DEFORMATION OF SOLIDS

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Deformation :-

Property by which the change in dimensions and shape of a solid is studied due to compressive or tensile force.

Types of deformation:-

- Elastic deformation
- Plastic deformation

Elastic deformation:- Solid return to its original dimensions and shape when deforming force is removed from it due to its elastic property.

Plastic deformation:- Solid can never return to its original dimensions and shape if deforming force is removed from it. Solid is permanently stretched or compressed if enter into plastic region.

Stress:-

Def: Force per unit cross-sectional area is stress.

Symbol: σ (Sigma)

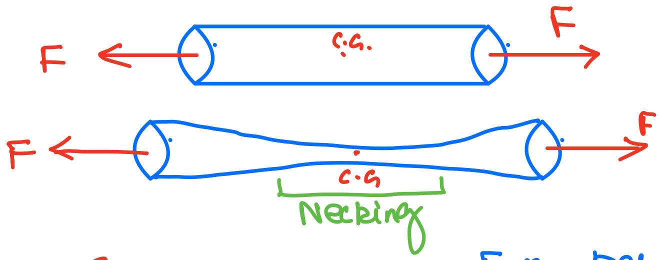
Formula: $\sigma = \frac{F}{A}$

Units: $N\,m^{-2} = kg\,m^{-1}\,s^{-2}$ = Pascal (Pa)

P.S: Scalar

Types:

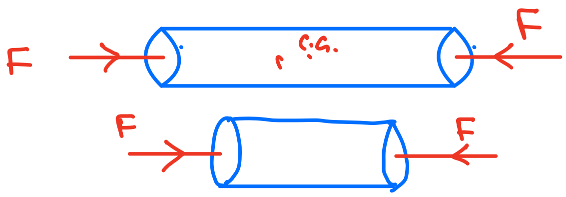
i) **Tensile stress:-** Force per unit cross-sectional area used to increase length of a solid



Length is increased but volume is kept constant.

$V = (A)(L)$
$\quad\;\; \downarrow \;\; \uparrow$

(ii) Compressive stress: Force per unit cross-sectional area used to decrease length of a solid



Length is decreased but volume is kept constant

$V = (A)(L)$
$\quad\;\; \uparrow \;\; \downarrow$

Strain :-

Def Change in length per unit original length is strain

Symbol ε (Epsilon)

Formula: $\varepsilon = \dfrac{\Delta L}{L}$

Units: No units because it is the ratio b/w two similar physical quantities

P.S Scalar

Types:

(a) Compressive strain $= \dfrac{\text{Decrease in length}}{\text{original length}} = \dfrac{\text{compression}}{\text{original length}}$

(b) Tensile strain $= \dfrac{\text{Increase in length}}{\text{original length}} = \dfrac{\text{extension}}{\text{original length}}$

Hooke's law :-

Statement: Within the elastic limit, the force applied is directly proportional to the extension/

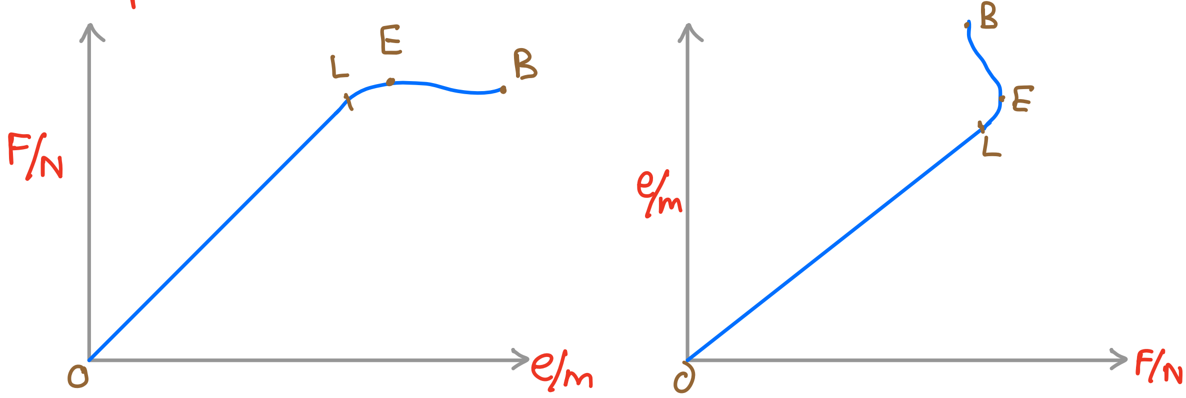
Compression produced.

Mathematical form:- $F \propto e$

$$F = Ke$$

└ Spring constant or Elastic or stiffness constant.

Graph:-



L - Limit of proportionality (end point of straight line from origin)

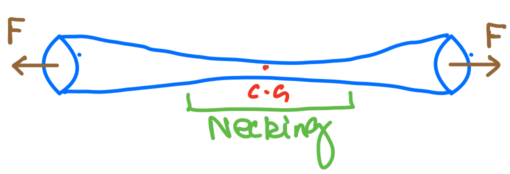
OL - Hooke's law

E - Elastic limit (end point from where a solid regain its original dimensions/shape when applied stress/Force is removed)

OE - Elastic region

Beyond E - Plastic region (solid never return to its original dimensions/shape if enter into this region)

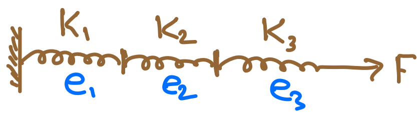
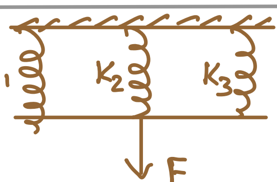
Ductile material:- Materials which undergo plastic deformation before breaking i.e metals



Necking is the property of Ductile region.

B - Breaking point i.e material/solid breaks/snaps at this point.

Combination of springs:-

Combination	Arrangement	Total spring constant	Total extension
Series		decreases $\frac{1}{K_T} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}$	increases $e_T = e_1 + e_2 + e_3$
Force acting on each spring is same	For identical n-springs each of spring constant K	$K_T = \frac{K}{n}$	$e_T = ne$
Parallel		increases $K_T = K_1 + K_2 + K_3$	decreases $\frac{1}{e_T} = \frac{1}{e_1} + \frac{1}{e_2} + \frac{1}{e_3}$
Force acting on each spring is different	For identical n-springs each of spring constant K	$K_T = nK$	$e_T = \frac{e}{n}$

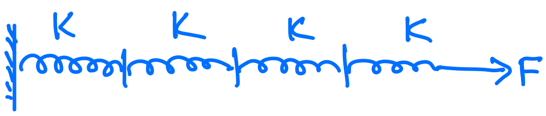
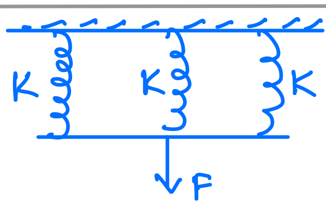
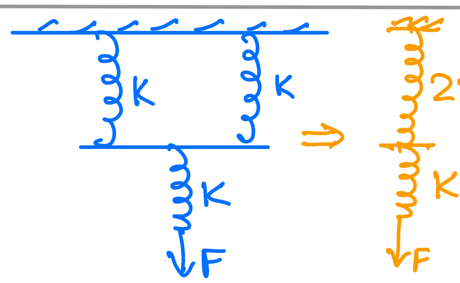
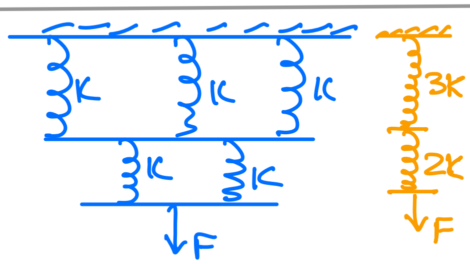
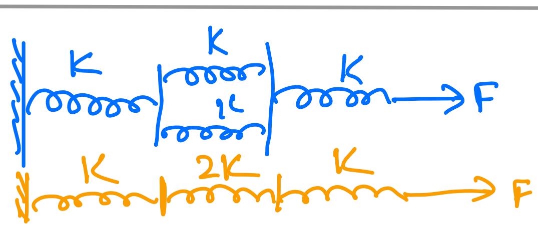
For two springs in series:-

$$\frac{1}{K_T} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{K_2 + K_1}{K_1 K_2}$$

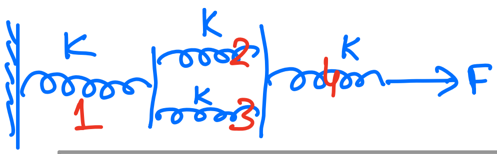
$$\boxed{K_T = \frac{K_1 K_2}{K_1 + K_2}}$$

i.e Total spring constant = $\frac{\text{Product of spring constants}}{\text{Sum of spring constants}}$

Q) A force F causes an extension e in a spring having spring constant K. Express the total spring constant (K_T) and total extension (e_T) in terms of K and e respectively in following arrangements of identical springs.

Arrangement	K_T	e_T
	$K_T = \frac{K}{n}$ $= \frac{K}{4}$	$e_T = ne$ $= 4e$ $F = K_T e_T$ $\cancel{K}e = (\frac{\cancel{K}}{4}) e_T$ $e_T = 4e$
	$K_T = nK$ $= 3K$	$e_T = \frac{e}{n}$ $= \frac{e}{3}$
	$K_T = \frac{(2K)(K)}{2K+K}$ $= \frac{2K}{3}$	$F = K_T e_T$ $F = (\frac{2K}{3})(\frac{3e}{2})$ So $e_T = \frac{3e}{2}$
	$K_T = \frac{(3K)(2K)}{3K+2K}$ $= \frac{6K}{5}$	$F = K_T e_T$ $F = (\frac{6K}{5})(\frac{5}{6}e)$ So $e_T = \frac{5e}{6}$
	$\frac{1}{K_T} = \frac{1}{K} + \frac{1}{2K} + \frac{1}{K}$ $\frac{1}{K_T} = \frac{2+1+2}{2K}$ $K_T = \frac{2K}{5}$	$F = K_T e_T$ $F = (\frac{2K}{5})(\frac{5}{2}e)$ So $e_T = \frac{5}{2}e$

Q)



Greatest force	Greatest extension
Spring 1 $\longrightarrow$ F Spring 2 or 3 $\longrightarrow$ Divide i.e $\frac{F}{2}$ Spring 4 $\longrightarrow$ F Spring (1) and (4)	Spring 1 $\longrightarrow$ e Spring 2 or 3 $\longrightarrow$ $\frac{e}{2}$ Spring 4 $\longrightarrow$ e Spring (1) and (4)
Smallest force	**Smallest extension**
Spring 1 $\longrightarrow$ F Spring 2 or 3 $\longrightarrow$ Divide i.e $\frac{F}{2}$ Spring 4 $\longrightarrow$ F Spring 2 or 3	Spring 1 $\longrightarrow$ e Spring 2 or 3 $\longrightarrow$ $\frac{e}{2}$ Spring 4 $\longrightarrow$ e Spring 2 or 3

Young Modulus:-

Concept:

$$\text{Stress} \propto \text{Strain}$$

$$\text{Stress} = (\text{Young Modulus})\ \text{Strain}$$

$$\text{Young Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

Def: Stress per unit strain is Young modulus.

Symbol: E

Formula:

$$E = \frac{\sigma}{\varepsilon} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \Rightarrow \boxed{E = \frac{FL}{A\Delta L}}$$

If $\Delta L = e$ (extension)

$$\boxed{E = \frac{FL}{Ae}}$$

Units: $Nm^{-2} = kg\,m^{-1}s^{-2}$ = Pascal (Pa)

P.S : Scalar

Dependance: Nature of solids i.e. its crystal structure or bonding.

$E \uparrow$ for hard materials ie metals

e.g greater than 10^{12} Pascal.

Relationship b/w Young Modulus (E) and Elastic or spring Constant (K):-

Since $F = Ke$ ------ (1)

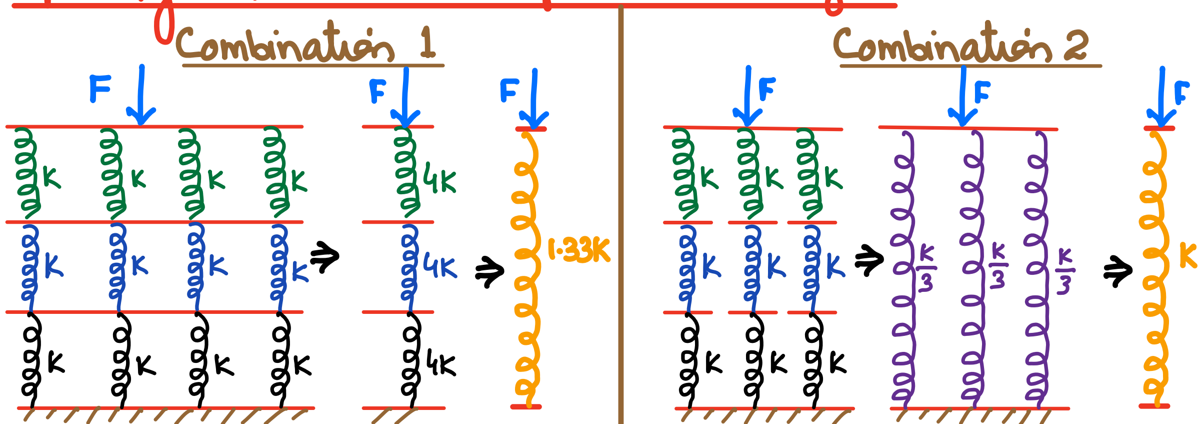
Also $E = \dfrac{FL}{Ae}$

$F = \left(\dfrac{EA}{L}\right)e$ -------- (2)

Comparing (1) and (2)

$$\boxed{K = \frac{EA}{L}}$$

Spring mattress compression analysis:-



$K_T = \dfrac{K}{n} = \dfrac{4K}{3}$

$K_T = nK = 3\left(\dfrac{K}{3}\right) = K$

$F = Ke \quad | \quad F = K_T e_T$

$$Ke = K_T e_T$$

$$\cancel{K}e = \left(\frac{4\cancel{K}}{3}\right)(e_T)$$

$$e_T = \frac{3}{4}e = 0.75e$$

$F = Ke \quad | \quad F = K_T e_T$

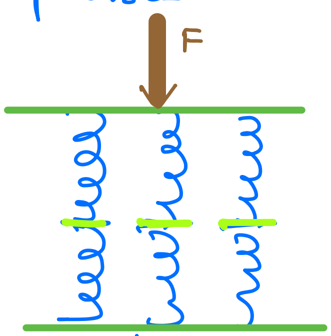
$$Ke = K_T e_T$$

$$Ke = (K)e_T$$

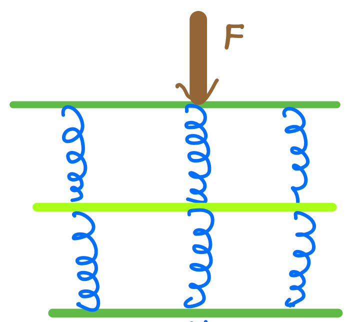
$$e_T = e$$

Result: Combination 2 mattress is more soft as compared to arrangement shown in 1 due to its greater compression.

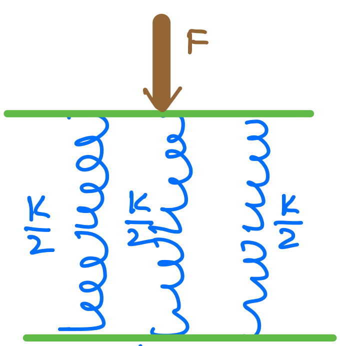
Q) Which one is the softer mattress ie which compresses more for same rise of force.



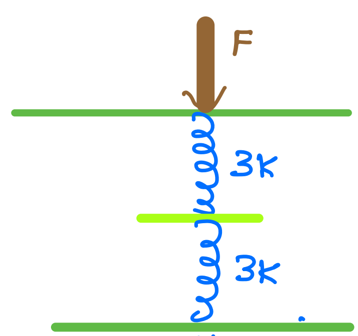
mattress A



mattress B



mattress A



mattress B

$$K_T = \frac{K}{2} + \frac{K}{2} + \frac{K}{2}$$

$$K_T = \frac{3K}{2}$$

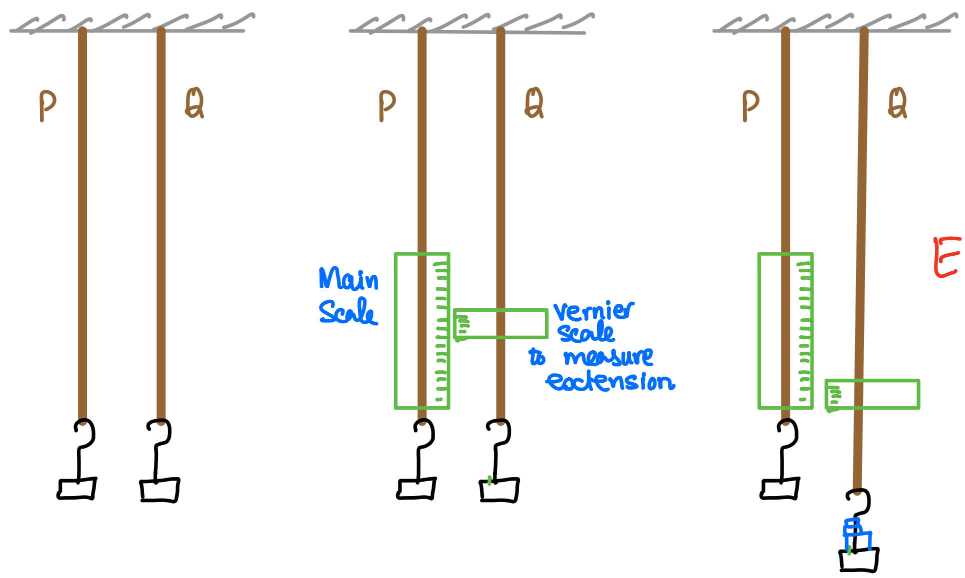
$$K_T = \frac{(3K)(3K)}{3K + 3K}$$

$$K_T = \frac{9K^2}{6K} = \frac{3K}{2}$$

Since Force and total spring constants are same so both arrangement of springs equally compress for the same rise of force

June 2011/22/Q.No 4 [10 marks]

Experimental determination of Young Modulus of a metallic wire:-



$$E = \frac{FL}{Ae}$$

Design features:-

S.No.	Design feature	Reason
1.	Identical wires ie made of same material with similar dimensions.	For better comparison because of same Young Modulus as they are made of same material.
2.	Larger length of both wires is taken i.e. approximately 1m	to reduce percentage error for higher degree of accuracy $\{(\frac{\Delta L}{L})100\}\downarrow$ if $L\uparrow$

3.	Heavy masses are initially suspended from their ends.	To make them free of kinks so that accurate length can be taken

Measurements:

1) Measure length of a wire by using metre rule : L
2) Measure several diameters along the length of a wire and get their mean diameter to reduce random error.

$$d = \frac{d_1 + d_2 + d_3}{3}$$

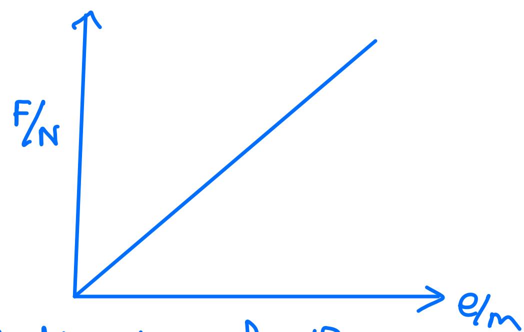
Procedure:

1) Add mass one by one to the hanger suspended with wire Q and measure the corresponding extension.
2) Also remove/unload masses from hanger one by one to check that elastic limit of wire is not exceeded. Now get the mean extension.

Analysis:

1) Plot the graph of F/N (defined by loading the hanger of wire Q) and mean extension. The graph should be a straight line and this plotting of graph reduce random

error if any.



(2) Get the gradient of this graph to eliminate systematic error if any.

Gradient $= \dfrac{F}{e}$

Calculation:

(1) Calculate the cross-sectional Area of wire using mean diameter value.

$$A = \pi r^2 \Rightarrow A = \pi\left(\frac{d}{2}\right)^2 \Rightarrow A = \frac{\pi d^2}{4}$$

(2) Young Modulus: $E = \dfrac{FL}{Ae}$

$$E = \left(\frac{F}{e}\right)\left(\frac{L}{\frac{\pi d^2}{4}}\right) \Rightarrow E = \left(\frac{F}{e}\right)\left(\frac{4L}{\pi d^2}\right)$$

$$\boxed{E = \frac{4\,(\text{Gradient})(\text{original length})}{(\pi)(\text{diameter})^2}}$$

Elastic Potential energy / strain energy:

Already done in work, energy and Power chapter.

$E_p =$ Area of F/N - e/m graph along with extension axis

If graph is a straight line

$$E_p = \frac{1}{2}(F)(e) \Rightarrow E_p = \frac{1}{2}(Ke^2) \Rightarrow E_p = \frac{F^2}{2K}$$

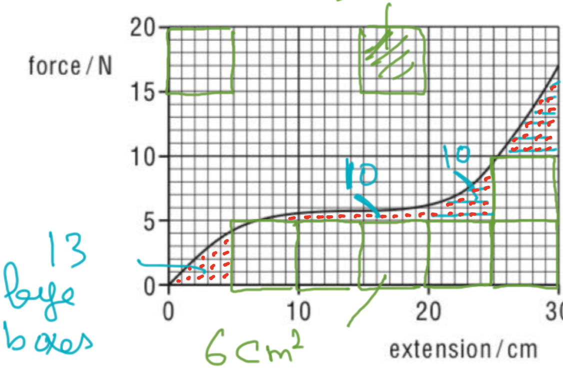
21 A rubber band is stretched by hanging weights on it and the force-extension graph is plotted from the results.

25 small boxes

Small boxs

13+10+10+16
49 Small box

$1\,cm^2 = \frac{49}{25} = 1.96$ large boxes



$1\,cm^2$ area $= (5)(5\times10^{-2})$
$= 0.25\,J$

25 small boxes area $= 0.25$

1 small box $= \frac{0.25}{25} = 0.01\,J$

$[(6 + 1.96)$ large boxes$]\,cm^2$
$= 7.96 \times 0.25$
$= 1.99\,J$

What is the best estimate of the strain energy stored in the rubber band when it is extended 30 cm?

A 2.0 J　　**B** 2.6 J　　**C** 5.1 J　　**D** 200 J

(b) The wire in **(a)** is now extended beyond its elastic limit. The forces causing the extension are then removed.
The variation with extension x of the tension F in the wire is shown in Fig. 4.1.

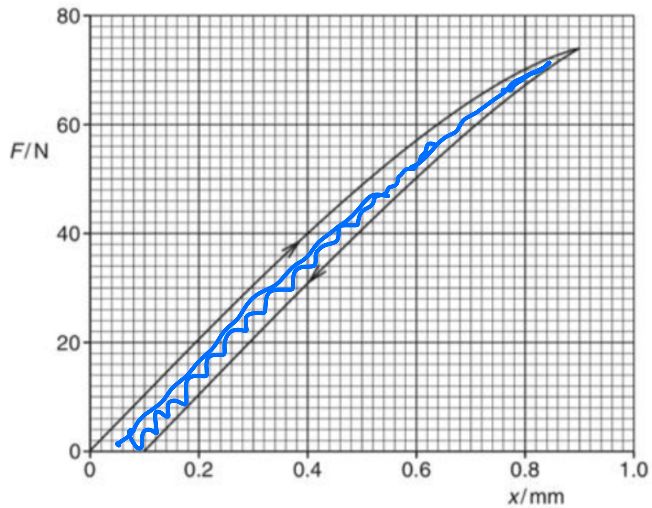


Fig. 4.1

Energy E_S is expended to cause a permanent extension of the wire.

(i) On Fig. 4.1, shade the area that represents the energy E_S. [1]

(ii) Use Fig. 4.1 to calculate the energy E_S.

$1\,cm^2$ area $= (10)(0.1 \times 10^{-3}) = 1.0 \times 10^{-3}\,J$

25 small boxes $= 1.0 \times 10^{-3}\,J$

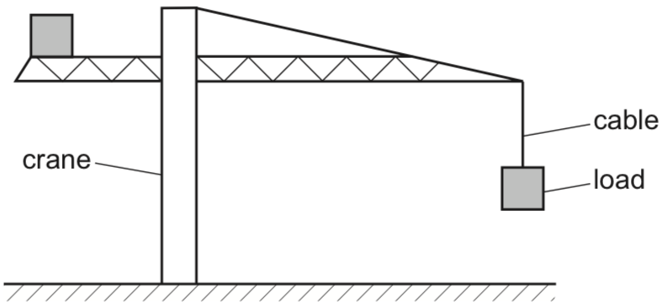
1 small box $= 4.0 \times 10^{-5}\,J$

E_S = ... mJ [3]

SOLUTIONS OF PROBLEMATIC QUESTIONS

Akhtar

19 The diagram shows a large crane on a construction site lifting a cube-shaped load.



A model is made of the crane, its load and the cable supporting the load.

The material used for each part of the model is the same as that in the full-size crane, cable and load. The model is one tenth full-size in all linear dimensions.

$F = ke \Rightarrow mg = ke$

$L_m = \frac{L_c}{10} \Rightarrow L_c = 10L_m$

What is the ratio $\dfrac{\text{extension of the cable on the full-size crane}}{\text{extension of the cable on the model crane}}$?

A 10^0 **B** 10^1 **C** 10^2 **D** 10^3

Same material means same Young Modulus

$e \uparrow$ and exceed its elastic limit

$$E_{model} = E_{crane}$$

$$\frac{F_m L_m}{A_m e_m} = \frac{F_c L_c}{A_c e_c}$$

$$\frac{\cancel{\rho g} L_m^2}{e_m} = \frac{\cancel{\rho g} L_c^2}{e_c}$$

$$\frac{e_c}{e_m} = \frac{L_c^2}{L_m^2}$$

$$\frac{e_c}{e_m} = \frac{(10L_m)^2}{L_m^2} = \frac{10^2 \cancel{L_m^2}}{\cancel{L_m^2}}$$

$$\frac{e_c}{e_m} = 10^2$$

$$\frac{FL}{Ae} = \frac{mgL}{Ae} = \frac{\rho V g L}{Ae} = \frac{\rho \cancel{A} L g L}{\cancel{A} e} = \frac{\rho g L^2}{e}$$

Oct/Nov 10/12 variant

22 To determine the mass of food in a pan, a scale is used that has high sensitivity for small masses but low sensitivity for large masses.

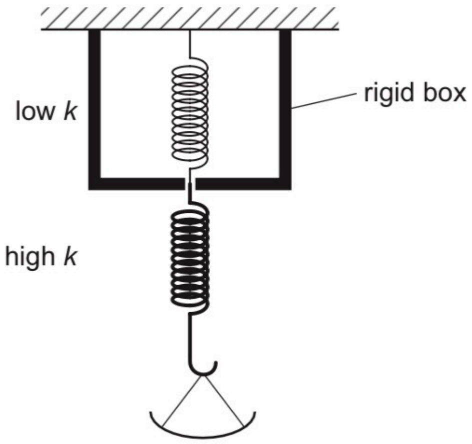
$F = ke \Rightarrow mg = ke$

$k = \frac{mg}{e}$

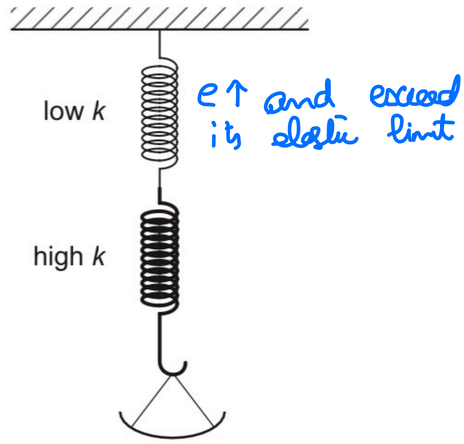
To do this, two springs are used, each with a different spring constant k. One of the springs has a low spring constant and the other has a high spring constant.

Which arrangement of springs would be suitable?

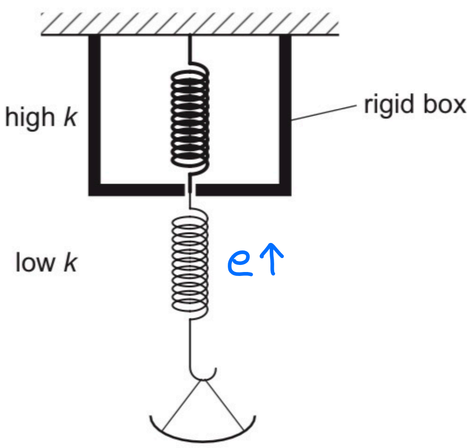
A ✓



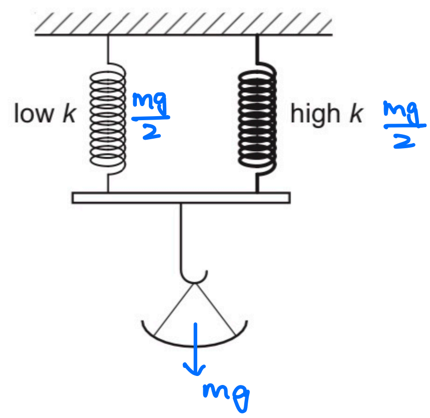
B ✓



C ✗



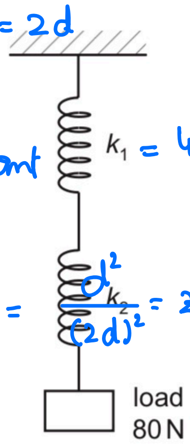
D



$A = \frac{\pi d^2}{4}$

Nov 14 / 13 variant

25 Two springs, one with spring constant k_1 = 4 kN m^{-1} and the other with spring constant k_2 = 2 kN m^{-1}, are connected as shown.



Since springs are connected in series, so total spring constant is

$K_T = \frac{K_1 K_2}{K_1 + K_2}$

$= \frac{(4000)(2000)}{4000 + 2000}$

$= 1333.3$ N m^{-1}

$F = ke$

$80 = (1333.3)\, e$

$e = 0.060$ m

$e = 6$ cm

$D = 2d$ | d

$\frac{\frac{T}{A_W}}{\frac{T}{A_N}} = \frac{A_N}{A_W} = \frac{\frac{\pi d^2}{4}}{\frac{\pi D^2}{4}}$

$= \frac{d^2}{D^2} = \frac{d^2}{(2d)^2} = \frac{d^2}{4d^2} = \frac{1}{4} = 0.25$

What is the total extension of the springs when supporting a load of 80 N?

A 1.3 cm **B** 4 cm **C** 6 cm **D** 60 cm

June 15 / 13 variant / June 21/12 variant / Q.19

$A = \pi r^2 = \pi \left(\frac{d}{2}\right)^2$

$A = \frac{\pi d^2}{4}$

22 A steel bar of circular cross-section is under tension T, as shown.

The diameter of the wide portion is double the diameter of the narrow portion.

What is the value of $\frac{\text{stress in the wide portion}}{\text{stress in the narrow portion}}$?

$\frac{\frac{\cancel{T}}{A_W}}{\frac{\cancel{T}}{A_N}} = \frac{A_N}{A_W} = \frac{\frac{\cancel{\pi} d^2}{\cancel{4}}}{\frac{\cancel{\pi} D^2}{\cancel{4}}}$

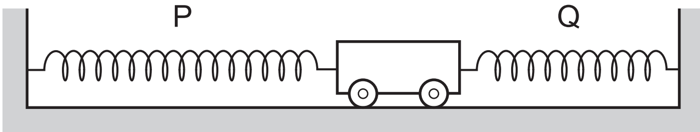
A 0.25 **B** 0.50 **C** 2.0 **D** 4.0

$\frac{\sigma_W}{\sigma_N} = \frac{d^2}{D^2} = \frac{d^2}{(2d)^2} = \frac{\cancel{d^2}}{4\cancel{d^2}} = \frac{1}{4} = 0.25$

$F = Ke$

$e = \frac{F}{K}$

24 A trolley is held at rest between two steel springs.



$$\frac{e_Q}{e_P} = \frac{\frac{\cancel{F}}{K_Q}}{\frac{\cancel{F}}{K_P}} = \frac{K_P}{K_Q}$$

Each spring has an unstretched length of 0.10 m.

Spring P has spring constant 60 N m^{-1}. $= K_P$
Spring Q has spring constant 120 N m^{-1}. $= K_Q$

Spring P has an extension of 0.40 m. $= e_P$

$$\frac{e_Q}{0.40} = \frac{60}{120} \Rightarrow e_Q = 0.20\,\text{m}$$

What is the extension of spring Q? $= e_Q = ?$

A 0.10 m **B** 0.20 m **C** 0.30 m **D** 0.80 m

9702/13/O/N/12

25 A lift is supported by two steel cables, each of length 10 m and diameter 0.5 cm.

The lift drops 1 mm when a man of mass 80 kg steps into the lift.

What is the best estimate of the value of the Young modulus of the steel?

A 2×10^{10} N m^{-2}

B 4×10^{10} N m^{-2}

C 2×10^{11} N m^{-2}

D 4×10^{11} N m^{-2}

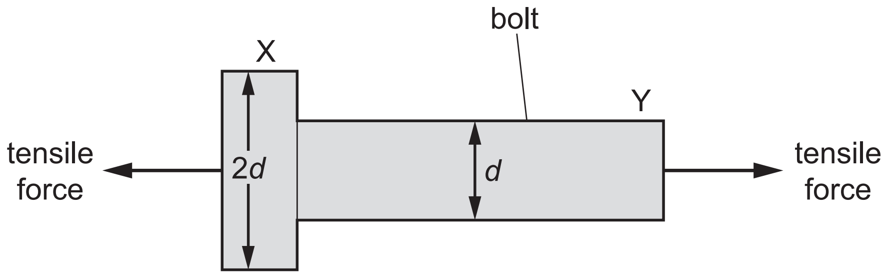
$$E = \frac{FL}{Ae} = \frac{mgL}{\left(\frac{\pi d^2}{4}\right)(e)}$$

$$E = \frac{4mgL}{\pi d^2 e} \Rightarrow E = \frac{4(40)(9.81)(10)}{(3.14)(0.5\times10^{-2})^2(1\times10^{-3})}$$

Space for working

$$E = 1.999 \times 10^{11}$$
$$E = 2 \times 10^{11}\,\text{Pa}$$

20 A bolt is subjected to a tensile force, as shown.



$A = \pi r^2$
$= \pi \left(\frac{d}{2}\right)^2$
$= \frac{\pi d^2}{4}$

The bolt has a circular cross-section. At end X the diameter is 2*d*. At end Y the diameter is *d*.

What is the ratio $\frac{\text{stress at Y}}{\text{stress at X}}$? $= \frac{\cancel{F}/A_Y}{\cancel{F}/A_X} = \frac{A_X}{A_Y} = \frac{\cancel{\pi} d_X^2/\cancel{4}}{\cancel{\pi} d_Y^2/\cancel{4}} = \frac{d_X^2}{d_Y^2} = \frac{(2d)^2}{d^2} = 4$

A 0.25 **B** 0.50 **C** 2.0 **D** 4.0

9702/12/O/N/17

21 A rectangular block of steel supporting a very large component of a bridge has a height of 15 cm and a cross-section of 20 cm × 12 cm. It is designed to compress 1 mm when under maximum, evenly distributed, load.

The Young modulus of steel is $2.0 \times 10^{11}\,\text{N m}^{-2}$.

What is the maximum load it can support?

A 32 MN **B** 56 GN **C** 720 GN **D** 32 TN

$E = \frac{FL}{Ae} \Rightarrow F = \frac{EAe}{L}$

$F = \frac{(2.0\times10^{11})(20\times12\times10^{-4})(1\times10^{-3})}{15\times10^{-2}}$

$F = 32\times10^{6}\,\text{N}$

$F = 32\,\text{MN}$

9702/13/M/J/20

20 The diagram shows a simplified model of a building with four identical heavy floors.

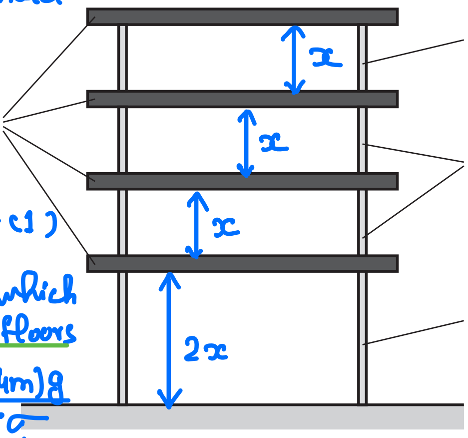
Stress in top rods which hold the weight of top floor

$\sigma = \dfrac{mg}{\frac{\pi d_t^2}{4}}$

$d_t^2 = \dfrac{4mg}{\pi\sigma}$ ....(1)

Stress in bottom rods which hold the weight of Four floors

$\sigma = \dfrac{m_2 g}{\frac{\pi d_b^2}{4}} \Rightarrow d_b^2 = \dfrac{4(4m)g}{\pi\sigma}$ (2)



Divide (2) by (1)

$\dfrac{d_b^2}{d_t^2} = \dfrac{\frac{16\cancel{mg}}{\cancel{\pi\sigma}}}{\frac{4\cancel{mg}}{\cancel{\pi\sigma}}}$

$\dfrac{d_b^2}{d_t^2} = \dfrac{16}{4} = 4$

$\dfrac{d_b}{d_t} = \sqrt{4} = 2$

The spacing of the bottom floor from the ground is twice that of the spacing between the floors. Between each floor are equal numbers of vertical steel supporting rods of negligible mass compared with the floors. The rods are of different diameters so that the stress in each rod is the same.

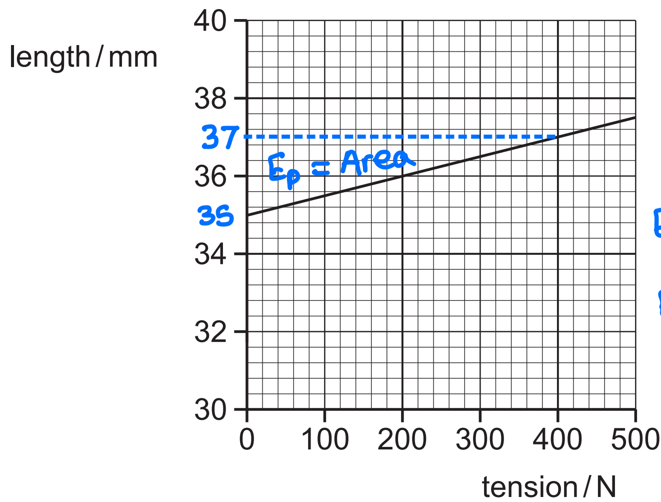
What is the ratio $\dfrac{\text{diameter of bottom rods}}{\text{diameter of top rods}}$?

A 2 **B** 4 **C** 8 **D** 16

9702/13/M/J/20

21 The Achilles tendon in a rabbit's leg is stretched when the rabbit jumps.

The graph shows the variation with tension of the length of the tendon.



Ep = Area of graph along with extension or Length axis

$E_p = \frac{1}{2}[(37-35)10^{-3}][400]$

$E_p = 0.40\,J$

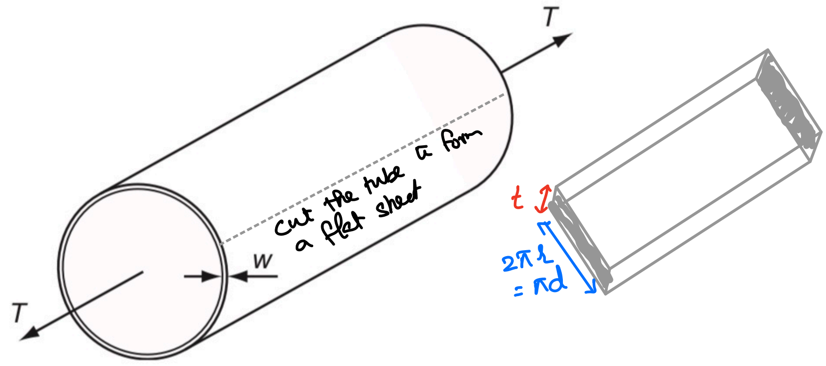
What is the strain energy in the tendon when the tension is 400 N?

A 0.40 J **B** 0.80 J **C** 2.4 J **D** 7.4 J

June 15/12 Variant

23 The diagram represents a steel tube with wall thickness *w* which is small in comparison with the diameter of the tube.

A*



The tube is under tension, caused by a force *T*, parallel to the axis of the tube. To reduce the stress in the material of the tube, it is proposed to thicken the wall.

The tube diameter and the tension being constant, which wall thickness gives half the stress?

A $\frac{w}{2}$ **B** $\sqrt{2}\,w$ **C** $2w$ **D** $4w$

$$\frac{\sigma_2}{\sigma_1} = \frac{\frac{F}{A_2}}{\frac{F}{A_1}} = \frac{A_1}{A_2} = \frac{(t_1)(\cancel{2\pi r})}{(t_2)(\cancel{2\pi r})}$$

$$\frac{\sigma_2}{\sigma_1} = \frac{t_1}{t_2} \Rightarrow \frac{\frac{\cancel{\sigma}}{2}}{\cancel{\sigma}} = \frac{w}{t_2}$$

$$\frac{1}{2} = \frac{w}{t_2} \Rightarrow \boxed{t_2 = 2w}$$

9702/11/O/N/17

20 A spring is loaded with weights. When the weights are removed, the spring returns to its original length. Elastically deformed

$E = \frac{FL}{Ae}$

The spring is then loaded with heavier weights. When the weights are removed, the spring is longer than it was originally. Plastically deformed

Which types of deformation are shown by this experiment?

A both elastic and plastic deformation

B elastic deformation only

C neither elastic nor plastic deformation

D plastic deformation only

$$\frac{FL}{e} = EA = \text{Constant (same)}$$

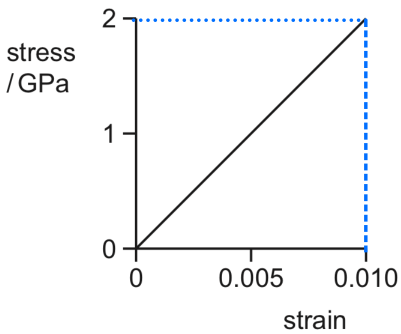
$$\frac{Ae}{L} = \frac{F}{E} = \frac{\text{Different}}{\text{same}} = \text{Different}$$

$$\frac{E}{FL} = \frac{1}{Ae} = \frac{1}{(\text{same})(\text{same})} = \text{same}$$

9702/11/O/N/17

21 The stress-strain graph for a metal is shown.

Since
$E_p = \frac{1}{2}(F)(e)$



$$\frac{E_p}{V} = \frac{\frac{1}{2}Fe}{AL}$$
$$= \frac{1}{2}\left(\frac{F}{A}\right)\left(\frac{e}{L}\right)$$
$$= \frac{1}{2}(\text{stress})(\text{strain})$$
$$\frac{E_p}{V} = \frac{1}{2}(2\times10^9)(0.010)$$
$$= 10\times10^6\ \text{J m}^{-3}$$
$$= 10\ \text{MJ m}^{-3}$$

What is the strain energy per unit volume of a rod made from this metal when the strain of the rod is 0.010?

A $10\,\text{kJ m}^{-3}$ **B** $100\,\text{kJ m}^{-3}$ **C** $1.0\,\text{MJ m}^{-3}$ **D** $10\,\text{MJ m}^{-3}$

9702/13/M/J/17

18 Two wires with the same Young modulus *E* and cross-sectional area *A*, but different lengths *L*, are subject to different tensile forces *F*. The extension *e* of each wire is the same.

The column headings in the table show four different quantities.

$E = \frac{FL}{Ae}$

Which quantities have the same value and which quantities have different values for the two wires?

	$\frac{FL}{e}$	$\frac{Ae}{L}$	$\frac{E}{FL}$
A	different	different	same
B	different	same	same
C	same	different	different
D	same	different	same

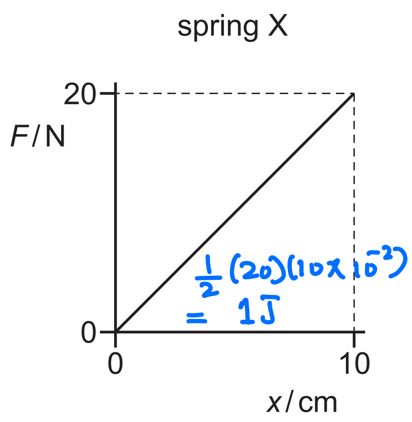
$\frac{FL}{e} = EA$ = Constant (Same)

$\frac{Ae}{L} = \frac{F}{E} = \frac{\text{Different}}{\text{Same}}$ = Different

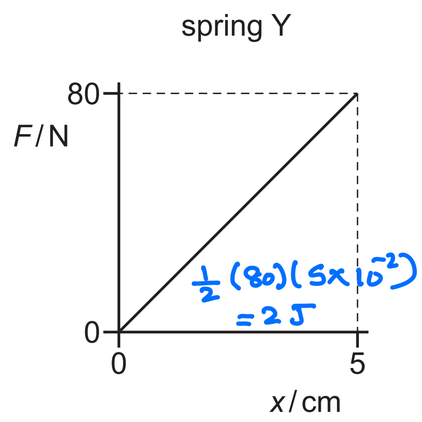
$\frac{E}{FL} = \frac{1}{Ae} = \frac{1}{(\text{Same})(\text{Same})}$
= Same

9702/13/M/J/17

19 Two springs X and Y stretch elastically. The graphs show the variation with extension x of the force F applied to each spring.



$\frac{1}{2}(20)(10\times10^{-2})$
$= 1J$



$\frac{1}{2}(80)(5\times10^{-2})$
$= 2J$

Which statement is correct?

A When each spring is given the same extension, the energy stored in Y is 4 times the energy stored in X. $E_x = \frac{1}{2}(10)(5\times10^{-2}) = 0.25\,J$ and $E_Y = 2J$ So $E_Y = 8\,E_x$

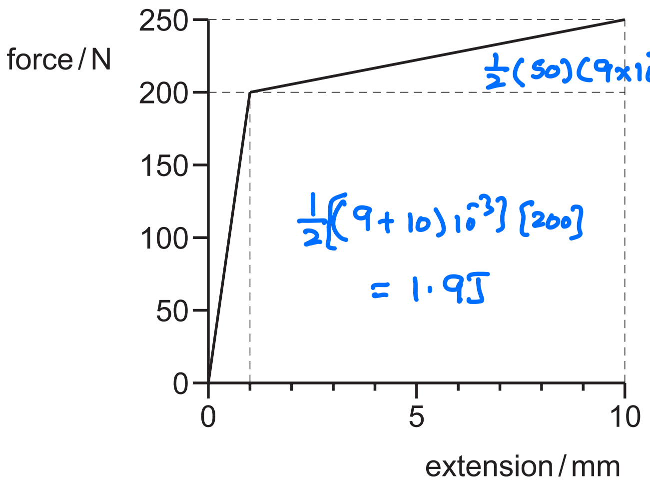
B When each spring is given the same extension, the energy stored in Y is 8 times the energy stored in X.

C When the same force is applied to each spring, the energy stored in Y is 4 times the energy stored in X.

D When the same force is applied to each spring, the energy stored in Y is 8 times the energy stored in X.

9702/13/M/J/17

20 The diagram shows the force-extension graph for a steel wire, up to its breaking point.



$\frac{1}{2}(50)(9\times10^{-3}) = 0.225\,J$

$\frac{1}{2}[(9+10)10^{-3}][200]$
$= 1.9J$

What is the best estimate of the work done to break the wire? = Area under graph

A 2.1 J **B** 2.3 J **C** 2.4 J **D** 2.5 J

$1.9 + 0.225$
$= 2.125J$

March 17/12

20 Two wires X and Y are made of different metals. The Young modulus of wire X is twice that of wire Y. The diameter of wire X is half that of wire Y.

The wires are extended with the same strain and obey Hooke's law.

What is the ratio $\dfrac{\text{tension in wire X}}{\text{tension in wire Y}}$?

A $\frac{1}{8}$ **B** $\frac{1}{2}$ **C** 1 **D** 8

$E = \dfrac{FL}{Ae} \Rightarrow F = EA\left(\dfrac{e}{L}\right)$

$F = EA\varepsilon$

$F = EA\varepsilon \Rightarrow F = (E)\left(\dfrac{\pi d^2}{4}\right)(\varepsilon)$ = same on both wires

$\dfrac{F_x}{F_Y} = \dfrac{E_x\,\frac{\pi d_x^2}{4}\,\cancel{\varepsilon}}{E_Y\,\frac{\pi d_Y^2}{4}\,\cancel{\varepsilon}} \Rightarrow \dfrac{F_x}{F_Y} = \dfrac{E_x d_x^2}{E_Y d_Y^2}$

$d_x = d$, $d_y = 2d$, $\varepsilon_x = \varepsilon$, $\varepsilon_Y = \varepsilon$

$\dfrac{F_x}{F_Y} = \dfrac{(2\cancel{E})(d^2)}{(\cancel{E})(4d^2)} \Rightarrow \dfrac{F_x}{F_Y} = \dfrac{2}{4} \Rightarrow \dfrac{F_x}{F_Y} = \dfrac{1}{2}$

March 17/12

21 A weight of 120 kN is placed on top of a metal column. The length of the column is compressed by 0.25 mm. The column obeys Hooke's law when compressed.

How much energy is stored in the compressed column?

A 15 J **B** 30 J **C** 15 kJ **D** 30 kJ

$E = \frac{1}{2}Fe$

$= \frac{1}{2}(120\times10^{3})(0.25\times10^{-3})$

$= 15\,J$

March 16/12

19 The Young modulus of steel is twice that of copper.

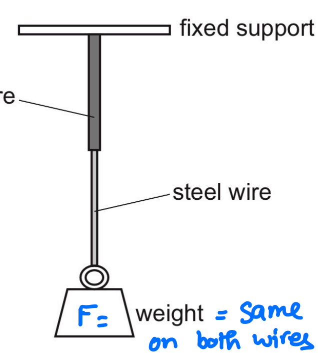
A 50 cm length of copper wire of diameter 2.0 mm is joined to a 50 cm length of steel wire of diameter 1.0 mm, making a combination wire of length 1.0 m, as shown.

$E_s = 2E_c$

$\dfrac{\cancel{F}L_s}{A_s e_s} = 2\left(\dfrac{\cancel{F}L_c}{A_c e_c}\right)$

$\dfrac{L_s}{\left(\frac{\pi d_s^2}{4}\right)e_s} = 2\left[\dfrac{L_c}{\left(\frac{\pi d_c^2}{4}\right)(e_c)}\right]$

$\dfrac{L_s}{d_s^2 e_s} = \dfrac{2L_c}{d_c^2 e_c}$



$\dfrac{50}{(1)^2 e_s} = 2\left[\dfrac{50}{(2)^2 e_c}\right]$

$\dfrac{1}{e_s} = \dfrac{2}{4e_c}$

$e_s = 2e_c$

$\dfrac{e_s}{e_c} = 2$

The combination wire is stretched by a weight added to its end. Both the copper and the steel wires obey Hooke's law.

What is the ratio $\dfrac{\text{extension of steel wire}}{\text{extension of copper wire}}$?

A 4 **B** 2 **C** 1 **D** 0.5

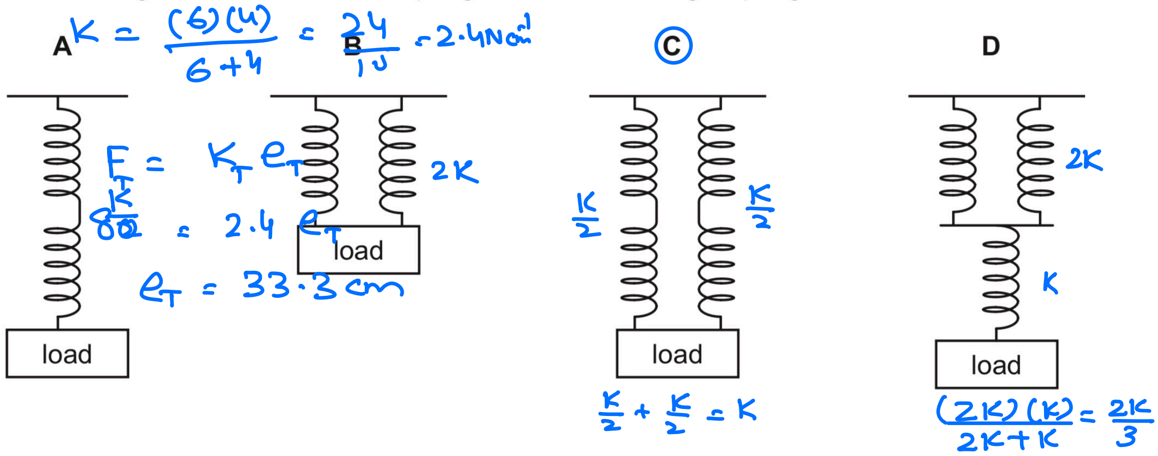
June 16/11

20 A number of identical springs are joined in four arrangements.

Identical springs in
(i) series → $K_T = \frac{K}{n}$
(ii) Parallel → $K_T = nK$

Total spring constant:

Which arrangement has the same spring constant as a single spring?



$K = \frac{(6)(4)}{6+4} = \frac{24}{10} = 2.4\,\text{N cm}^{-1}$

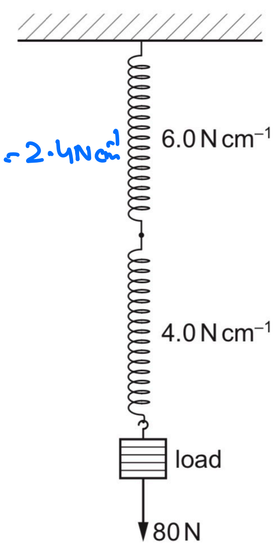
$F_T = K_T e_T$

$80 = 2.4\, e_T$

$e_T = 33.3\,\text{cm}$

March 19/12

20 A spring has a spring constant of 6.0 N cm $^{-1}$. It is joined to another spring whose spring constant is 4.0 N cm^{-1}. A load of 80 N is suspended from this composite spring.



Total spring constant:

$K = \frac{(6)(4)}{6+4} = \frac{24}{10} = 2.4\,\text{N cm}^{-1}$

$F_T = K_T e_T$

$80 = 2.4\, e_T$

$e_T = 33.3\,\text{cm}$

What is the extension of this composite spring?

A 8.0 cm **B** 16 cm **C** 17 cm **D** 33 cm

9702/13/O/N/14

maximum ↑ Stretching $\frac{F}{A}$

23 What is meant by the ultimate tensile stress of a material?

A the maximum force that can be applied to a bar of the material before it bends

B the maximum inter-atomic force before the atomic bonds of the material break

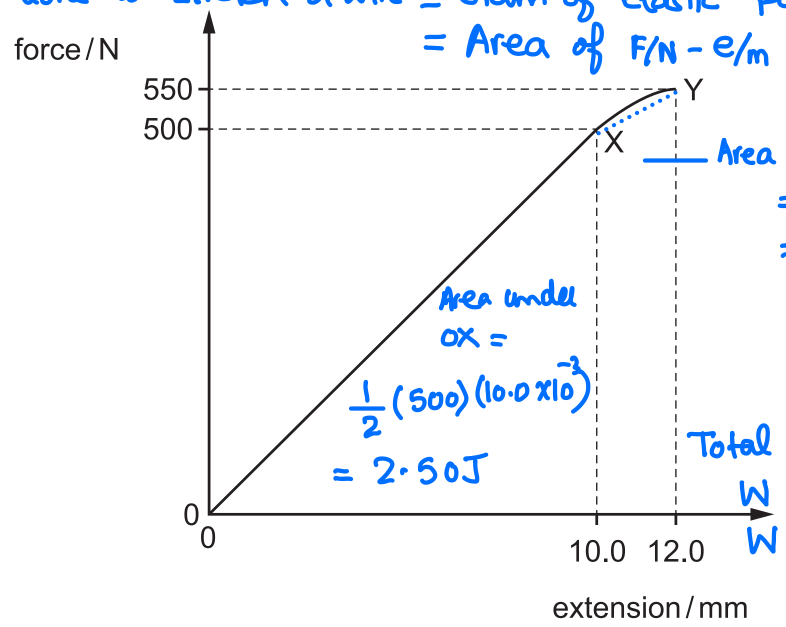
C the maximum stretching force per unit cross-sectional area before the material breaks

D the maximum tensile force in a wire of the material before it breaks

9702/13/O/N/14

24 The graph shows the behaviour of a sample of a metal when it is stretched until it starts to undergo plastic deformation.

Work done to stretch a wire = Gain of Elastic potential energy
= Area of F/N – e/m graph along with extension axis



Area under XY
$= \frac{1}{2}(500+550)(2.0\times10^{-3})$
$= 1.05\,\text{J}$

Area under OX =
$\frac{1}{2}(500)(10.0\times10^{-3})$
$= 2.50\,\text{J}$

Total work done = Area (OX + XY)
$W = 2.50 + 1.05$
$W = 3.55\,\text{J}$

What is the total work done in stretching the sample from zero to 12.0 mm extension?
Simplify the calculation by treating the curve XY as a straight line.

A 3.30 J **B** 3.55 J **C** 3.60 J **D** 6.60 J

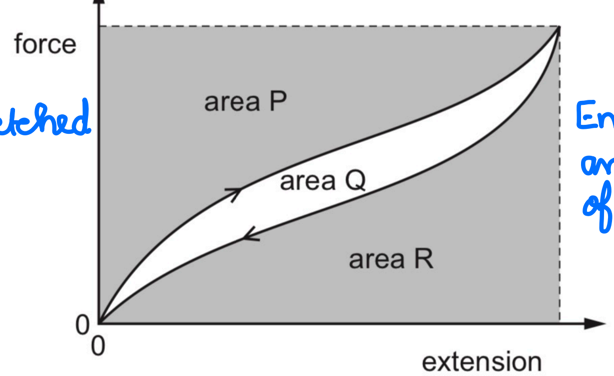
Space for working

9702/13/M/J/16

20 The diagram shows the force-extension graph for a sample of material. The sample is stretched and then returns to its original length.

Work done to stretch sample = Area (Q + R)

Energy recovered from stretched material = Area R



Energy which can not be recovered and becomes the internal energy of material = Area Q

Which area represents the work done to stretch the sample?

A P + Q **B** P only **C** Q + R **D** R only

Nov 11/11/Q.26

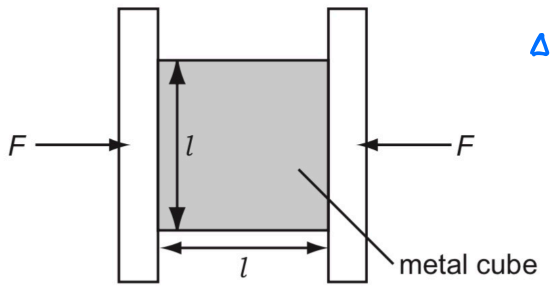
26 A metal cube of side l is placed in a vice and compressed elastically by two opposing forces F.

$E = \frac{FL}{A\,\Delta L}$

$E = \frac{F\ell}{\ell^2\,\Delta\ell}$

$E = \frac{F}{\ell\,\Delta\ell}$

$\Delta\ell = \left(\frac{F}{E}\right)\left(\frac{1}{\ell}\right)$



$\Delta\ell = (\text{Constant})\left(\frac{1}{\ell}\right)$

$\Delta\ell \propto \frac{1}{\ell}$

How will Δl, the amount of compression, relate to l?

A $\Delta l \propto \frac{1}{l^2}$ **B** $\Delta l \propto \frac{1}{l}$ **C** $\Delta l \propto l$ **D** $\Delta l \propto l^2$

June 19/12/Q.20

20 A wire X is stretched by a force and gains elastic potential energy E.

The same force is applied to wire Y of the same material, with the same initial length but twice the diameter of wire X. Both wires obey Hooke's law.

What is the gain in elastic potential energy of wire Y?

A $0.25E$ **B** $0.5E$ **C** $2E$ **D** $4E$

	wire X	wire Y
diameter	$d_X = d$	$d_Y = 2d$
P.E.	$E_X = E$	$E_Y = ?$

Same material means same Young Modulus

$$E_X = E_Y$$

$$\frac{F\cancel{L}}{A_X e_X} = \frac{F\cancel{L}}{A_Y e_Y} \Rightarrow A_X e_X = A_Y e_Y$$

$$\left(\frac{\cancel{\pi} d_X^2}{\cancel{4}}\right) e_X = \left(\frac{\cancel{\pi} d_Y^2}{\cancel{4}}\right) e_Y \Rightarrow d^2 e_X = 4d^2 e_Y$$

$$e_Y = \frac{e_X}{4} \Rightarrow e_X = 4e_Y$$

Elastic potential energy $= E = \frac{1}{2}Fe$

$$\frac{E_Y}{E_X} = \frac{\cancel{\frac{1}{2}}(\cancel{F})(e_Y)}{\cancel{\frac{1}{2}}(\cancel{F})(e_X)} \Rightarrow \frac{E_Y}{E} = \frac{\cancel{e_Y}}{4\cancel{e_Y}} \Rightarrow \frac{E_Y}{E} = \frac{1}{4}$$

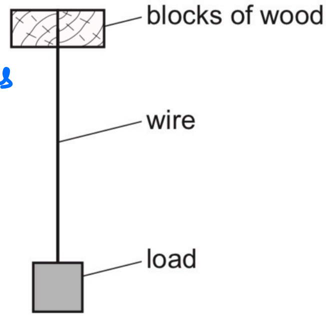
$$E_Y = 4E$$

June 18/11/Q 18

18 The diagram shows a wire of diameter D and length L that is firmly clamped at one end between two blocks of wood. A load is applied to the wire which extends its length by x.

Same material means same Young Modulus

$$E = \frac{FL}{Ae}$$



	wire 1	wire 2
Same material	E	E
Diameter	$d_1 = D$	$d_2 = 2D$
Length	$L_1 = L$	$L_2 = 3L$
Force	$F_1 = F$	$F_2 = F$

A second wire is made of the same material, but of diameter $2D$ and length $3L$. Both wires obey Hooke's law.

What is the extension of the second wire when the same load is applied?

A $\frac{2}{3}x$ **B** $\frac{3}{4}x$ **C** $\frac{4}{3}x$ **D** $\frac{3}{2}x$

$$\frac{F_1 L_1}{A_1 e_1} = \frac{F_2 L_2}{A_2 e_2} \Rightarrow \frac{(\cancel{F})(\cancel{L})}{\left(\frac{\cancel{\pi} d_1^2}{\cancel{4}}\right) e_1} = \frac{(\cancel{F})(3\cancel{L})}{\left(\frac{\cancel{\pi} d_2^2}{\cancel{4}}\right) e_2}$$

$$\frac{1}{d_1^2 e_1} = \frac{3}{d_2^2 e_2} \Rightarrow e_2 = \frac{3 d_1^2 e_1}{d_2^2}$$

$$e_2 = \frac{3(D)^2 x}{(2D)^2} \Rightarrow e_2 = \frac{3\cancel{D}^2 x}{4\cancel{D}^2} \Rightarrow \boxed{e_2 = \frac{3}{4}x}$$

June 18/12/Q.20, June 14/13/Q.23

20 An elastic material with Young modulus E is subjected to a tensile stress S. Hooke's law is obeyed.

$$E = \frac{\text{Stress}}{\text{Strain}} \Rightarrow E = \frac{FL}{Ae} \Rightarrow E = \frac{SL}{e} \Rightarrow \frac{L}{e} = \frac{E}{S}$$

What is the expression for the elastic energy stored per unit volume of the material?

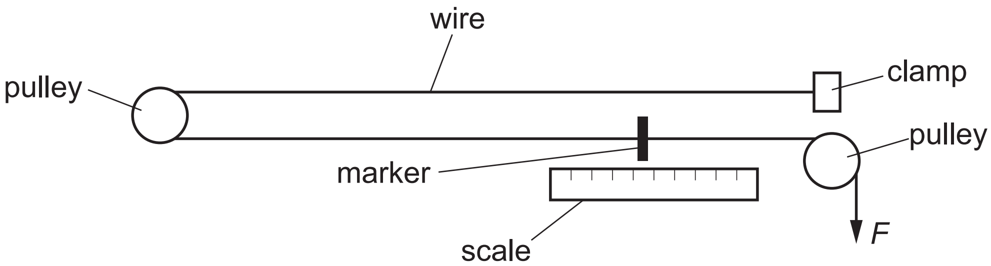
$$\frac{e}{L} = \frac{S}{E}$$

A $\frac{E}{2S^2}$ **B** $\frac{2E}{S^2}$ **C** $\frac{S^2}{E}$ **D** $\frac{S^2}{2E}$

$$\frac{E_p}{V} = \frac{\frac{1}{2}(F)(e)}{V} \Rightarrow \frac{E_p}{V} = \frac{\frac{1}{2}(F)(e)}{(A)(L)}$$

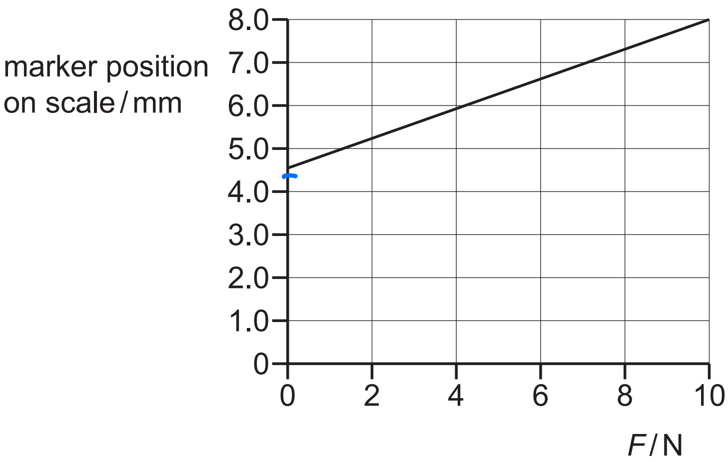
$$\frac{E_p}{V} = \frac{1}{2}(S)\left(\frac{S}{E}\right) \Rightarrow \boxed{\frac{E_p}{V} = \frac{S^2}{2E}}$$

19 In an experiment to measure the Young modulus of a metal, a wire of the metal of diameter 0.25 mm is clamped, as shown.



The wire passes from a clamp, around a frictionless pulley, and then to a second frictionless pulley where loads F are applied to it. A marker is attached to the wire so that the total length of wire between the clamp and the marker is initially 3.70 m. A scale is fixed near to this marker.

The graph shows how the reading on the scale varies with F.



What is the Young modulus of the metal?

A 5.5×10^{10} Pa

B 9.4×10^{10} Pa

C 1.6×10^{11} Pa

D 2.2×10^{11} Pa

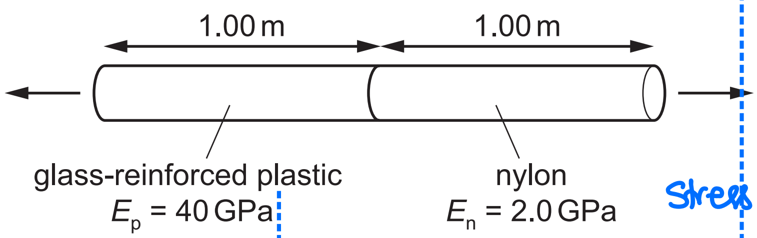
$$E = \frac{FL}{Ae} = \frac{FL}{\left(\frac{\pi d^2}{4}\right) e}$$

$$E = \frac{4FL}{\pi d^2 e}$$

$$E = \frac{4(10)(3.70)}{(3.14)(0.25 \times 10^{-3})^2 (8.0 - 4.6) 10^{-3}}$$

$$E = 2.21 \times 10^{11} \text{ Pa}$$

21 A composite rod is made by attaching a glass-reinforced plastic rod and a nylon rod end to end, as shown.



$E = \frac{FL}{Ae}$

$e = \frac{(\text{Stress})L}{E}$

Stress on both rods is same

The rods have the same cross-sectional area and each rod is 1.00 m in length. The Young modulus E_p of the plastic is 40 GPa and the Young modulus E_n of the nylon is 2.0 GPa.

The composite rod will break when its total extension reaches 3.0 mm.

What is the greatest tensile stress that can be applied to the composite rod before it breaks?

A 7.1×10^{-14} Pa

B 7.1×10^{-2} Pa

C 5.7×10^{6} Pa

D 5.7×10^{9} Pa

Total extension = $e_{Plastic} + e_{Nylon}$

$3.0 \times 10^{-3} = \frac{(\text{Stress})(1.00)}{40 \times 10^{9}} + \frac{(\text{Stress})(1.00)}{2.0 \times 10^{9}}$

Stress $= 5.71 \times 10^{6}$

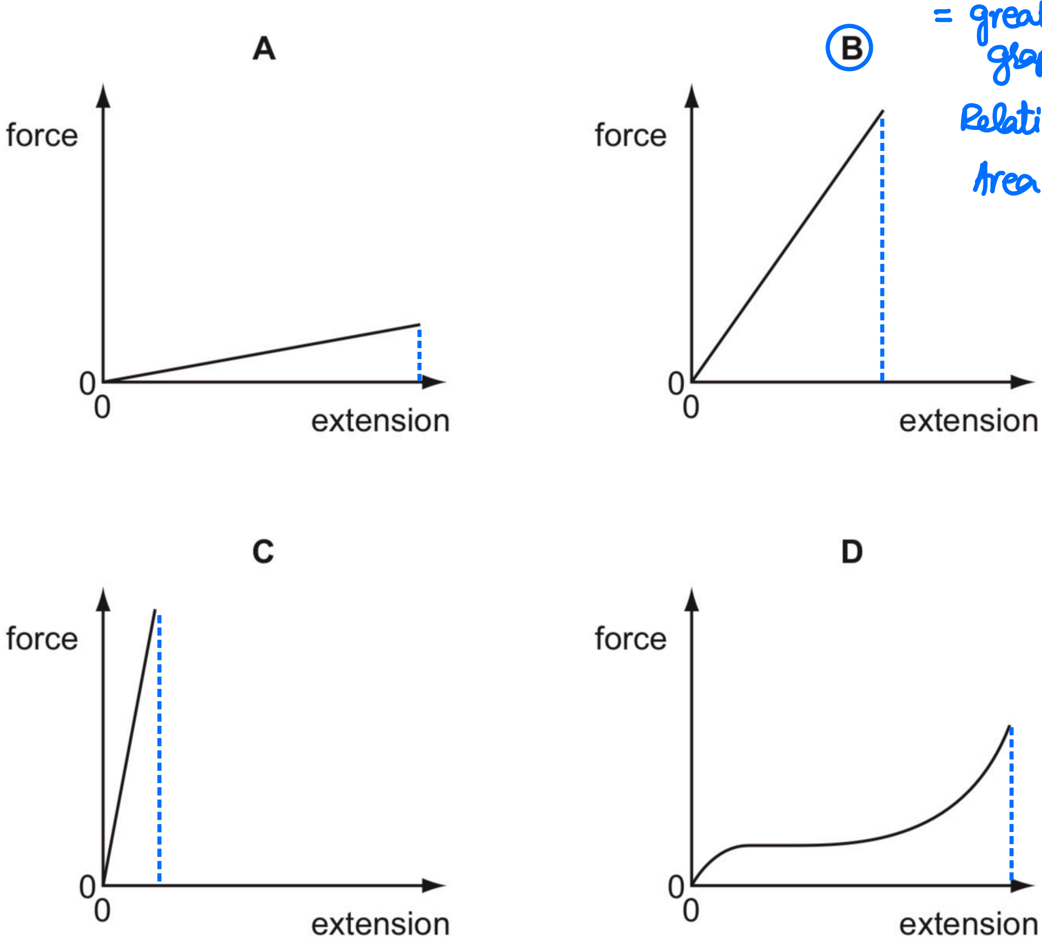
Nov.11/12/Q23

23 The following force-extension graphs are drawn to the same scale.

Which graph represents the deformed object with the greatest amount of elastic potential energy?

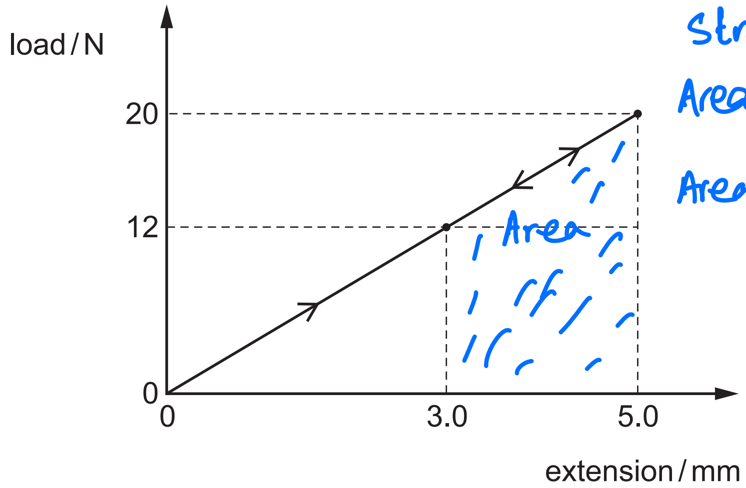
= greatest Area under graph

Relative order of Area [B > D > (A = C)]



ne 20/12/Q19

21 A metal wire is attached at one end to a fixed point and a load is hung from the other end so that the wire hangs vertically. The load is increased from zero to 20 N. This causes the wire to extend elastically by 5.0 mm. The load is then reduced to 12 N and the extension decreases to 3.0 mm.



Strain energy = Area under graph

Area = $\frac{1}{2}(12+20)(2.0\times10^{-3})$

$= 3.2\times10^{-2}$ J

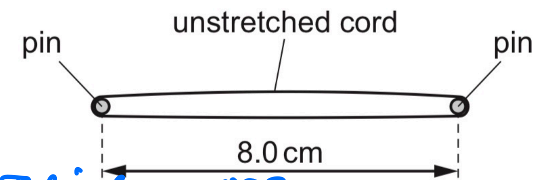
How much strain energy is released during the unloading process?

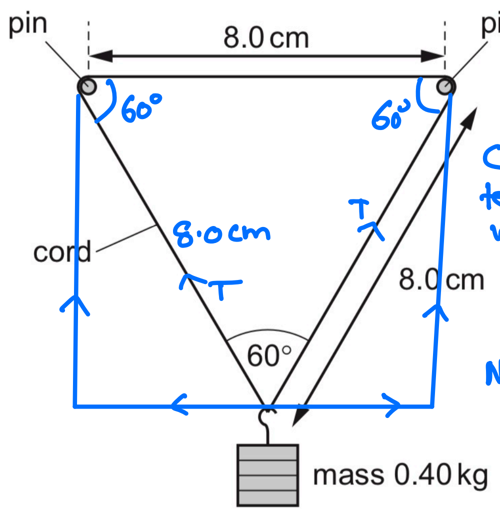
A 0.8×10^{-2} J **B** 1.8×10^{-2} J **C** 2.4×10^{-2} J **D** 3.2×10^{-2} J

June 20/12/Q18

18 An elastic cord of unstretched total length 16.0 cm and cross-sectional area 2.0×10^{-6} m^2 is held horizontally by two smooth pins a distance 8.0 cm apart.

The cord obeys Hooke's law. A load of mass 0.40 kg is suspended centrally on the cord. The angle between the two sides of the cord supporting the load is 60°.





Component of tension against weight = T sin 60

New length = 24.0 cm

extension = 24 − 16 = 8.0 cm

$2T\sin 60 = mg$

$T = \frac{(0.40)(9.81)}{2\sin 60} = 2.27$ N

$E = \frac{FL}{Ae} = \frac{TL}{Ae}$

$E = \frac{(2.27)(16.0\times10^{-2})}{(2.0\times10^{-6})(8.0\times10^{-2})}$

$E = 3.92\times10^{6}$ Pa

What is the Young modulus of the cord material?

A 5.7×10^{5} Pa **B** 1.1×10^{6} Pa **C** 2.3×10^{6} Pa **D** 3.9×10^{6} Pa

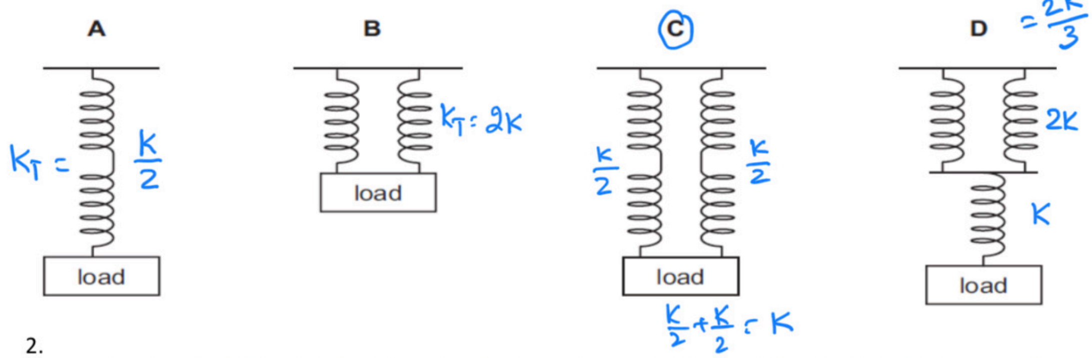
Deformation of solids

1.

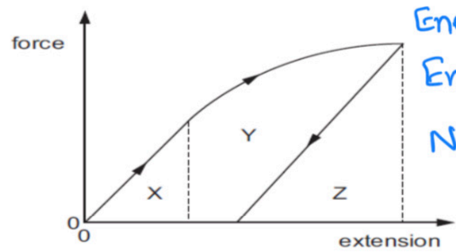
A number of identical springs are joined in four arrangements.

Which arrangement has the same spring constant as a single spring?

$\frac{(2K)(K)}{2K+K} = \frac{2K}{3}$



2.
A sample of material is stretched by a tensile force to a point beyond its elastic limit. The tensile force is then reduced to zero. The graph of force against extension is shown below.



Energy supplied = X + Y + Z

Energy recovered = Z

Net Energy = X + Y + Z − Z

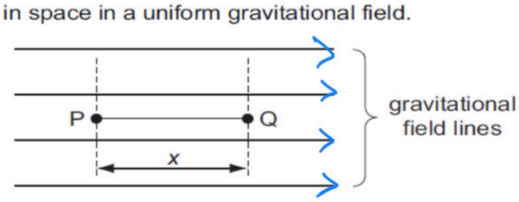
= X + Y

Which area represents the net work done on the sample?

A X **B** X + Y **C** Y + Z **D** Z

3.

A mass *m* is situated in space in a uniform gravitational field.



When the mass moves through a displacement *x*, from P to Q, it loses an amount of potential energy *E*.

Which row correctly specifies the magnitude and the direction of the acceleration due to the gravity in this field?

	magnitude	direction
A	$\frac{E}{mx}$	→
B	$\frac{E}{mx}$	←
C	$\frac{E}{x}$	→
D	$\frac{E}{x}$	←

$E = mgx \Rightarrow g = \frac{E}{mx}$

→

4.

The Young modulus of steel is determined using a length of steel wire and is found to have the value E.

Another experiment is carried out using a wire of the same steel, but of half the length and half the diameter.

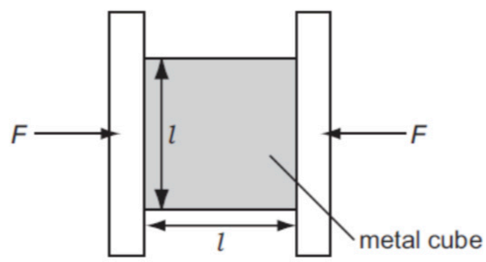
What value is obtained for the Young modulus in the second experiment?

A $\frac{1}{2}E$ **B** E **C** $2E$ **D** $4E$

Young Modulus depends upon nature of material.

5.

A metal cube of side l is placed in a vice and compressed elastically by two opposing forces F.



$E = \frac{FL}{Ae}$

$E = \frac{Fl}{(l^2)(\Delta l)}$

$\Delta l = \left(\frac{F}{E}\right)\frac{1}{l}$

$\Delta l = (\text{constant})\frac{1}{l}$

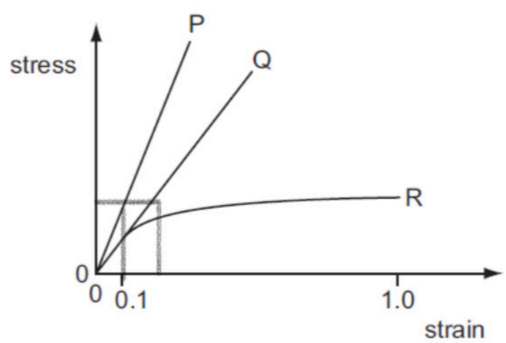
$\Delta l \propto \frac{1}{l}$

How will Δl, the amount of compression, relate to l?

A $\Delta l \propto \frac{1}{l^2}$ **B** $\Delta l \propto \frac{1}{l}$ **C** $\Delta l \propto l$ **D** $\Delta l \propto l^2$

6.

The graph shows the relationship between stress and strain for three wires of the same linea dimensions but made from different materials.



Which statements are correct?

1 The extension of P is approximately twice that of Q for the same stress.

2 The ratio of the Young modulus for P to that of Q is approximately two.

3 For strain less than 0.1, R obeys Hooke's law.

A 1, 2 and 3 **B** 1 and 3 only **C** 2 and 3 only **D** 2 only

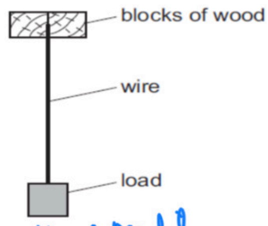
7.

The diagram shows a wire of diameter D and length L that is firmly clamped at one end between two blocks of wood. A load is applied to the wire which causes it to extend by an amount x.

$\frac{YL_1}{\cancel{\pi} D_1^2 e_1} = \frac{YL_2}{\cancel{\pi} D_2^2 e_2}$

$\frac{L}{D^2 x} = \frac{3L}{4D^2 e_2}$

$e_2 = \frac{3x}{4}$



$L_1 = L$, $D_1 = D$ | $L_2 = 3L$, $D_2 = 2D$

$E = E$

$\frac{FL_1}{A_1 e_1} = \frac{FL_2}{A_2 e_2}$

Young Modulus

By how much would a wire of the same material, but of diameter $2D$ and length $3L$, extend when the same load is applied?

A $\frac{2}{3}x$ **B** $\frac{3}{4}x$ **C** $\frac{4}{3}x$ **D** $\frac{3}{2}x$

8.

The behaviour of a wire under tensile stress may be described in terms of the Young modulus E of the material of the wire and of the force per unit extension k of the wire.

For a wire of length L and cross-sectional area A, what is the relation between E and k?

$F = ke$ — (1)

A $E = \frac{A}{kL}$ **B** $E = \frac{kA}{L}$ **C** $E = \frac{kL}{A}$ **D** $E = \frac{L}{kA}$

$E = \frac{FL}{Ae}$

$F = \left(\frac{EA}{L}\right)e$ — (2)

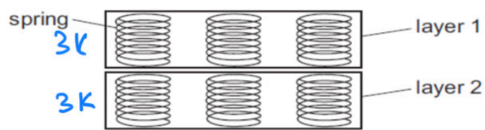
$K = \frac{EA}{L}$

$E = \frac{KL}{A}$

9.

The diagram shows the structure of part of a mattress.

$\frac{3}{2}K$



3K

3K

K↓

The manufacturer wants to design a softer mattress (one which will compress more for the same load).

Which change will **not** have the desired effect?

A using more layers of springs

B using more springs per unit area

C using springs with a smaller spring constant ✓

D using springs made from wire with a smaller Young modulus ✓

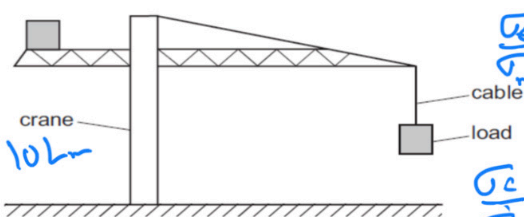
σ

10.

The diagram shows a large crane on a construction site lifting a cube-shaped load.

$\rho = \frac{m}{V}$

$m = \rho V$



$L_m = \frac{L_c}{10} \Rightarrow L_c = 10L_m$

$\frac{\sigma_c}{\sigma_m} = \frac{\frac{m_c g}{A_c}}{\frac{m_m g}{A_m}} = \frac{\frac{\rho V_c}{A_c}}{\frac{\rho V_m}{A_m}}$

$\frac{\sigma_c}{\sigma_m} = \frac{(\cancel{A_c})(L_c)}{\cancel{A_c}} \times \frac{\cancel{A_m}}{(\cancel{A_m})(L_m)}$

A model is made of the crane, its load and the cable supporting the load.

The material used for each part of the model is the same as that in the full-size crane, cable and load. The model is one tenth full-size in all linear dimensions.

$= \frac{L_c}{L_m}$

What is the ratio $\frac{\text{stress in the cable on the full-size crane}}{\text{stress in the cable on the model crane}}$?

$= \frac{10\cancel{L_m}}{\cancel{L_m}}$

A 10^0 **B** 10^1 **C** 10^2 **D** 10^3

$= 10$