110° \times x$	$x \times 4.5$
$180x = 110\pi$	$\pi x = 180 \times 4.5$
$x = \frac{110\pi}{180} = 1.919$ rad	$x = \frac{180 \times 4.5}{\pi}$
	$x = 257.831$

VALUES TO MEMORIZE

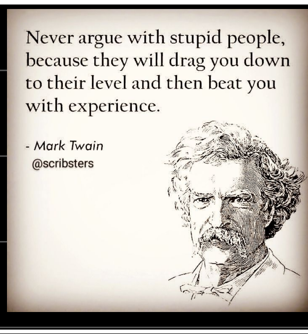
Degrees	0	30	45	60	90
RADIANS	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞ undefined

Check to see if your answer is required in exact form. In a trigonometry question you will need to use exact values of sin 60°, for example, to obtain an exact answer. Make sure you know the exact values of sin, cos and tan of 30°, 45° and 60° as they are not provided in the examination.

"EXACT ANSWER"

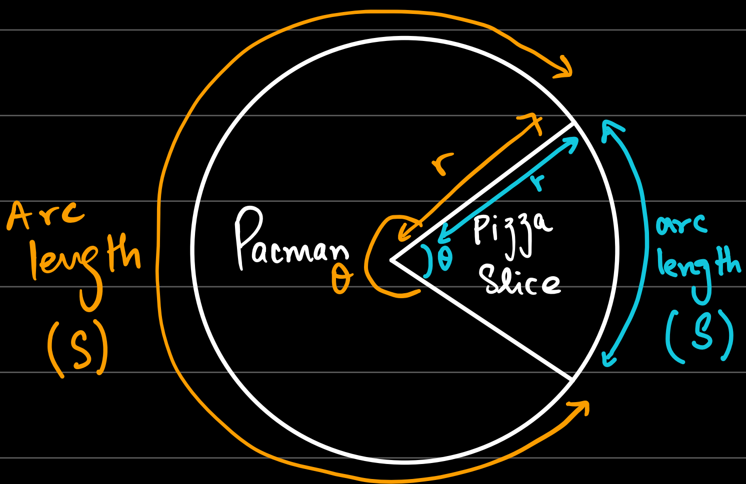
DO NOT USE CALCULATOR IN THIS QUESTION.

Some questions ask for answers in exact form. In these questions you must **not** use your calculator to evaluate answers and you must show the steps in your working. Exact answers may include fractions or square roots and you should simplify them as far as possible.



CIRCULAR MEASURE

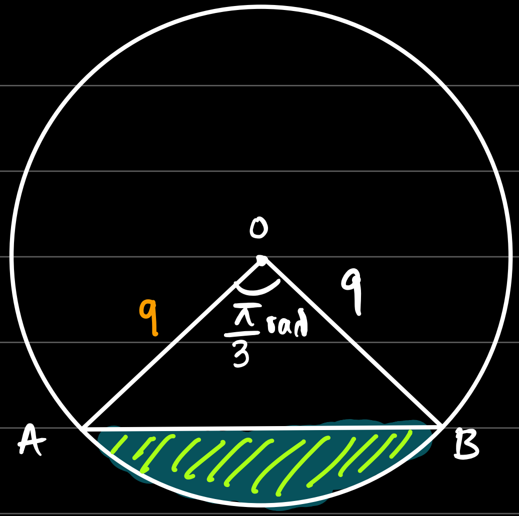
SECTORS



	Angle in Degrees (θ)	Angle in Radian (θ)
ARC LENGTH	Arc length $= \frac{\theta^\circ}{360} \times 2\pi r$	$s = r\theta$ (s → Arc length, r → radius, θ → angle in radians.)
AREA OF SECTOR	Area of Sector $= \frac{\theta^\circ}{360} \times \pi r^2$	$A = \frac{1}{2} r s$ ($s = r\theta$)

$$A = \frac{1}{2} r^2 \theta$$

Q:



Find the area of shaded region.

SHADED = SECTOR − TRIANGLE

$$\left[\frac{1}{2}r^2\theta\right] - \left[\frac{1}{2}\square\,\square\sin\bigcirc\right]$$

$$= \left[\frac{1}{2}(9)^2\left(\frac{\pi}{3}\right)\right] - \left[\frac{1}{2}\,\boxed{9}\,\boxed{9}\sin\left(\pi/3\right)\right]$$

mode change

$$= 42.4115 - 35.074$$

$$= 7.33747$$

ONLY CHANGE CALCULATOR MODE IF YOU ARE USING FOLLOWING

sin ☐	$\sin^{-1}$ ☐
cos ☐	$\cos^{-1}$ ☐
tan ☐	$\tan^{-1}$ ☐

and angle is in Radian.

(ES) SHIFT MODE 4

(EX) SHIFT MENU ANGLEUNITS

(MS) MODE MODE

OL TRIG
- Pythagoras
- sin/cos/tan
- Sine rule/cos rule.
- Area of triangle = $\frac{1}{2}bh = \frac{1}{2}$ ☐☐ $\sin\theta$ (☐☐ → neighboring sides; θ → Angle)

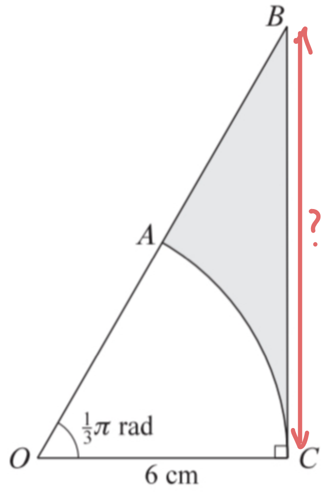
2

SHADED = TRIANGLE − SECTOR

$= \frac{1}{2}(OC)(CB) - \frac{1}{2}r^2\theta$

$= \frac{1}{2}(6)(6\sqrt{3}) - \frac{1}{2}(6)^2\left(\frac{\pi}{3}\right)$

SHADED = $18\sqrt{3} - 6\pi$



$\tan\left(\frac{1}{3}\pi\right) = \frac{BC}{6}$

$\sqrt{3} = \frac{BC}{6}$

$BC = 6\sqrt{3}$

In the diagram, AC is an arc of a circle, centre O and radius 6 cm. The line BC is perpendicular to OC and OAB is a straight line. Angle $AOC = \frac{1}{3}\pi$ radians. Find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [5]

exact form (no calculator)

6

$\sin\frac{\pi}{3} = \frac{OC}{12}$

$\frac{\sqrt{3}}{2} = \frac{OC}{12}$

$OC = \frac{\sqrt{3}}{2} \times 12$

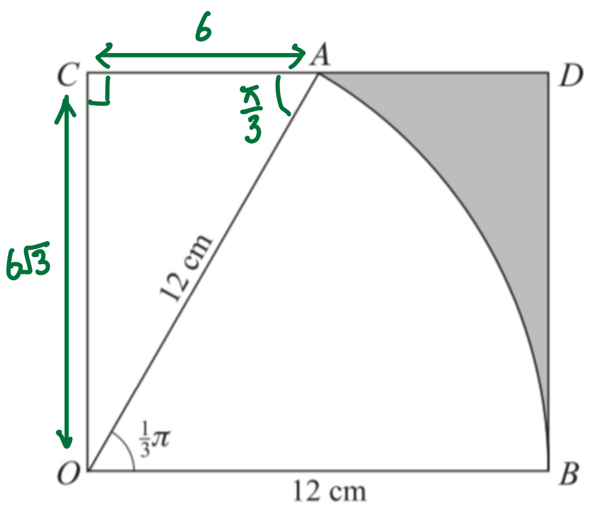
$OC = 6\sqrt{3}$

$\cos\frac{\pi}{3} = \frac{AC}{12}$

$\frac{1}{2} = \frac{AC}{12}$

$AC = \frac{1}{2} \times 12$

$AC = 6$



In the diagram, AOB is a sector of a circle with centre O and radius 12 cm. The point A lies on the side CD of the rectangle $OCDB$. Angle $AOB = \frac{1}{3}\pi$ radians. Express the area of the shaded region in the form $a(\sqrt{3}) - b\pi$ stating the values of the integers a and b. [6]

exact form

SHADED AREA = RECTANGLE − TRIANGLE − SECTOR

$= (OB)(OC) - \frac{1}{2}(OC)(AC) - \frac{1}{2}r^2\theta$

$= (12)(6\sqrt{3}) - \frac{1}{2}(6\sqrt{3})(6) - \frac{1}{2}(12)^2\left(\frac{\pi}{3}\right)$

$= 72\sqrt{3} - 18\sqrt{3} - 24\pi$

$= 54\sqrt{3} - 24\pi$

$a\sqrt{3} - b\pi$

$a = 54$, $-b = -24$

$b = 24$

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$OT^2 = 5^2 + 12^2$

$OT^2 = 169$

$OT = 13$

$\tan\theta = \frac{12}{5}$

$\theta = \tan^{-1}\left(\frac{12}{5}\right)$

$\theta = 1.17$

Perimeter:

$PT = 12$

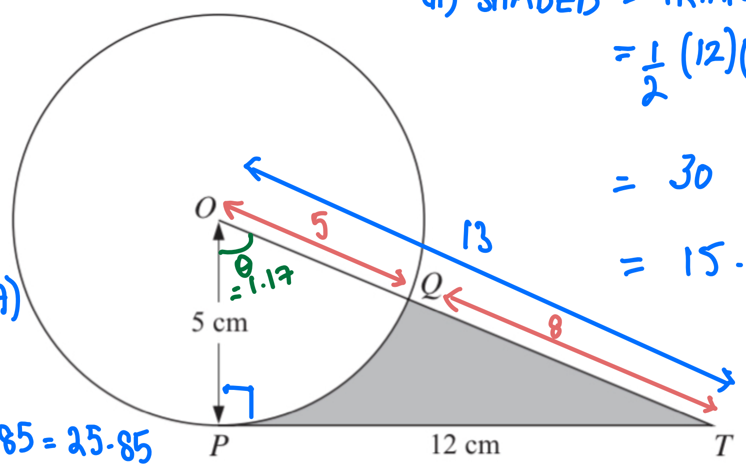
$QT = 8$

Arc $PQ = r\theta$

$= 5(1.17)$

$= 5.85$

$P = 12 + 8 + 5.85 = 25.85$



(ii) SHADED = TRIANGLE − SECTOR

$= \frac{1}{2}(12)(5) - \frac{1}{2}(5)^2(1.17)$

$= 30 - 14.625$

$= 15.375.$

The diagram shows a circle with centre O and radius 5 cm. The point P lies on the circle, PT is a tangent to the circle and $PT = 12$ cm. The line OT cuts the circle at the point Q.

(i) Find the perimeter of the shaded region. [4]

(ii) Find the area of the shaded region. [3]

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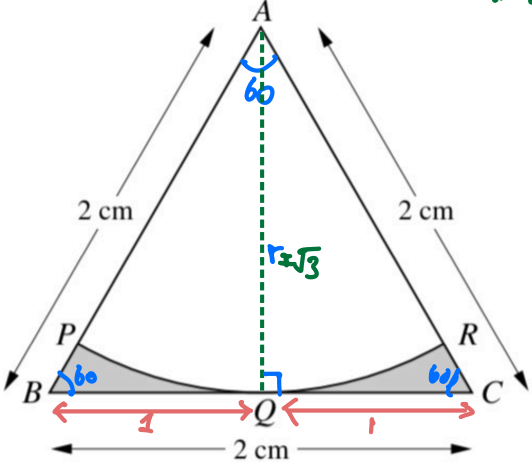
Pythagoras

$2^2 = 1^2 + r^2$

$4 = 1 + r^2$

$r^2 = 3$

$r = \sqrt{3}$



SHADED = TRIANGLE − SECTOR

$= \frac{1}{2}(2)(\sqrt{3}) - \left[\frac{60}{360} \times \pi(\sqrt{3})^2\right]$

$= \sqrt{3} - \frac{1}{6}\pi(3)$

$= \boxed{\sqrt{3} - \frac{\pi}{2}}$

In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q. An arc of a circle with centre A touches BC at Q, and meets AB at P and AC at R. Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

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$6^2 + x^2 = 10^2$

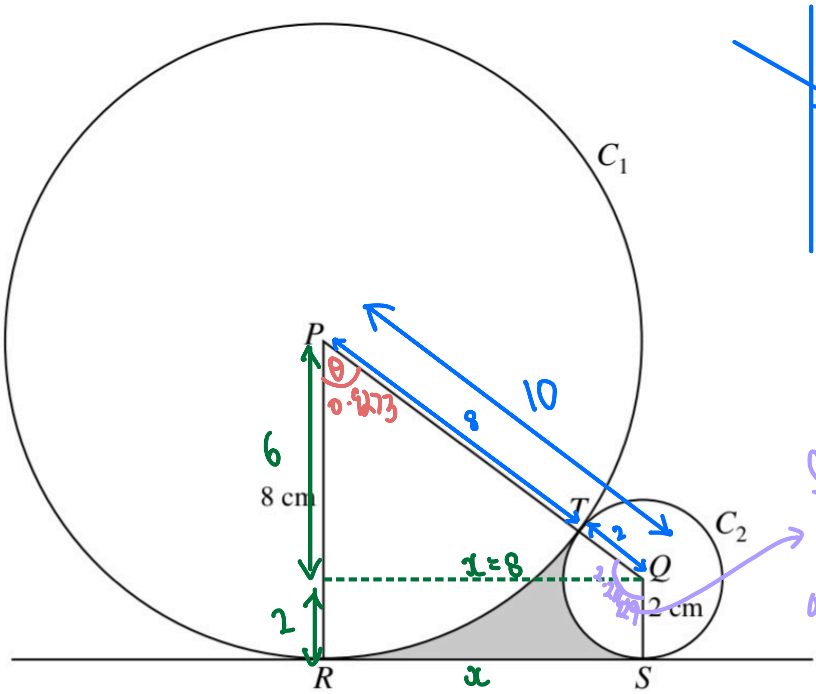
$36 + x^2 = 100$

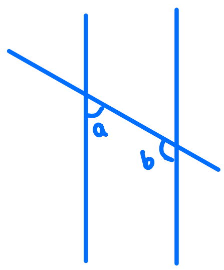
$x^2 = 64$

$x = 8$

$\tan\theta = \frac{8}{6}$

$\theta = \tan^{-1}\left(\frac{8}{6}\right) = 0.9273$





$a + b = 180$

$a + b = \pi$

Supplementary

$P + Q = \pi$

$0.9273 + Q = \pi$

$Q = 2.214297$

The diagram shows two circles, C_1 and C_2, touching at the point T. Circle C_1 has centre P and radius 8 cm; circle C_2 has centre Q and radius 2 cm. Points R and S lie on C_1 and C_2 respectively, and RS is a tangent to both circles.

(i) Show that $RS = 8$ cm. [2]

(ii) Find angle RPQ in radians correct to 4 significant figures. [2]

(iii) Find the area of the shaded region. [4]

SHADED AREA = TRAPEZIUM − BIG SECTOR − SMALL SECTOR

$= \frac{1}{2}(8)(2+8) - \frac{1}{2}(8)^2(0.9273) - \frac{1}{2}(2)^2(2.214297)$

$=$

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$BC^2 = r^2 + r^2$

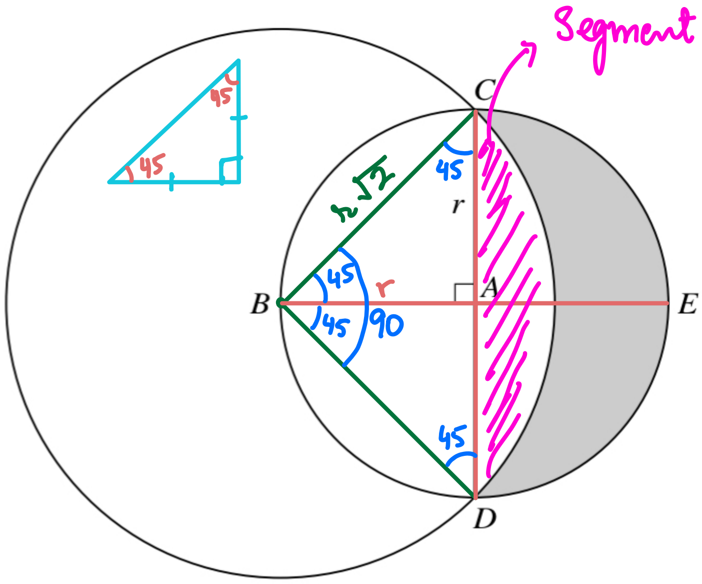
$BC^2 = 2r^2$

$BC = \sqrt{2r^2}$

$= \sqrt{2}\sqrt{r^2}$

$= \sqrt{2}\, r$

$BC = r\sqrt{2}$



HARD

The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(i) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

(ii) Find the area of the shaded region in terms of r. [6]

SHADED REGION = SMALL SEMICIRCLE − SEGMENT MINOR

SHADED REGION = SMALL SEMICIRCLE − [PIZZA − TRIANGLE]

$= \frac{\pi r^2}{2} - \left[\frac{90}{360} \times \pi (r\sqrt{2})^2 - \frac{1}{2}(2r)(r)\right]$

$= \frac{\pi r^2}{2} - \left[\frac{1}{4} \times \pi (r^2 \times 2) - r^2\right]$

$= \frac{\pi r^2}{2} - \left[\frac{\pi r^2}{2} - r^2\right]$

$= \frac{\pi r^2}{2} - \frac{\pi r^2}{2} + r^2 = \boxed{r^2}$

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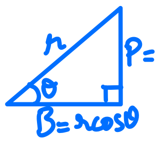
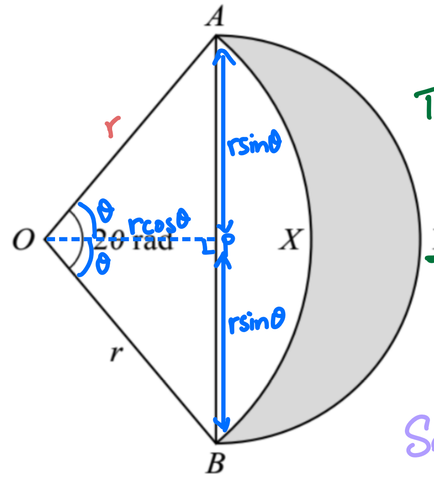


$\sin\theta = \frac{P}{r}$ | $\cos\theta = \frac{B}{r}$

$P = r\sin\theta$ | $B = r\cos\theta$



Semicircle $= \frac{\pi (r\sin\theta)^2}{2}$

Triangle OAB $= \frac{1}{2} \times b \times h$

$= \frac{1}{2} \times 2r\sin\theta \times r\cos\theta$

Triangle $= \frac{1}{2}(r)(r)\sin 2\theta$

Sector $= \frac{1}{2} r^2 (2\theta)$

In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r. Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ, for the area of the shaded region. [4]

SHADED REGION = SMALL SEMICIRCLE − SEGMENT MINOR

SHADED REGION = SMALL SEMICIRCLE − [PIZZA − TRIANGLE]

$$= \frac{\pi (r\sin\theta)^2}{2} - \left[\frac{1}{2} r^2 (2\theta) - \frac{1}{2}(r)(r)\sin 2\theta\right]$$

$$= \frac{\pi r^2 \sin^2\theta}{2} - \frac{r^2 2\theta}{2} + \frac{r^2 \sin 2\theta}{2}$$

$(\sin\theta)^2$

$\sin^2\theta$

$$= \frac{r^2}{2}\left[\pi \sin^2\theta - 2\theta + \sin 2\theta\right]$$

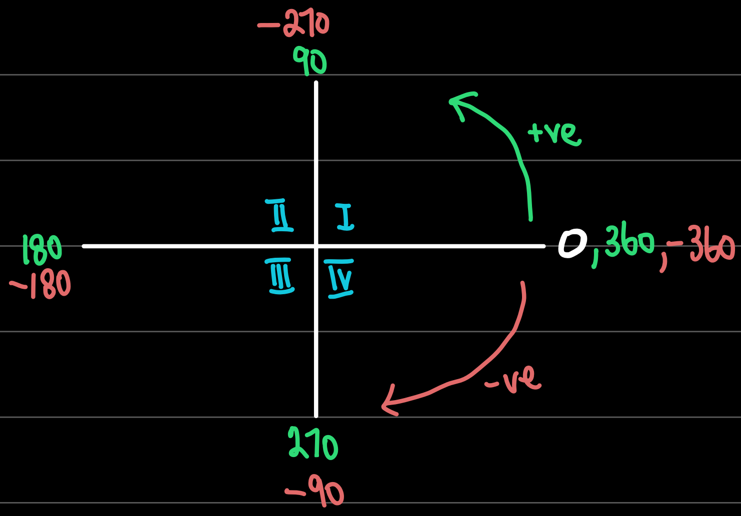
OL: 1) Pythagoras

2) sin/cos/tan

3) Sine Rule / Cos Rule

4) Area of Triangle $= \frac{1}{2} bh = \frac{1}{2} \square \square \sin \bigcirc$

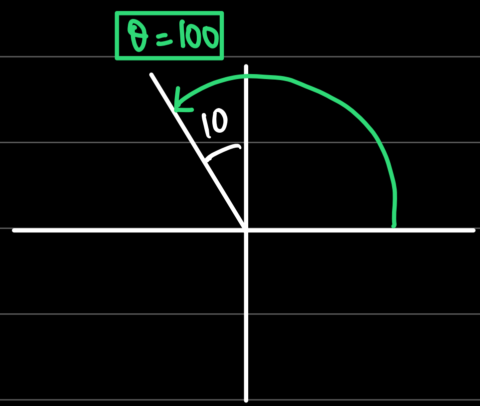
TRIGONOMETRY

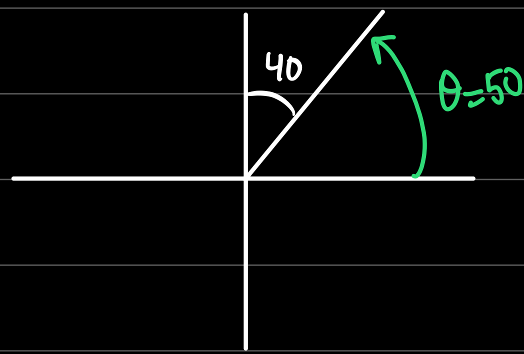


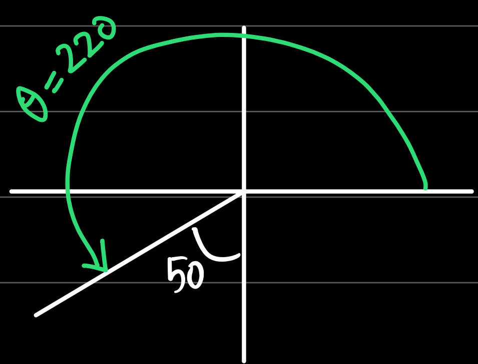
Sin +ve cos } -ve tan }	All +ve
Tan +ve sin } -ve cos }	Cos +ve sin } -ve tan }

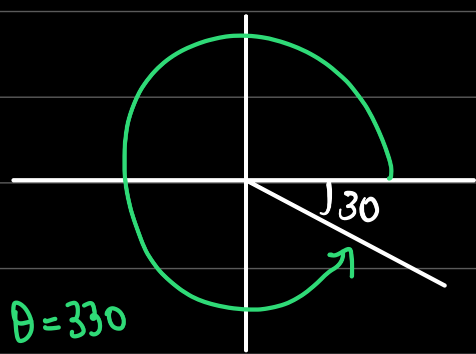
Aa Sajna Tur Chaliye

All Science Teachers Crazy





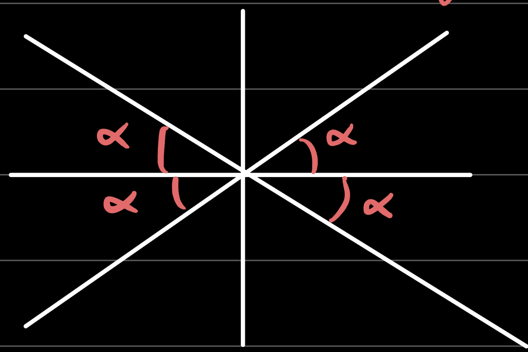


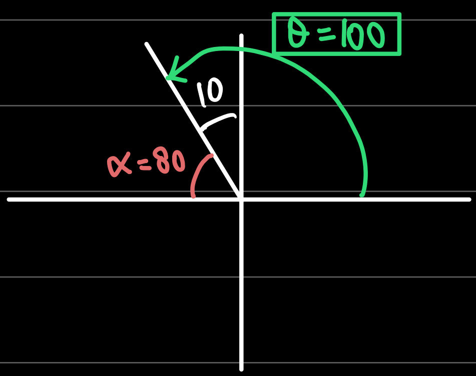


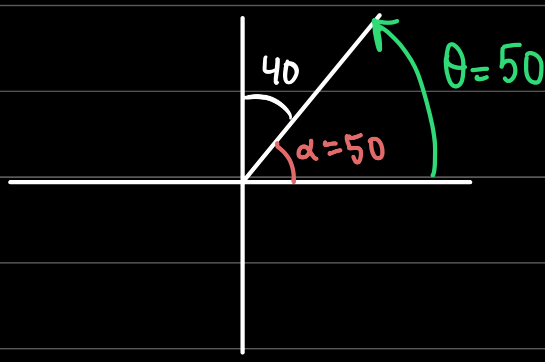
THERE ARE TWO MAIN VALUES FOR EACH ANGLE

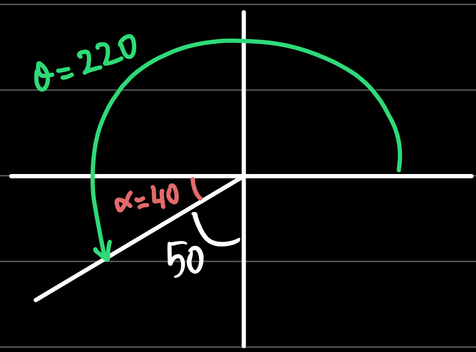
θ Value = original angle = Starts from Zero and go ACW. for +ve angles, CW for −ve angles.

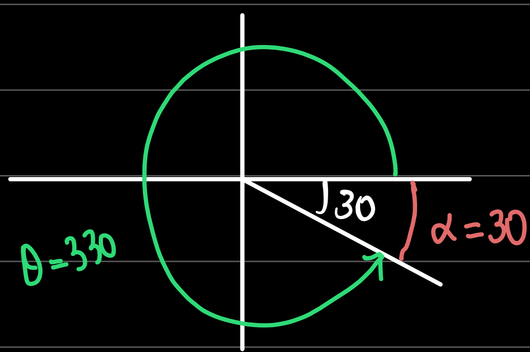
α Value = Basic angle = Acute angle with x-axis. (Basic angle is always +ve)







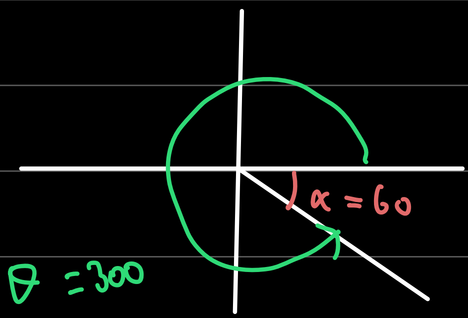




TRIG RATIO OF ANY ANGLE (θ) [sin/cos/tan] = TRIG RATIO OF ITS BASIC ANGLE (α) [sin/cos/tan] AFTER ADJUSTING +/− QUADRANT SIGNS

Q: Without using calculator evaluate:

(a) $\sin 300^\circ$



$\sin 300$

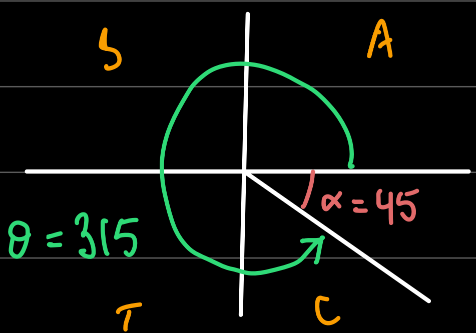
↓

$\sin 60 = \dfrac{\sqrt{3}}{2}$

↓

$\boxed{\sin 300 = -\dfrac{\sqrt{3}}{2}}$

2) $\cos 315^\circ$



$\cos 315$

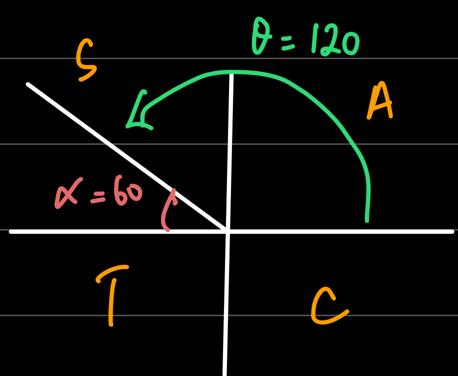
↓

$\cos 45 = \dfrac{1}{\sqrt{2}}$

↓

$\boxed{\cos 315 = +\dfrac{1}{\sqrt{2}}}$

3) $\tan 120^\circ$



$\tan 120$

↓

$\tan 60 = \sqrt{3}$

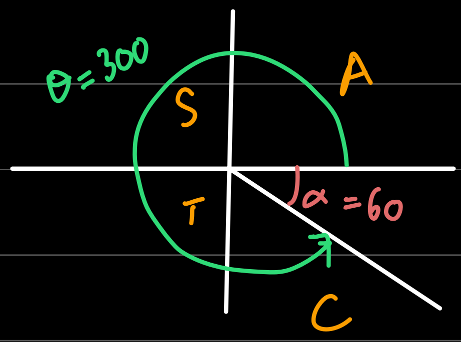
↓

$\boxed{\tan 120 = -\sqrt{3}}$

4) $\cos\left(\frac{5\pi}{3}\right)$

$5 \times 60 = 300$

$\cos 300$



$\cos 300$

$\downarrow$

$\cos 60 = \frac{1}{2}$

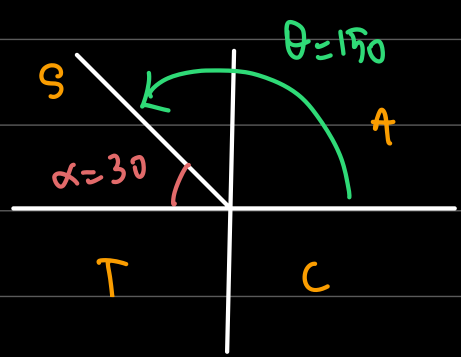
$\downarrow$

$\cos 300 = +\frac{1}{2}$

5) $\tan\left(\frac{5\pi}{6}\right)$

$5 \times 30 = 150$

$\tan 150$



$\tan 150$

$\downarrow$

$\tan 30 = \frac{1}{\sqrt{3}}$

$\downarrow$

$\tan 150 = -\frac{1}{\sqrt{3}}$

EQUATION SOLVING

① permission to take inverse

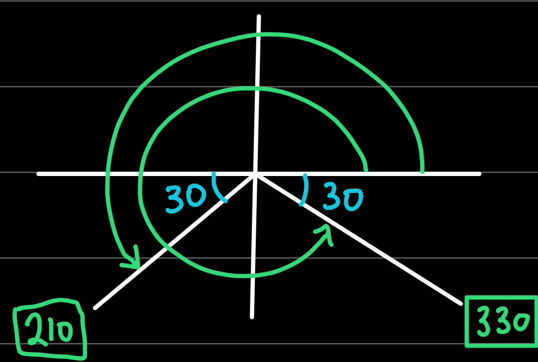
1 $\sin x = -\frac{1}{2}$ $\qquad 0 < x < 360$

Quadrant

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

③ Basic angle

② ignore all +/- signs while taking inverse



$x = 210, 330$

① permission to take inverse.

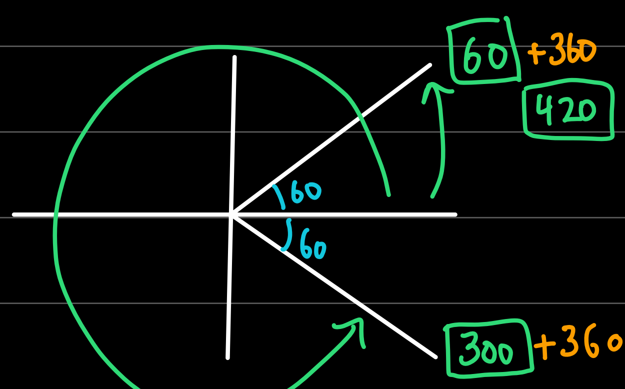
2 $\cos x = +\frac{1}{2}$ $\qquad 0 < x < 720$

Quadrants

$\alpha = \cos^{-1}\left(\frac{1}{2}\right)$

$\alpha = 60$

$x = 60, 300, 420, 660$

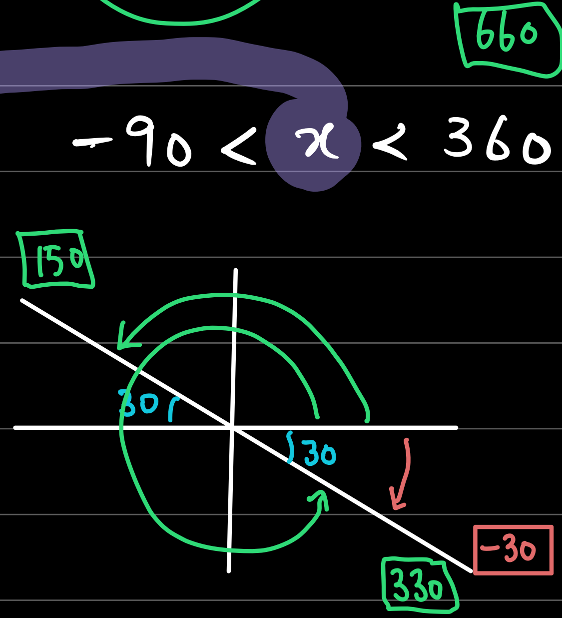


3 $\tan x = -\frac{1}{\sqrt{3}}$ $-90 < x < 360$

permission of inverse

Quad.

$\alpha = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30$



$x = -30, 150, 330$

v.imp

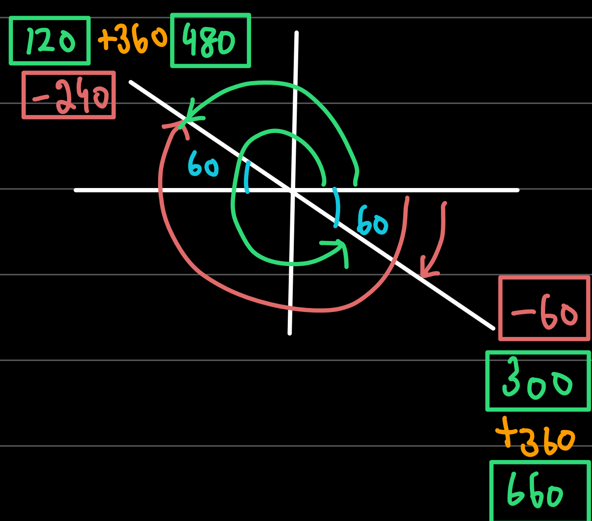
Rule: Whenever range is available in negative always find the negative angles first by going CW.

Q. $\tan\theta = -\sqrt{3}$ $-360 < \theta < 720$

permission for inverse.

$\alpha = \tan^{-1}(\sqrt{3})$

$\alpha = 60^\circ$



$x = -240, -60, 120, 300, 480, 660$

IF THERE IS NO PERMISSION FOR INVERSE

RANGE CHANGE

→ Super imp for P1 / P3.

NO PERMISSION FOR INVERSE

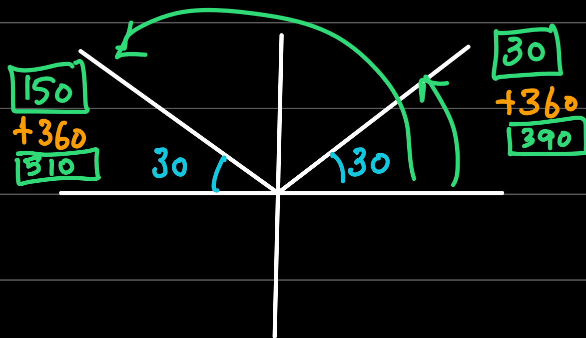
1) $\sin(2x) = \frac{1}{2}$ $\qquad 0 < x < 360$

$\times 2$

$2x = A$ $\qquad 0 < 2x < 720$

$\sin A = \oplus \frac{1}{2}$ $\qquad 0 < A < 720$

$\alpha = \sin^{-1}\left(\frac{1}{2}\right) = 30$



$A = 30, 150, 390, 510$

$2x = 30, 150, 390, 510$

$x = 15, 75, 195, 255$

2) $\cos(2x + 70) = \frac{1}{2}$ $\qquad 0 < x < 180$

$\times 2$

$0 < 2x < 360$

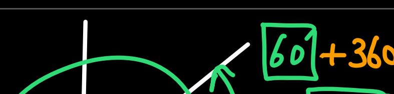
$+70$

$2x + 70 = A$ $\qquad 70 < 2x+70 < 430$

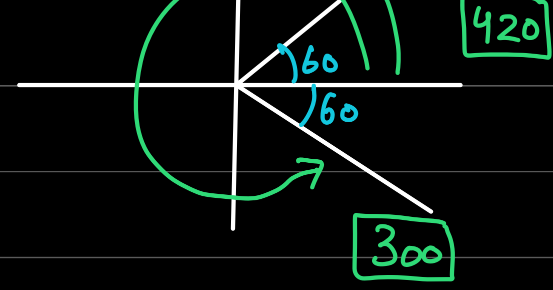
$\cos A = \oplus \frac{1}{2}$ $\qquad 70 < A < 430$

Quad

$\alpha = \cos^{-1}\left(\frac{1}{2}\right) = 60$



(2)



out of range

$A = \cancel{60},\ 300,\ 420$

$2x + 70 = 300,\ 420$

$2x = 230,\ 350$

$\boxed{x = 115,\ 175}$

3) $\cos(2x - 80) = \dfrac{\sqrt{3}}{2}$ $\qquad 0 < x < 180$

$\times 2$

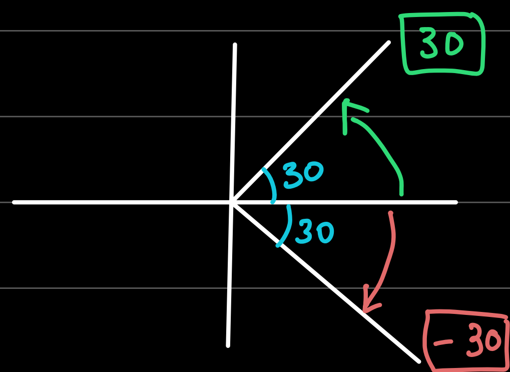
$0 < 2x < 360$

-80

$2x - 80 = A \qquad -80 < 2x - 80 < 280$

$\cos A = \dfrac{\sqrt{3}}{2} \qquad -80 < A < 280$

$\alpha = \cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$



$\alpha = 30$

$A = -30,\ 30$

$2x - 80 = -30,\ 30$

$2x = 50,\ 110$

$x = 25,\ 55$

IDENTITIES (17 + 8 = 25)

(P1 + P3)

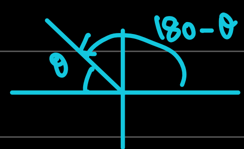
RECIPROCAL
$\frac{1}{\sin x} = \text{cosec}\,x$
$\frac{1}{\cos x} = \sec x$
$\frac{1}{\tan x} = \cot x$

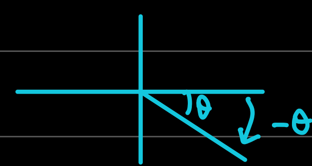
$\tan x = \frac{\sin x}{\cos x}$	$\cot x = \frac{\cos x}{\sin x}$

$$\sin^2 x + \cos^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$1 + \cot^2 x = \text{cosec}^2 x$$

$(\sin x)^2 \downarrow \sin^2 x$ | $\sin(x^2) \downarrow \sin x^2$

$$\sin^2 x = (\sin x)^2$$
$$\sin x^2 = \sin(x^2)$$





Second Quadrant

$$\sin(180-\theta) = \sin\theta$$
$$\cos(180-\theta) = -\cos\theta$$
$$\tan(180-\theta) = -\tan\theta$$

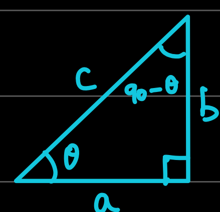
Fourth Quadrant.

$$\sin(-\theta) = -\sin\theta$$
$$\cos(-\theta) = \cos\theta$$
$$\tan(-\theta) = -\tan\theta$$

Right angled Triangles

used in M1 forces

$$\sin(90-\theta) = \cos\theta$$
$$\cos(90-\theta) = \sin\theta$$
$$\tan(90-\theta) = \frac{1}{\tan\theta}$$



$\sin(90-\theta) = \frac{a}{c}$, $\cos\theta = \frac{a}{c}$

$\cos(90-\theta) = \frac{b}{c}$, $\sin\theta = \frac{b}{c}$

$\tan(90-\theta) = \frac{a}{b}$, $\tan\theta = \frac{b}{a}$

PROVING IDENTITIES

Rule: You are allowed to solve only one side of identity. You cannot solve both sides and come to a midway proof.

QUESTION

- SQUARES
  - $\sin^2\theta + \cos^2\theta = 1$
  - $1 + \tan^2\theta = \sec^2\theta$
  - $1 + \cot^2\theta = \csc^2\theta$
  - $a^2 - b^2 = (a+b)(a-b)$
- NO SQUARES
  - Bring everything to (sin) and (cos) only and then simplify.

$\equiv$ Identity

1) We cannot shift terms from one side to other while "Proving the identity"
2) This we will discuss in P3.

13 Prove the identity

$$\frac{1+\sin x}{\cos x} + \frac{\cos x}{1+\sin x} \equiv \frac{2}{\cos x}. \quad [4]$$

$$\frac{(1+\sin x)(1+\sin x) + (\cos x)(\cos x)}{\cos x(1+\sin x)}$$

$(\cos x)(\cos x)$

$(\cos x)^2$

$\cos^2 x$

$$\frac{1+\sin x+\sin x+\sin^2 x+\cos^2 x}{\cos x(1+\sin x)}$$

$$1 + 2\sin x + 1$$

$\cos x\,(1+\sin x)$

$$\frac{2+2\sin x}{\cos x\,(1+\sin x)}$$

$$\frac{2\,(1+\sin x)}{\cos x\,(1+\sin x)}$$

$$\frac{2}{\cos x}$$

SHOW THAT

AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

14 Prove the identity $\dfrac{\sin x}{1-\sin x} - \dfrac{\sin x}{1+\sin x} \equiv 2\tan^2 x.$ [3]

LHS

$$\frac{\sin x(1+\sin x) - \sin x\,(1-\sin x)}{(1-\sin x)(1+\sin x)}$$

$$\frac{\sin x + \sin^2 x - \sin x + \sin^2 x}{(1-\sin x)(1+\sin x)}$$

$$\frac{2\sin^2 x}{\cos^2 x}$$

$2\tan^2 x$ (Q.E.D)

$\sin^2 x + \cos^2 x = 1$

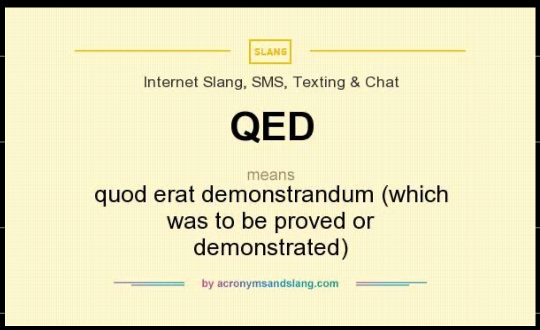
$\cos^2 x = 1 - \sin^2 x$

$(1-\sin x)(1+\sin x)$

$(1)^2 - (\sin x)^2$

$1 - \sin^2 x$

$\cos^2 x$


SLANG
Internet Slang, SMS, Texting & Chat
QED
means
quod erat demonstrandum (which was to be proved or demonstrated)
by acronymsandslang.com

27 **(i)** Prove the identity $\left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)^2 \equiv \frac{1-\cos\theta}{1+\cos\theta}$. [3]

$$\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$\left(\frac{1-\cos\theta}{\sin\theta}\right)^2$$

$$\frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$\frac{(1-\cos\theta)^{\not 2}}{(1+\cos\theta)\cancel{(1-\cos\theta)}}$$

$$\frac{1-\cos\theta}{1+\cos\theta}$$

$\sin^2\theta$

$1 - \cos^2\theta$

$(1)^2 - (\cos\theta)^2$

$(1+\cos\theta)(1-\cos\theta)$