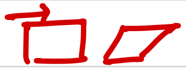
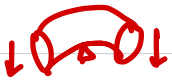
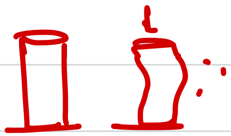
DEFORMATION

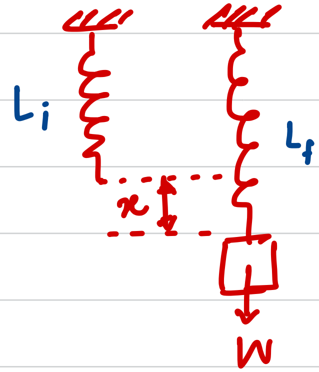
FORCE : The rate of change of momentum is called force

 Shear ← Deformation → Stretch 

 Bending ← **Deformation** → Compress 

Buckling ← Deformation → Twist 





EXTENSION : Change in length due to the application of force

$$x = L_f - L_i$$

$x = +ve$ — inc. in length

$x = -ve$ — dec. in length

HOOK'S LAW : Within the limit of proportionality, the force applied is directly proportional to the extension.

$$F \propto x$$

$$\boxed{F = kx}$$

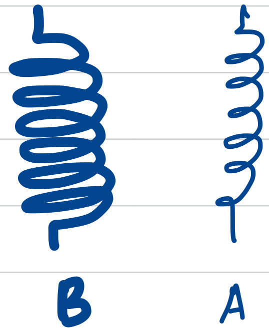
F : force $\quad$ x : extension

k : spring constant (Nm^{-1})

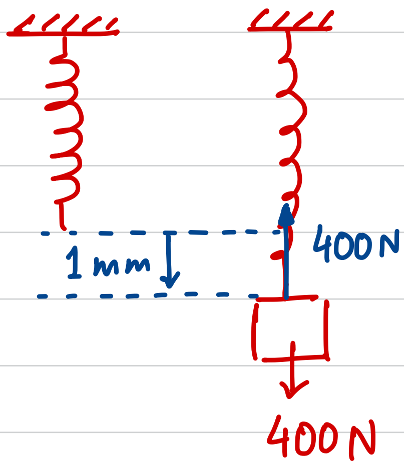
Spring constant tells about the stiffness of the body. i.e. how difficult it is to stretch or compress that body. Value of k depends upon both, design of object and material.

k_A : 400 $N\,mm^{-1}$
k_B : 1000 $N\,mm^{-1}$

More value of spring constant → More stiffness!



400 $N\,mm^{-1}$ means that 400N of force is needed to stretch the spring by 1 mm!

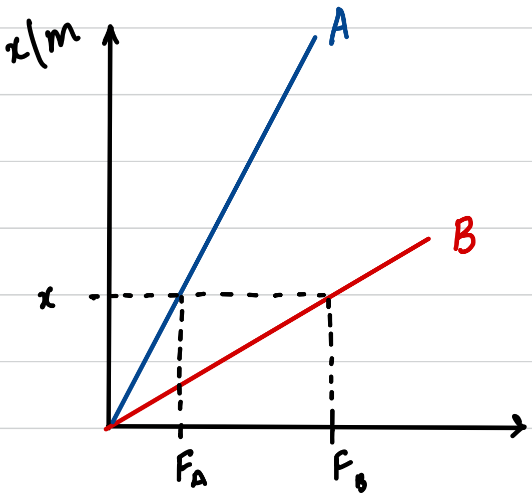


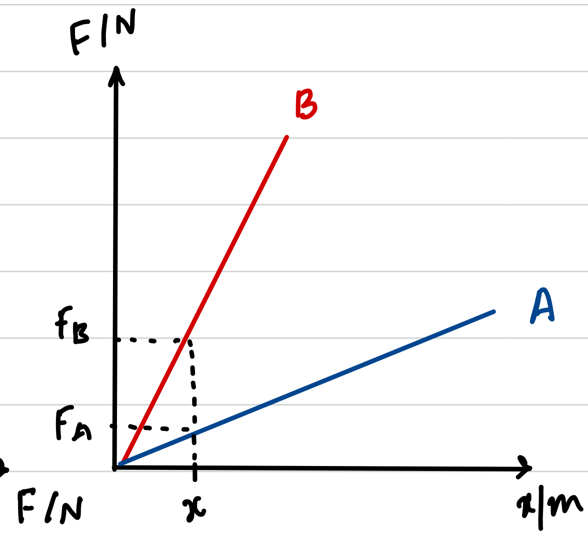
When the spring stretches by 1mm, the induced force in the spring is 400 N.

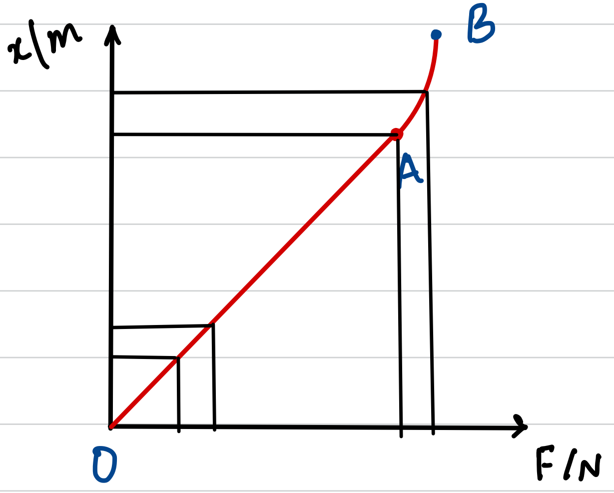
Hook's Law can also be understood in the similar way of induced force. i.e.

the extension in the spring is directly proportional to the induced force, within the limit of proportionality.

For the same value of extension, the force in B is more than in A so B is the stiffer spring with greater 'k'.







$O \rightarrow A$: spring constant is same so $F \propto x$ and $F = kx$ is a valid equation

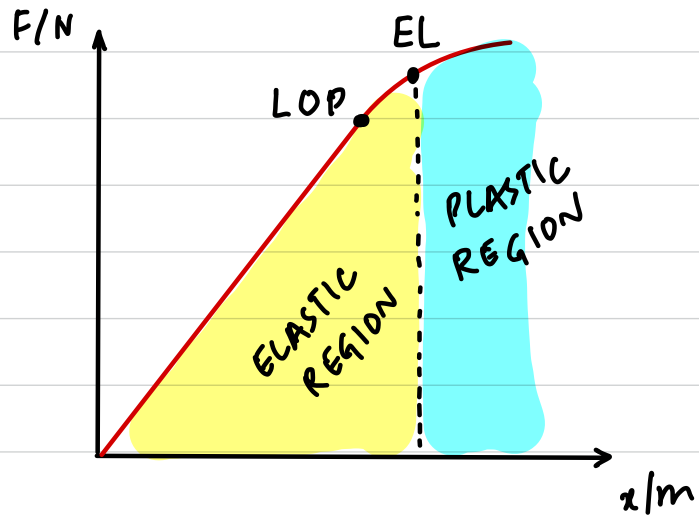
$A \rightarrow B$: the curve says that spring constant changes so F is no longer proportional to x. But $F = kx$ is still valid. The value of k is now diff.

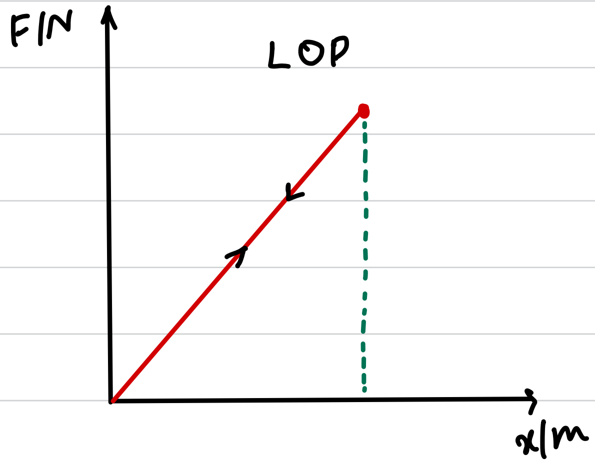
Spring constant decreases beyond the limit of proportionality as it get easier to deform the spring. For the same increment in force, the increment in extension is more than before!

LIMIT OF PROPORTIONALITY (LOP) VS ELASTIC LIMIT (EL)

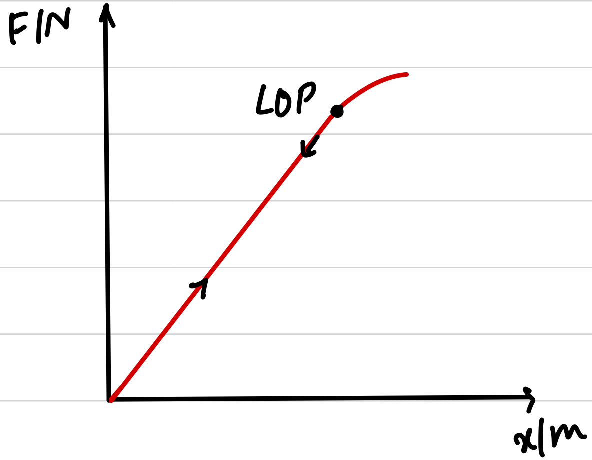
LOP: The point till which $F \propto x$. The $F-x$ graph is a straight line.

EL: The point beyond which permanent deformation occurs.

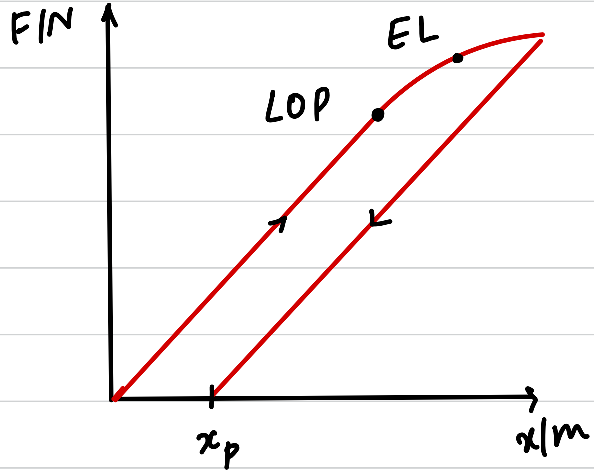




If force is applied till LOP, object returns to original length

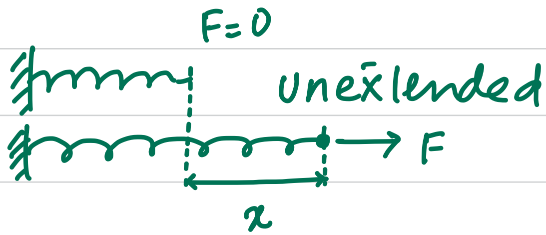


Still returns as EL is not crossed



If force is applied beyond EL, permanent deformation occurs.

ELASTIC POTENTIAL ENERGY (Strain Energy)



Workdone on the spring = Gain in Elastic Potential Energy

Definition

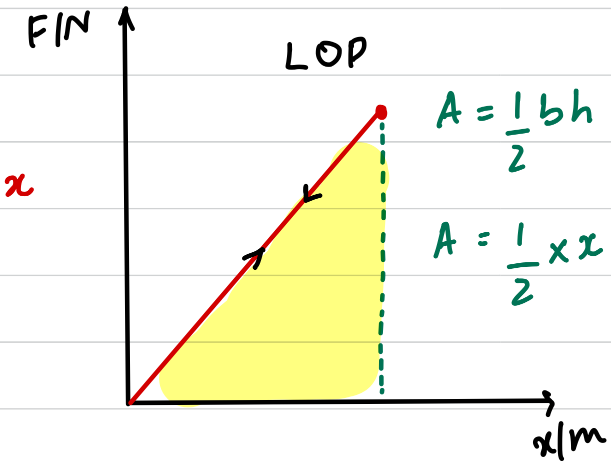
Energy stored in a body due to deformation

avg.

$$E.P.E = F \times s$$

$$= \left(\frac{0+F}{2}\right) \times x$$

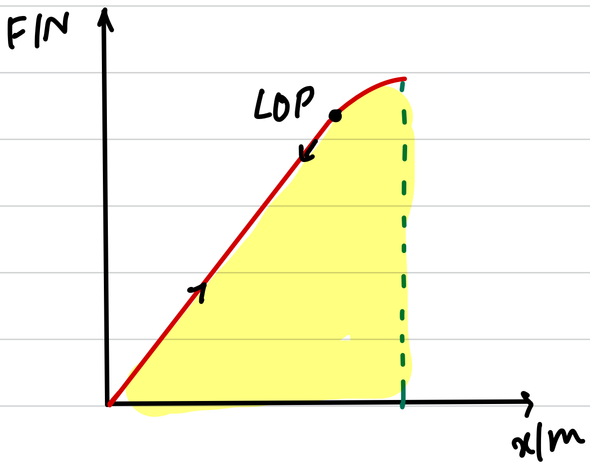
$$\boxed{EPE = \frac{1}{2}Fx}$$

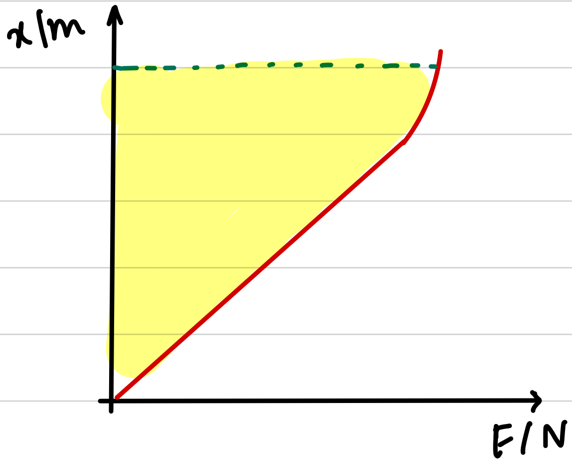


$A = \frac{1}{2}bh$

$A = \frac{1}{2} \times x \times F$

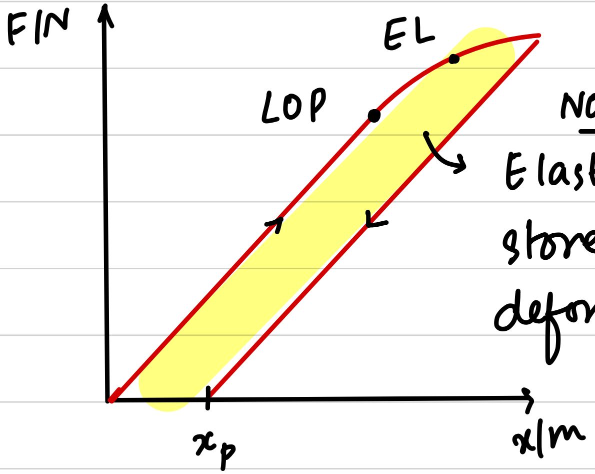
AREA UNDER F-x GRAPH = ELASTIC POTENTIAL ENERGY





Area is supposed to be between the graph and extension axis

$EPE = \frac{1}{2}Fx$ is applicable until the LOP as the graph curves beyond that. Beyond LOP, use area under graph!



NON-RECOVERABLE ENERGY

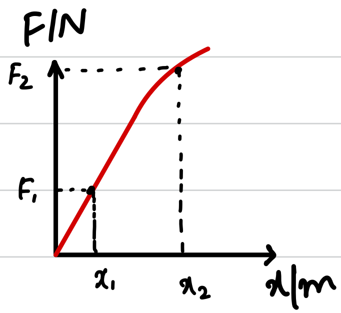
Elastic Potential Energy stored due to permanent deformation. Lost as heat from the body.

$EPE = \frac{1}{2}Fx$ as $F = kx$ so

$EPE = \frac{1}{2}(kx)x$

$$\boxed{EPE = \frac{1}{2}kx^2}$$

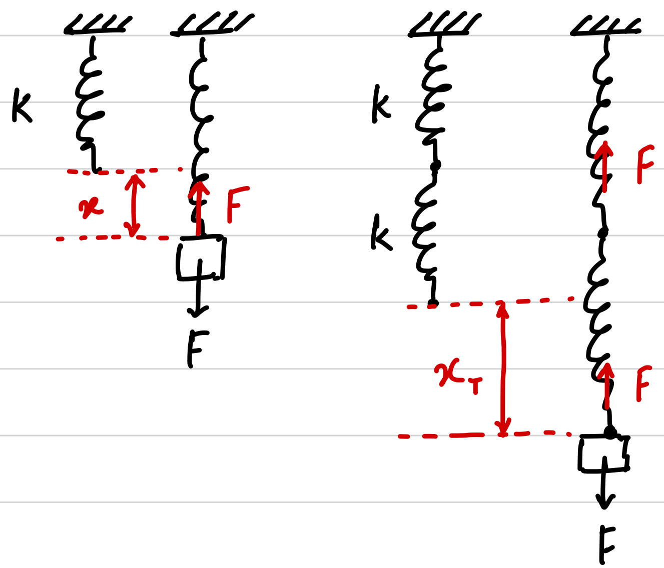
Beyond LOP, as the graph curves, the value of 'k' is no longer constant. k is calculated by ratio of F and x at any point on graph.



$\frac{F_1}{x_1} = k_1$ $\frac{F_2}{x_2} = k_2$

NO GRADIENT!

SPRINGS IN SERIES



When attached in series, both springs experience the same force. This causes both the springs to extend based on their spring constant. As a result,

→ the total extension of the combination inc.

→ The combination behave like a soft spring so the spring constant of combination dec.

Total Extension of Combination

$$x_T = x_1 + x_2 + x_3 \dots + x_n$$

(Similar Spring) $x_T = x + x + x \dots$ or $\boxed{x_T = nx}$

no. of springs

$$x_T = x_1 + x_2 + x_3$$

as $F = kx$ so $x = \frac{F}{k}$

$$\frac{F}{k_T} = \frac{F}{k_1} + \frac{F}{k_2} + \frac{F}{k_3}$$

$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$$

Total Spring Constant of Combination

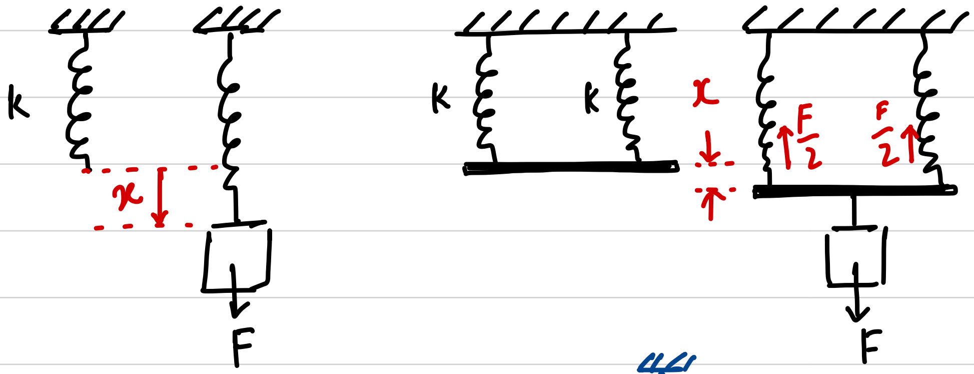
$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \dots + \frac{1}{k_n}$$

if all springs were of same "k" (similar springs)

$$\frac{1}{k_T} = \frac{1}{k} + \frac{1}{k} + \frac{1}{k} \dots \text{ or } \frac{1}{k_T} = \frac{n}{k} \text{ or } \boxed{k_T = \frac{k}{n}}$$

no. of springs

SPRINGS IN PARALLEL



- When springs are attached in parallel, the applied force is divided among the springs attached across one another.
- As the springs are of equal values of "k" (similar springs), the force is divided equally among springs.

$$F_{spring} = \frac{F_T}{n}$$

force in each spring! ; F_T → applied force ; n → no. of springs

e.g. $F = 100\,N$ $k = 20\,N/mm$

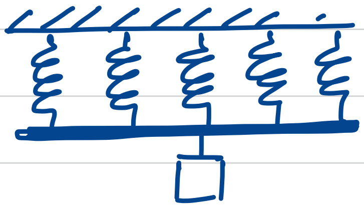
find x?

$F = kx$

$100 = 20 \times x$

$x = 5\,mm$ (1 spring)

if 5 such springs were attached in parallel



$x_n = \frac{x}{n}$

$x_n = \frac{5}{5} = 1\,mm$.

They are working as $5 \times 20 = 100\,Nmm^{-1}$!

- $F = kx$ $F \propto k$. $F \propto x$ $k \propto \frac{1}{x}$

- The overall extension of the system

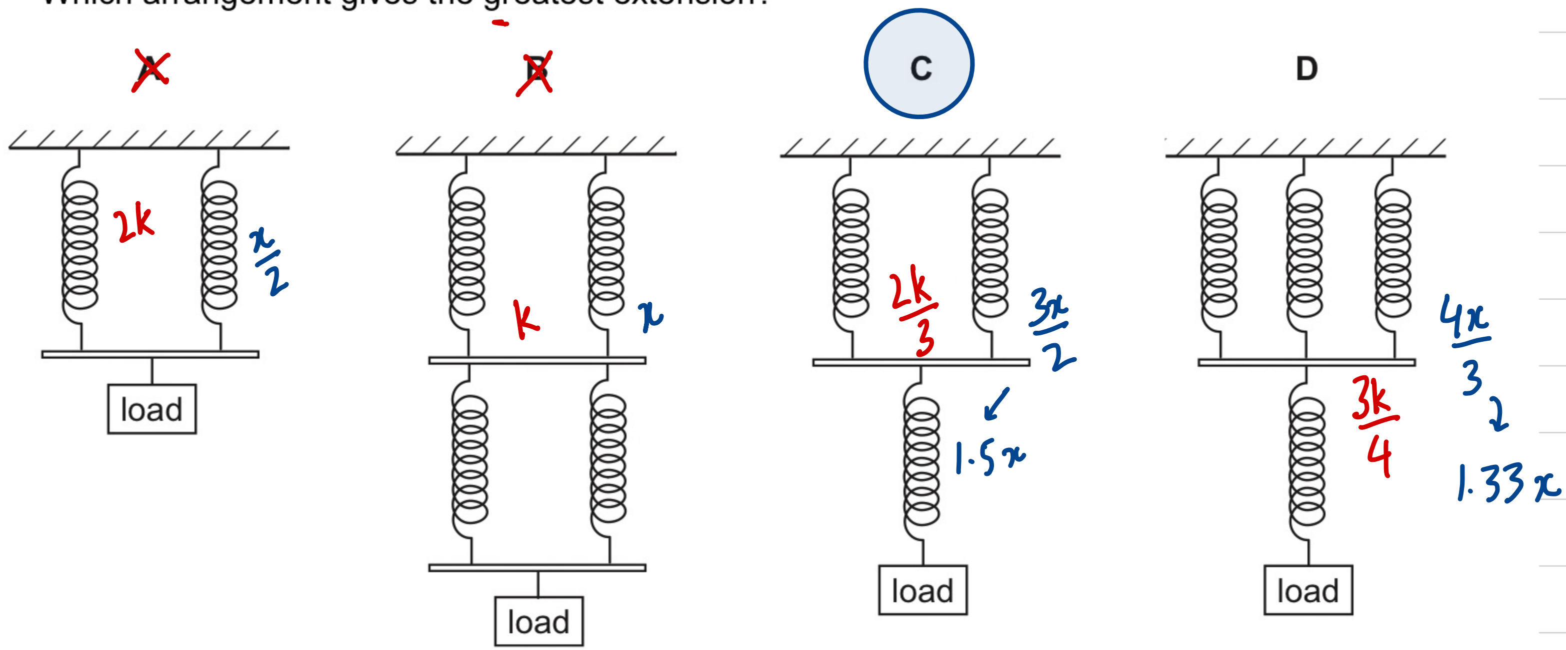
$\boxed{x_n = \frac{x}{n}}$; x → if 1 spring was attached ; n → # of springs.

$\boxed{k_n = nk}$

D plastic and ductile

21 A number of similar springs, each having the same spring constant, are joined in four arrangements. The same load is applied to each.

Which arrangement gives the greatest extension?



- Jitnay spring parallel main, utni kam extension.
- Jitnay spring series main, utni ziada extension

STRESS (σ)

The force per unit cross sectional area.

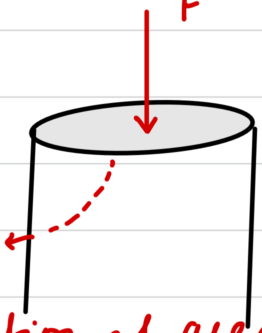
→ Stress tells about the distribution of force inside the body.

SI Unit: Pascal (Pa)

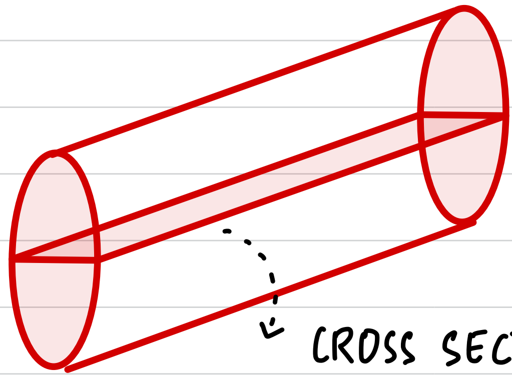
$$\boxed{\sigma = \frac{F}{A}}$$

Pressure = $\dfrac{\text{force applied}}{\text{surface area}}$

Stress = $\dfrac{\text{force induced}}{\text{cross-sectional area}}$



surface area & cross sectional area are same.



STRAIN (ϵ)

The change in length per unit length

OR

The ratio of change in length to original length

$$\boxed{\epsilon = \frac{x}{L}}$$

x → extension

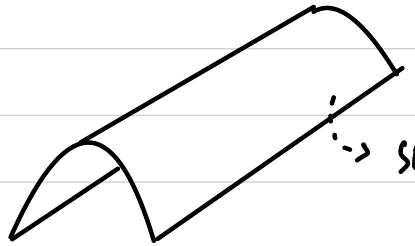
L → initial length

SI Unit: NO UNIT

mm/m

percentage strain

$$\epsilon\% = \frac{x}{L} \times 100$$



$x = 4\text{mm}$

$L = 5\text{cm}$

$$\epsilon\% = \frac{4\times10^{-3}}{5\times10^{-2}} \times 100 = 8\%$$

YOUNG'S MODULUS (E)

The ratio of stress to strain.

$$\boxed{E = \frac{\sigma}{\varepsilon} = \frac{\text{stress}}{\text{strain}}}$$

SI Unit: Pascals (Pa)

- Stress and strain helped in making the quantities independent of design.

$E = \sigma \div \varepsilon$

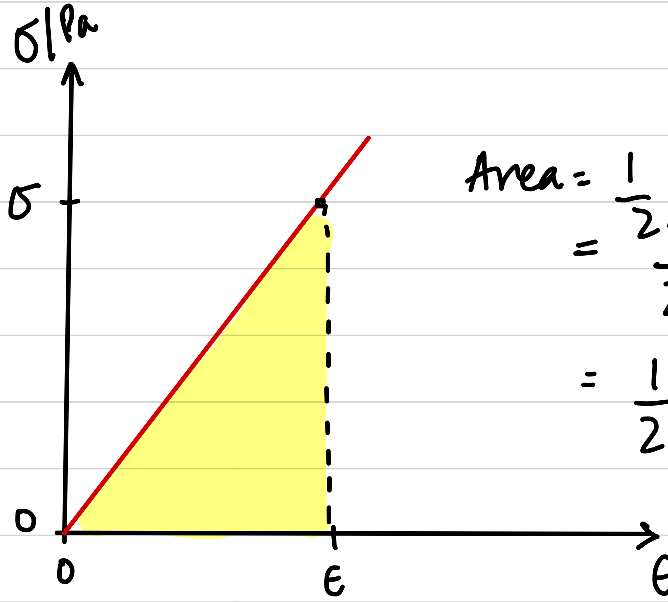
$E = \frac{F}{A} \div \frac{x}{L}$

$$\boxed{E = \frac{FL}{Ax}}$$

e.g. Young's Modulus of Steel is around 200 GPa.

Hence Young's Modulus is also independent of design i.e. it is a MATERIAL PROPERTY

To find out the value of Young's Modulus of a body from graph, take value of σ & ε and apply $E = \frac{\sigma}{\varepsilon}$.



Area $= \frac{1}{2} b h$

$= \frac{1}{2} \varepsilon \sigma$

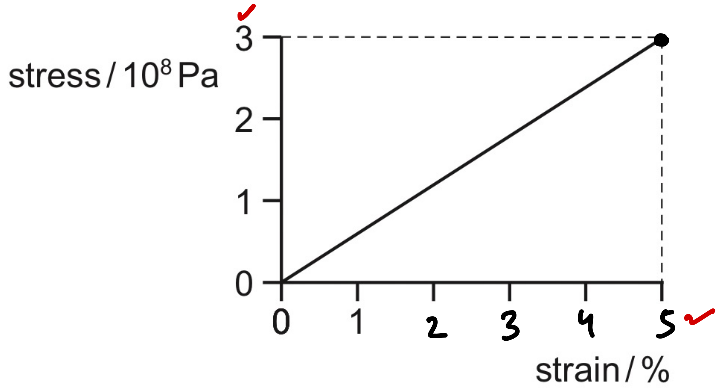
$= \frac{1}{2} \cdot \frac{x}{L} \cdot \frac{F}{A}$

Area $= \frac{1}{2} \frac{Fx}{V}$ $(V = A \times L)$

$$\boxed{\text{Area} = \frac{\text{Strain Energy}}{\text{Volume}}}$$

Area under σ-ε graph tells strain energy density i.e. energy stored per unit volume

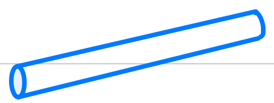
19 In stress-strain experiments on metal wires, the stress axis is often marked in units of 10^8 Pa and the strain axis is marked as a percentage. This is shown for a particular wire in the diagram.



What is the value of the Young modulus for the material of the wire?

A 6.0×10^7 Pa **B** 7.5×10^8 Pa **C** 1.5×10^9 Pa **D** 6.0×10^9 Pa

- $E = \dfrac{\sigma}{\varepsilon} = \dfrac{3 \times 10^8}{0.05} = 6 \times 10^9$. $\dfrac{x}{L} \times 100 = 5\%$. So $\dfrac{x}{L} = 0.05$



- Strain Energy = ? Area = $\dfrac{\text{Strain Energy}}{\text{Volume}}$

$d = 5.0 \times 10^{-3}$ m

$L = 1$ m

Strain Energy = A x Vol.

Strain Energy = $\left(\frac{1}{2} \times 0.05 \times 3 \times 10^8\right) \times \left(1.96 \times 10^{-5}\right)$

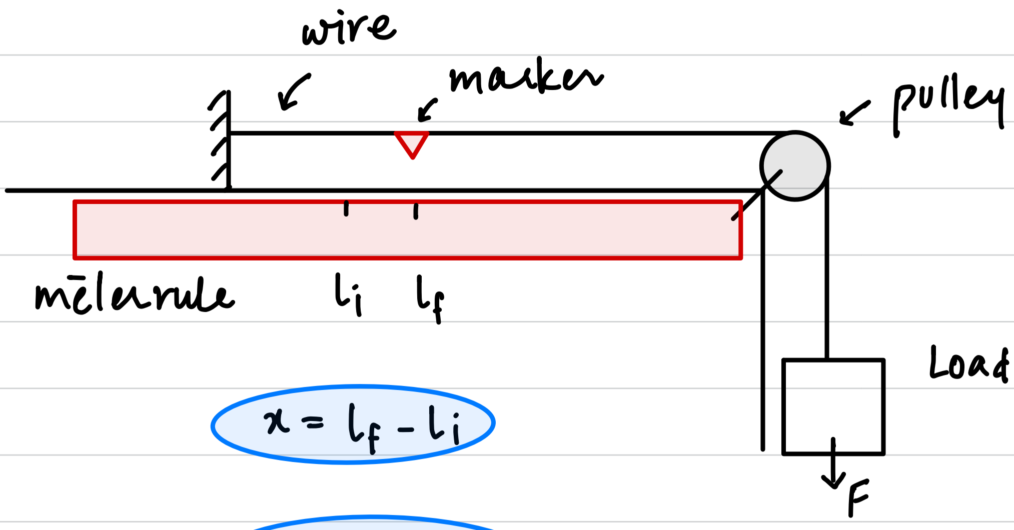
$\boxed{\text{Strain Energy} = 147 \text{ J}}$

$V = \pi r^2 h$

$V = \pi \left(\frac{5 \times 10^{-3}}{2}\right)^2 (1)$

$V = 1.96 \times 10^{-5}\ m^3$

Experiment to determine Young's Modulus

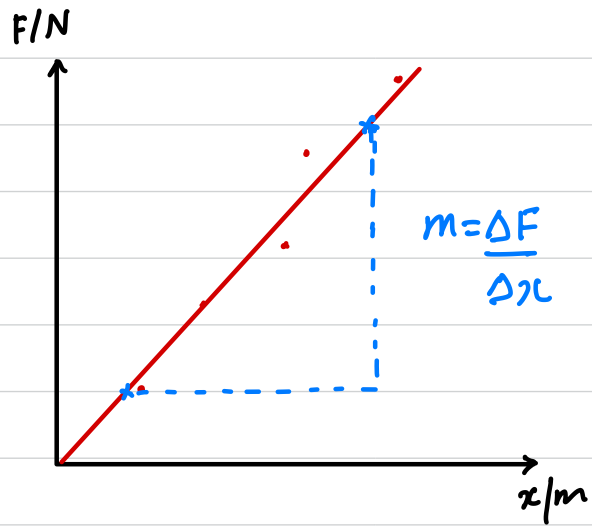


$$x = l_f - l_i$$

$$F = W = mg$$

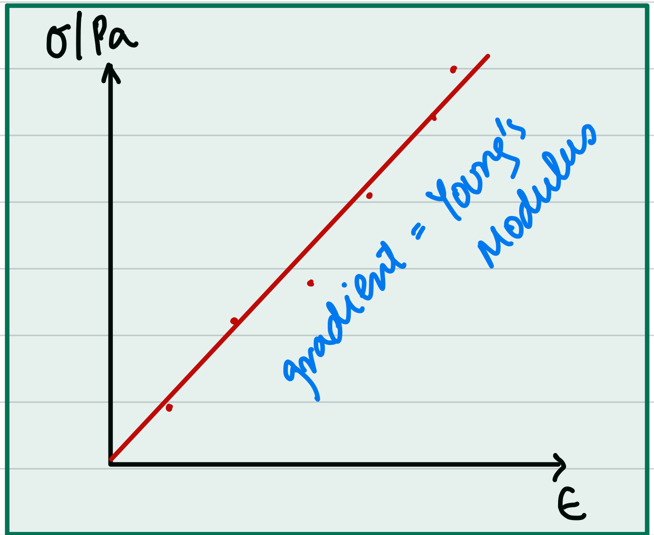
$$E = \frac{\sigma}{\epsilon} = \frac{FL}{Ax}$$

$$E = \frac{F}{x} \cdot \frac{L}{A}$$



for an average value of F/x, determine the gradient

$$\boxed{E = \text{gradient} \times \frac{L}{A}}$$



DIRECT APPROACH

L = initial length of wire

(determine before exp) (meter rule or measuring tape)

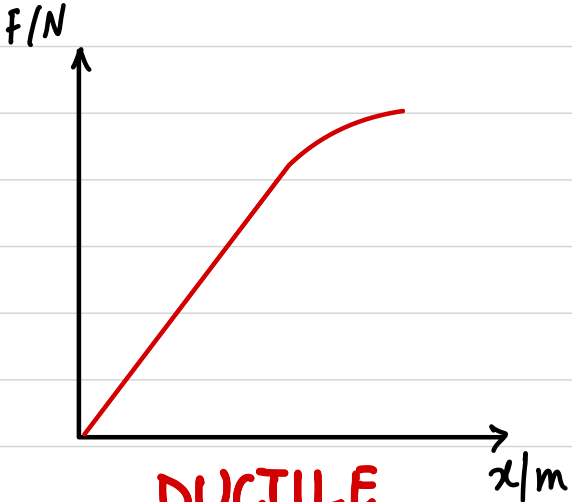
A = cross sectional area of wire

$A = \frac{\pi d^2}{4}$ d: dia of wire

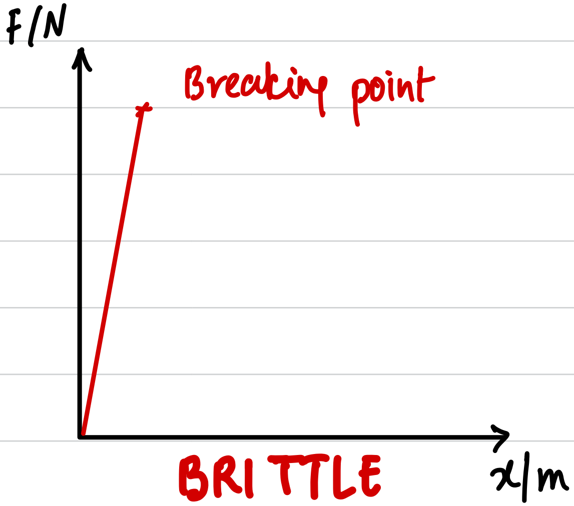
(Micrometer Screw Gauge)

Note "d" from diff points on wire and determine an average.

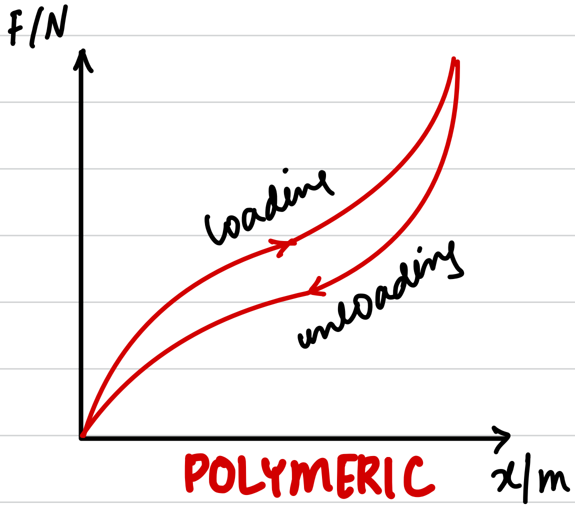
F-x graphs of different material



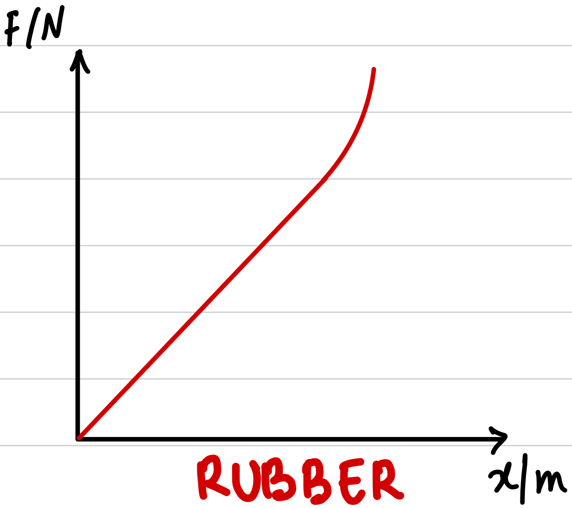
DUCTILE
(metal wires)



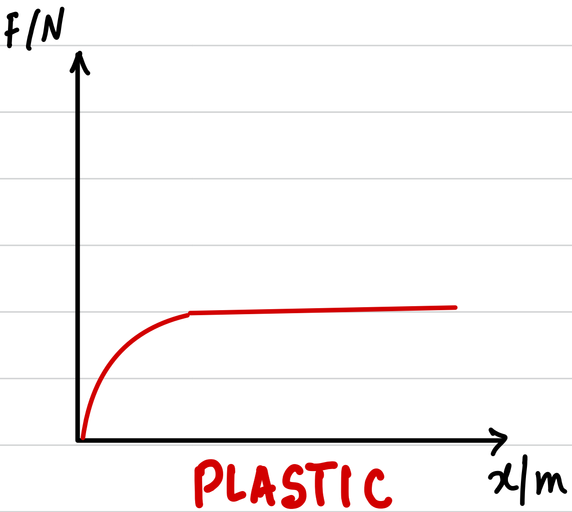
BRITTLE
(glass)



POLYMERIC

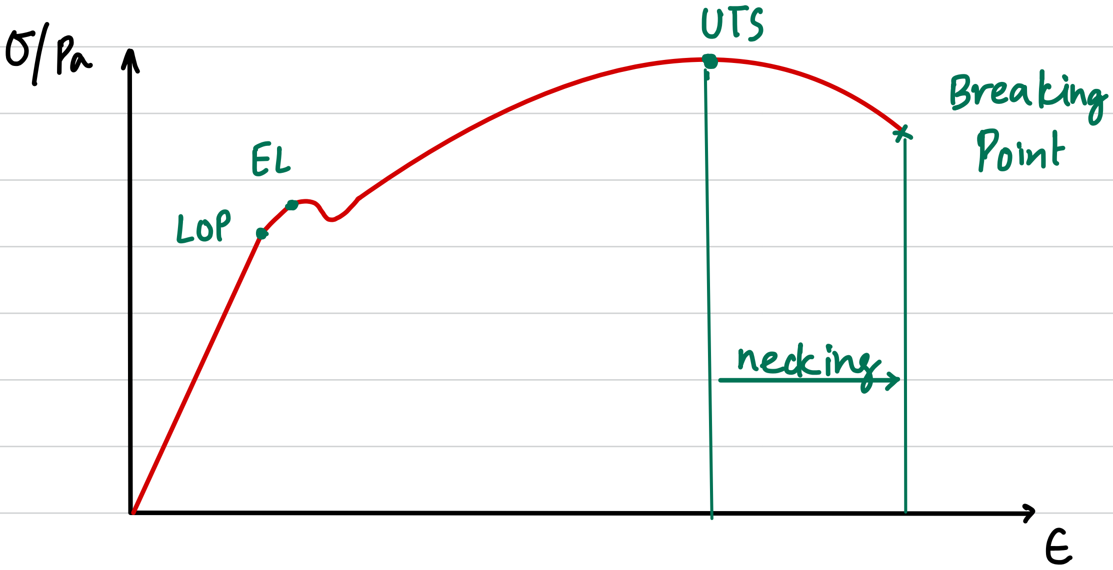


RUBBER



PLASTIC
(polythene)

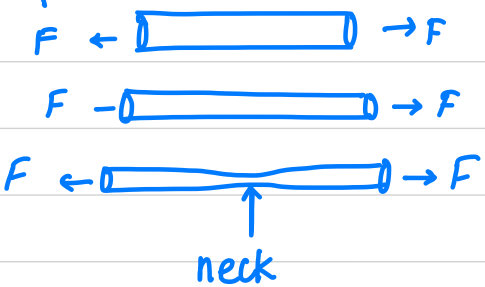
STRESS- STRAIN GRAPH OF DUCTILE MATERIAL



UTS (Ultimate Tensile Stress)

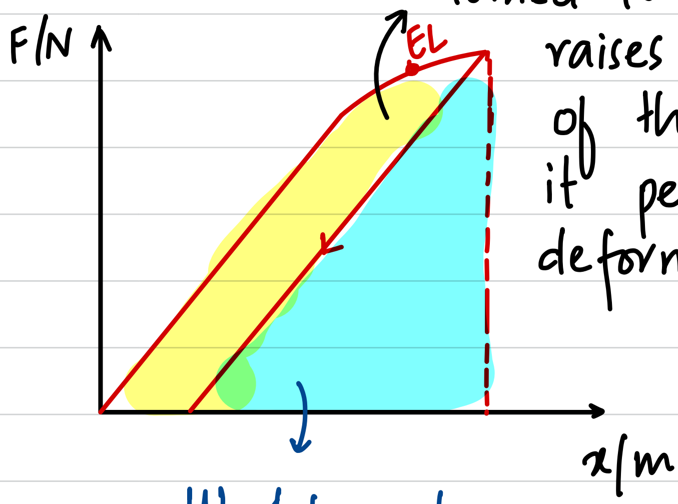
The maximum stress that a material can endure before failure.

Necking



Formation of a neck in a material due to excessive strain.

Workdone on stretching the spring



turned to heat and raises the temp of the body as it permanently deforms.

Workdone by the spring as it relaxes.

↳ Maximum work that can be done by a spring as it contracts after crossing elastic limit.

Stopped! no more contraction!