

KINEMATICS

Difference between distance and displacement:

S.No.	Property	Distance	Displacement (s)
1.	Definition	Actual path travelled by a body.	Straight directed distance from the starting point to the ending point.
2.	P.S.	Scalar	Vector and is directed towards ending point.

Conceptual questions

Q.No.	Problem	d/m	s/m
1.		$100 + 70 = 170$	$100 - 70 = 30$ toward C (0°)
2.		$4 + 3 = 7$	$\sqrt{(4)^2 + (3)^2} = 5$ at $\theta = \tan^{-1}(\frac{3}{4}) = 37^\circ$
3.		$10 + 10 = 20$	10 at 60°
4.		$= 1.5 (\text{circumference})$ $= 1.5 (2\pi r)$ $= 3(3.14)(10)$ $= 94.2$	diameter i.e. 20 at 180°

Difference between speed and velocity:

	Speed	Velocity
Def.	Distance travelled per unit time	Displacement per unit time
P.S.	Scalar	Vector
	Uniform speed = $\frac{\text{Equal distance moved}}{\text{Equal time interval}}$	Uniform velocity = $\frac{\text{Equal displacement}}{\text{Equal time interval}}$
	Variable speed = $\frac{\text{Unequal distance moved}}{\text{Equal time interval}}$	Variable velocity = $\frac{\text{Unequal displacement}}{\text{Equal time interval}}$
	Average speed = $\frac{\text{Total distance moved}}{\text{Total time taken}}$	Average velocity = $\frac{\text{Total displacement}}{\text{Total time taken}}$
	Instantaneous speed = Gradient of distance time graph at any instant OR Rate of change of distance	Instantaneous velocity = Gradient of displacement time graph at any instant OR Rate of change of displacement

Note :- If an object
 (i) start from rest then its initial speed/velocity is zero and
 (ii) comes to rest then its final velocity is zero.

Acceleration:-

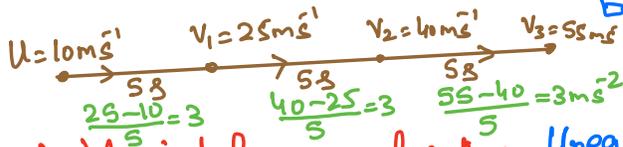
Def: change of velocity per unit time.

Symbol: a

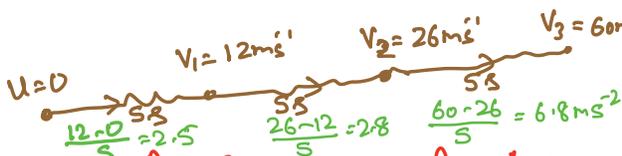
Formula: $a = \frac{\Delta v}{\Delta t} \Rightarrow a = \frac{v-u}{t}$

P.S.: Vector

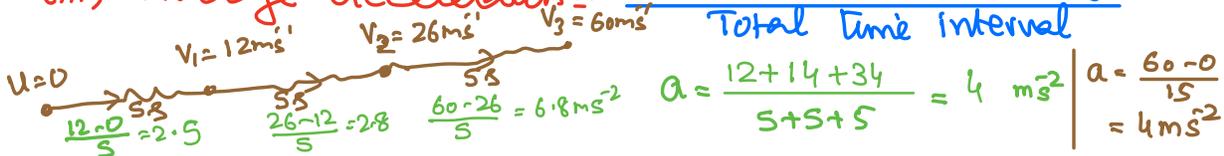
(i) Uniform acceleration = $\frac{\text{Equal change of velocity}}{\text{Equal time interval}}$



(ii) Variable acceleration = $\frac{\text{Unequal change of velocity}}{\text{Equal time interval}}$



(iii) Average acceleration = $\frac{\text{Total change of velocity}}{\text{Total time interval}}$



(iv) Instantaneous Acceleration = Gradient of velocity time graph at any instant.

Rate of change ^{OR} of velocity

Time derivative ^{OR} of velocity.

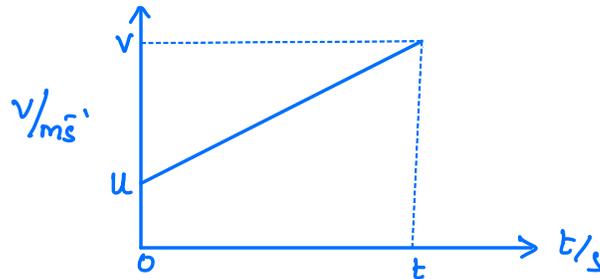
Note :-

	Situation	Velocity	Acceleration
1.	object starts from rest	$u = 0$	$a \neq 0$ as resultant force brings it into motion
2.	object moves with uniform velocity	$v = \text{Constant}$	Zero as $v = u$ by $a = \frac{v-u}{t}$
3.	object comes to rest	$v = 0$	$a \neq 0$ if it approaches to rest and $a = 0$ if it comes to rest.
4.	If $v > u$		Accelerate
5.	If $v < u$		Retard/Decelerate

Note: Decreasing acceleration is not known as retardation or deceleration.

Proof of equation of motion:-

Suppose an object move with an initial velocity u . After time t , its velocity becomes ' v ' and travel a displacement s with uniform acceleration a .



First eq.: $v = u + at$

Change of velocity = $v - u$
Rate of " " " = $\frac{v - u}{t}$
By definition of acceleration
 $a = \frac{v - u}{t}$
 $at = v - u$

Acceleration = Gradient of graph

$$a = \frac{v - u}{t - 0}$$

$$at = v - u$$

$$\boxed{v = u + at}$$

Second eq.: $s = ut + \frac{1}{2}at^2$

Average velocity = $\frac{\text{Total displacement}}{\text{Total time}}$

$$\frac{v + u}{2} = \frac{s}{t}$$

$$s = \frac{(v + u)t}{2}$$

To eliminate final velocity v , use first eq.

$$s = \left(\frac{u + at + u}{2} \right) t$$

$$s = \frac{2ut}{2} + \frac{at^2}{2} \Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$$

Displacement = Area under graph

$$s = \frac{(u + v)t}{2}$$

3rd eq. of motion: $2as = v^2 - u^2$

Average Velocity = $\frac{\text{Total displacement}}{\text{Total time}}$ | Displacement = Area under graph

$$\frac{v+u}{2} = \frac{s}{t}$$

$$s = \left(\frac{v+u}{2}\right)t$$

To eliminate time, t , use first eq. of motion $v = u + at$
 $t = \frac{v-u}{a}$

$$s = \frac{(u+v)t}{2}$$

$$s = \left(\frac{v+u}{2}\right)\left(\frac{v-u}{a}\right) \Rightarrow \boxed{2as = v^2 - u^2}$$

4th eq. of motion: $s = vt - \frac{1}{2}at^2$

Average Velocity = $\frac{\text{Total displacement}}{\text{Total time}}$ | Displacement = Area under graph

$$\frac{v+u}{2} = \frac{s}{t}$$

$$s = \left(\frac{v+u}{2}\right)t$$

To eliminate initial velocity u , use first eq. $v = u + at$
 $u = v - at$

$$s = \frac{(u+v)t}{2}$$

$$s = \left(\frac{v+v-at}{2}\right)t$$

$$s = \frac{2vt}{2} - \frac{at^2}{2} \Rightarrow \boxed{s = vt - \frac{1}{2}at^2}$$

Acceleration due to gravity:

All objects irrespective of their masses fall freely due to gravitational pull of Earth and move with a uniform acceleration, also known as acceleration due to Gravity.

Symbol: g

Value: 9.81 m s^{-2}

Sign:

Here ($v < u$)

$$v = 0$$
$$a = \frac{v - u}{t}$$
$$a = -g = -9.81 \text{ m s}^{-2}$$

$u = \text{Max}$

upward motion

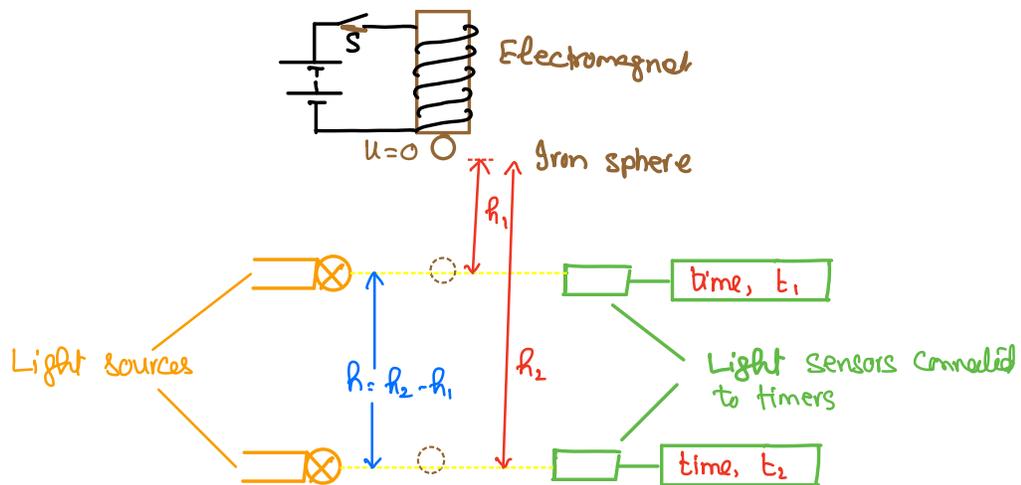
Here ($v > u$)

$$u = 0$$
$$a = \frac{v - u}{t}$$
$$a = +g = +9.81 \text{ m s}^{-2}$$

$v = \text{Max}$

Downward motion

Experiment to determine g:-



For r_1

$$S = ut + \frac{1}{2} at^2$$

$$r_1 = (0)(t_1) + \frac{1}{2}(a)(t_1^2)$$

$$r_1 = \frac{a t_1^2}{2} \quad \text{--- (1)}$$

Subtract (1) from (2) to eliminate systematic error

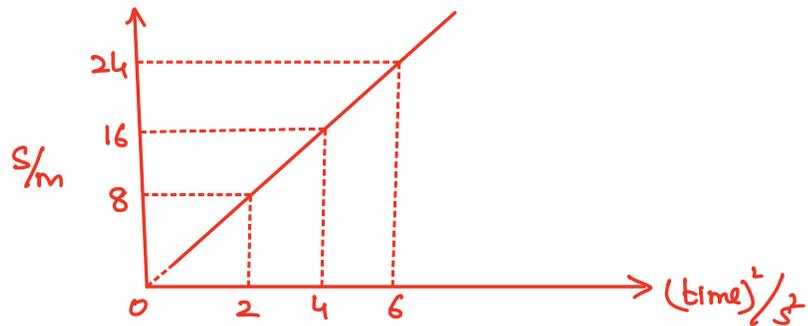
if any

$$r_2 - r_1 = \frac{a}{2} (t_2^2 - t_1^2)$$

$$a = \frac{2(r_2 - r_1)}{t_2^2 - t_1^2} \Rightarrow a = g = \frac{2r}{t_2^2 - t_1^2}$$

Note: Acceleration = $2 \left(\begin{array}{l} \text{Gradient of distance against (time)}^2 \text{ graph} \\ \text{if object starts from rest} \end{array} \right)$

Q) The displacement against $(\text{time})^2$ of an object initially started from rest is shown.



(a) Calculate gradient of graph. [1]

$$\text{gradient} = \frac{24-0}{6-0} = 4.0 \text{ m s}^{-2}$$

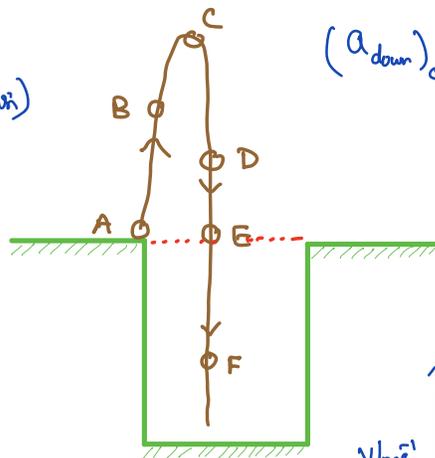
(b) Calculate acceleration of object [2]

$$\begin{aligned} \text{acceleration} &= 2 (\text{Gradient}) \\ &= 2 (4.0) \\ &= 8.0 \text{ m s}^{-2} \end{aligned}$$

Representation of vector quantities in Kinematics' graph

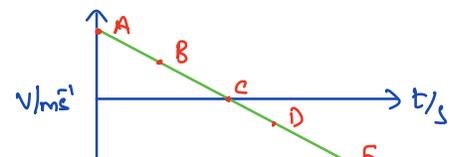
Example.

$$\begin{aligned} (a_{\text{up}})_c &= \frac{(v-u)}{t} \text{ (Direction)} \\ &= (-ve)(+ve) \\ &= -ve \\ &= -9.81 \text{ m s}^{-2} \end{aligned}$$



$$\begin{aligned} (a_{\text{down}})_c &= \frac{(v-u)}{t} \text{ (Direction)} \\ &= (+ve)(-ve) \\ &= -ve \\ &= -9.81 \text{ m s}^{-2} \end{aligned}$$

So acceleration at c is -9.81 m s^{-2}



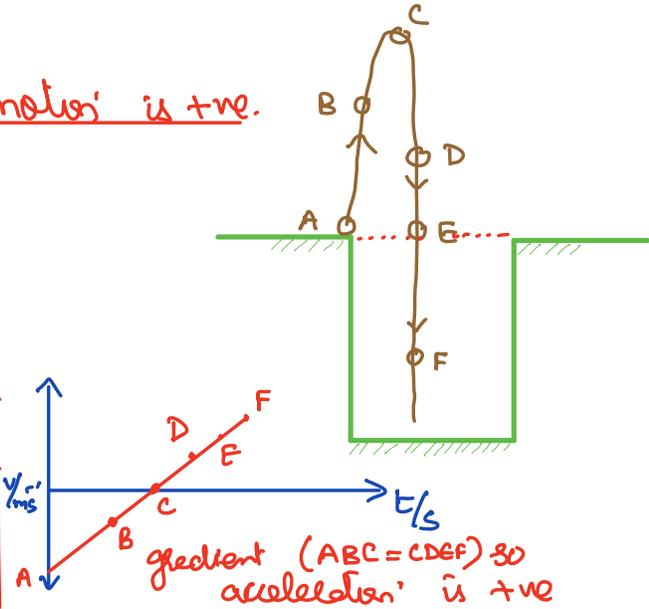
Assumption: upward motion is +ve



s.no.	Physical Qty	A	B	C	D	E	F	Result
1.	Displacement	Zero —	Increase upward	Max upward	Decrease upward	Zero —	Increase Downward	Displacement is always taken from a reference position/point and is independent of assumption.
2.	Velocity	Max +ve	Decrease +ve	Zero —	Increase -ve	Max ($V_E = V_A$) -ve	Max ($V_F > V_A$) -ve	velocity under assumption is +ve and against assumption is -ve.
3.	Acceleration $a = (\text{Mag})(\text{Dir})$ $a = \left(\frac{V-U}{T}\right)(\text{Assumption})$	Max (-ve)(+ve) -ve	(-ve)(+ve) = -ve = -9.81 m/s^2	-ve = -9.81 m/s^2	(+ve)(-ve) = -ve = -9.81 m/s^2	(+ve)(-ve) = -ve = -9.81 m/s^2	(+ve)(-ve) = -ve = -9.81 m/s^2	Both magnitude and acceleration as per assumption are considered.

Assumption: Downward motion is +ve.

Displacement	velocity	Acceleration
No change as displacement is independent of assumption	Max -ve	(-ve)(-ve) = +ve = 9.81 m/s^2
	decrease -ve	(-ve)(-ve) = +ve = 9.81 m/s^2
	Zero —	+ve = 9.81 m/s^2
	Increase +ve	(+ve)(+ve) = +ve = 9.81 m/s^2
	Max $V_E = V_A$ +ve	(+ve)(+ve) = +ve = 9.81 m/s^2
	Maximu $V_F > V_E$ +ve	(+ve)(+ve) = +ve = 9.81 m/s^2



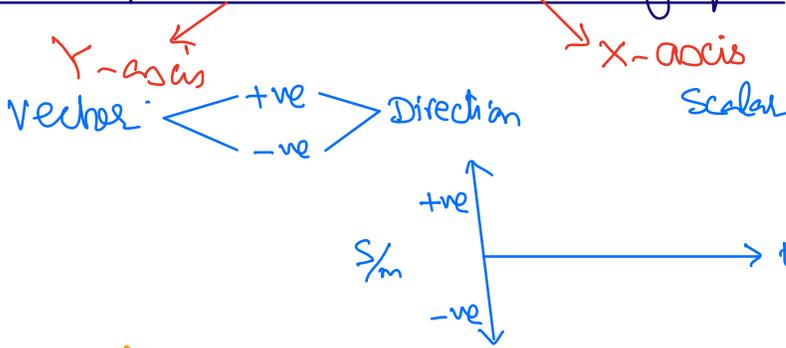
Result:

- (1) Displacement - time graph remains in first or 4th quadrant if the object does not cross its starting position.
- (2) Velocity - time graph is in first and 4th quadrant due to change of direction as per

assumption.

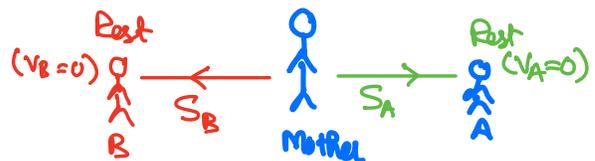
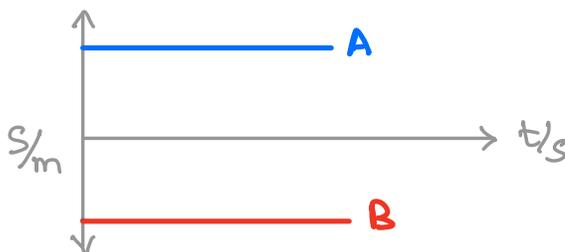
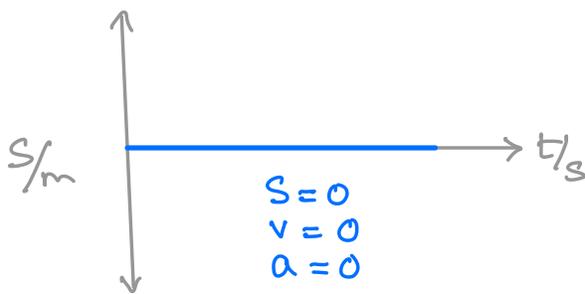
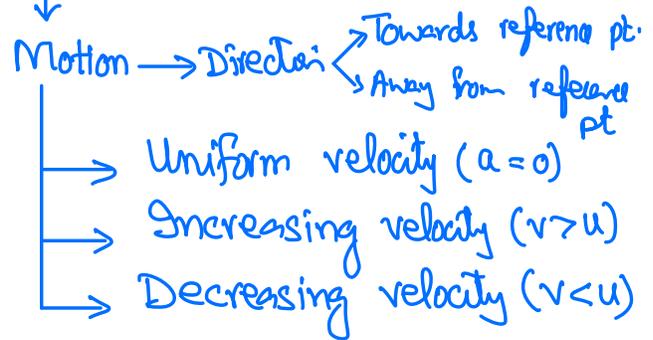
(3) Acceleration-time graph is either in first
or 4th quadrant depending upon assumption.

Displacement-time graph

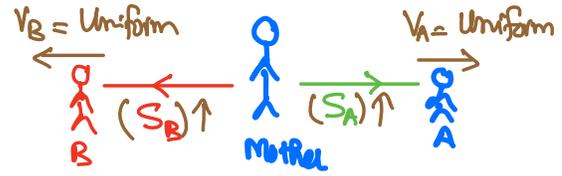
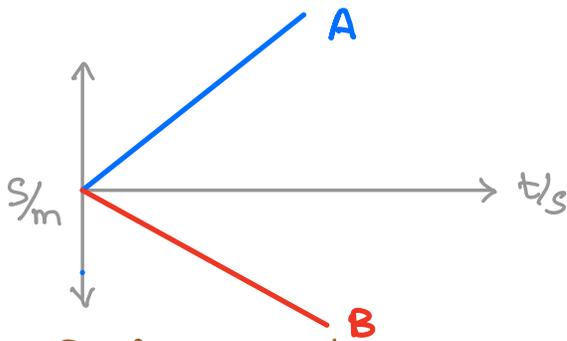


Results:

- (i) Instantaneous displacement $\rightarrow Y$ -axis
- (ii) Instantaneous velocity \rightarrow Gradient of graph
- (iii) Analysis of graph \rightarrow Rest position ($v=0$)

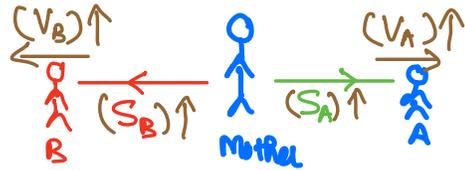
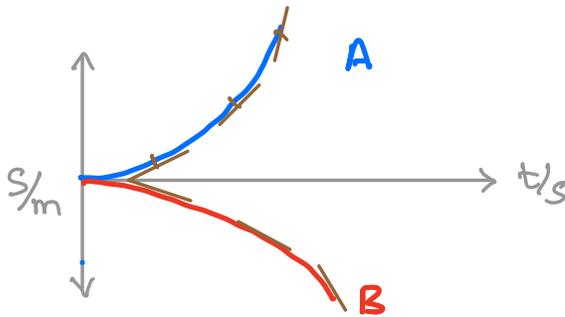


(3)



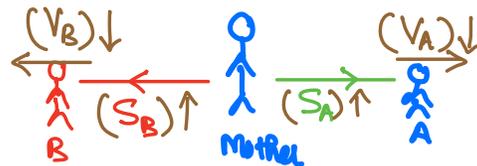
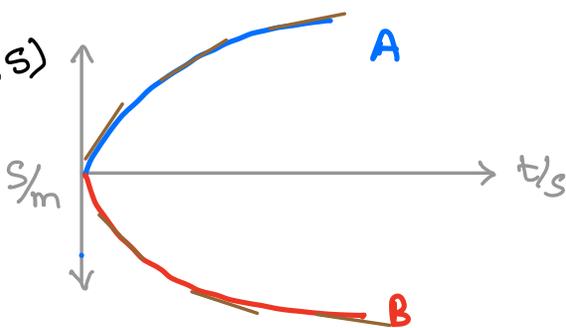
Both A and B move with uniform velocity in opposite directions away from their mother

(4)



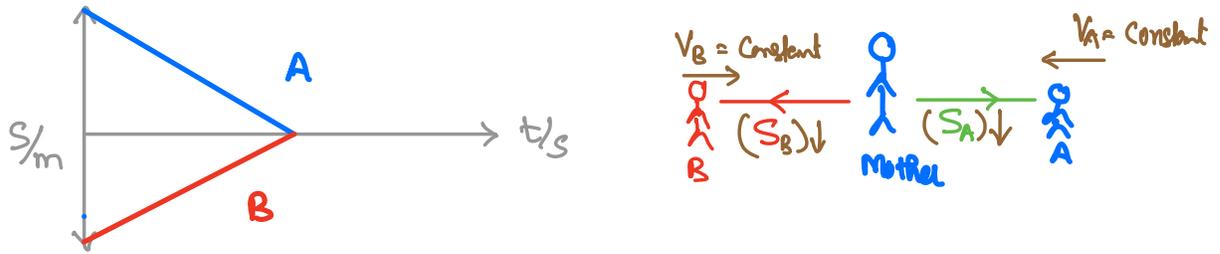
Both A and B move with increasing velocity (accelerate) in opposite directions away from mother.

(5)



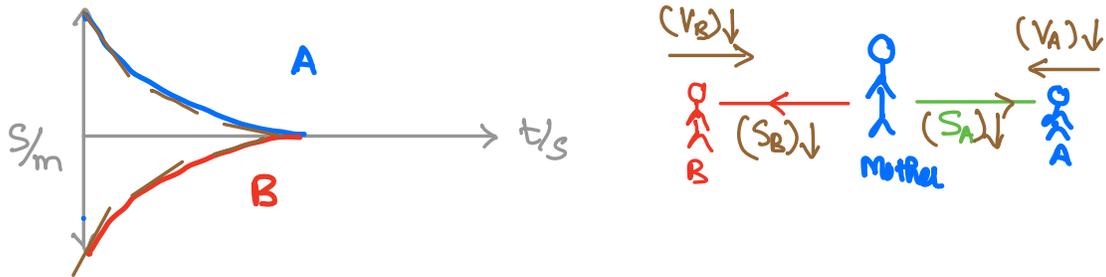
Both A and B move with decreasing velocity (decelerate) in opposite directions away from mother.

(6)



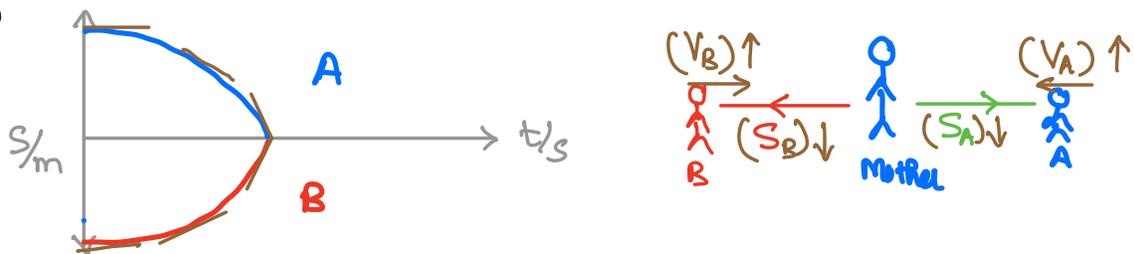
Both A and B move with uniform velocity in opposite directions towards mother

(7)

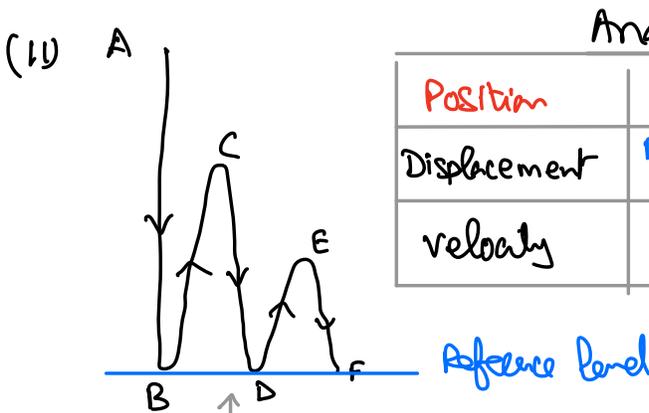
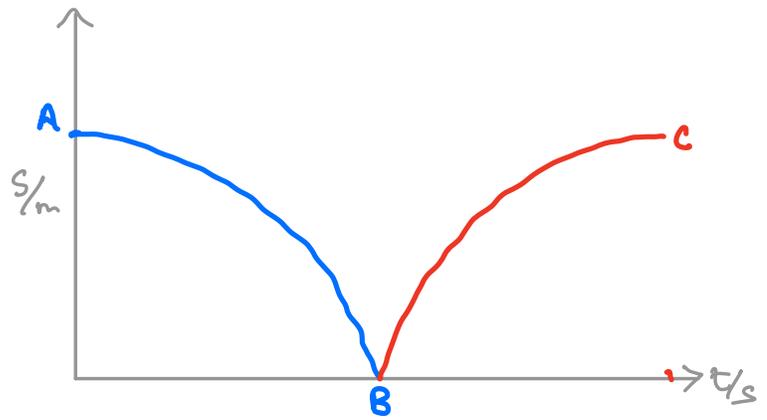
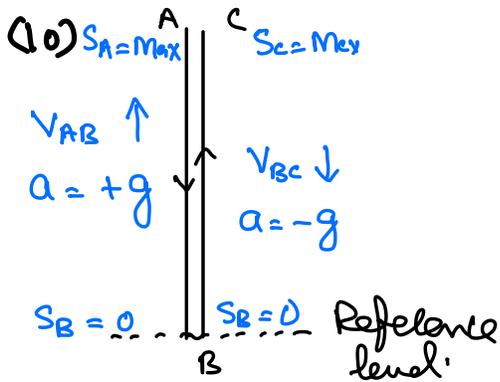
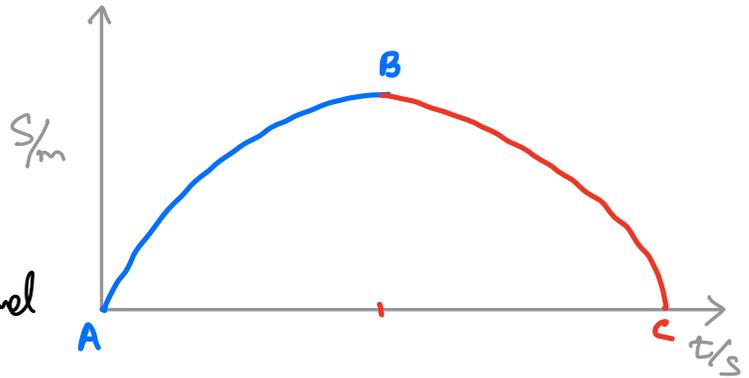
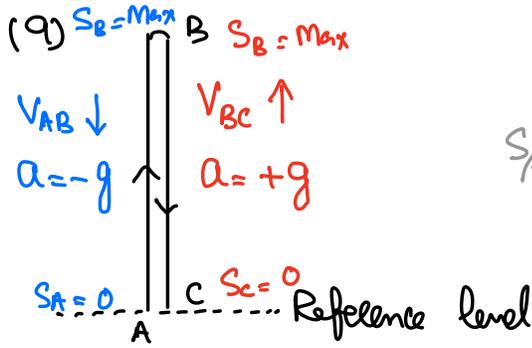


Both A and B move with decreasing velocity (decelerate) in opposite directions towards their mother.

(8)

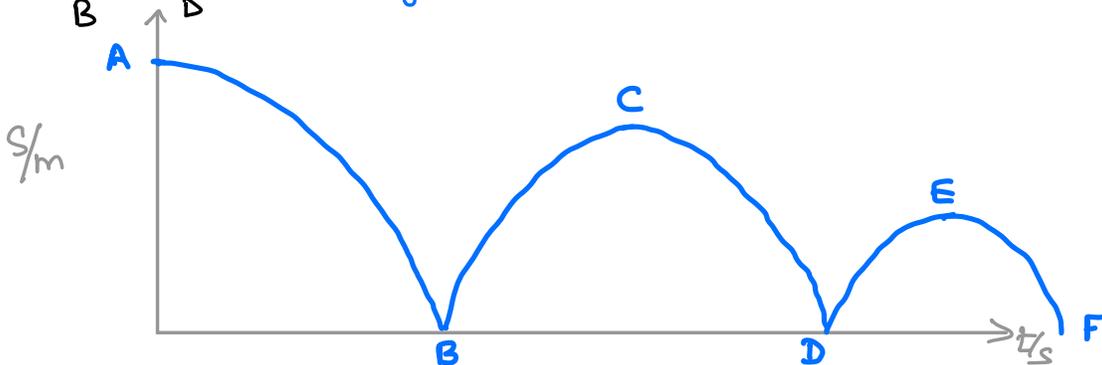


Both A and B move with increasing velocity (accelerate) in opposite directions towards mother.

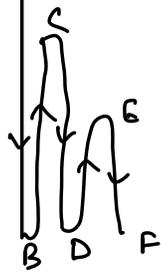


Analysis

Position	A to A	AB	BC	CD	DE	Ef
Displacement	Max	decrease	Increase	decrease	Inc.	Dec
velocity	Zero	Increase	Dec	Inc	Dec	Inc

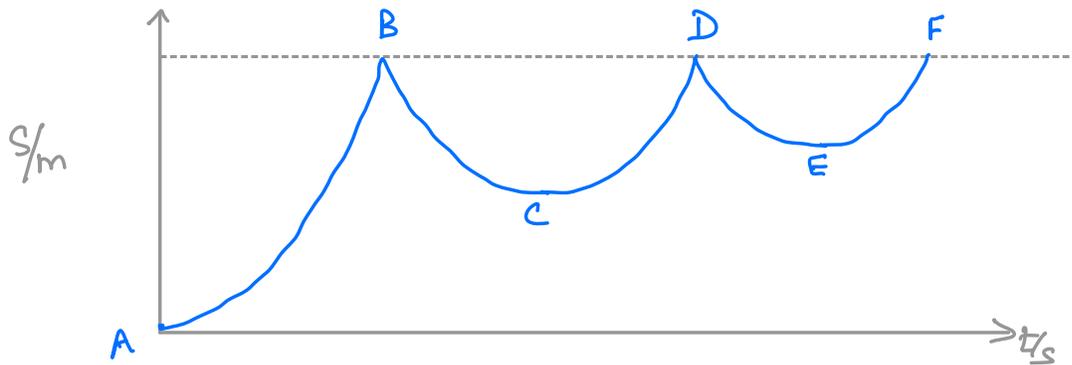


(12) - A --- Reference level.

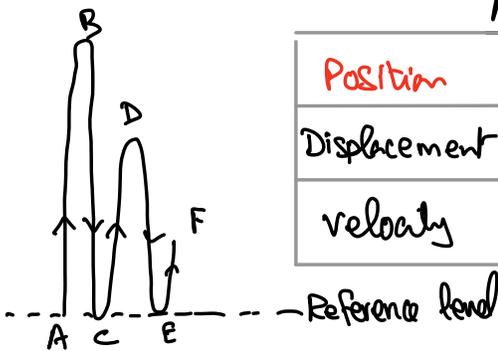


Analysis

Position	At A	AB	BC	CD	DE	Ef
Displacement	Zero	Increase	Decrease	Inc	Dec	Inc
velocity	Zero	Increase	Dec	Inc	Dec	Inc

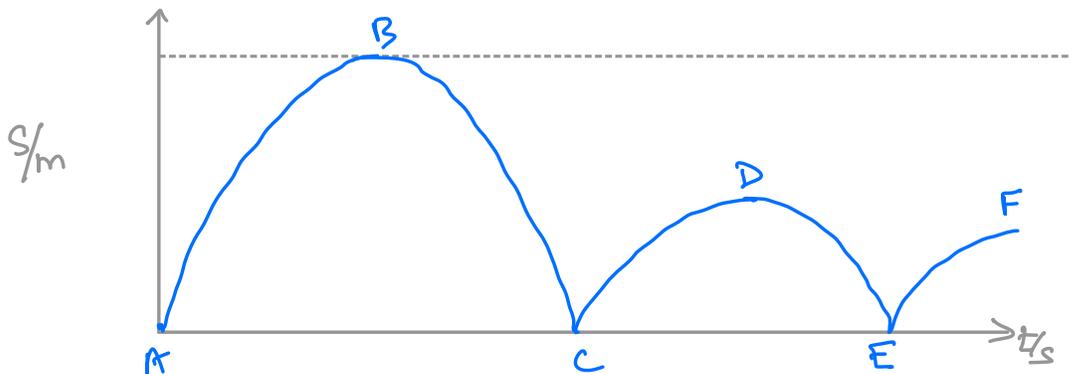


(13)



Analysis

Position	At A	AB	BC	CD	DE	Ef
Displacement	Zero	Increase	decrease	Increase	Dec	Inc.
velocity	Max	Dec	Inc	Dec	Inc	Dec.



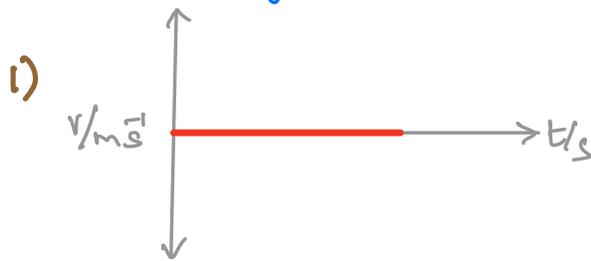
Velocity-time graphs

Dependent qty (Y-axis)

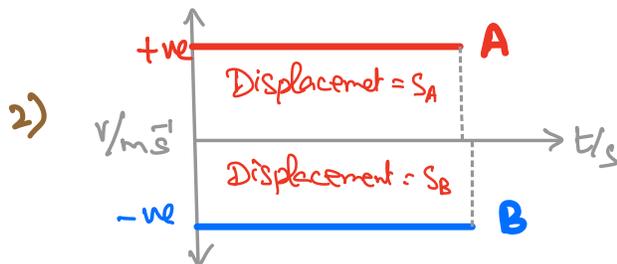
Independent qty (X-axis)

Results:

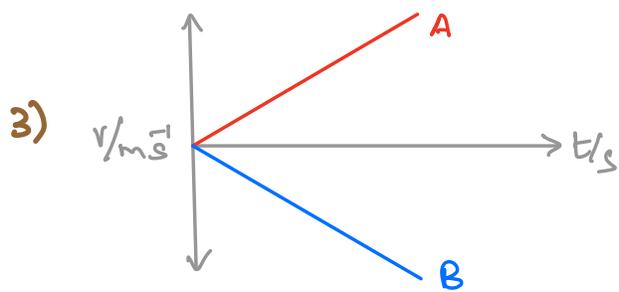
- (i) Instantaneous velocity \rightarrow Y-axis
- (ii) " acceleration \rightarrow Gradient of graph
- (iii) " displacement \rightarrow Area of graph along with time axis
- (iv) Analysis of graph \rightarrow Rest position ($v=0, a=0$)
 - Motion \rightarrow Direction of motion as per assumption
 - Uniform velocity ($a=0$)
 - Increasing velocity ($v > u$)
 - Uniform acceleration
 - Increasing acceleration
 - Decreasing acceleration
 - Decreasing velocity ($v < u$)
 - Uniform deceleration
 - Increasing deceleration
 - Decreasing deceleration



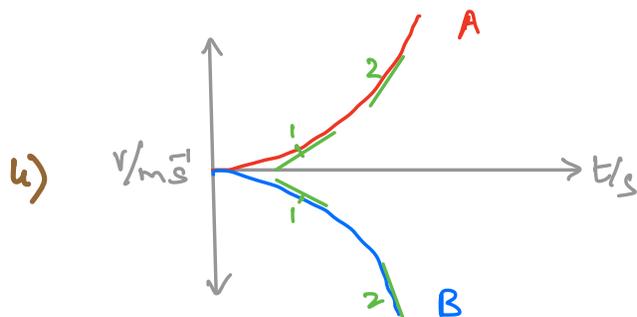
Rest position ($v=0, s=0, a=0$)



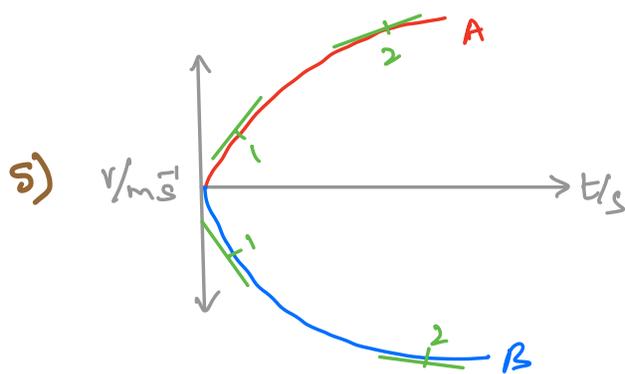
Both A and B move with uniform velocity but in opposite direction.



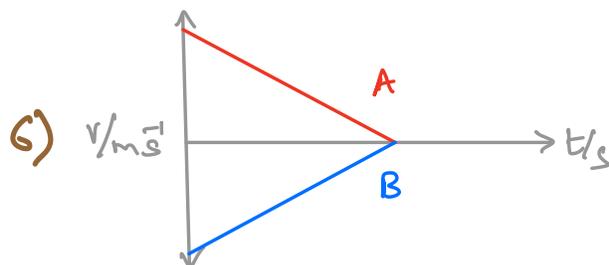
Both start from rest ($u=0$) and move with uniformly increasing velocity (uniform acceleration) but in opposite directions.



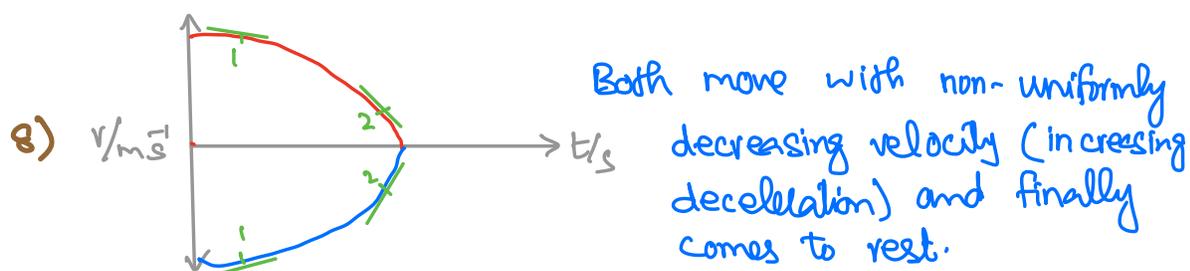
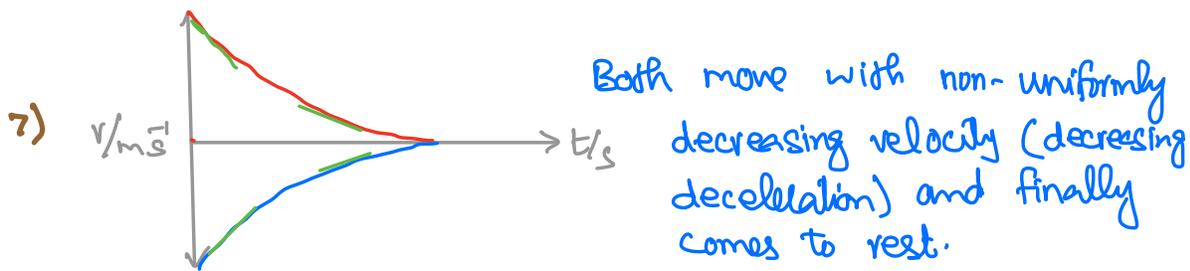
Both start from rest ($u=0$) and move with non-uniformly increasing velocity (increasing acceleration i.e. $a_1 < a_2$) but in opposite directions.



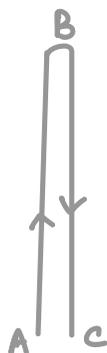
Both start from rest ($u=0$) and move with non-uniformly increasing velocity (decreasing acceleration i.e. $a_1 > a_2$) but in opposite directions.



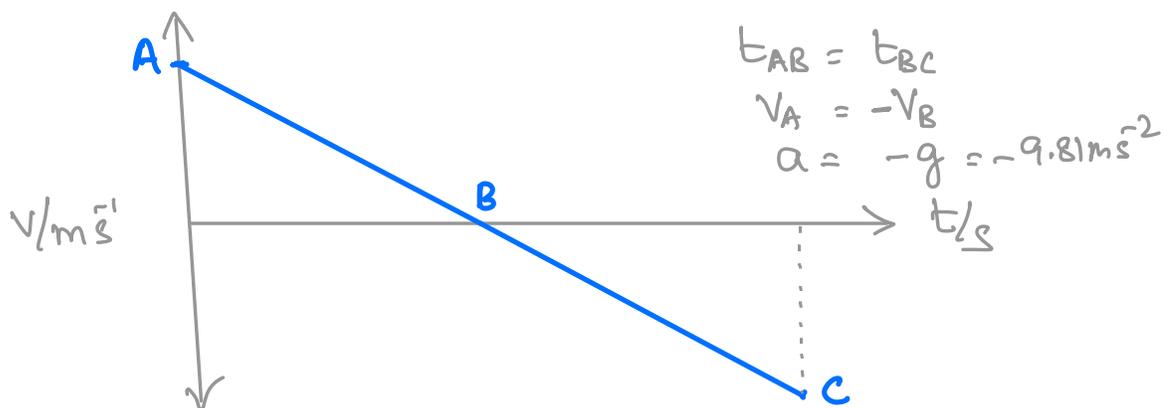
Both move with uniformly decreasing velocity (uniform deceleration) and finally comes to rest.



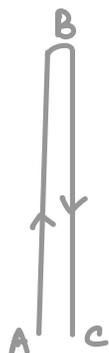
(9) Assumption: upward motion is +ve
Analysis



Position	At A	A to B	At B	B to C	At C	
Magnitude of velocity	Max	decrease	Zero	Increase	Max	.
Direction of velocity	+ve	+ve	-	-ve	-ve	
Acceleration / ms^{-2}	-g	-g	-g	-g	-g	

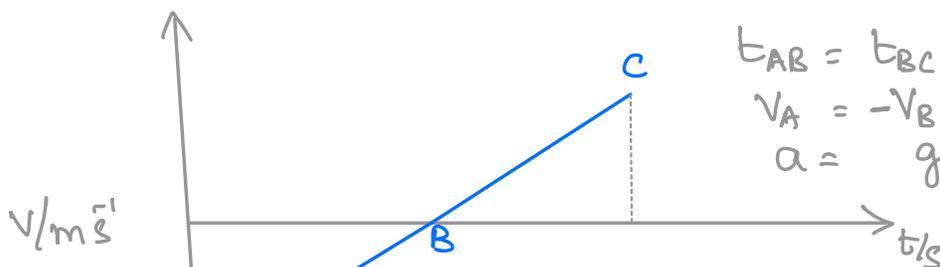


(10)



Assumption: upward motion is -ve
Analysis

Position	At A	At B	At B	B to C	At C	
Magnitude of velocity	Max	decrease	Zero	Increase	Max	.
Direction of velocity	-ve	-ve	-	+ve	+ve	
Acceleration/ m/s^2	+g	+g	+g	+g	+g	



$$t_{AB} = t_{BC}$$

$$v_A = -v_B$$

$$a = g = 9.81 m/s^2$$

Assumption: upward motion is +ve
Analysis

Position	At A	At B	At B	B to C	At C	
Magnitude of velocity	Max	decrease	Zero	Increase	Max	.
Direction of velocity	+ve	+ve	-	-ve	-ve	
Acceleration/ m/s^2	-g	-g	-g	-g	-g	



$$t_{AB} = t_{BC}$$

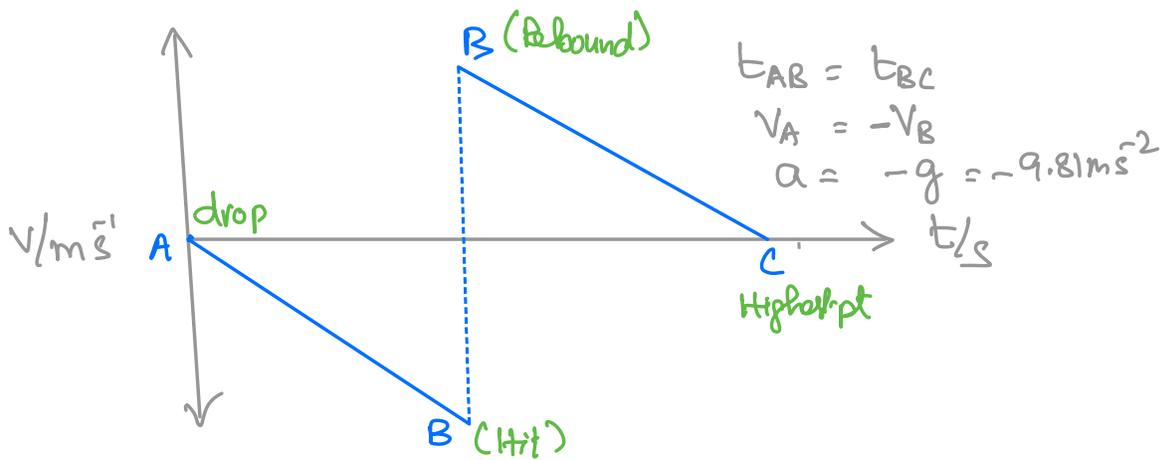
$$v_A = -v_B$$

$$a = -g = -9.81 m/s^2$$

Assumption: upward motion is +ve
Analysis

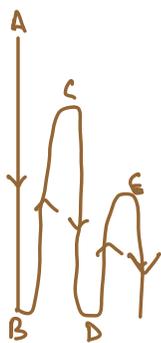


Position	At A	At B	At B	B to C	At C	
Magnitude of velocity	Zero	Increase	Max	Decrease	Zero	.
Direction of velocity	-	-ve	-ve	+ve	+ve	
Acceleration/ m/s^2	-g	-g	-g	-g	-g	

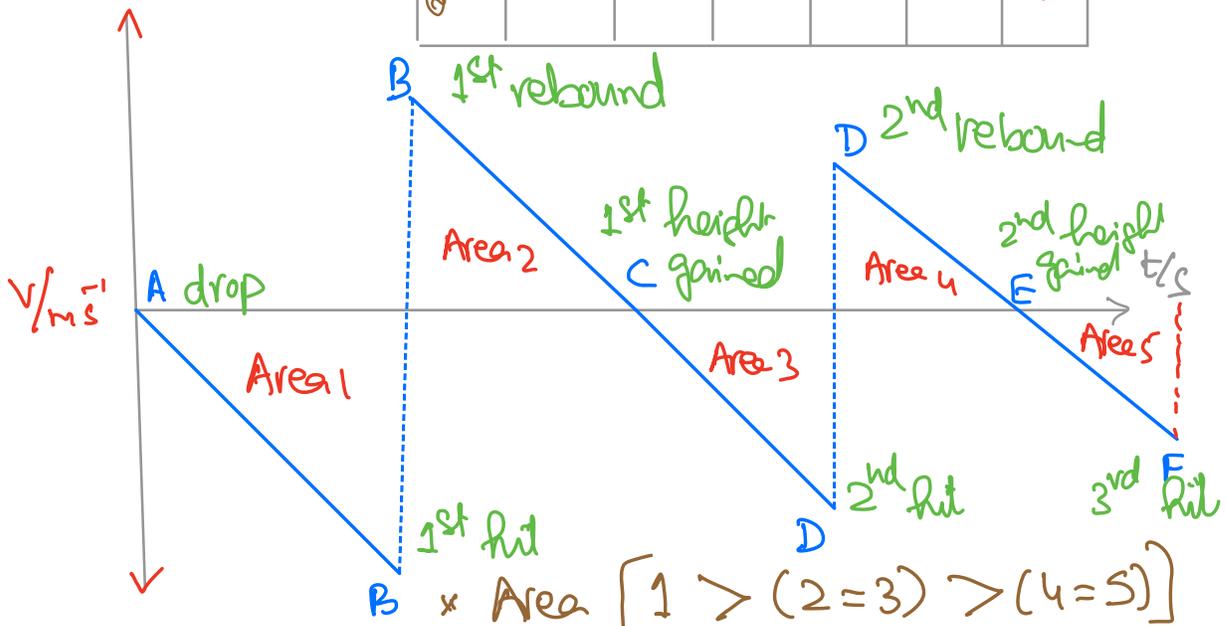


(12)

Assumption: Upward motion is +ve



Position	A-A	A-B	B-C	C-D	D-E	E-F
Magnitude of velocity	Zero	Increase	Decrease	Inc	Dec	Inc
Direction of velocity	-	-ve	+ve	-ve	+ve	-ve
Gradient	-	4 th	1 st	4 th	1 st	4 th



Q)



Total time of flight = 3.4 s

Note: Height gained is not provided.

Method 1

Time to go up and come down is 3.4 s. So time to go up = $\frac{3.4}{2} = 1.7$ s

$$v = u + at$$

$$0 = u + (-9.81)(1.7)$$

$$u = 16.6 \text{ m s}^{-1}$$

Method 2

Total displacement = 0

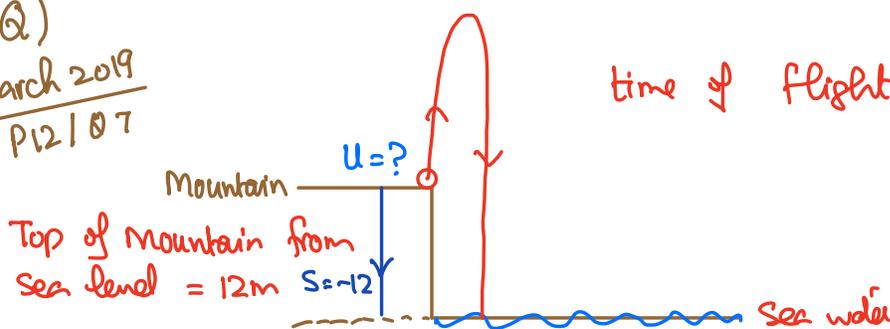
$$S = ut + \frac{1}{2}at^2$$

$$0 = (u)(3.4) + \frac{1}{2}(-9.81)(3.4)^2$$

$$u = 16.6 \text{ m s}^{-1}$$

Q)

March 2019
P12/07



time of flight = 3.4 s

Calculate initial velocity of projection.

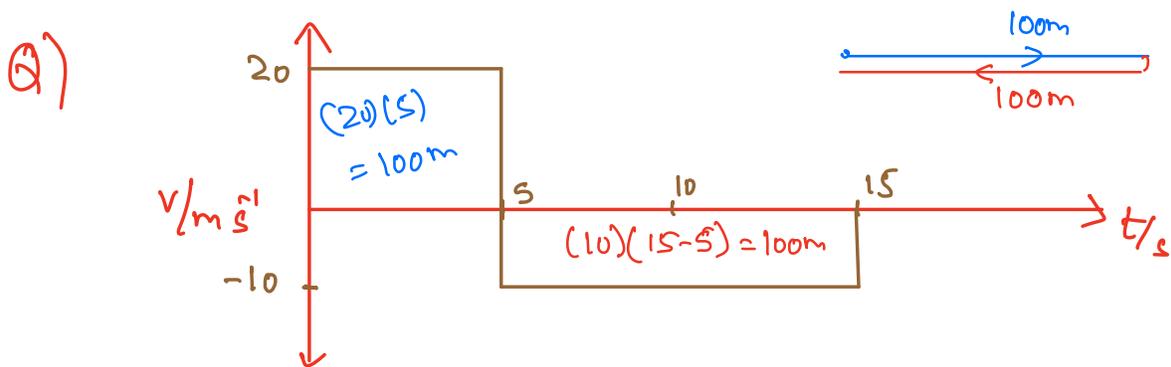
- (A) 3.5 m s^{-1} (B) 6.6 m s^{-1} (C) 13 m s^{-1} (D) 20 m s^{-1}

Displacement is in downward direction taken from top of mountain to the sea water. Now velocity is in upward direction and displacement is in

downward direction:

$$S = ut + \frac{1}{2} at^2$$
$$-12 = (u)(3.4) + \frac{1}{2} (-9.81)(3.4)^2$$

$$u = 13 \text{ m s}^{-1}$$

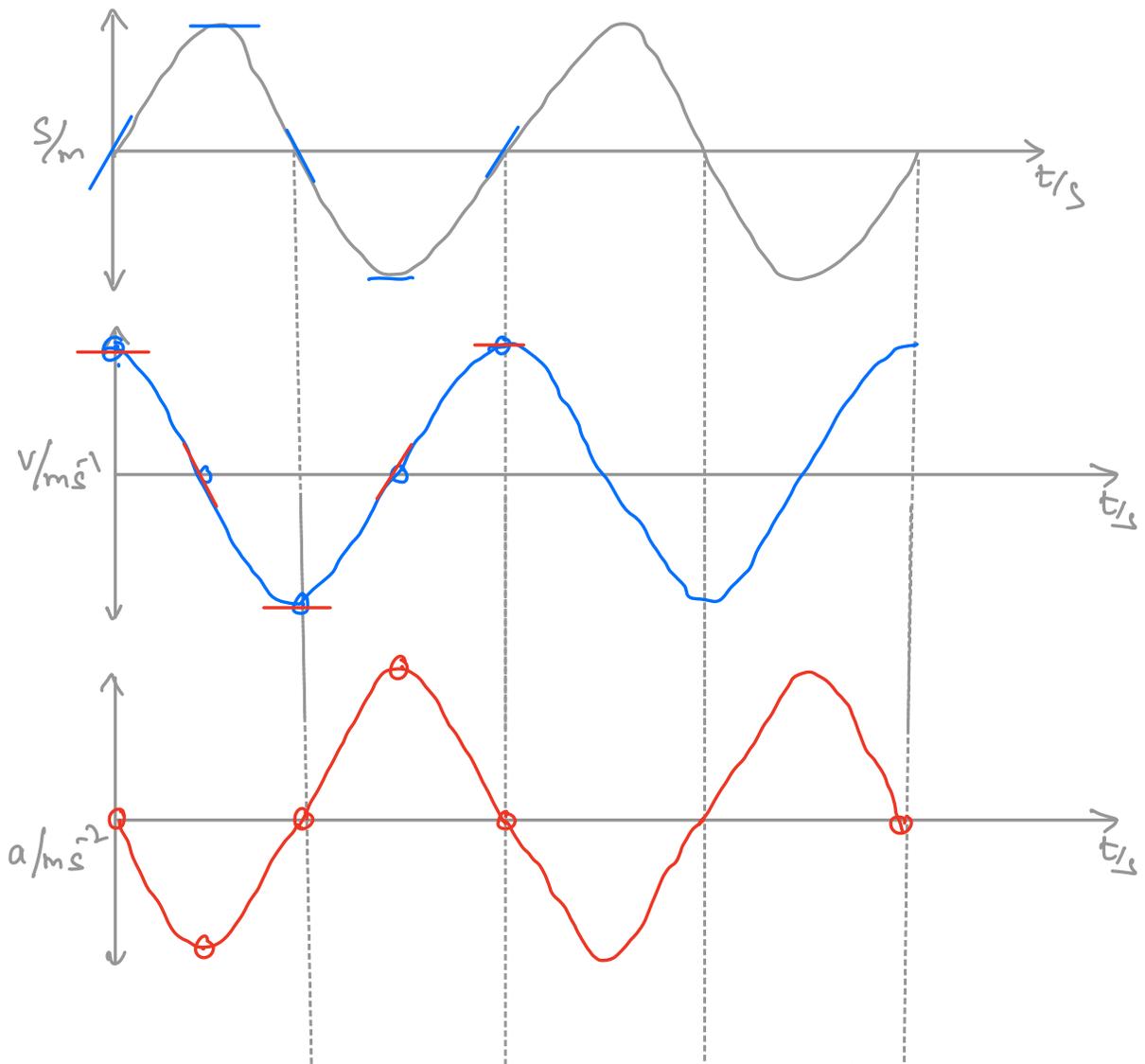


Calculate average velocity of object.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time}}$$
$$= \frac{100 + (-100)}{15} = 0$$

Q) The displacement-time graph of a vibrating object is given.

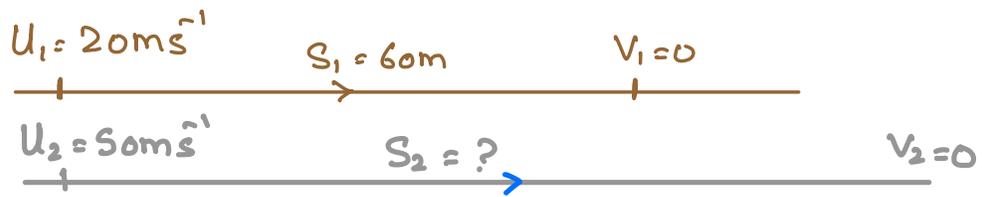
- (a) Sketch the corresponding
- velocity-time graph
 - acceleration-time graph.



(b) Using above graphs, define a relationship b/w displacement and acceleration.

Acceleration \propto - (Displacement)

Q)



What distance should 2nd object covers if same braking force is applied in both case.

Hint: Same braking force means same deceleration / retardation by $F = ma$

Kinematic's approach

$$2as = v^2 - u^2$$

ie

$$\frac{2a S_2}{2a S_1} = \frac{v_2^2 - u_2^2}{v_1^2 - u_1^2}$$

$$\frac{S_2}{60} = \frac{(0)^2 - (50)^2}{(0)^2 - (20)^2}$$

Dynamics' approach

Work done against motion = Loss of Kinetic energy

$$FS = \frac{1}{2} m u^2$$

i.e

$$\frac{FS_2}{FS_1} = \frac{\frac{1}{2} m u_2^2}{\frac{1}{2} m u_1^2} \Rightarrow \frac{S_2}{S_1} = \frac{u_2^2}{u_1^2}$$

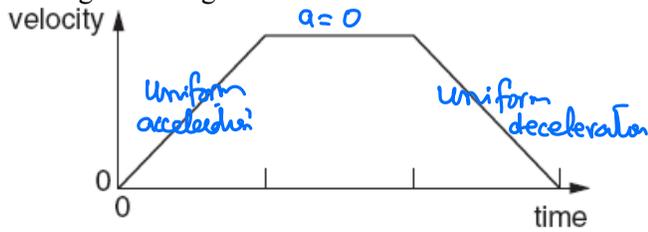
$$\frac{S_2}{60} = \frac{(50)^2}{(20)^2}$$

$$S_2 = 375 \text{ m}$$

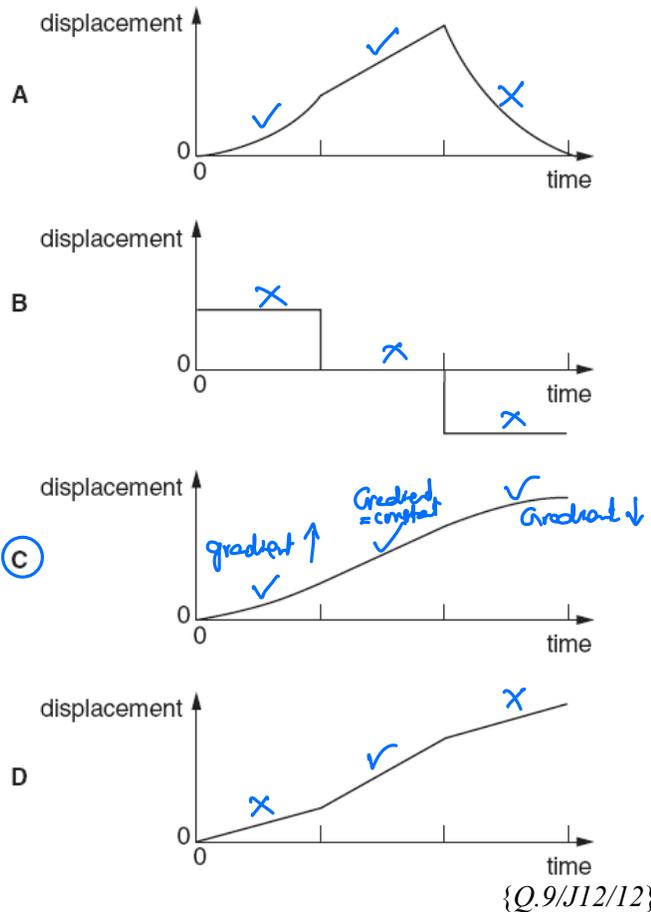
DISPLACEMENT-TIME GRAPHS

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

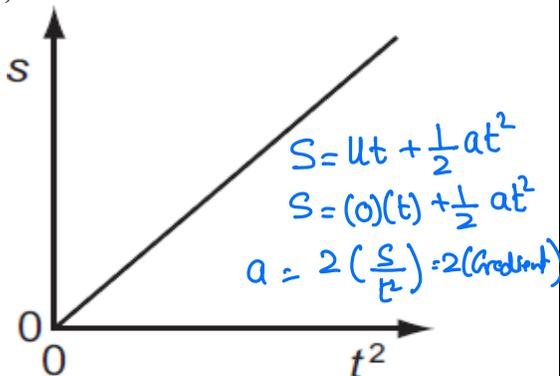
1. The graph of velocity against time for an object moving in a straight line is shown.



Which of the following is the corresponding graph of displacement against time?



2. At time $t = 0$, a body moves from rest with constant acceleration in a straight line. At time t , the body is distance s from its rest position. A graph is drawn of s against t^2 , as shown.

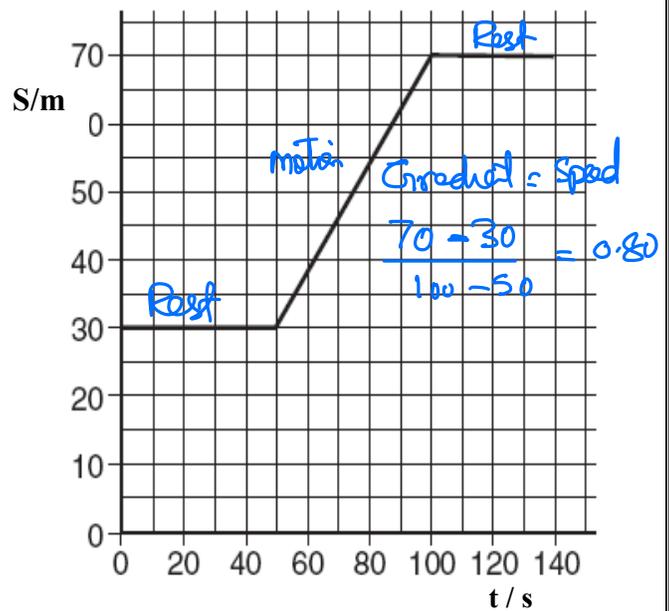


Which statement describes the acceleration of the body?

- A It is equal to half the value of the gradient of the graph.
- B It is equal to the value of the gradient of the graph.
- C** It is equal to twice the value of the gradient of the graph.
- D It is equal to the reciprocal of the gradient of the graph.

{Q.8/J13/13}

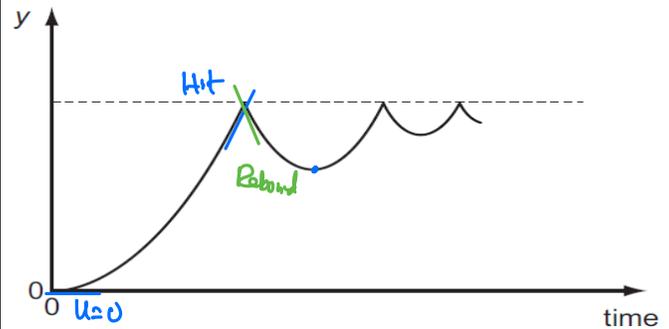
3. A car at rest in a traffic queue moves forward in a straight line and then comes to rest again. The graph shows the variation with time of its displacement.



What is its speed while it is moving?

- A 0.70ms^{-1}
- B** 0.80ms^{-1}
- C 1.25ms^{-1}
- D 1.40ms^{-1}

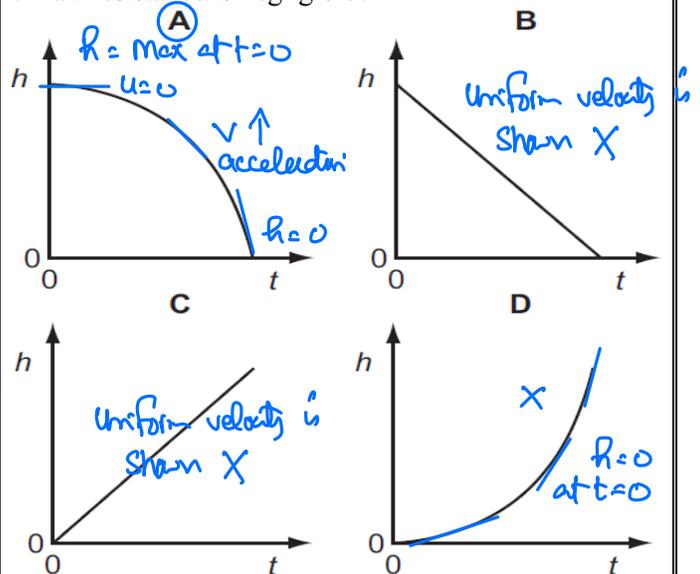
4. A ball is released from rest above a horizontal surface and bounces several times. The graph shows how, for this ball, a quantity y varies with time.



What is the quantity y ?

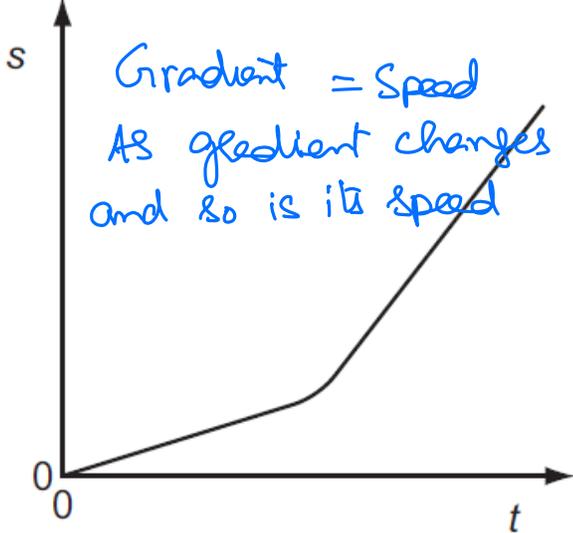
- A acceleration
 - B** displacement
 - C kinetic energy
 - D velocity
- {Q.8/J13/12, Q.6/N09/11}

5. A brick is dislodged from a building and falls vertically under gravity. Which graph best represents the variation of its height h above the ground with time t if air resistance is negligible?



{Q.9/J12/12, Q.9/J10/11}

6. The variation with time t of the distance s moved by a body is shown below.



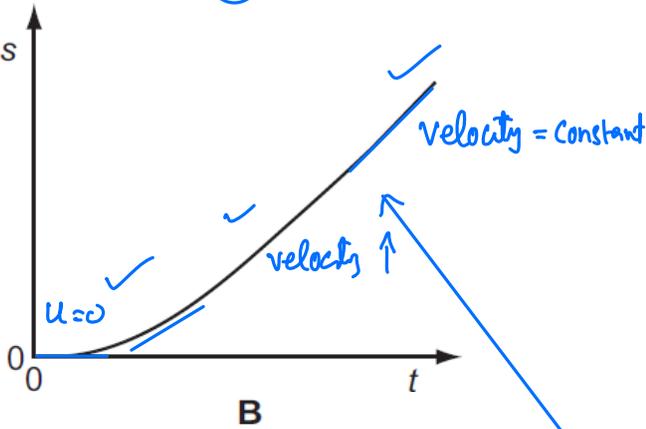
What can be deduced from the graph about the motion of the body?

- A It accelerates continuously.
- B It starts from rest.
- C The distance is proportional to time.
- D** The speed changes.

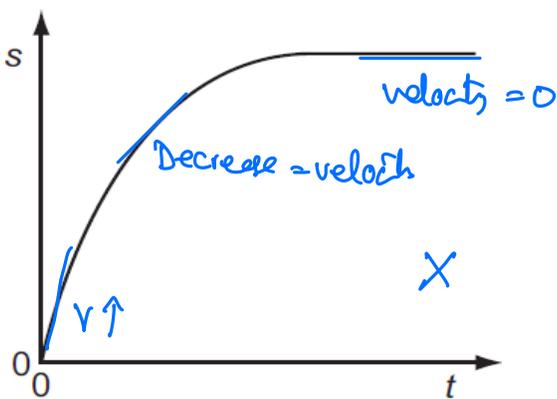
{Q.8/N-11/12}

7. A tennis ball falls freely, in air, from the top of a tall building. Which graph best represents the variation of distance s fallen with time t ?

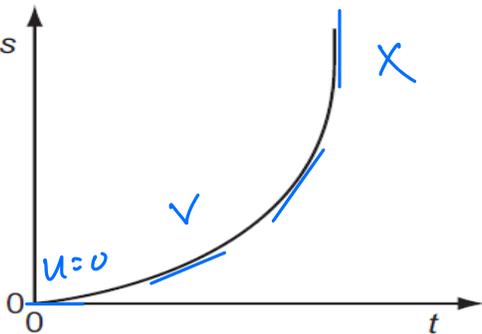
A



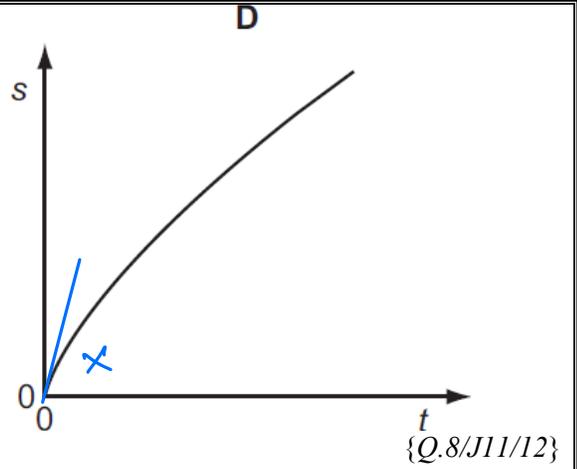
B



C

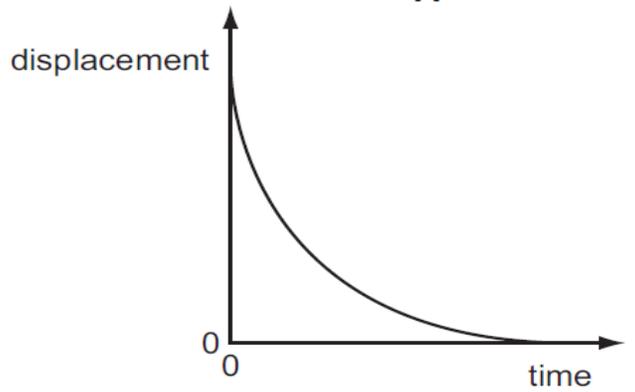


8.

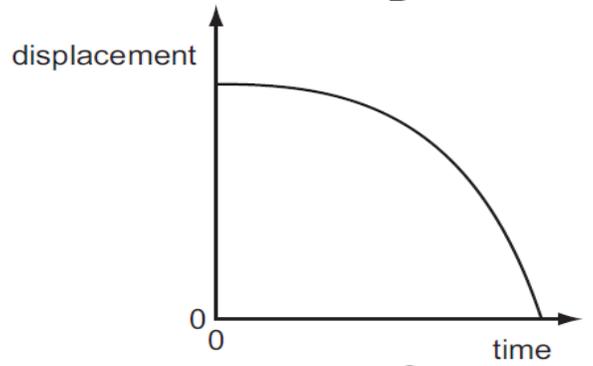


Which displacement-time graph best represents the motion of a falling sphere, the initial acceleration of which eventually reduces until it begins to travel at constant terminal velocity?

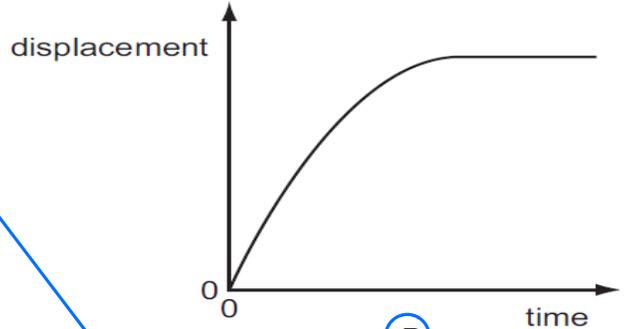
A



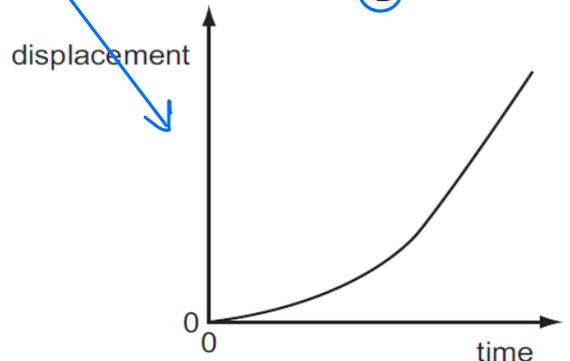
B



C



D

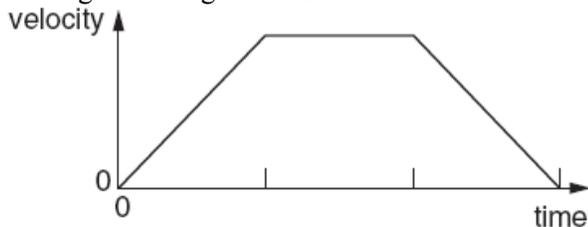


{Q.5/J09}

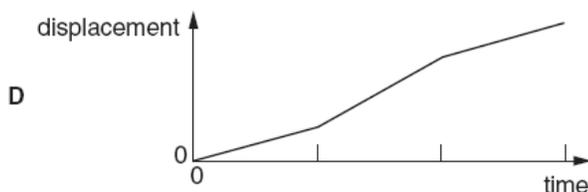
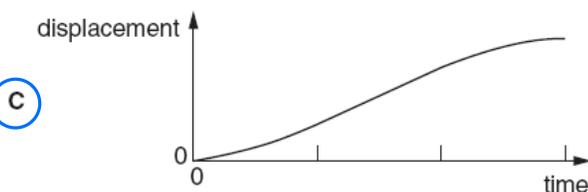
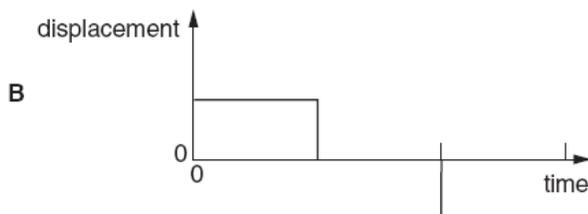
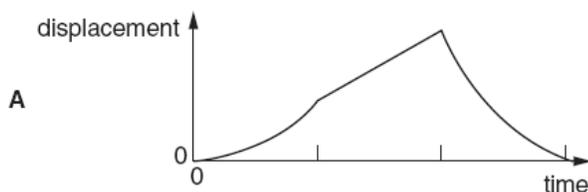
KINEMATIC'S GRAPHS

Akhtar Mahmood (0333-4281759)
M.Sc.(Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

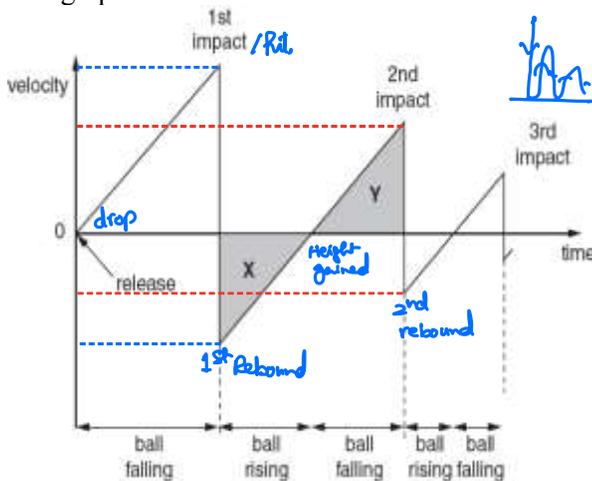
1. The graph of velocity against time for an object moving in a straight line is shown.



Which of the following is the corresponding graph of displacement against time?



2. A ball is released from rest above a horizontal surface. The graph shows the variation with time of its velocity.

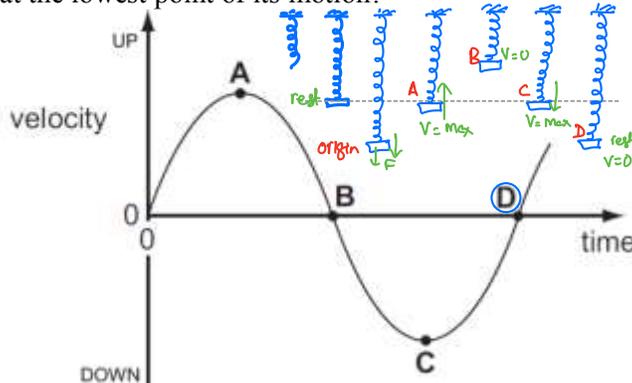


Areas X and Y are equal.

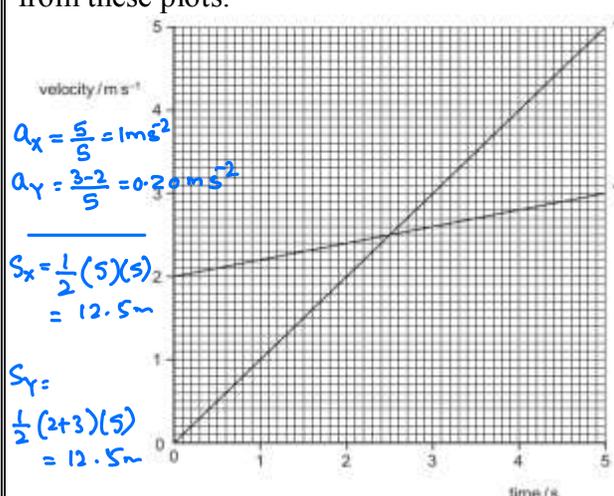
This is because

- A the ball's acceleration is the same during its upward and downward motion. ✓
- B the speed at which the ball leaves the surface after an impact is equal to the speed at which it returns to the surface for the next impact. ✓
- C for one impact, the speed at which the ball hits the surface equals the speed at which it leaves the surface. ✗
- D** the ball rises and falls through the same distance between impacts. ✓

3. The diagram shows a velocity-time graph for a mass moving up and down on the end of a spring. Which point represents the velocity of the mass when at the lowest point of its motion?



4. The graph shows velocity-time plots for two vehicles X and Y. The accelerations and distances travelled by the two vehicles can be estimated from these plots.

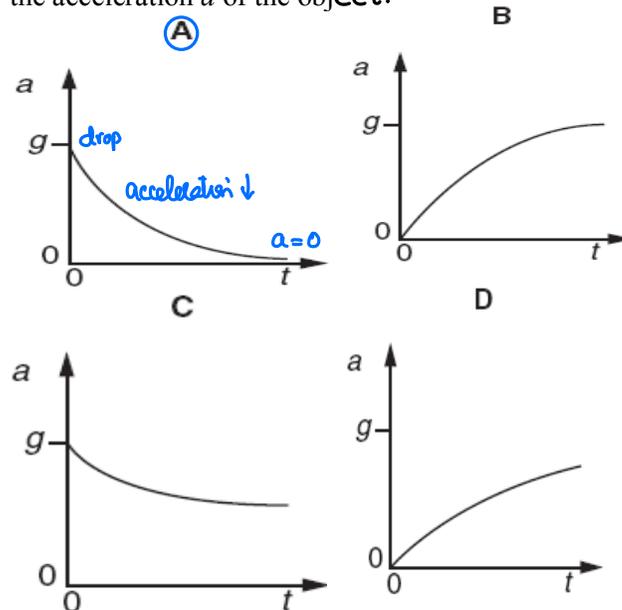


Which statement is correct?

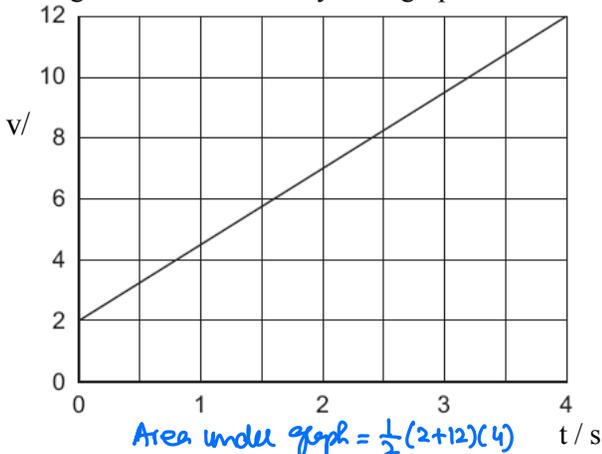
- A The accelerations of X and Y are the same at 2.5 s. ✗
- B The initial acceleration of Y is greater than that of X. ✗
- C The distance travelled by X is ^{same} greater than that travelled by Y in the 5 s period. ✗
- D** The distances travelled by X and Y in the 5 s period are the same.

5. An object is dropped from a great height and falls through air of uniform density. The acceleration of free fall is g .

Which graph could show the variation with time t of the acceleration a of the object?



6. The diagram shows a velocity-time graph for a car.



What is the distance travelled between time $t = 0$ and $t = 4$ s?

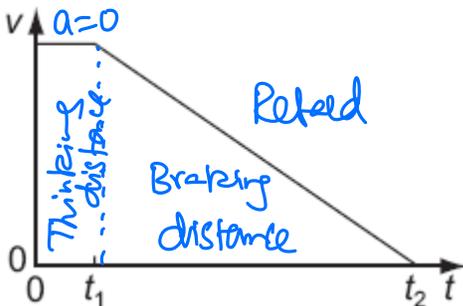
- A 2.5 m B 3.0 m C 20 m **D 28 m**

7. A boy throws a ball vertically upwards. It rises to a maximum height, where it is momentarily at rest, and falls back to his hands.

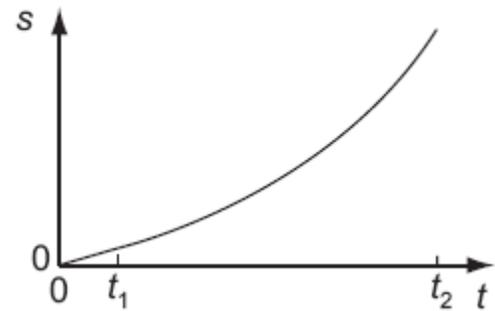
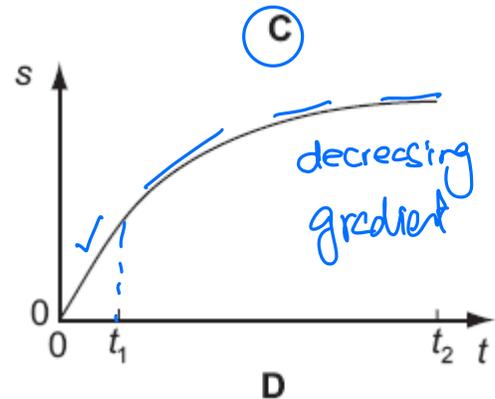
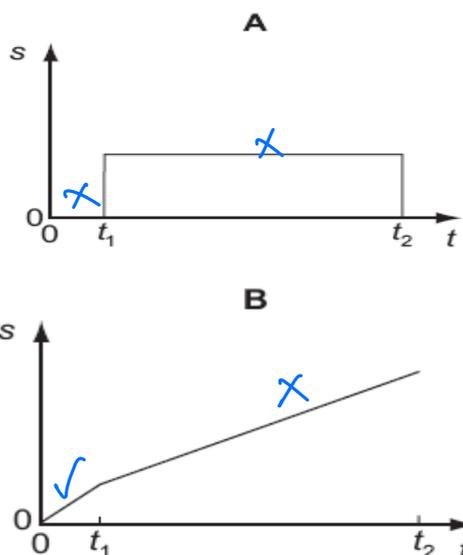
Which of the following gives the acceleration of the ball at various stages in its motion? Take vertically upwards as positive. Neglect air resistance.

	rising	at maximum height	falling
A	-9.81 m s^{-2}	0	$+9.81 \text{ m s}^{-2}$
B	-9.81 m s^{-2}	-9.81 m s^{-2}	-9.81 m s^{-2}
C	$+9.81 \text{ m s}^{-2}$	$+9.81 \text{ m s}^{-2}$	$+9.81 \text{ m s}^{-2}$
D	$+9.81 \text{ m s}^{-2}$	0	-9.81 m s^{-2}

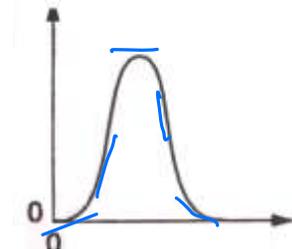
8. When a car driver sees a hazard ahead, she applies the brakes as soon as she can and brings the car to rest. The graph shows how the speed v of the car varies with time t after the hazard is seen.



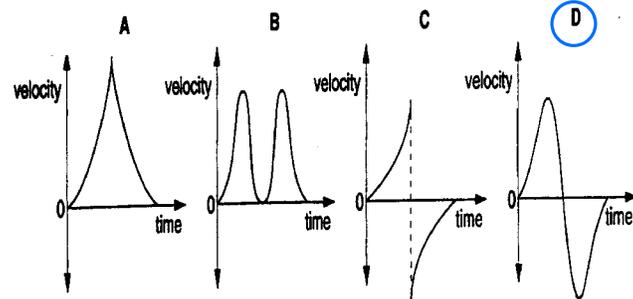
Which graph represents the variation with time t of the distance s travelled by the car after the hazard has been seen?



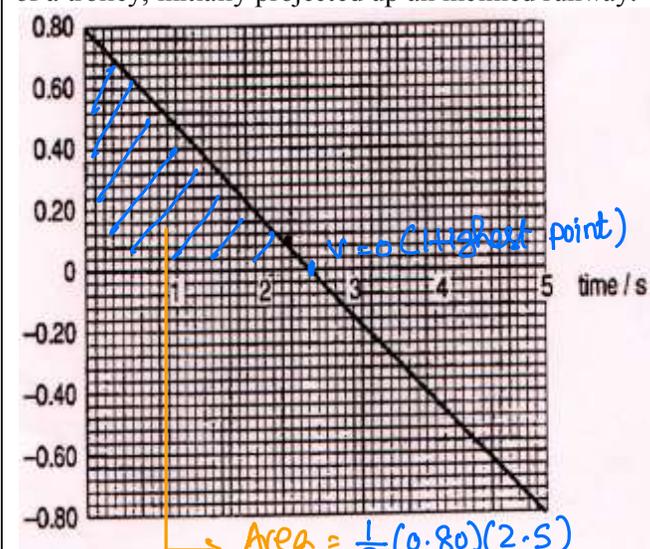
9. The graph shows the variation with time of the displacement for a particle moving along a straight track.



Which graph best shows the variation with time of the velocity of the particle?



10. The graph shows the variation with time of the velocity of a trolley, initially projected up an inclined runway.



What is the maximum distance up the slope reached by the trolley?

- A 0.80 m **B 1.0 m** C 2.0 m D 4.0 m

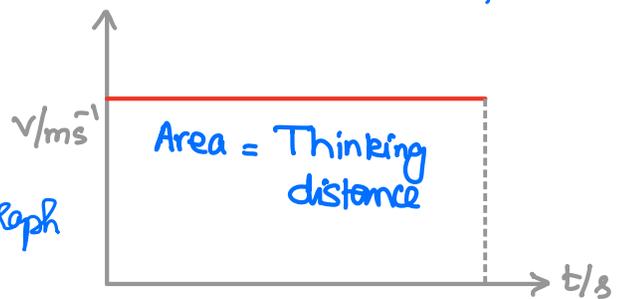
Thinking, braking and stopping distance:

Thinking distance:

Def. It is the distance travelled by a vehicle during the driver's reaction/decision time.

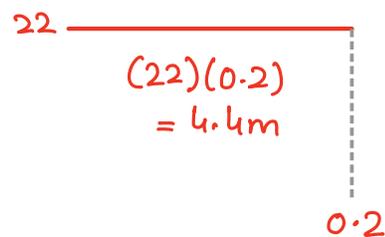
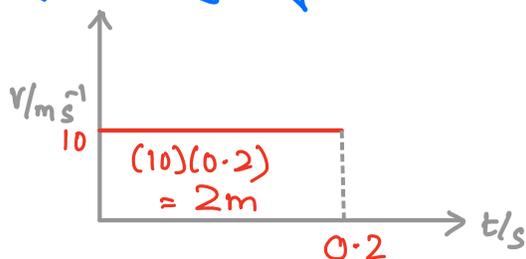
Note: Thinking distance is travelled with uniform velocity.

Thinking distance = Area under straight line graph

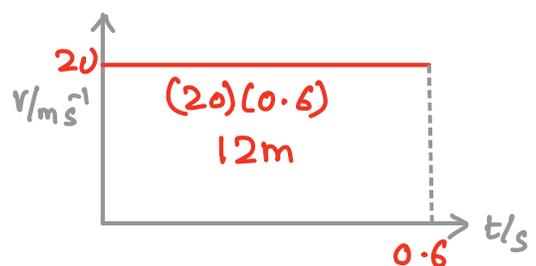
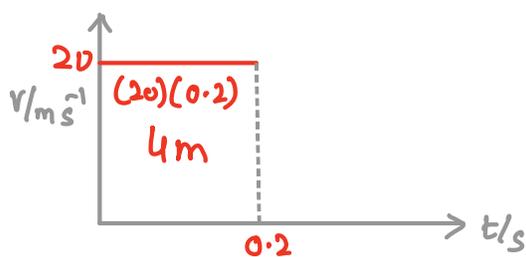


Dependence: (Thinking distance) \uparrow if

1- (velocity of vehicle) \uparrow



2- driver is not alert i.e (reaction time) \uparrow

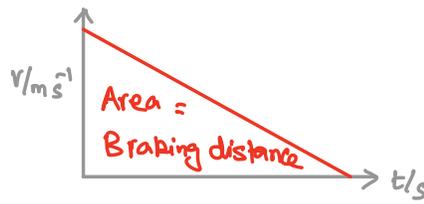


Braking distance:

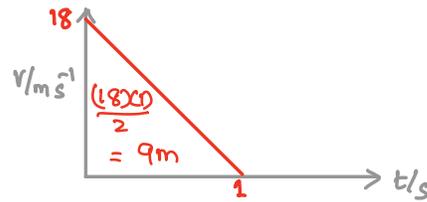
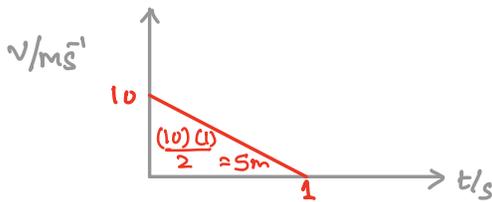
Def. It is the path travelled by a vehicle when brakes are applied.

Note: Braking distance is travelled with uniform deceleration or retardation.

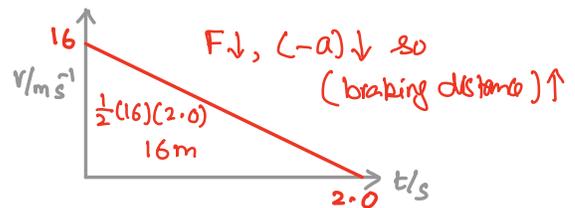
Graph:



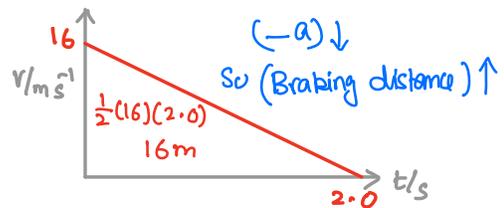
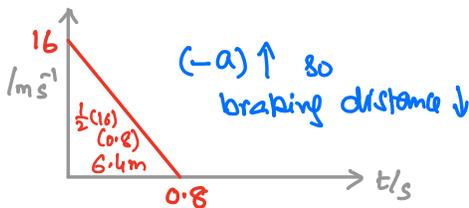
Dependance: (Braking distance) \uparrow if
 1- initial velocity of vehicle is high



2- Less force is applied on brakes



3- Less friction between road and tyres:-

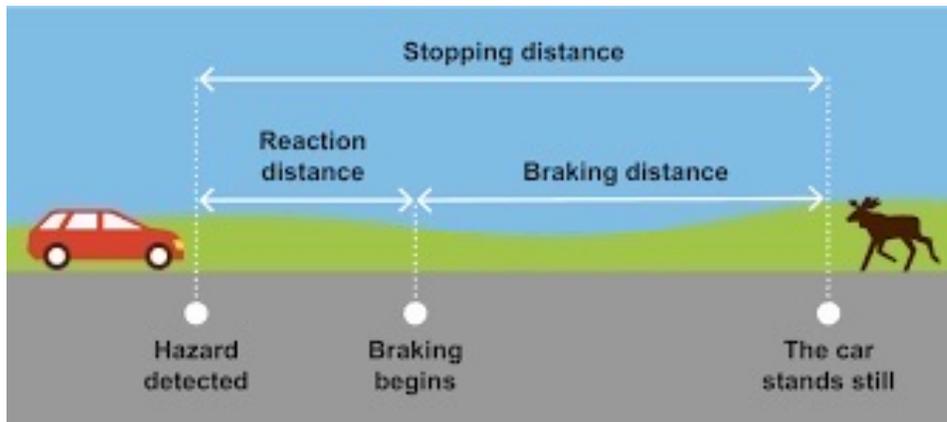
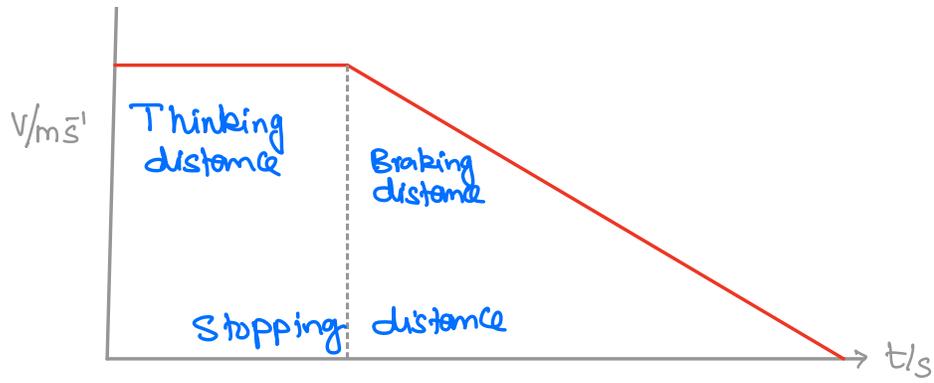


4- Greater mass and hence inertia of vehicle:

Stopping distance:

Def. It is the sum of thinking and braking distance.

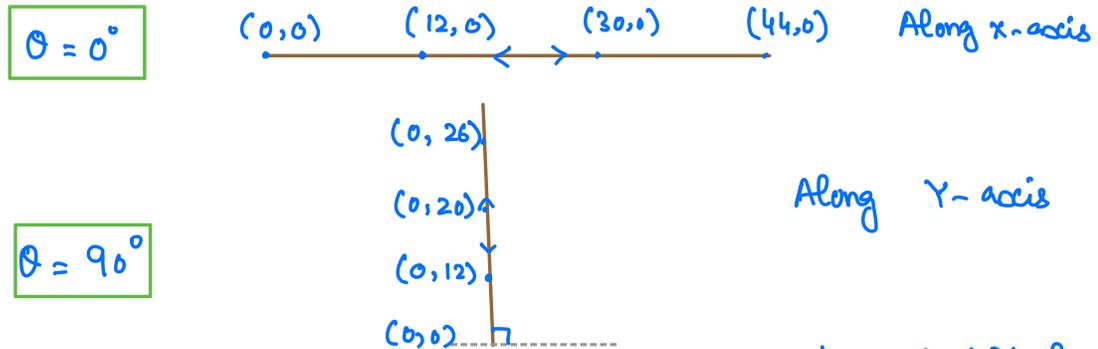
\uparrow



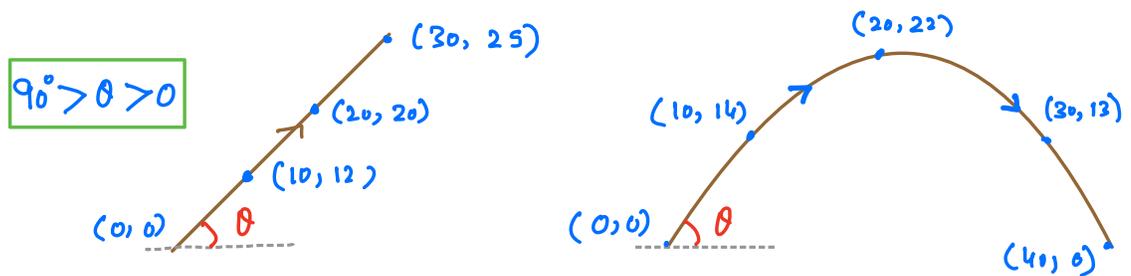
Acceleration- time graph:-

Motion in two dimension:

(a) One dimensional motion:- Always in a straight line.



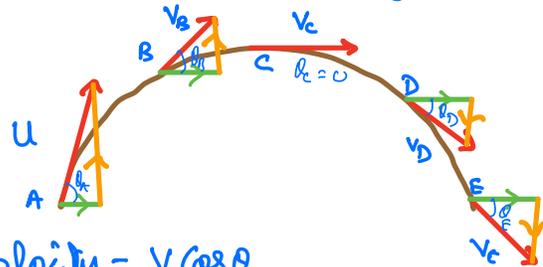
(b) Two dimensional motion:- Either in a straight line or follows a curved path.



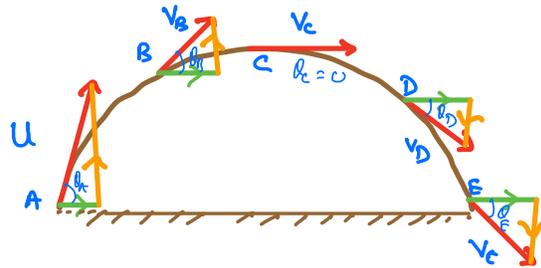
Projectile:- When an object is thrown in such a way that it makes an angle θ ($90^\circ > \theta > 0$) with the horizontal then it moves along a curved path due to Gravitational pull of Earth. Such a motion in two dimensions is called projectile and curved path traced by projectile is trajectory.

At any position,
vertical component of velocity
 $= V \sin \theta$

Horizontal component of velocity $= V \cos \theta$



$\theta / ^\circ$	0	30	45	60	90	\uparrow
$\sin(\theta / ^\circ)$	0	0.5	0.707	0.866	1	\uparrow
$\cos(\theta / ^\circ)$	1	0.866	0.707	0.5	0	\downarrow

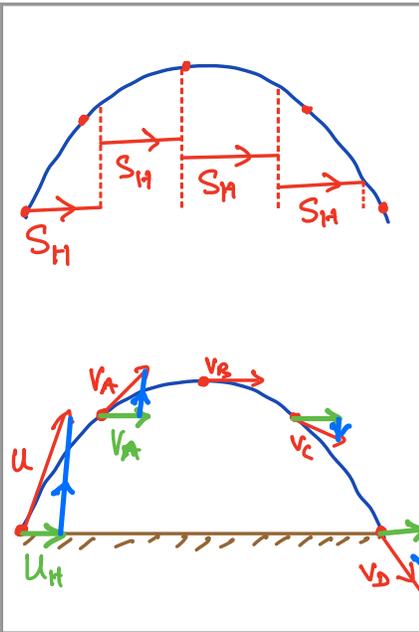


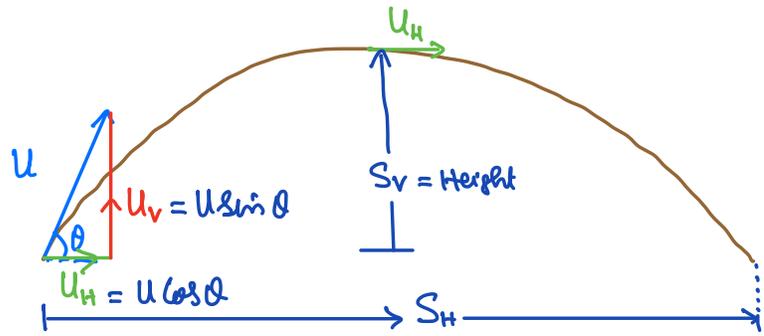
Analysis:

Position	A	B	C	D	E	Result
Vertical velocity motion	$U \sin \theta_A$ Max (Max)(Max) = Max	$V_B \sin \theta_B$ $\downarrow \quad \downarrow$ (\downarrow)(\downarrow) = Decrease	$V_C \sin \theta_C$ Min $\underline{0}$ (Min)(0) = Zero	$V_D \sin \theta_D$ $\uparrow \quad \uparrow$ (\uparrow)(\uparrow) = Increase	$V_E \sin \theta_E$ Max \underline{Max} (Max)(Max) = Max	Vertical component of velocity varies due to gravitational pull of Earth
Horizontal velocity motion	$U \cos \theta_A$ Max \underline{Min} (Max)(Min) = Constant	$V_B \cos \theta_B$ $\downarrow \quad \uparrow$ (\downarrow)(\uparrow) = Constant	$V_C \cos \theta_C$ Min \underline{Max} (Min)(Max) = Constant	$V_D \cos \theta_D$ $\uparrow \quad \downarrow$ (\uparrow)(\downarrow) = Constant	$V_E \cos \theta_E$ Max \underline{Min} (Max)(Min) = Constant	Horizontal component of velocity remain constant as no force acts horizontally

Conclusions:-

Motion	Diagram	Displacement travelled per unit time	Velocity	Acceleration	Force
Vertical motion		Decreases when moving upwards and increases when moving in downward direction	Decreases when moving upwards and becomes zero at the highest point. It increases again when moving in downward direction	upward motion $a = -g$ Highest Point $a = g$ Downward motion $a = +g$	$F = W = mg$

Horizontal motion		Remain constant	Remain constant	Remain zero	Remain zero i.e. no force is acting along horizontal direction.
-------------------	---	-----------------	-----------------	-------------	---



Height gained by projectile :-

Consider vertical motion i.e. vertical component of velocity

$$2 a s_v = v_v^2 - u_v^2$$

$$2(-g) S_v = (0)^2 - (u \sin \theta)^2$$

$$-2g S_v = -u^2 \sin^2 \theta$$

Loss of $E_k = \text{Gain of } G.P$

$$\frac{1}{2} m u_v^2 = m g h$$

$$\frac{(u \sin \theta)^2}{2} = g (S_v)$$

$$S_v = \frac{u^2 \sin^2 \theta}{2g}$$

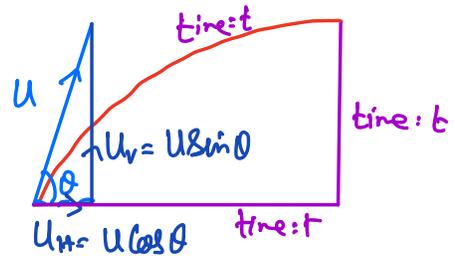
Time taken to reach the highest point:-

Consider vertical motion

$$v_v = u_v + a_v t$$

$$0 = u \sin \theta + (-g)t$$

$$t = \frac{u \sin \theta}{g}$$



Time of flight:- This is the total time to go up and come back.

$$T = 2t$$

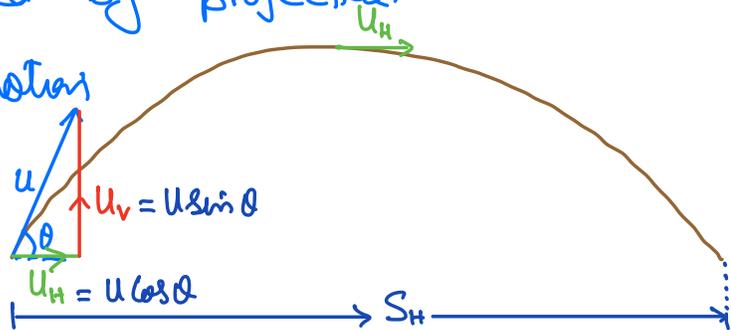
$$T = \frac{2u \sin \theta}{g}$$

Range of projectile:- This is the horizontal distance travelled by projectile.

Consider horizontal motion

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$u_h = \frac{s_h}{T}$$



$$s_h = (u_h)(T)$$

$$= \frac{(u \cos \theta)(2u \sin \theta)}{g} = \frac{u^2}{g} (2 \sin \theta \cos \theta)$$

$$\left[\text{Identity: } \sin 2\theta = 2 \sin \theta \cos \theta \right. \\ \left. \text{i.e. } \sin 60 = 2 \sin 30 \cos 30 \right]$$

$$S_H = \frac{u^2}{g} (\sin 2\theta)$$

Angle for maximum range:- Initial speed or kinetic energy of stone is constant,
Since

$$S_H = \frac{u^2}{g} \sin 2\theta$$

$$S_H = (\text{Constant}) \sin 2\theta$$

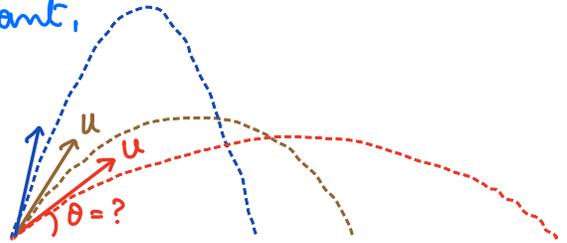
S_H is maximum if

$$\sin 2\theta = \text{maximum}$$

$$\sin 2\theta = 1$$

$$2\theta = \sin^{-1}(1) \Rightarrow 2\theta = 90^\circ$$

$$\theta = 45^\circ$$



PROJECTILE (Motion in two dimensions)

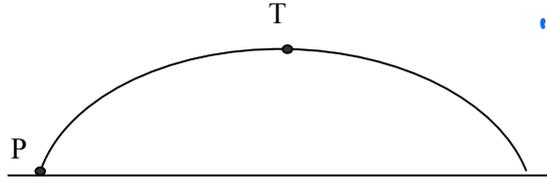
Akhtar Mahmood (0333-4281759)
M.Sc. (Physics), MCS, MBA-IT, B.Ed.
MIS, DCE, D AS/400e (IBM), OCP (PITB)

1. In the absence of air resistance, a stone is thrown from P and follows a parabolic path in which the higher point reached is T.

$$a_H = 0$$

$$a_V = g$$

$$a = g$$



'g' is constant at all positions through out flight

The vertical component of acceleration of the stone is

A zero at T

B greatest at T

C greatest at P

D the same at P as at T.

2. A motorcycle stunt-rider moving horizontally takes off from a point 1.25 m above the ground, landing 10 m away as shown.

Consider vertical motion to calculate time

$$S_V = u_V t + \frac{1}{2} a t^2$$

$$1.25 = (0)(t) + \frac{1}{2} (9.81) t^2$$

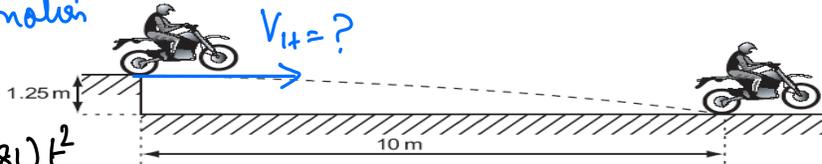
What was the speed at take-off?
 $t = 0.502$

A 5 m s^{-1}

B 10 m s^{-1}

C 15 m s^{-1}

D 20 m s^{-1}



Now consider horizontal motion

$$S_H = V_H t$$

$$10 = (V_H)(0.502)$$

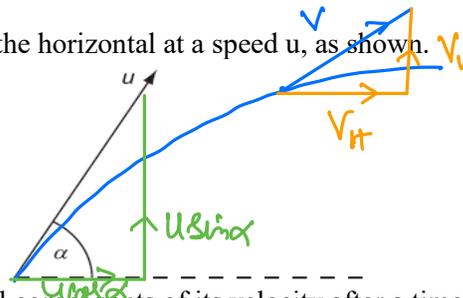
$$V_H = 20 \text{ m s}^{-1}$$

3. A projectile is fired at an angle α to the horizontal at a speed u , as shown.

vertical velocity

$$V_V = u_V + a_V t$$

$$V_V = u \sin \alpha + (-g)(t)$$



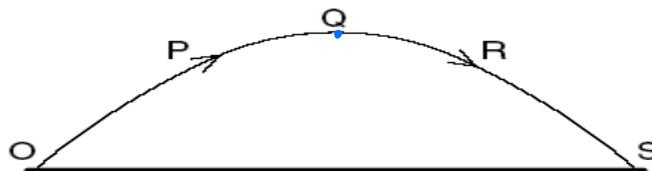
horizontal velocity

$$V_H = u_H = u \cos \alpha$$

What are the vertical and horizontal components of its velocity after a time t ? Assume that air resistance is negligible. The acceleration of free fall is g .

	vertical component	horizontal component
A	$u \sin \alpha$	$u \cos \alpha$ ✓
B	$u \sin \alpha - gt$ ✓	$u \cos \alpha - gt$
C	$u \sin \alpha - gt$ ✓	$u \cos \alpha$ ✓
D	$u \cos \alpha$	$u \sin \alpha - gt$

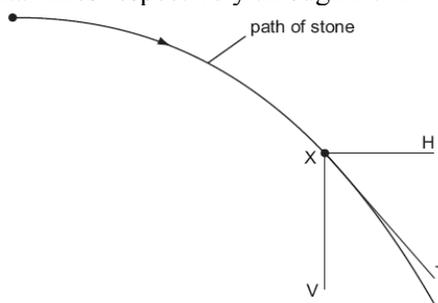
4. A projectile is launched at point O and follows the path OPQRS, as shown. Air resistance may be neglected.



Which statement is true for the projectile when it is at the highest point Q of its path?

- A** The horizontal component of the projectile's acceleration is zero. ✓ due to constant horizontal velocity
B The horizontal component of the projectile's velocity is zero. (non zero)
C The kinetic energy of the projectile is zero. (non zero)
D The momentum of the projectile is zero. (non zero)

5. A stone is projected horizontally in a vacuum and moves along a path as shown. X is a point on this path. XV and XH are vertical and horizontal lines respectively through X. XT is the tangent to the path at X.



Only Gravitational pull of Earth acts in downward direction in vacuum.

Along which direction or directions do forces act on the stone at X?

A XV

B XH

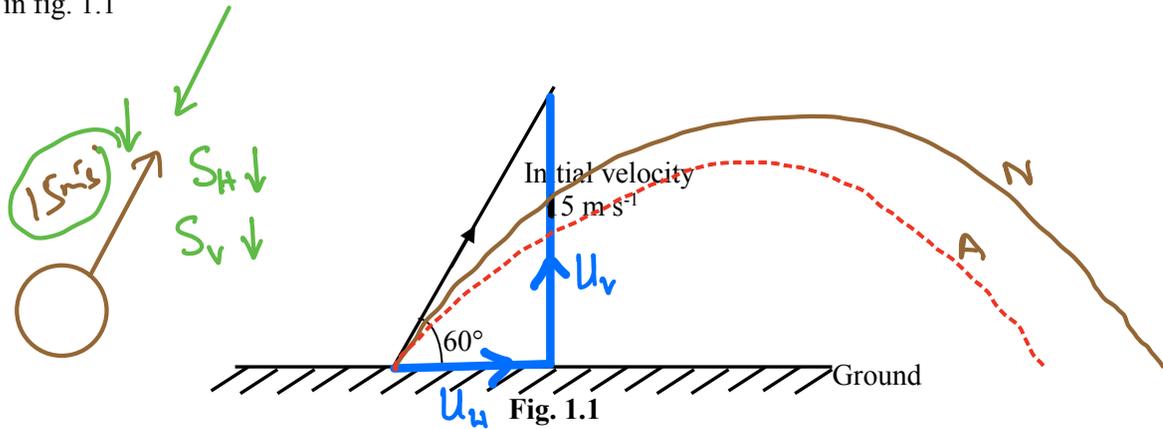
C XV and XH

D XT

PROJECTILE

Akhtar Mahmood (0333-4281759)
MSc.(Physics), MCS, MBA-IT, Bed.
MIS, DCE, D AS/400e(IBM), OCP(PITB)
teacher_786@hotmail.com

Q. 1 A ball is thrown from horizontal ground with an initial velocity of 15 ms^{-1} at an angle of 60° to the horizontal, as shown in fig. 1.1



(i) Calculate, for this ball, the initial values of

1. the vertical component of the velocity

$$U_v = U \sin \theta$$

$$= 15 \sin 60 = 13.0 \text{ ms}^{-1}$$

2. the horizontal component of the velocity

$$U_h = U \cos \theta$$

$$= 15 \cos 60 = 7.5 \text{ ms}^{-1}$$

[3]

(ii) Assuming that air resistance can be neglected, use your answers in (i) to determine

1. the maximum height to which the ball rises,

Consider vertical components only

$$2as_v = v_v^2 - u_v^2$$

$$2(-9.81)h = (0)^2 - (13.0)^2$$

$$h = \frac{-169}{-19.6} = 8.61 \text{ m}$$

$$\frac{1}{2} m u_v^2 = m g h$$

$$\frac{(13.0)^2}{2} = (9.81) h$$

2. the time of flight, i.e. the time interval between the ball being thrown and returning to ground level,

Consider vertical motion

$$v_v = u_v + a_v t \Rightarrow 0 = 13.0 + (-9.81)t$$

$$t = \frac{13.0}{9.81} = 1.325 \text{ s}$$

Time of flight: $T = 2t = 2(1.325) = 2.65 \text{ s}$

3. the horizontal distance between the point from which the ball was thrown and the point where it strikes the ground.

Consider horizontal motion only

$$S_h = U_h T$$

$$S_h = (7.5)(2.65) = 19.9 \text{ m}$$

(iii) Use your answers to (ii) to sketch the path of the ball, assuming air resistance is negligible. Label it with N [6]

1. On your sketch in (iii), draw the path of the ball, assuming that air resistance cannot be neglected. Label this path A. [2]

2. Suggest an explanation for any difference between the two paths N and A.

Both the vertical height gained and horizontal distance travelled decrease in air resistance. [6]

Q.2 A steel ball is projected horizontally from a catapult near the ceiling of a laboratory. Photographs of the position of the ball are taken at 0.10 s intervals. The photographs are superimposed and the result is illustrated in Fig. 2.1

horizontal distance / m

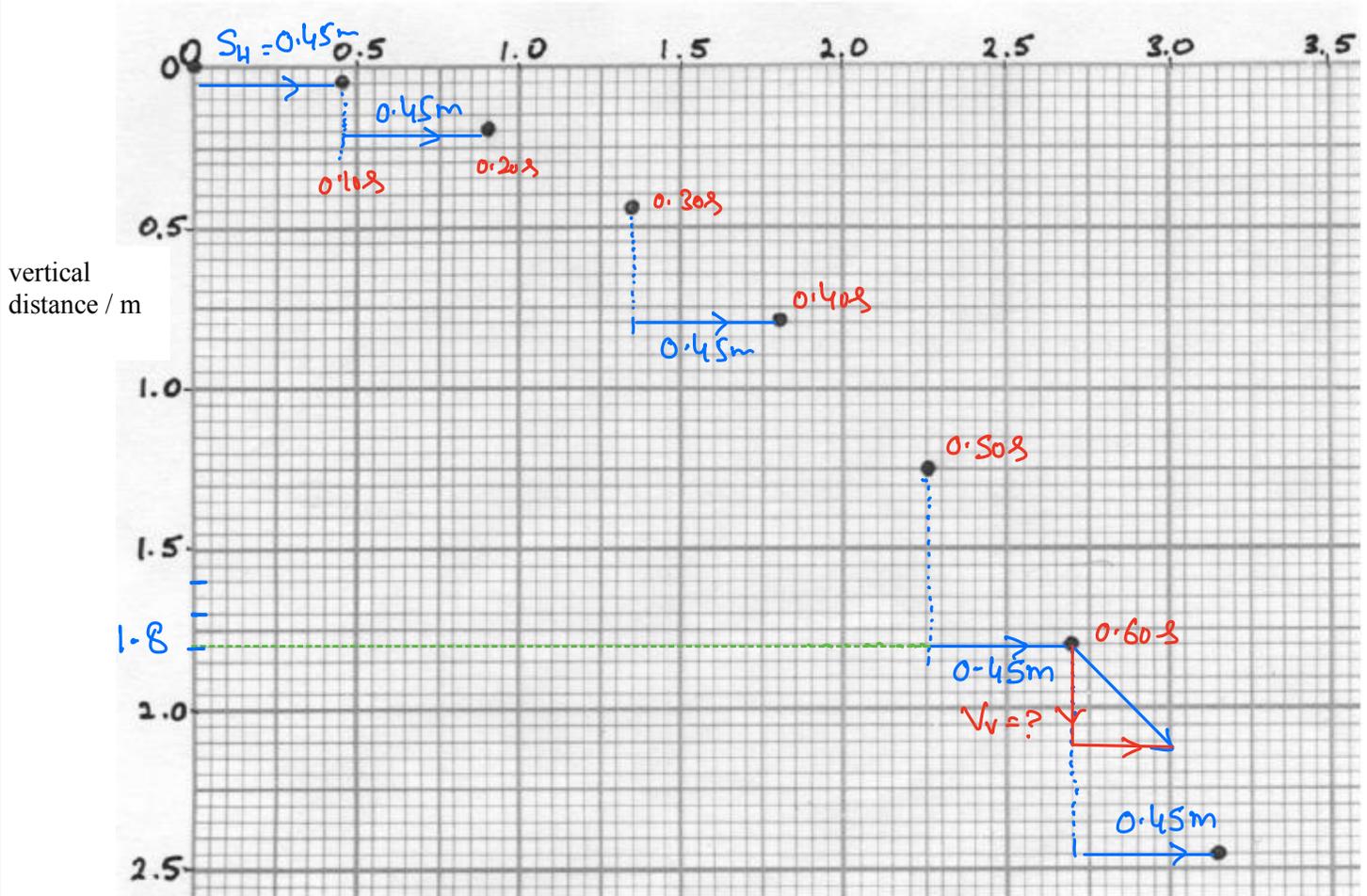


Fig. 2.1

The first photograph is taken at the instant when the ball is projected.

(a) (i) Use Fig. 2.1 to show, by considering the horizontal motion of the ball, that air resistance is negligible.

Equal horizontal displacement of 0.45m is travelled in equal time interval of 0.10s. So horizontal velocity is constant and no force acts horizontally.

(ii) Hence calculate the horizontal velocity of the ball.

$$S_H = V_H t$$

$$0.45 = (V_H)(0.10)$$

horizontal velocity = 4.5 m s⁻¹[3]

(b) (i) From figure 2.1, read off the vertical distance through which the ball falls during the first 0.60 s of its motion.

distance = 1.8 m

(ii) Use your answer to (i) to calculate the vertical speed of the ball 0.60 s after projection.

$2as_v = v_v^2 - u_v^2$ $2(9.81)(1.8) = v_v^2 - (0)^2$	$S_v = v_v t - \frac{1}{2} a_v t^2$ $1.8 = (v_v)(0.60) - \frac{1}{2}(9.81)(0.60)^2$	$\frac{1}{2} v^2 = v_y g h$ $v = \sqrt{2gh} = \sqrt{2(9.81)(1.8)}$
--	---	--

speed = m s⁻¹
 [5]

Q. 3 A rugby ball is kicked towards the goal posts shown in Fig. 3.1 from a position directly in front of the posts. The ball passes over the cross-bar and between the posts.

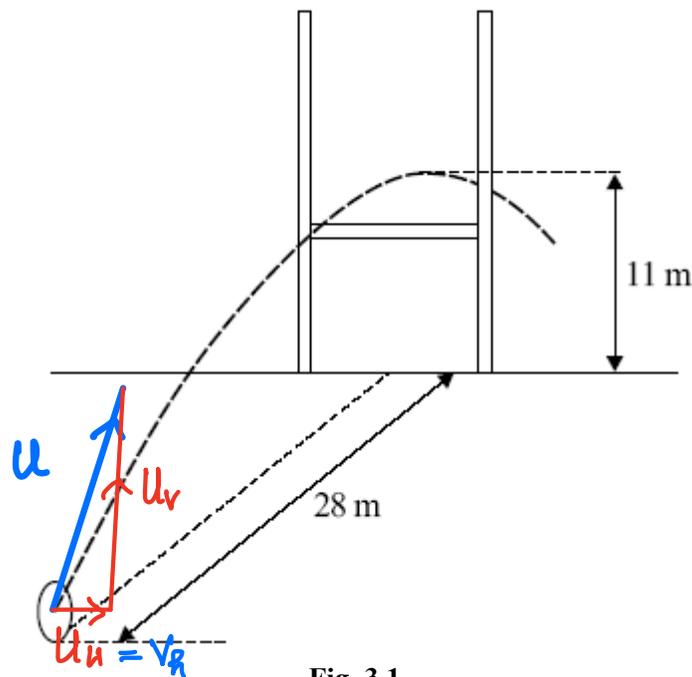


Fig. 3.1

- (a) The ball takes 1.5 s to reach a point vertically above the cross-bar of the posts.
 (i) Calculate the ball's horizontal component of velocity, V_h . Ignore air resistance.

Consider horizontal motion

$$S_h = V_h t$$

$$28 = (V_h)(1.5)$$

V_h 18.7 m s^{-1} [2]

- (ii) The ball reaches its maximum height at the same time as it passes over the cross-bar. State the vertical component of velocity when the ball is at its maximum height.

Zero

..... [1]

- (iii) The ball's maximum height is 11 m. Calculate, V_v , the vertical component of velocity of the ball immediately after it has been kicked. Ignore the effects of air resistance. acceleration due to gravity, $g = 9.8 \text{ m s}^{-2}$

Initial get vertical velocity

$$2as_v = v_v^2 - u_v^2$$

$$2(-9.8)(11) = (0)^2 - u_v^2$$

$$S_v = u_v t + \frac{1}{2} a t^2$$

$$11 = (u_v)(1.5) + \frac{1}{2}(-9.8)(1.5)^2$$

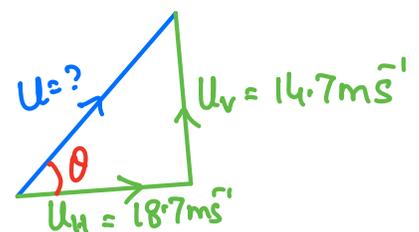
$$\frac{1}{2} m v^2 = m g h$$

$$v_v = \sqrt{2 g h} = \sqrt{2(9.8)(11)}$$

V_v 14.7 m s^{-1} [3]

- (b) (i) Determine the magnitude of the initial velocity, v , of the ball immediately after it is kicked.

$$u = \sqrt{(18.7)^2 + (14.7)^2}$$



V [3]

- (ii) Determine the angle above the horizontal at which the ball was kicked.

$$\tan \theta = \frac{14.7}{18.7} \Rightarrow \theta = \tan^{-1} \left(\frac{14.7}{18.7} \right)$$

Angle [1]

- (c) State and explain at what instant the ball will have its maximum kinetic energy.

$E_k = \frac{1}{2} m v^2$, Since velocity is maximum when ball is kicked or when it hits the ground and so is the kinetic energy.

Q.4 A stone is thrown with a horizontal velocity of 20 m s^{-1} from the top of a cliff 15 m high. The path of the stone is shown in Fig. 4.1.

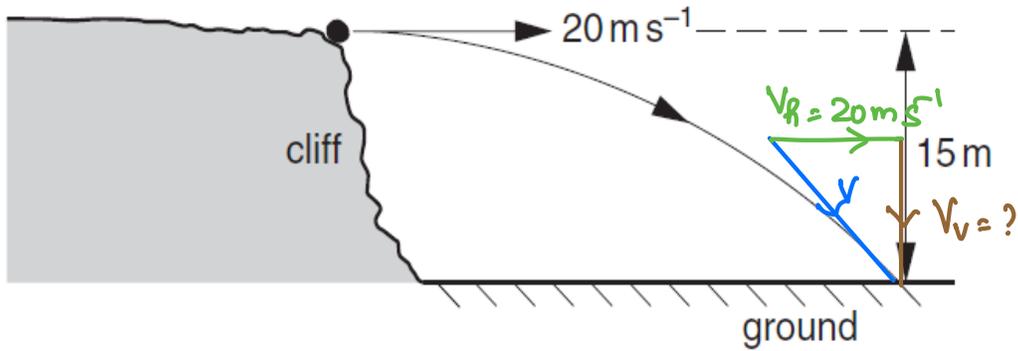


Fig. 4.1

Air resistance is negligible.

For this stone,

(i) calculate the time to fall 15 m ,

Consider vertical motion

$$S_v = u_v t + \frac{1}{2} a_v t^2$$

$$15 = (0)(t) + \frac{1}{2} (9.81) t^2$$

$$t = \sqrt{\frac{2(15)}{9.81}}$$

time = 1.75 s [2]

(ii) calculate the magnitude of the resultant velocity after falling 15 m ,

Consider vertical motion

$$V_v = u_v + a_v t$$

$$V_v = 0 + (9.81)(1.75)$$

$$V_v = 17.2 \text{ m s}^{-1}$$

$$S_v = v_v t - \frac{1}{2} a_v t^2$$

$$15 = (V_v)(1.75) - \frac{1}{2}(9.81)(1.75)^2$$

$$2a_v S_v = v_v^2 - u_v^2$$

$$2(9.81)(15) = v_v^2 - 0$$

$$\frac{1}{2} m v^2 = mgh$$

$$v_v = \sqrt{2gh}$$

$$V_v = \sqrt{2(9.81)(15)}$$

$$V = \sqrt{V_{Ht}^2 + V_v^2} \Rightarrow V = \sqrt{(20)^2 + (17.2)^2}$$

resultant velocity = m s⁻¹ [3]

(iii) describe the difference between the displacement of the stone and the distance that it travels.

Shortest directed distance is displacement and is a vector while real path travelled by object is distance and is a scalar. [2]

{Q.1/9702/22/M/J/11}

MCQ (KINEMATICS)

*A

A moving body undergoes uniform acceleration while travelling in a straight line between points X, Y and Z. The distances XY and YZ are both 40 m . The time to travel from X to Y is 12 s and from Y to Z is 6.0 s .

What is the acceleration of the body?

A 0.37 m s^{-2}

B 0.49 m s^{-2}

C 0.56 m s^{-2}

D 1.1 m s^{-2}

{Q.8/9702/12/O/N/10}

A moving body undergoes uniform acceleration while travelling in a straight line between points X, Y and Z. The distances XY and YZ are both 40 m. The time to travel from X to Y is 12 s and from Y to Z is 6.0 s.

What is the acceleration of the body?

A 0.37 m s^{-2}

B 0.49 m s^{-2}

C 0.56 m s^{-2}

D 1.1 m s^{-2}

{Q.8/9702/12/O/N/10}



Here initial velocity at X is also unknown.

For XY

$$S_{xy} = u_x t_{xy} + \frac{1}{2} a t_{xy}^2$$

$$40 = (u_x)(12) + \frac{1}{2}(a)(12)^2$$

$$40 = 12u_x + 72a \dots (1)$$

For XZ

$$S_{xz} = u_x t_{xz} + \frac{1}{2} a t_{xz}^2$$

$$80 = (u_x)(18) + \frac{1}{2}(a)(18)^2$$

$$80 = 18u_x + 162a \dots (2)$$

Solve (1) and (2) simultaneously to get acceleration (a) i.e. $a = 0.37 \text{ m s}^{-2}$

Q.5 A ball is thrown from a point P, which is at ground level, as illustrated in Fig. 5.1.

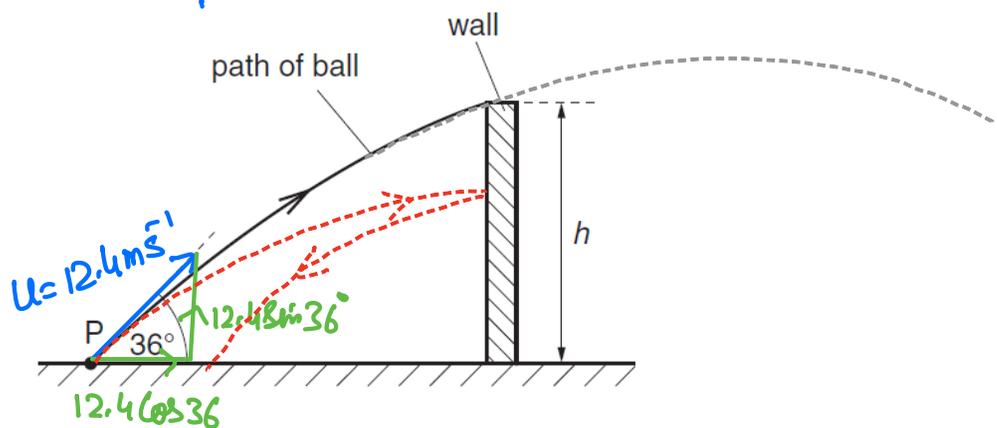


Fig.5.1

The initial velocity of the ball is 12.4 m s^{-1} at an angle of 36° to the horizontal. The ball just passes over a wall of height h . The ball reaches the wall 0.17 s after it has been thrown.

- (a) Assuming air resistance to be negligible, calculate
 (i) the horizontal distance of point P from the wall,

Consider horizontal motion

$$S_H = V_H t$$

$$S_H = (12.4 \cos 36)(0.17)$$

distance = 1.71 m [2]

- (ii) the height h of the wall.

$$S_V = U_V t + \frac{1}{2} a_V t^2$$

$$h = (12.4 \sin 36)(0.17) + \frac{1}{2} (-9.81)(0.17)^2$$

$$h =$$

$h =$ m [3]

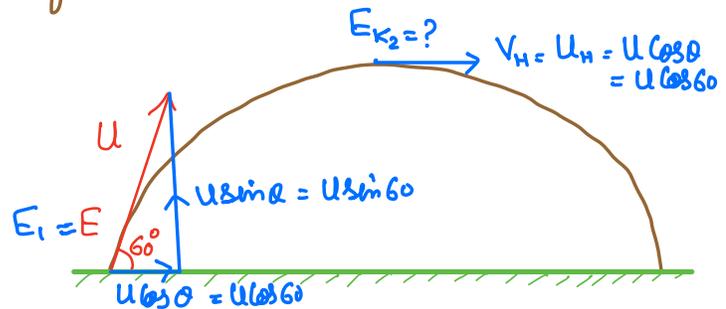
- (b) A second ball is thrown from point P with the same velocity as the ball in (a). For this ball, air resistance is not negligible.

This ball hits the wall and rebounds.

On Fig. 5.1, sketch the path of this ball between point P and the point where it first hits the ground. [2]

{Q.2/9702/22/O/N/10}

Q) A ball of mass m is thrown at 60° with horizontal with a speed u and kinetic energy E . What is its kinetic energy at the highest point in terms of E ?



$$\frac{E_{K_2}}{E_{K_1}} = \frac{\frac{1}{2} m u_H^2}{\frac{1}{2} m u^2}$$

$$\frac{E_{K_2}}{E} = \frac{(u \cos 60)^2}{u^2}$$

$$\frac{E_{K_2}}{E} = \frac{u^2 (\cos 60)^2}{u^2}$$

$$\frac{E_{K_2}}{E} = \left(\frac{1}{2}\right)^2$$

$$\boxed{E_{K_2} = \frac{E}{4}}$$

Acceleration - time graph:-

Dependent Qty (Y-axis)

Independent Qty (x-axis)

Results:-

(i) Instantaneous acceleration \rightarrow Y-axis

(ii) Analysis of graph \rightarrow Equilibrium ($a=0$)

Rest
move with
Uniform velocity

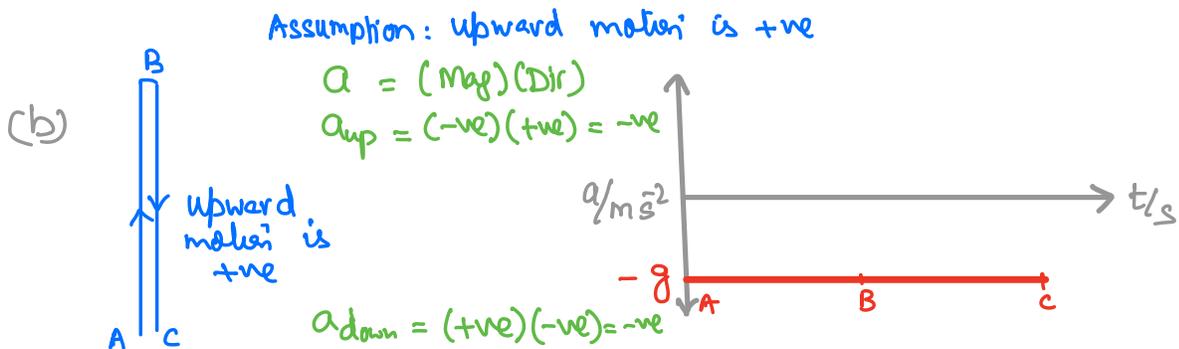
Non-uniform velocity ($a \neq 0$)

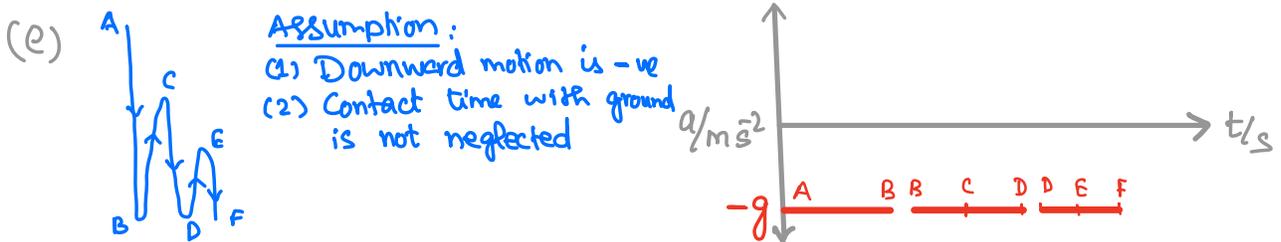
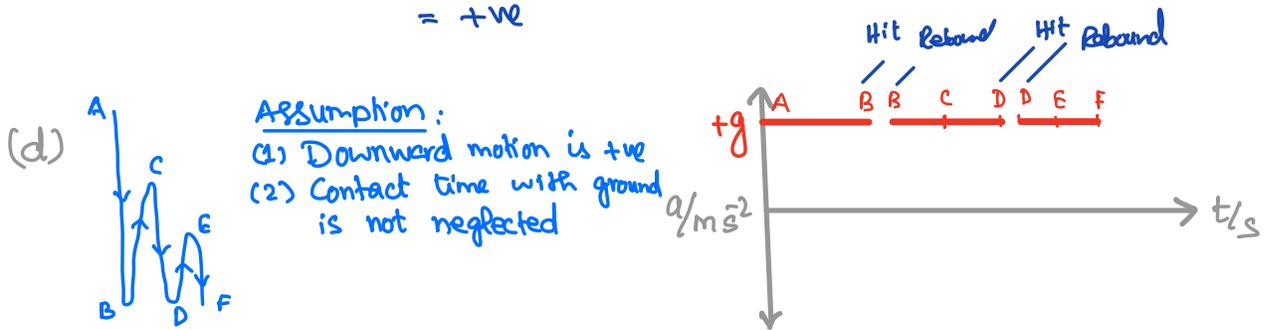
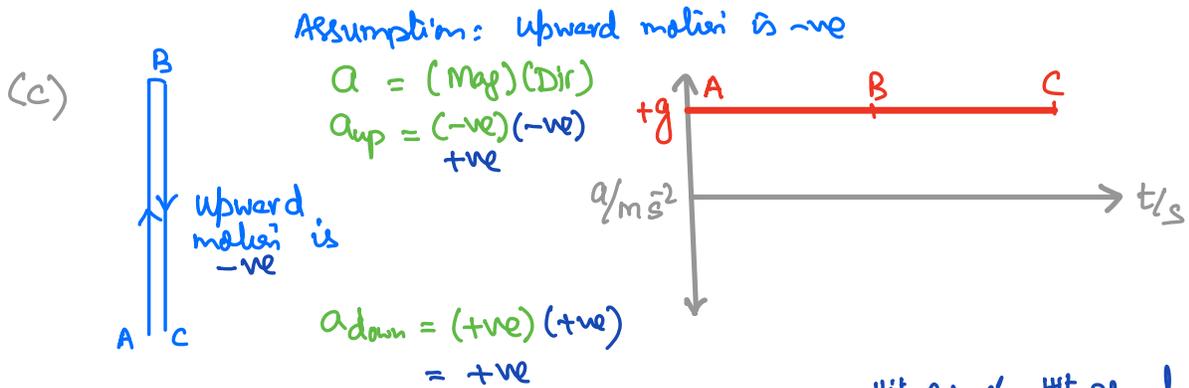
Direction of motion depends upon assumption.

Uniform Acc
Non-uniform Acceleration due to air resistance

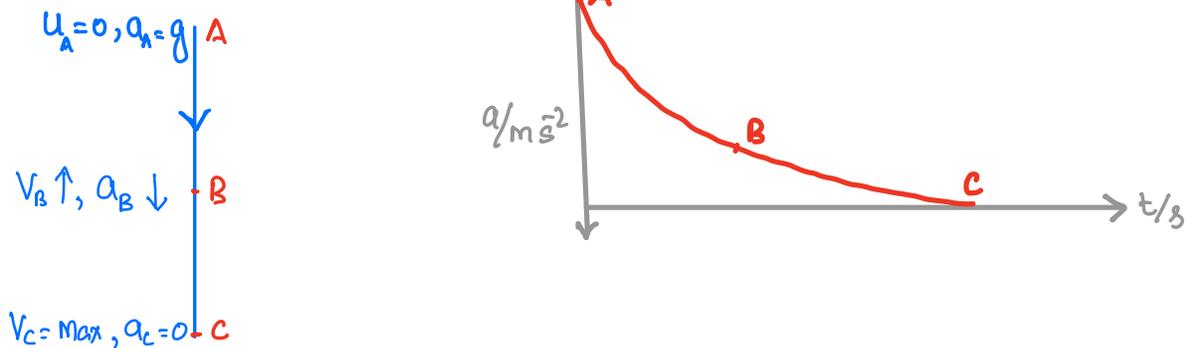


Equilibrium position is either at rest or move with uniform velocity.



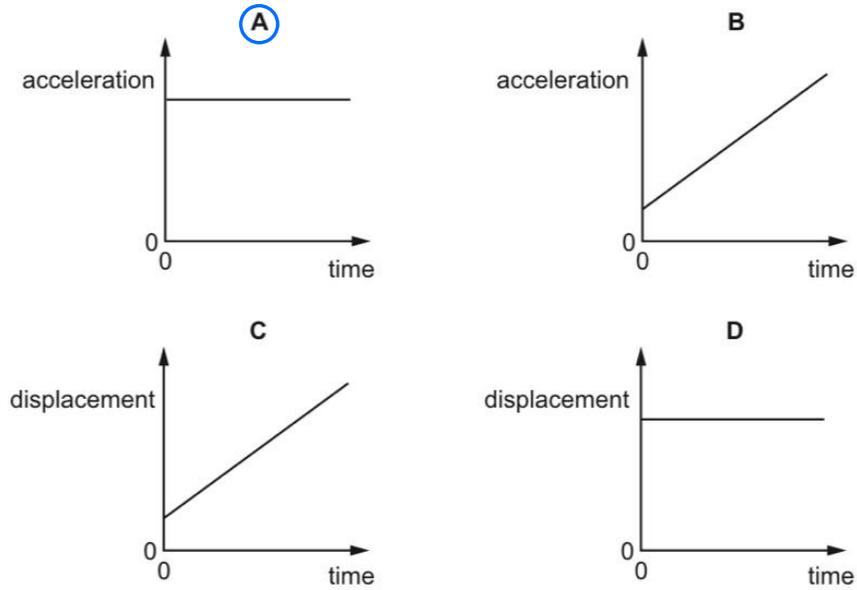


(f) An object is dropped from a great height and air resistance is not neglected so that terminal velocity is achieved.



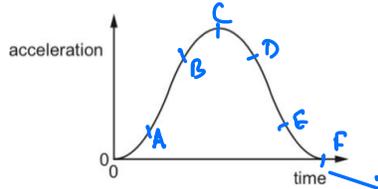
Nov 14
P11/Q6
P12/Q6
Q.1)

Which graph represents the motion of a car that is travelling along a straight road with a speed that increases uniformly with time? *no change of direction*
uniform acceleration



June 14
P11/Q8
June 11
P13/Q7
Q.2)

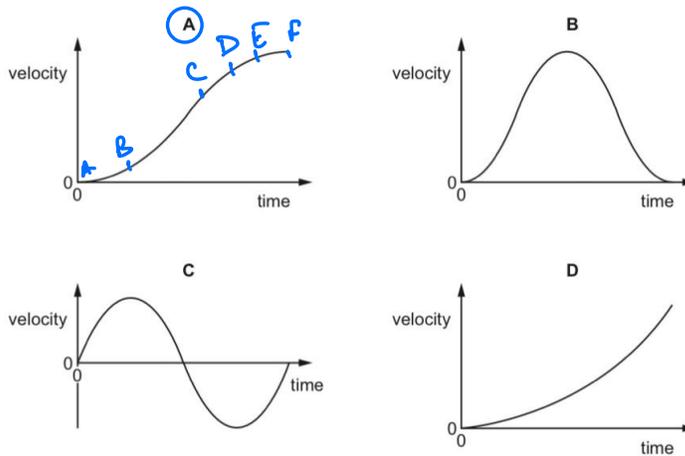
The graph shows how the acceleration of an object moving in a straight line varies with time.



The object starts from rest. $u = 0$

$a_F = 0$
unif/terminal velocity

Which graph shows the variation with time of the velocity of the object over the same time interval?



Nov. 10
P12/Q7

A student throws a ball in the positive direction vertically upwards.

The ball makes an elastic collision with the ceiling, rebounds and accelerates back to the student's hand in a time of 1.2 s.

(Q.3)

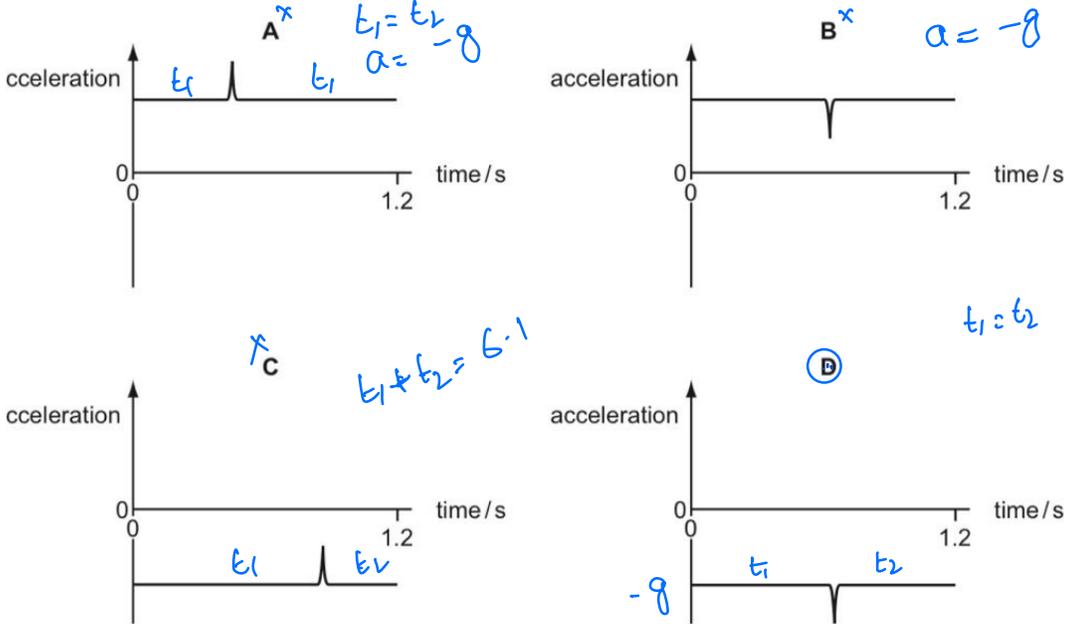
Which graph best represents the acceleration of the ball from the moment it leaves the hand to the instant just before it returns to the hand?

$a = (\text{Mag})(\text{Direction})$

$a_{\text{up}} = (-ve)(+ve) = -g$

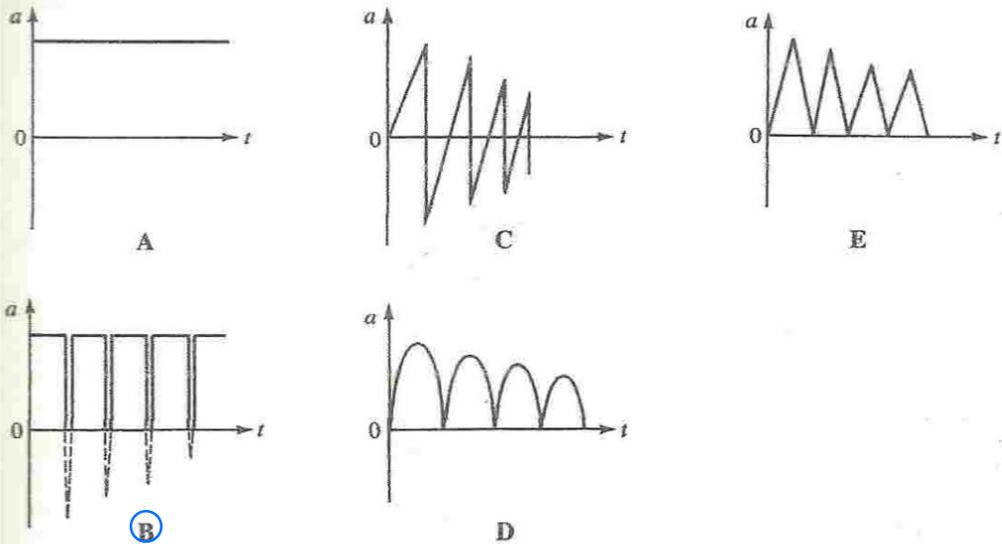
$a_{\text{down}} = (+ve)(-ve) = -g$

on hitting, $a \uparrow$ because hitting force is maximum



(Q.4)

4. A steel ball held above a horizontal table is released so that it falls onto the table and rebounds several times. Which one of the graphs below best represents the variation of the ball's acceleration a with t , if the collisions are inelastic?



Q) Give two situations in which velocity of a body is zero but it is accelerating.

(i) If a body start from rest.

(ii) At the highest point, velocity is zero but body is accelerating due to Gravitational pull of Earth.