



DYNAMICS (MOMENTUM)

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Kishan Pasid

Momentum

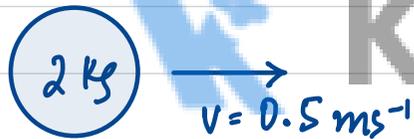
- Understanding : It is the amount of motion.
- Understanding : It is the strength of motion.

Definition: It is the product of mass and velocity of the body.

$$p = m \cdot v$$

mass
velocity

SI Unit: Ns or kgms^{-1} | Vector Quantity momentum



$$p = mv$$
$$p = (2)(0.5)$$
$$p = 1 \text{ Ns}$$

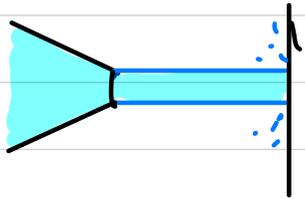


$$p = mv$$
$$p = (0.5)(2)$$
$$p = 1 \text{ Ns}$$

Two bodies can have the same momentum even if they have different masses & velocities.

Change of momentum

Due to change
in mass



$$\Delta p = p_f - p_i$$

$$\Delta p = m_f v - m_i v$$

$$\Delta p = (m_f - m_i) v$$

$$\Delta p = \Delta m \cdot v$$

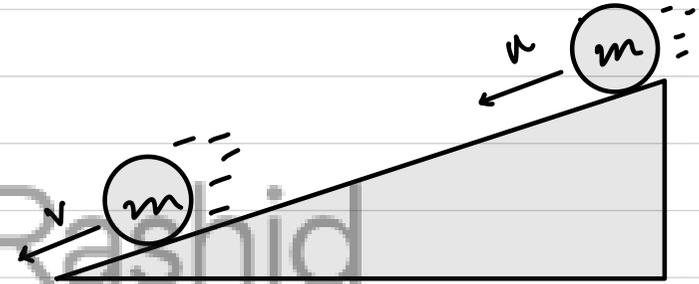
Application: flowing fluids,
fuel consumption in moving
bodies.

Due to change
in velocity

speed
up

slow
down

change in
direction



$$\Delta p = p_f - p_i$$

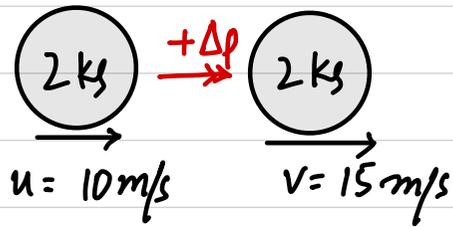
$$= mv - mu$$

$$\Delta p = m(v - u)$$

$$\Delta p = m \cdot \Delta v$$

Application: solid bodies in
motion

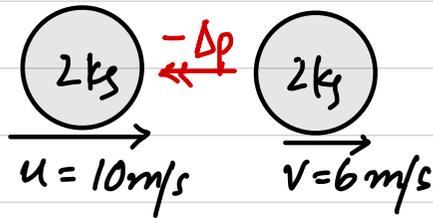
- ← ————— → +



$$\Delta p = m(v - u)$$

$$\Delta p = (2)(15 - 10)$$

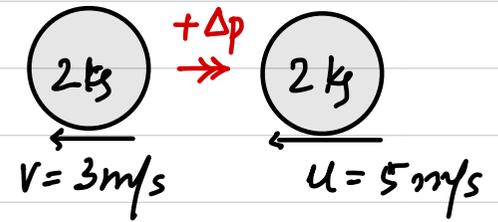
$$\Delta p = 10 \text{ Ns}$$



$$\Delta p = m(v - u)$$

$$= 2(6 - 10)$$

$$\Delta p = -8 \text{ Ns}$$



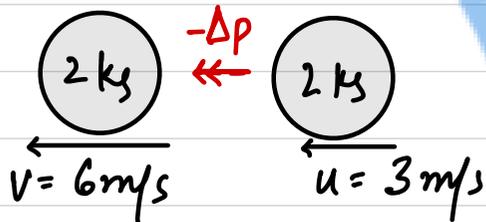
$$\Delta p = m(v - u)$$

$$= 2(-3 - (-5))$$

$$\Delta p = 4 \text{ Ns}$$



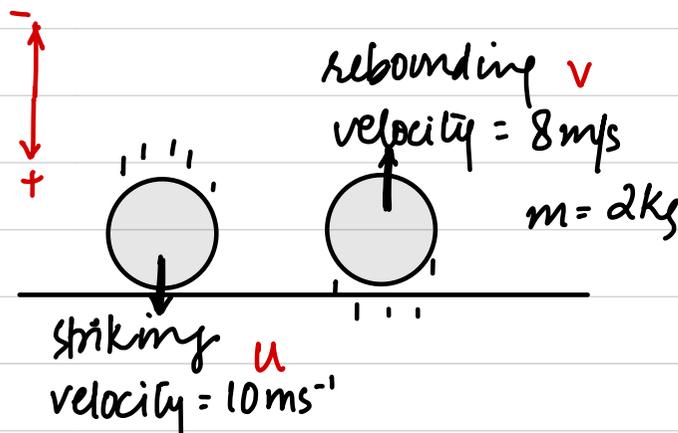
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$$\Delta p = m(v - u)$$

$$\Delta p = 2(-6 - (-3))$$

$$\Delta p = -6 \text{ Ns}$$



change of momentum

$$\Delta p = m(v - u)$$

$$= 2(-8 - 10)$$

$$\Delta p = -36 \text{ Ns}$$

Force

Rate of change of momentum is called force.

$$F = \frac{\Delta p}{\Delta t}$$

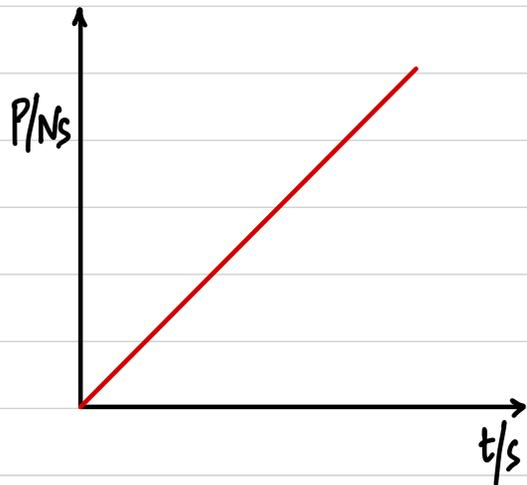
- For a greater change in momentum in the same time span, the force required is greater.
- For the same change of momentum but in a shorter span of time, we need greater force.

$$\Delta p = F \times \Delta t$$

↓ ↓ ↓
IMPULSE Force time of application of force.

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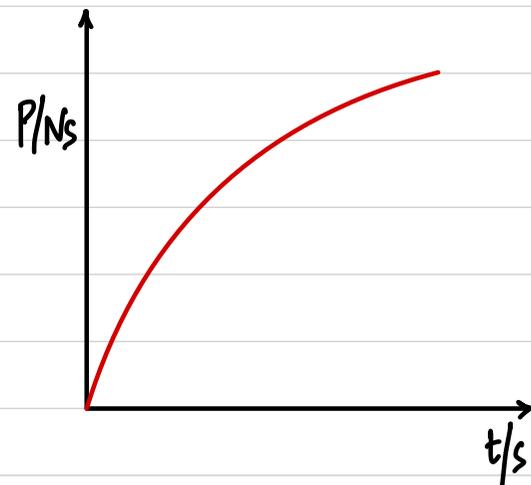
GRADIENT OF p-t graph = Force



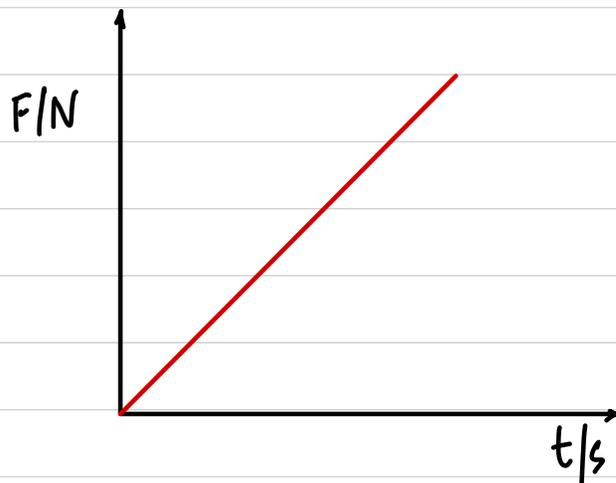
gradient: constant
Force: constant
Acceleration: constant



gradient: increases
Force: increases
Acceleration: increases



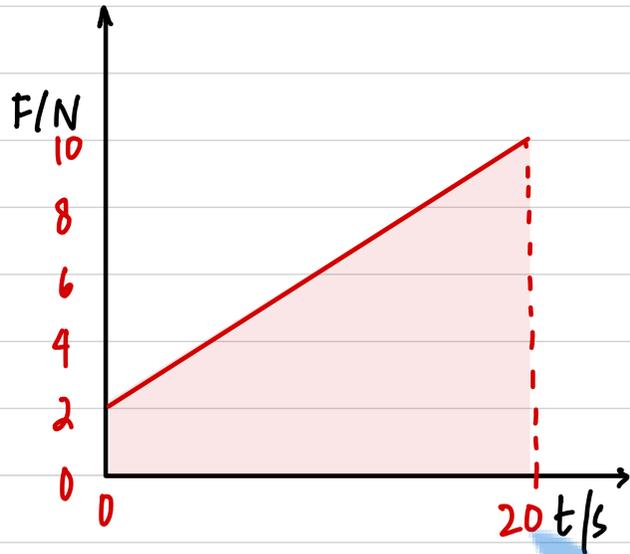
gradient: decreases
Force: decreases
Acceleration: decreases



$$\text{Area} = \frac{1}{2} b \times h$$

$$= \frac{1}{2} (t \times F)$$

Area = change in momentum



An object of mass 2 kg travelling at 2 ms^{-1} experiences a force as shown in the graph.

Determine the final velocity of the object after 20s.

$$A = \frac{1}{2}(p+q)h$$

$$A = \frac{1}{2}(2+10) \times 20$$

$$A = \Delta p = 120 \text{ Ns}$$

$$\Delta p = m(v-u)$$

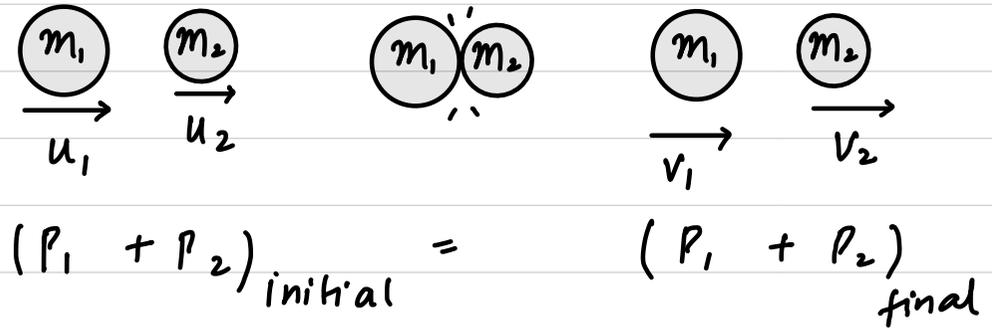
$$120 = 2(v-2)$$

$$v = 62 \text{ ms}^{-1}$$

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Law of Conservation of Momentum

- ✓ For an isolated system,
- ✓ the total momentum before collision is equal to the total momentum after collision.



$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

- ✓ For an isolated system,
- ✓ the momentum lost by one body equals the momentum gained by the other during collision.

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$m_1 (u_1 - v_1) = m_2 (v_2 - u_2)$$
$$-m_1 (v_1 - u_1) = m_2 (v_2 - u_2)$$

$$-\Delta p_1 = \Delta p_2$$

- ✓ For an isolated system,
- ✓ the change in momentum of the entire system is zero.

$$\Delta p_1 + \Delta p_2 = 0$$

$$\Delta p_{\text{sys}} = 0$$

Types of Collisions

ELASTIC (IDEAL)

- No loss of energy as heat
- Collision of small masses, subatomic particles e.g. neutrons, atoms, molecules, dust particles.

Momentum: conserved

Kinetic Energy: conserved

Total Energy: conserved

INELASTIC (REAL)

- Loss of energy as heat
- Collisions of large masses, explosions, objects stick after collision.

Momentum: conserved

Kinetic Energy: not conserved

Total Energy: conserved

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Checking whether the collision is elastic or inelastic

1. If the collision is elastic, then

Total K.E before collision = Total K.E after collision

$$(K.E_1 + K.E_2)_{\text{initial}} = (K.E_1 + K.E_2)_{\text{final}}$$

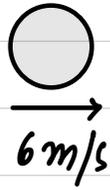
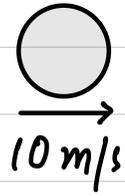
$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

2. If the collision is elastic, then

Relative speed of approach = Relative speed of separation

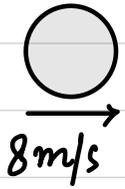
If the bodies have equal mass and the collision is elastic, the bodies exchange their velocities.



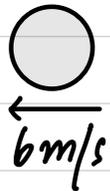
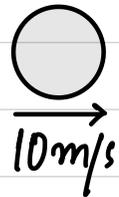


Relative Speed
of Approach = $10 - 6 = 4 \text{ m/s}^{-1}$

If bodies are moving
in the same direction,
you subtract speeds.

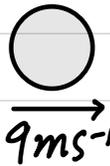
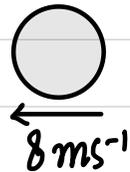


Relative speed
of Separation = $12 - 8 = 4 \text{ m/s}^{-1}$



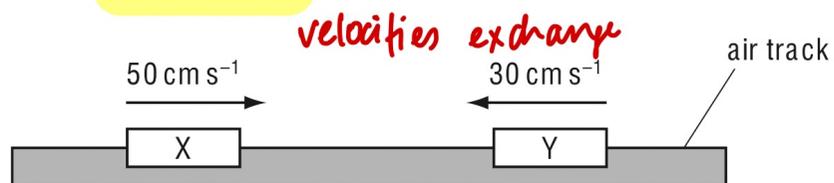
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Relative Speed
of Approach = $10 + 6 = 16 \text{ m/s}^{-1}$

If bodies are moving
in opposite direction,
you add speeds.



Relative Speed
of Separation = $8 + 9 = 17 \text{ m/s}^{-1}$

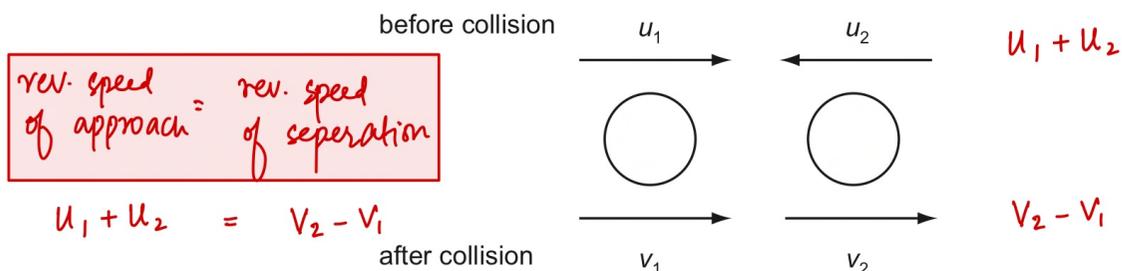
- 10 Two equal masses X and Y are moving towards each other on a frictionless air track as shown. The masses make an elastic collision. **DYNAMICS 2008**



Which row gives possible velocities for the two masses after the collision?

	velocity of X	velocity of Y
A	zero	20 cm s ⁻¹ to the right
B	10 cm s ⁻¹ to the right	10 cm s ⁻¹ to the right
C	20 cm s ⁻¹ to the left	zero
D	30 cm s ⁻¹ to the left	50 cm s ⁻¹ to the right

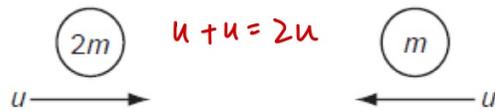
- 10 Two spheres approach each other along the same straight line. Their speeds are u_1 and u_2 before collision, and v_1 and v_2 after collision, in the directions shown below.



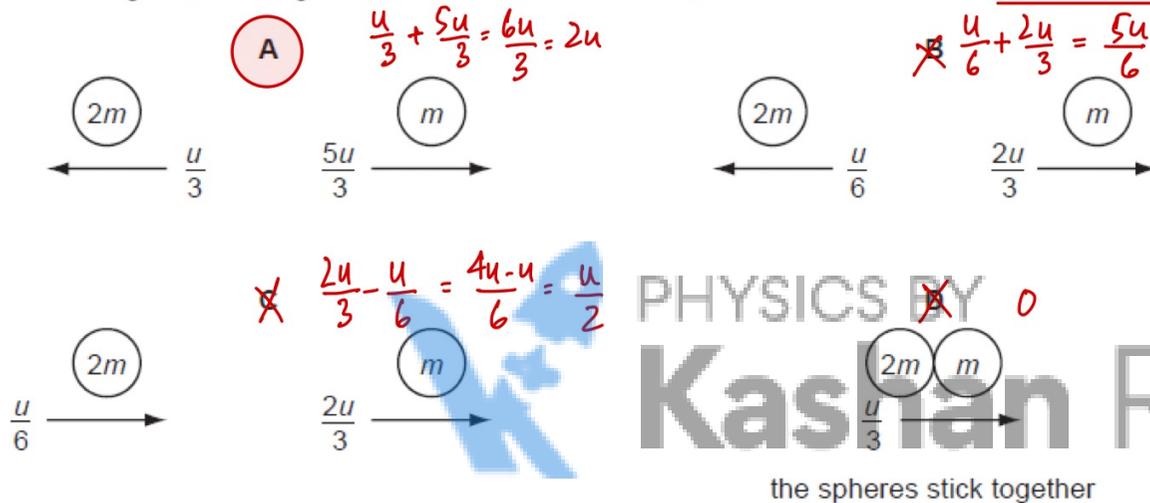
Which equation is correct if the collision is perfectly elastic?

- A** $u_1 - u_2 = v_2 + v_1$
- B** $u_1 - u_2 = v_2 - v_1$
- C** $u_1 + u_2 = v_2 + v_1$
- D** $u_1 + u_2 = v_2 - v_1$

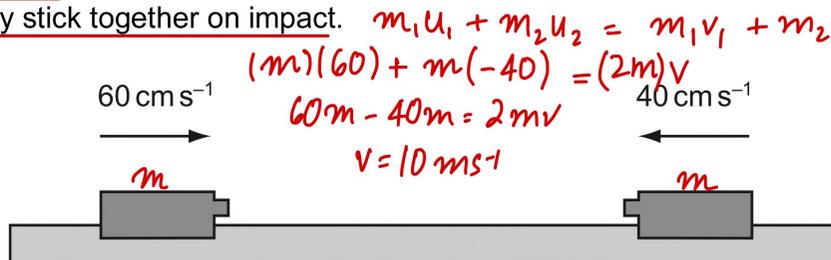
- 9 The diagram shows two spherical masses approaching each other head-on at an equal speed u . One has mass $2m$ and the other has mass m .



Which diagram, showing the situation after the collision, shows the result of an elastic collision?



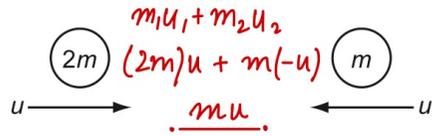
- 10 Two equal masses travel towards each other on a frictionless air track at speeds of 60 cm s^{-1} and 40 cm s^{-1} . They stick together on impact.



What is the speed of the masses after impact?

- A 10 cm s^{-1} B 20 cm s^{-1} C 40 cm s^{-1} D 50 cm s^{-1}

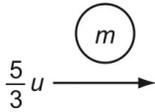
- 11 The diagram shows two spherical masses approaching each other head-on at an equal speed u . One is of mass m and the other of mass $2m$.



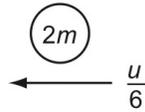
Which diagram, showing the situation after the collision, is not consistent with the principle of conservation of momentum?

$m_1 v_1 + m_2 v_2$
 $(2m)\left(-\frac{u}{3}\right) + m\left(\frac{5u}{3}\right)$
 $-\frac{2mu}{3} + \frac{5mu}{3}$
 \underline{mu}

A

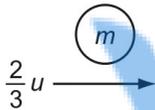
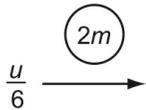


B

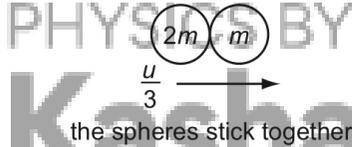


$m_1 v_1 + m_2 v_2$
 $(2m)\left(-\frac{u}{6}\right) + m\left(\frac{2u}{3}\right)$
 $-\frac{mu}{3} + \frac{2mu}{3}$
 $\frac{mu}{3}$
 after collision

C



D



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- 11 The diagram shows a man standing on a platform that is attached to a flexible pipe. Water is pumped through the pipe so that the man and platform remain at a constant height. *Equilibrium*

$F = \frac{\Delta p}{\Delta t}$

$F = \frac{\Delta m \cdot v}{\Delta t}$

$F = m \cdot \frac{\Delta v}{\Delta t}$

$F = ma$

$F = \dot{m}v$
flowing fluid

flexible pipe

water in →

platform

vertical jet of water out,
mass flow rate 40 kg s^{-1}

$F_{\text{fluid}} = W$

$\dot{m}v = mg$

$40 \times v = (96)(9.8)$

$v = 23.52 \text{ ms}^{-1}$

The resultant vertical force on the platform is zero. The combined mass of the man and platform is 96 kg. The mass of water that is discharged vertically downwards from the platform each second is 40 kg.

What is the speed of the water leaving the platform?

- A 2.4 ms^{-1} B 6.9 ms^{-1} **C 24 ms^{-1}** D 47 ms^{-1}

- 2 A ball B of mass 1.2 kg travelling at constant velocity collides head-on with a stationary ball S of mass 3.6 kg, as shown in Fig. 2.1.

For
Examiner's
Use

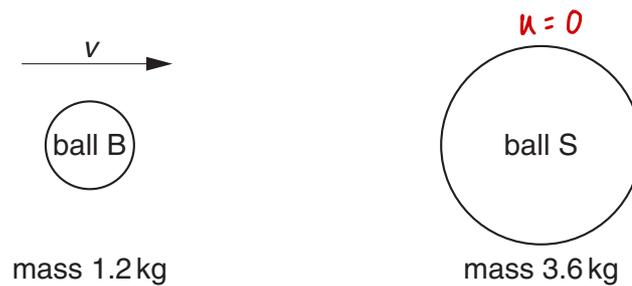


Fig. 2.1

Frictional forces are negligible.

The variation with time t of the velocity v of ball B before, during and after colliding with ball S is shown in Fig. 2.2.

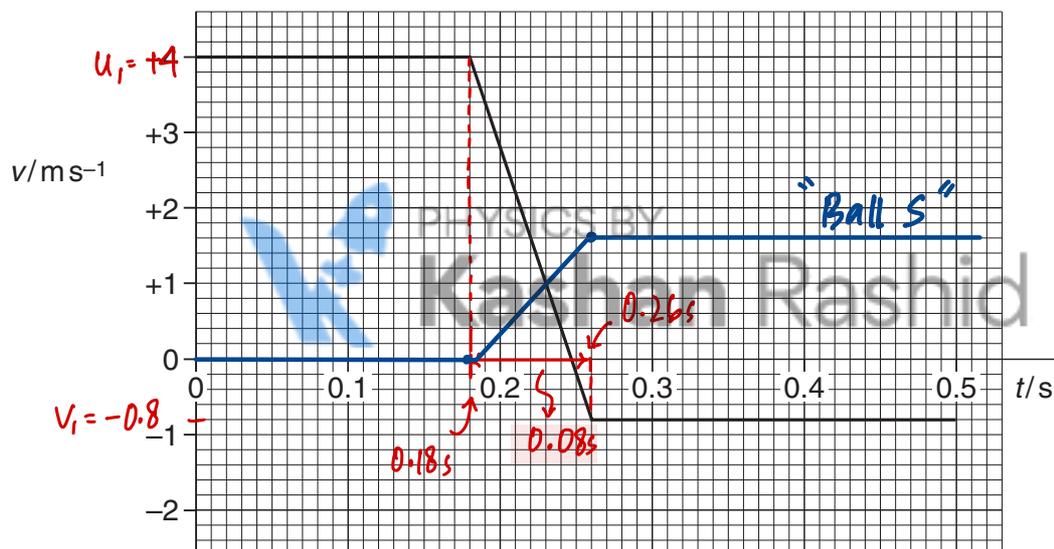


Fig. 2.2

- (a) State the significance of positive and negative values for v in Fig. 2.2.

It tells about the direction of motion.

[1]

(b) Use Fig. 2.2 to determine, for ball B during the collision with ball S,

For
Examiner's
Use

(i) the change in momentum of ball B,

$$\begin{aligned}\Delta p &= m(v-u) \\ &= 1.2(-0.8 - 4) \\ \Delta p &= -5.76\end{aligned}$$

change in momentum = -5.8 Ns [3]

(ii) the magnitude of the force acting on ball B.

$$\begin{aligned}F &= \frac{\Delta p}{\Delta t} \\ &= \frac{-5.76}{0.08} \\ F &= -72\text{ N}\end{aligned}$$



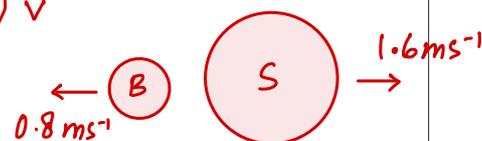
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force = 72 N [3]

(c) Calculate the speed of ball S after the collision.

$$\begin{aligned}m_1u_1 + m_2u_2 &= m_1v_1 + m_2v_2 \\ (1.2)(4) + (0) &= (1.2)(-0.8) + (3.6)v \\ v &= 1.6\text{ ms}^{-1}\end{aligned}$$

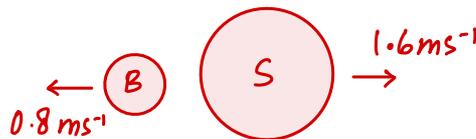


speed = 1.6 ms^{-1} [2]

- (d) Using your answer in (c) and information from Fig. 2.2, deduce quantitatively whether the collision is elastic or inelastic.

rel. speed
of approach : 4 ms^{-1}

rel. speed
of separation : $1.6 + 0.8$
 2.4 ms^{-1}



As rel. speed of approach and separation are
not equal, the collision is INELASTIC!

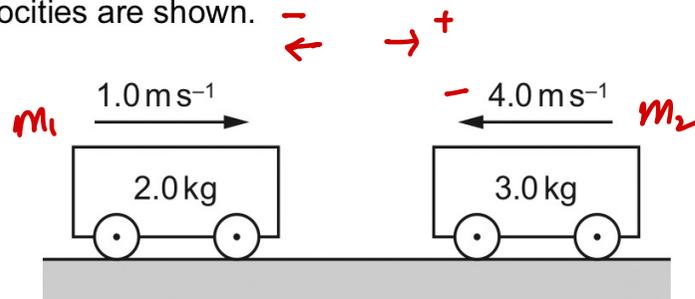
[2]

For
Examiner's
Use



Type #2. Bodies sticking after collision

- 11 Two frictionless trolleys are moving towards each other along the same horizontal straight line. Their masses and velocities are shown.



The trolleys collide and stick together.

What is the velocity of the trolleys after the collision?

- A 2.0 m s^{-1} to the left
- B 2.0 m s^{-1} to the right
- C 2.8 m s^{-1} to the left
- D 2.8 m s^{-1} to the right

$$m_1 u_1 + m_2 u_2 = M v$$



$$M = m_1 + m_2$$

combined mass

$$m_1 u_1 + m_2 u_2 = M v$$

$$(2)(1) + (3)(-4) = (2+3) v$$

$$v = -2 \text{ m s}^{-1}$$

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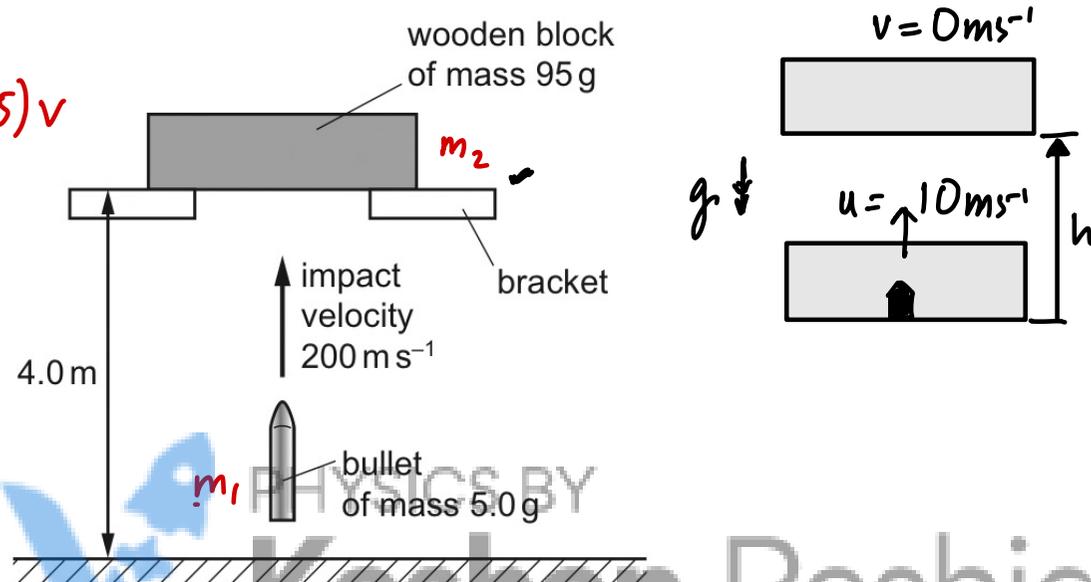
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MJ / 2015 / P13

13 A wooden block is freely supported on brackets at a height of 4.0 m above the ground, as shown.

$$m_1 u_1 + m_2 u_2 = Mv$$
$$(5)(200) + (95)(0) = (5+95)v$$
$$v = 10 \text{ m s}^{-1}$$

$$v^2 - u^2 = 2as$$
$$0^2 - 10^2 = 2(-9.8)s$$
$$s = \underline{5.1 \text{ m}}$$



A bullet of mass 5.0 g is shot vertically upwards into the wooden block of mass 95 g. It embeds itself in the block. The impact causes the block to rise above its supporting brackets.

The bullet hits the block with a velocity of 200 m s^{-1} . How far above the ground will the block be at the maximum height of its path?

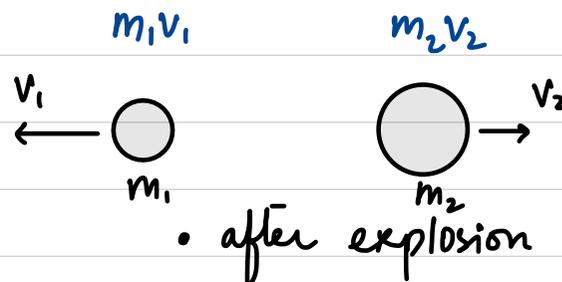
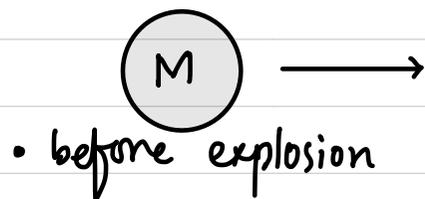
- A** 5.1 m **B** 5.6 m **C** 9.1 m **D** 9.6 m

Type #3

Explosion

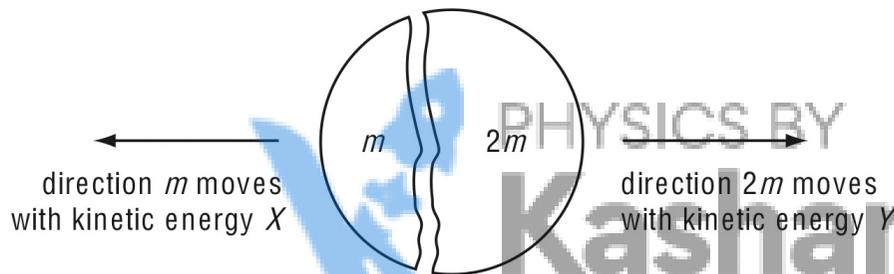
Mu

$$Mu = m_1 v_1 + m_2 v_2$$



10 A stationary body explodes into two components of masses m and $2m$.

The components gain kinetic energies X and Y respectively.



What is the value of the ratio $\frac{X}{Y}$?

- A $\frac{1}{4}$ B $\frac{1}{2}$ C $\frac{2}{1}$ D $\frac{4}{1}$

$$Mu = m_x v_x + m_y v_y$$

$$0 = (m)(-v_x) + (2m)(v_y)$$

$$mv_x = 2mv_y$$

$$v_x = 2v_y$$

THEORY QUESTION

0N/2009/P21/Q.3

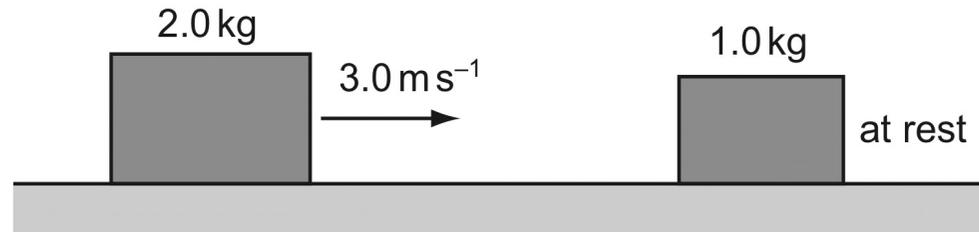
$$\frac{E_{kx}}{E_{ky}} = \frac{\frac{1}{2} m_x v_x^2}{\frac{1}{2} m_y v_y^2} = \frac{m_x v_x^2}{m_y v_y^2} = \frac{m \cdot v_x^2}{2m \cdot v_y^2}$$

$$\frac{E_{kx}}{E_{ky}} = \frac{v_x^2}{2v_y^2} = \frac{(2v_y)^2}{2v_y^2} = \frac{4v_y^2}{2v_y^2}$$

$$\frac{E_{kx}}{E_{ky}} = \frac{2}{1}$$

11 A 2.0 kg mass travelling at 3.0 ms^{-1} on a frictionless surface collides head-on with a stationary 1.0 kg mass. The masses stick together on impact.

initial $K.E = \frac{1}{2}mv^2$
 $= \frac{1}{2}(2)(3)^2$
 $K.E = 9 \text{ J}$



How much kinetic energy is lost on impact?

A zero

B 2.0 J

C 2.4 J

D 3.0 J

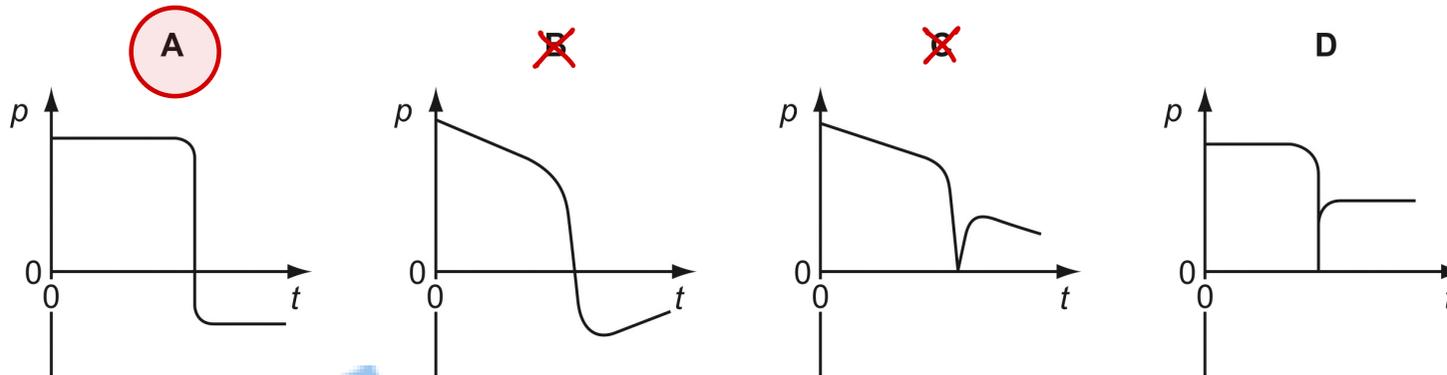
$m_1u_1 + m_2u_2 = Mv$
 $(2)(3) + (1)(0) = (2+1)v$
 $\frac{6}{3} = v$ $v = 2 \text{ ms}^{-1}$

$E_k = \frac{1}{2}mv^2$
 $= \frac{1}{2}(3)(2)^2$
 $E_k = 6 \text{ J}$
final

$$p = mv$$

- 12 An ice-hockey puck slides along a horizontal, frictionless ice-rink surface. It collides inelastically with a wall at right angles to its path, and then rebounds along its original path.

Which graph shows the variation with time t of the momentum p of the puck?



- 8 Water is pumped through a hose-pipe at a rate of 90 kg per minute. It emerges from the hose-pipe horizontally with a speed of 20 m s^{-1} .

Which force is required from a person holding the hose-pipe to prevent it moving backwards?

- A** 30 N **B** 270 N **C** 1800 N **D** 10 800 N

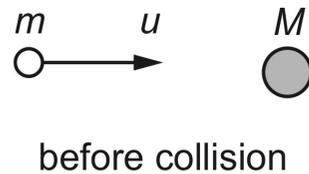
$$F = \dot{m}v$$

$$= \frac{\Delta m}{\Delta t} \times v$$

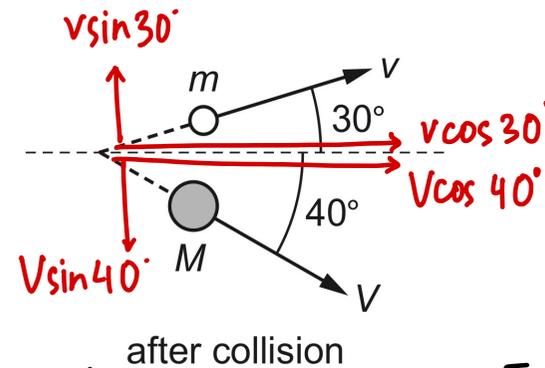
$$= \frac{90}{1 \times 60} \times 20 \quad \text{so } F = 30 \text{ N}$$

11 A ball of mass m travelling at velocity u collides with a stationary ball of mass M . After collision the two balls travel at velocities v and V respectively, in the directions shown.

$m = 2 \text{ kg}$
 $M = 3 \text{ kg}$
 $u = 15 \text{ ms}^{-1}$
 $v = ??$
 $V = ??$



not to scale



Design equations using law of conservation of momentum

x-axis

$$m_1 u_{1x} + m_2 u_{2x} = m_1 v_{1x} + m_2 v_{2x}$$

$$m u + M(0) = m(v \cos 30^\circ) + M(V \cos 40^\circ)$$

$$m u = m v \cos 30^\circ + M V \cos 40^\circ$$

$$(2)(15) = 2v \cos 30^\circ + 3V \cos 40^\circ$$

$$1.73v + 2.298V = 30 \quad \text{--- (1)}$$

$$v = 10.3 \text{ ms}^{-1}$$

y-axis

$$m_1 u_{1y} + m_2 u_{2y} = m_1 v_{1y} + m_2 v_{2y}$$

$$m(0) + M(0) = m(v \sin 30^\circ) + M(-V \sin 40^\circ)$$

$$0 = m v \sin 30^\circ - M V \sin 40^\circ$$

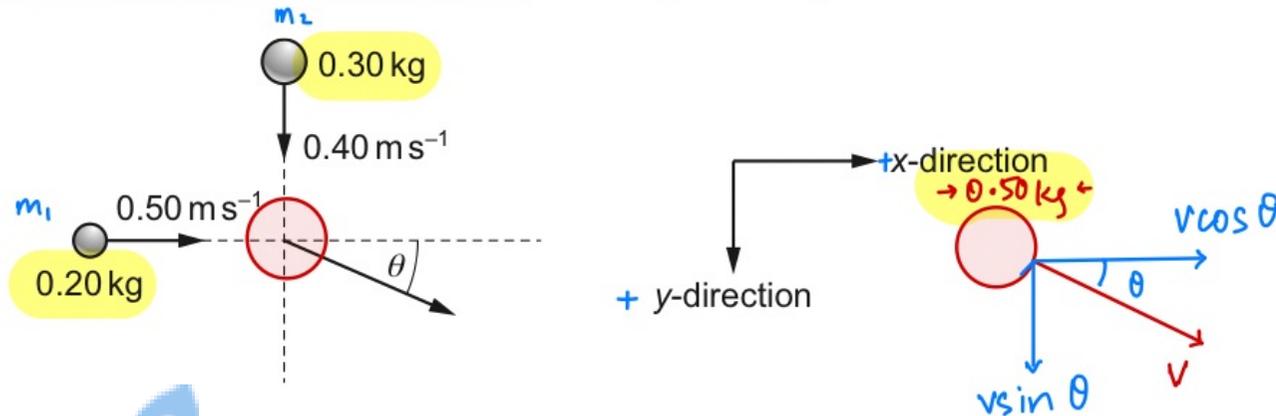
$$0 = (2)v \sin 30^\circ - 3V \sin 40^\circ$$

$$v - 1.93V = 0 \quad \text{--- (2)}$$

$$V = 5.32 \text{ ms}^{-1}$$

- 9 A ball of mass 0.20 kg , travelling in the x -direction at a speed of 0.50 m s^{-1} , collides with a ball of mass 0.30 kg travelling in the y -direction at a speed of 0.40 m s^{-1} .

The two balls stick together after the collision, travelling at an angle θ to the x -direction.



What is the value of θ ?

A 39°

B 40°

C 50°

D 51°

x-axis

$$m_1 u_1 + m_2 u_2 = MV$$

$$(0.2)(0.5) + (0.3)(0) = (0.5)(v \cos \theta)$$

$$0.1 = 0.5 v \cos \theta$$

$$v \cos \theta = 0.2$$

y-axis

$$m_1 u_1 + m_2 u_2 = MV$$

$$(0.2)(0) + (0.3)(0.4) = (0.5)(v \sin \theta)$$

$$\frac{0.12}{0.5} = v \sin \theta$$

$$v \sin \theta = 0.24$$

$$\frac{v \sin \theta}{v \cos \theta} = \frac{0.24}{0.2}$$

$$\tan \theta = 1.2$$

$$\theta = \tan^{-1}(1.2) \text{ so } \theta = 50^\circ$$