PROSPERITY ACADEMY

AS PHYSCIS 9702

Crash Course

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DEFORMATION IN SOLIDS

COMPLETE NOTES

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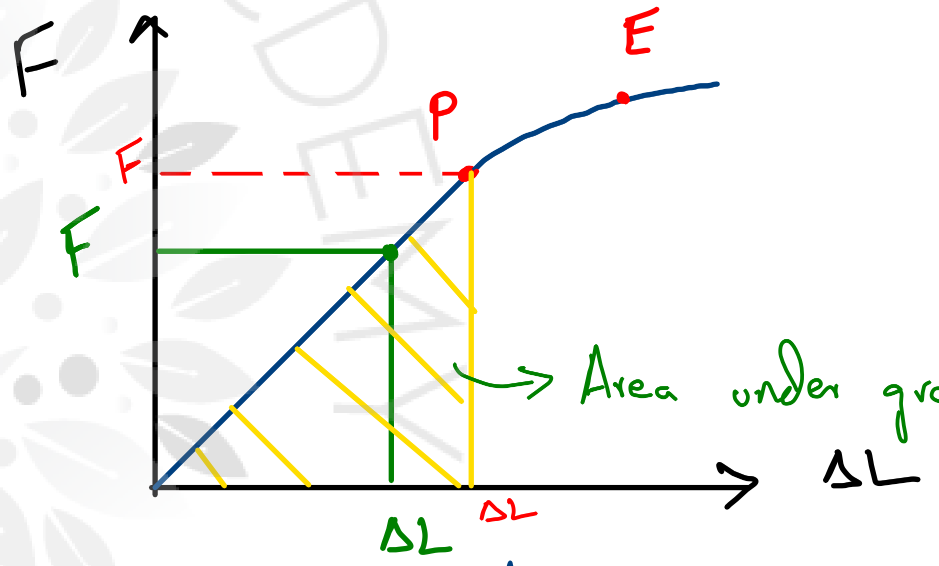
Deformation in Solids:- Change in Shape (AS content:- Extension / compression)

Hooke's law:-

The tensile force applied on an object under stress is directly proportional to the change in length of the object until the limit of proportionality is not exceeded.



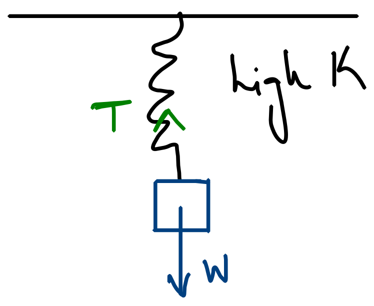
$$K = \frac{F}{\Delta L}$$

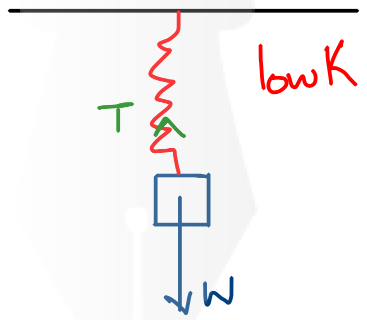
$$F \propto \Delta L \Rightarrow F = K \Delta L$$

K: spring's constant (constant for only 1 spring)

Spring's constant:- $\left(K = \frac{F}{\Delta L} \right)$ Ratio of the tensile force to the change in length of the spring.

Assume a constant force





$$\uparrow K \propto \frac{1}{\Delta L \downarrow} \qquad \downarrow K \propto \frac{1}{\Delta L \uparrow}$$

Spring's with a higher spring constant, extend less for the same amount of force as springs with low spring constants

Elastic potential energy:- It is the energy stored in an object by virtue of it's deformation.

$$\boxed{E.P.E = \frac{1}{2} \times \Delta L \times F}$$ or $\frac{1}{2} \times \Delta L \times K \times \Delta L$ (only works till limit of proportionality)

$$\boxed{E.P.E = \frac{1}{2} K \Delta L^2}$$

Limit of proportionality:- After this point, Hooke's law is not obeyed.

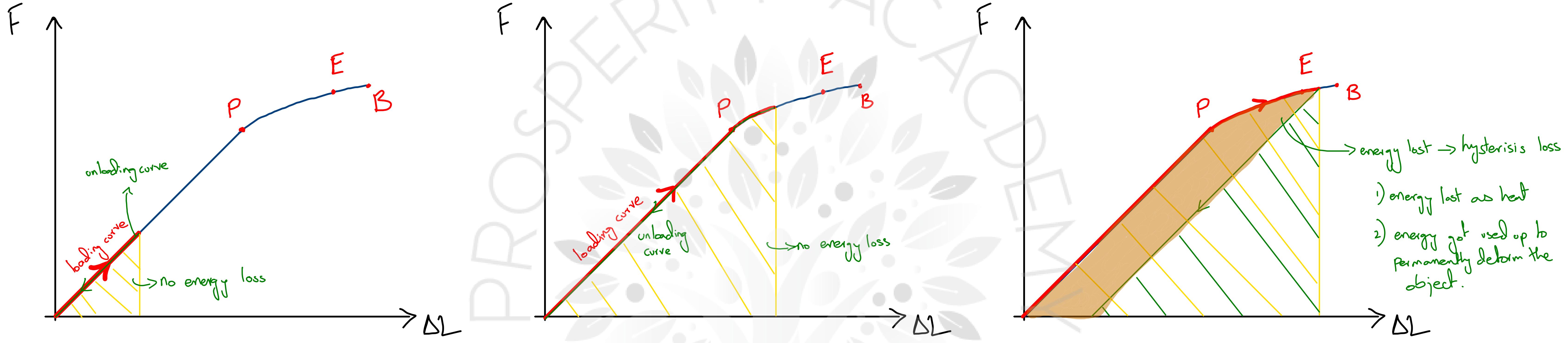
After this point, an increment in the force does not cause a proportional increment in change of length.

Elastic:- Once deformed, object returns to its original shape/length.

Elastic limit:- Point after which object does not return to its original shape/length or gets permanently deformed (plastic deformation)

Loading :- Applying / increasing force.

Unloading :- Decreasing / Removing force



Breaking point :- It is the point after which object ruptures

Stress:- Ratio of tensile force applied to the cross sectional area for a deformed solid. Scalar

Stress:- $\dfrac{\text{Force}}{\text{Area}}$ $\left(\dfrac{Kg\,m\,s^{-2}}{m^2} = Kg\,m^{-1}s^{-2} \right)$

Strain:- Ratio of change in length to the original length of a deformed solid. (Not a physical quantity)

Strain:- $\dfrac{\Delta L}{L_o}$ $\left(\dfrac{\cancel{m}}{\cancel{m}} = \text{no units} \right)$

$L_o \rightarrow$ original length

Young's Modulus:- Ratio of the stress to strain of an object. Scalar.

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \div \frac{\Delta L}{L_o} \Rightarrow \frac{F}{A} \times \frac{L_o}{\Delta L}$$

$$= \boxed{\frac{Kg\,m^{-1}s^{-2}}{\text{no units}} = Kg\,m^{-1}s^{-2}}$$

$$\boxed{E = \frac{F \times L_o}{A \times \Delta L}}$$

Constant for a type of material

K_1, E — copper spring

$L_o = 15cm$ $A = 1\times10^{-3}m^2$

K_2, E — copper spring

$L_o = 17cm$ $A = 5\times10^{-3}m^2$

E same (same material)

K will be different

The significance of Young Modulus :-

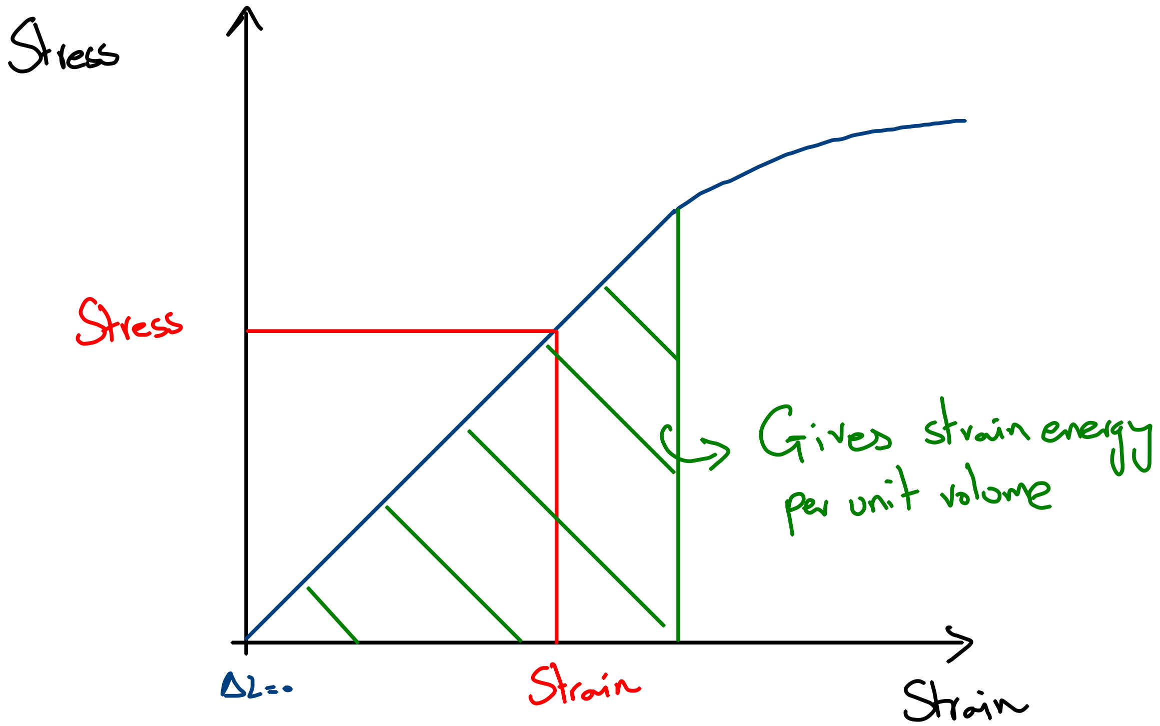
$$E = \frac{F \times L_o^{\;\to\,\text{const}}}{A \times \Delta L}$$

↳ approximately constant

$$\Rightarrow \downarrow\uparrow E \propto \frac{F}{\Delta L\downarrow\uparrow} \quad || \quad \downarrow\uparrow K = \frac{F}{\Delta L\downarrow\uparrow}$$

The greater the young modulus, the less the extension compared to a object with lower young modulus

Stress - Strain Graphs :-



$$E = \frac{\text{stress}}{\text{Strain}} = \frac{F \times L_o}{A \times \Delta L} \Rightarrow E \propto \frac{F}{\Delta L}$$

$$E \propto K$$

$$A = \frac{1}{2} \times \text{stress} \times \text{strain}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l_o}$$

→ E.p.e

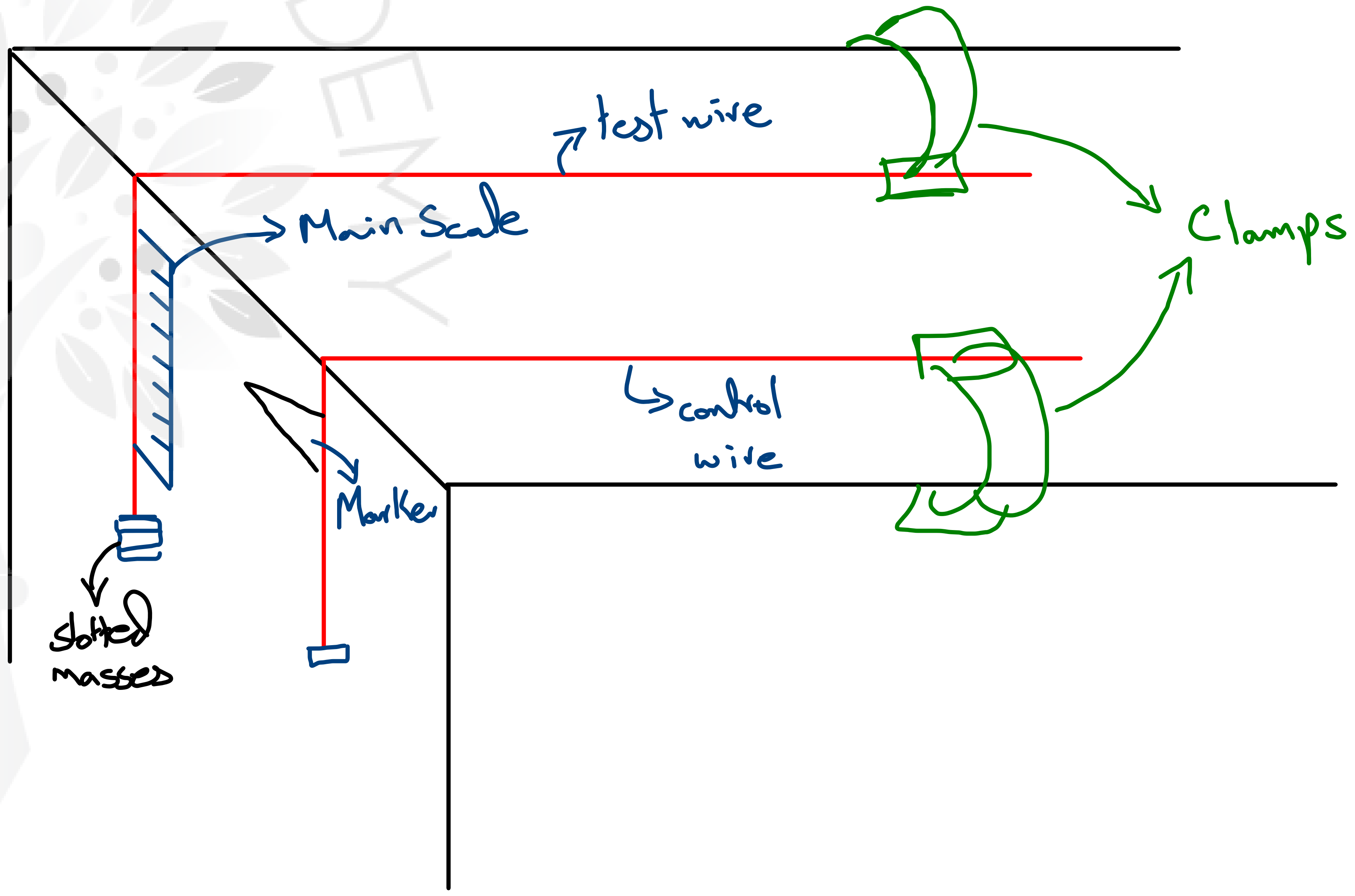
→ V

$A = E.p.e / V$ (Elastic potential energy/Volume)

Searle's Experiment:- To measure the Young Modulus.

Apparatus Required:-

1) Test wire → Young modulus of this is required
2) Control wire → this should be identical to test wire (material, length, cross sectional Area)
3) Slotted masses → for loading
4) Wooden blocks and clamps → to hold the wires in place
5) Main scale
6) Vernier scale / Marker → to work out ΔL
7) Metre rule → to measure l_0 of wires
8) Micrometer screw gauge → Measure the diameter of the wire ⇒ $\frac{\pi d^2}{4} = A$



1) Get a test wire and a control wire for the young modulus of the material you are wanting to measure for.
These 2 wires must be same in every sense (E same, L_o same, A same)

2) Measure L_o using metre rule (Avoid Parallax error)

3) Use a micrometer screw gauge to measure the diameter at atleast 3 different points along the wire. Average out the diameter.

Cross sectional area = πr^2 or $\frac{\pi d^2}{4}$

4) Setup the experiment as shown

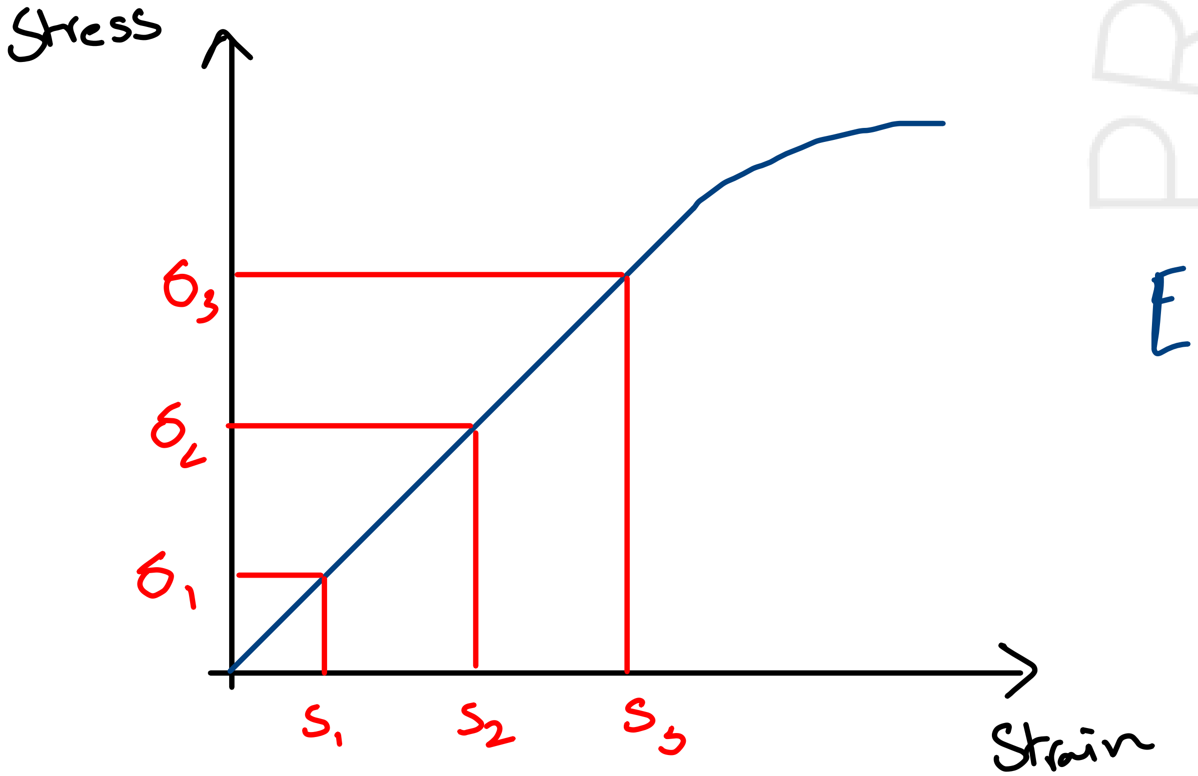
5) Attach small masses initially on both wires to make them taut (Removing coiling)

6) Add slotted masses on the test wire. Allow it to settle and take the reading using marker and main scale (Avoid parallax errors)

7) Make table

(Assuming $g = 10\,N\,Kg^{-1}$)

Mass added / Kg	Weight / N	Reading on main scale / cm	ΔL / cm	Stress	Strain
0	0	25	0	Use F/A	Use $\Delta L / L$
2	20	24.5	0.5		
3	30	24	1.0		
4	40	23	2.0		
5	50	22	3.0		

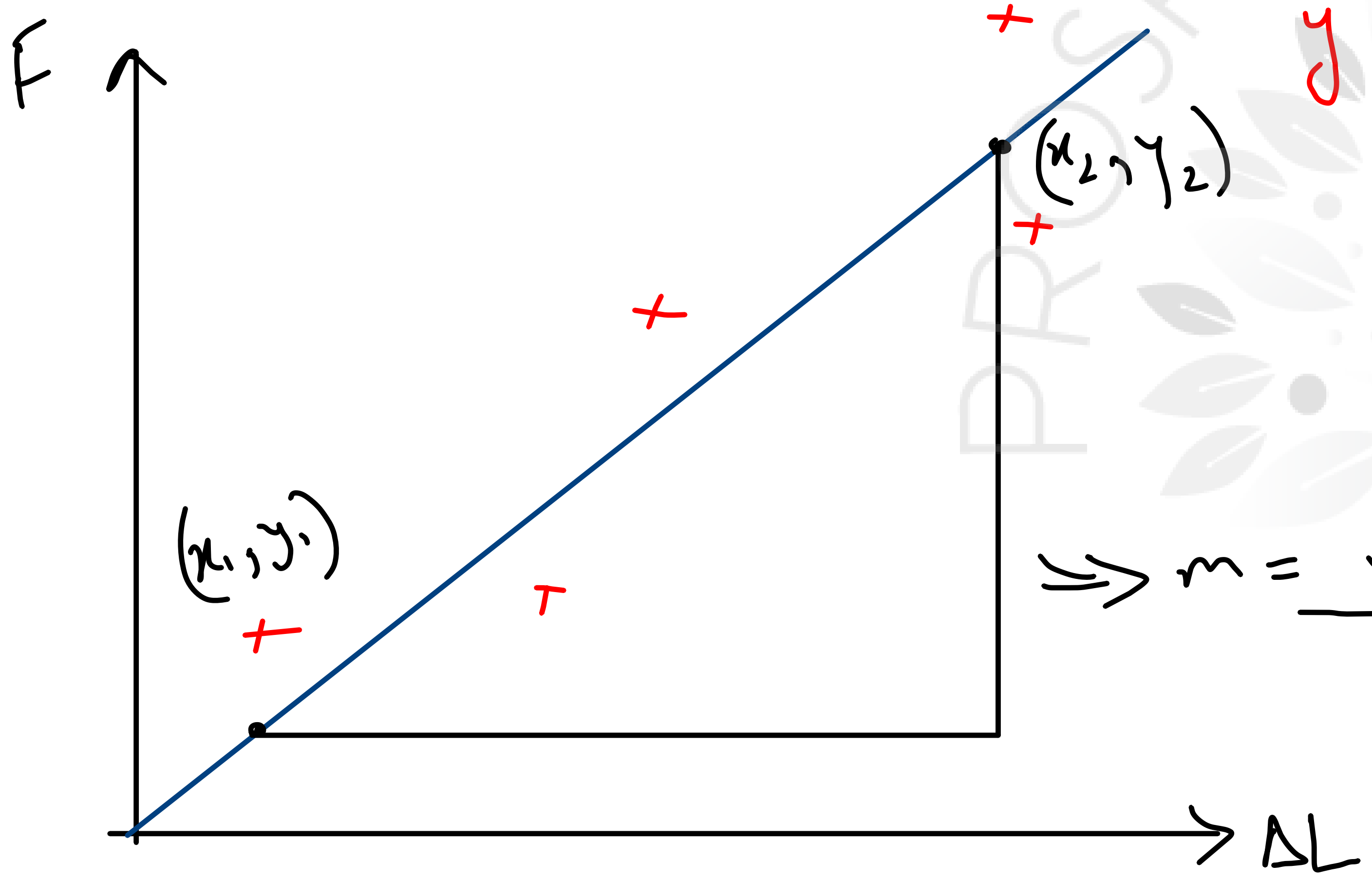


$$E_{avg} = \frac{\frac{\sigma_1}{s_1} + \frac{\sigma_2}{s_2} + \frac{\sigma_3}{3}}{3}$$

*) You can also use Force - extension graph

$$E = \frac{F \times L_0}{A \times \Delta L} \Rightarrow F = \frac{EA}{L_0} \times \Delta L$$

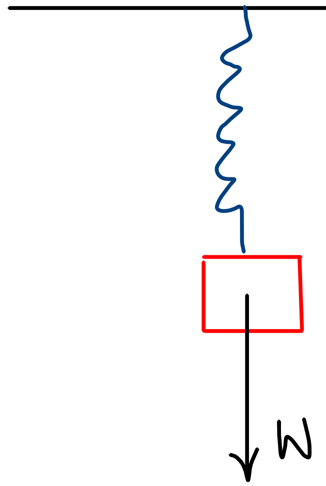
$$y = m \quad x$$



$$\Rightarrow m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{EA}{L_0}$$

Springs in series :-

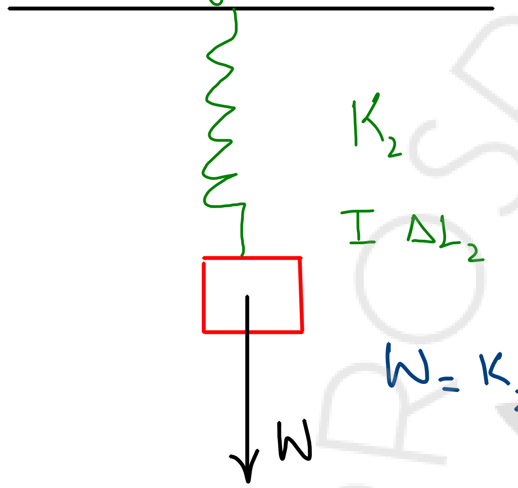
Spring 1



$W = K_1 \times \Delta L_1$

$\frac{1}{K_1} = \frac{\Delta L_1}{W}$

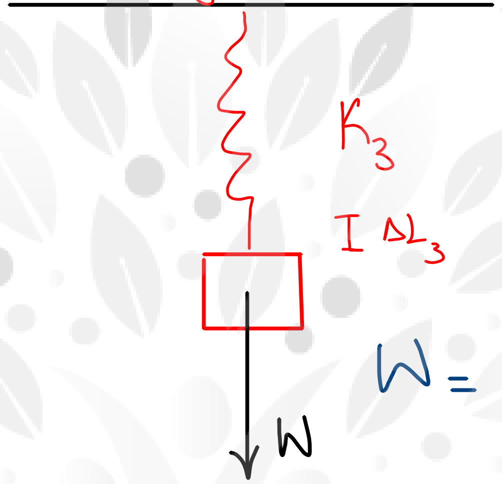
Spring 2



$W = K_2 \times \Delta L_2$

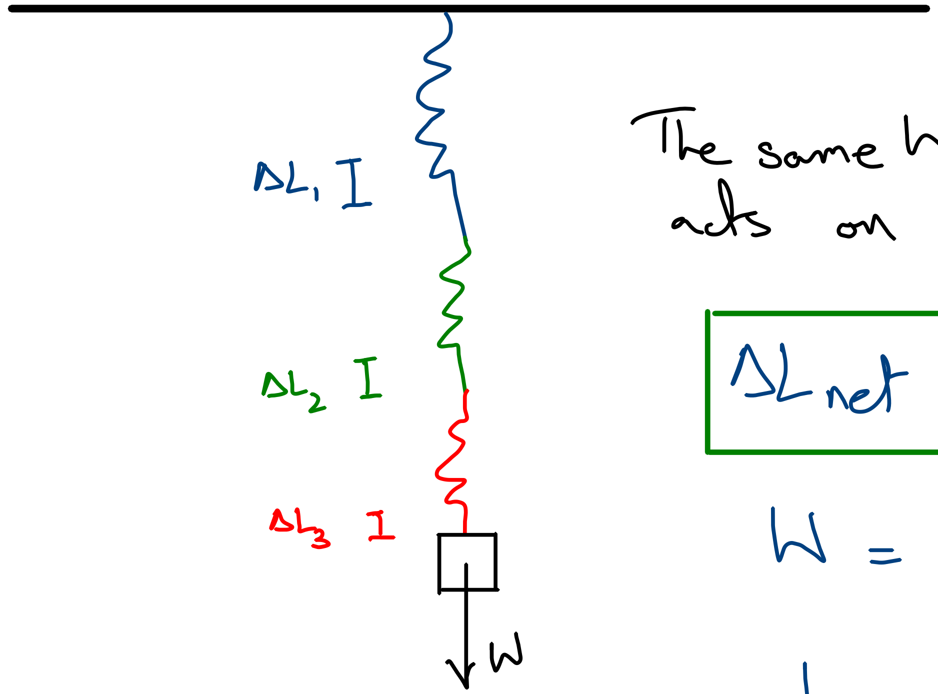
$\frac{1}{K_2} = \frac{\Delta L_2}{W}$

Spring 3



$W = K_3 \times \Delta L_3$

$\frac{1}{K_3} = \frac{\Delta L_3}{W}$



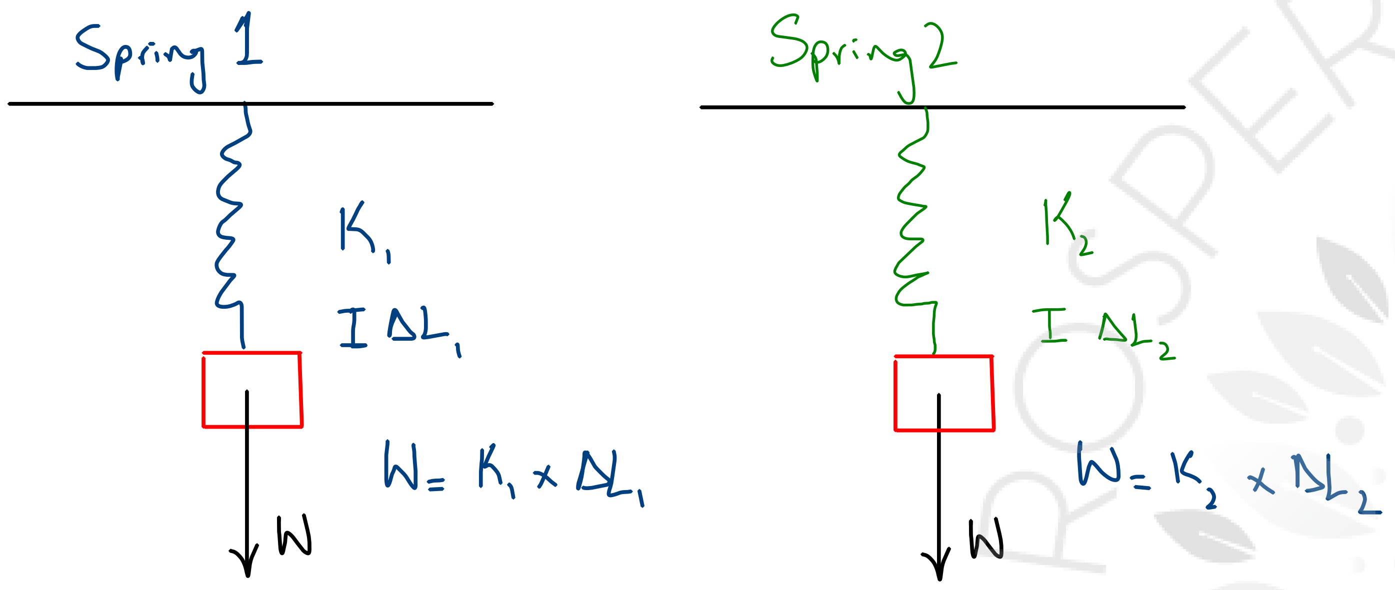
The same W acts on all 3 springs

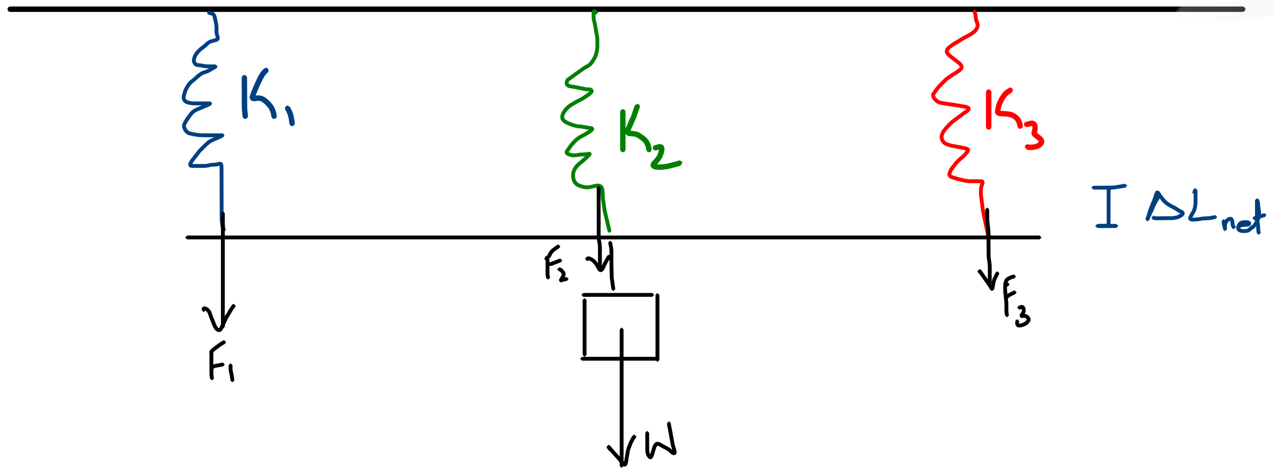
$\boxed{\Delta L_{net} = \Delta L_1 + \Delta L_2 + \Delta L_3}$ *

$W = K_{net} \times \Delta L_{net}$

$\frac{1}{K_{net}} = \frac{\Delta L_{net}}{W} \Rightarrow \overbrace{\frac{\Delta L_1}{W}}^{\frac{1}{K_1}} + \overbrace{\frac{\Delta L_2}{W}}^{\frac{1}{K_2}} + \overbrace{\frac{\Delta L_3}{W}}^{\frac{1}{K_3}} \Rightarrow \boxed{\frac{1}{K_{net}} = \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}}$ *

Springs in Parallel :-





All springs extend by same amount

$$W = K_{net} \times \Delta L_{net}$$

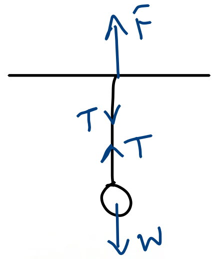
$$W = F_1 + F_2 + F_3$$

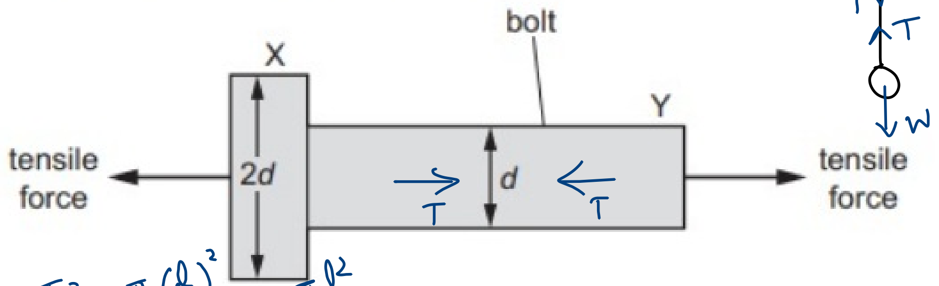
$$K_{net} \times \cancel{\Delta L_{net}} = (K_1 \times \cancel{\Delta L_{net}}) + (K_2 \times \cancel{\Delta L_{net}}) + (K_3 \times \cancel{\Delta L_{net}})$$

$$\boxed{K_{net} = K_1 + K_2 + K_3}\ *$$

w17 qp12 q20

20 A bolt is subjected to a tensile force, as shown.





$\pi r^2 \Rightarrow \pi\left(\frac{d}{2}\right)^2 \Rightarrow \frac{\pi d^2}{4}$

The bolt has a circular cross-section. At end X the diameter is 2*d*. At end Y the diameter is *d*.

What is the ratio $\frac{\text{stress at Y}}{\text{stress at X}}$? $= \frac{F}{A_Y} \div \frac{F}{A_X} \Rightarrow \frac{\cancel{F}}{A_Y} \times \frac{A_X}{\cancel{F}} = \frac{A_X}{A_Y}$

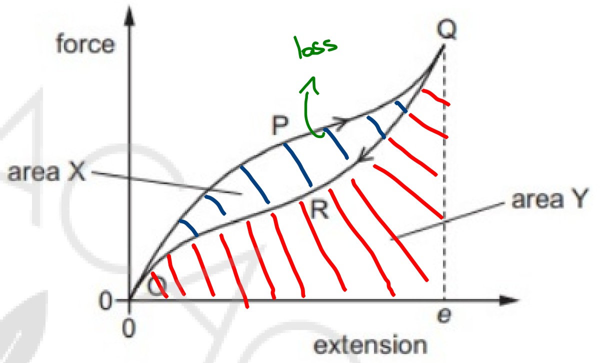
A 0.25 **B** 0.50 **C** 2.0 **D** 4.0

$= \frac{\pi d_x^2}{4} \div \frac{\pi d_y^2}{4}$

$= \frac{\cancel{\pi} \times 4\cancel{d^2}}{\cancel{4}} \times \frac{\cancel{4}}{\cancel{\pi}\cancel{d^2}} = \boxed{4}$

s17 qp12 q21

21 A rubber band is stretched and then relaxed to its original length. The diagram shows the force-extension graph for this process.



As the force is increased, the curve follows the path OPQ to extension *e*. As the force is reduced, the curve follows the path QRO to return to zero extension.

The area labelled X is between the curves OPQ and QRO. The area labelled Y is bounded by the curve QRO and the horizontal axis.

Which statement about the process is correct?

A Area X is the energy which heats the band as it is stretched to extension *e*. ✗

B (Area X + area Y) is the minimum energy required to stretch the band to extension *e*. ✓

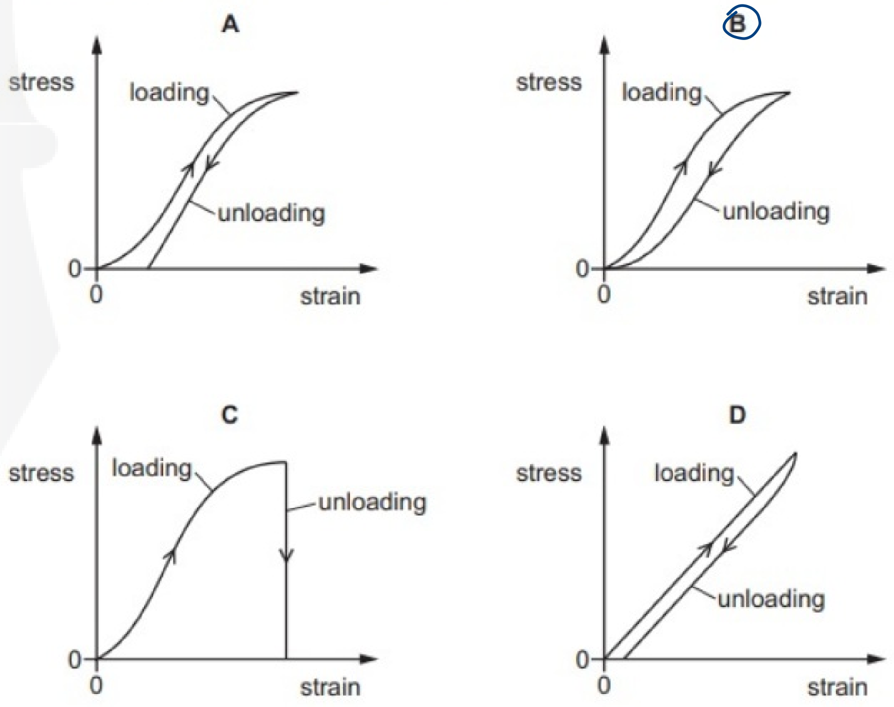
C Area X is the elastic potential energy stored in the band when it is stretched to extension *e*. ✗

D (Area Y – area X) is the net work done on the band during the process. ✗

s19 qp12 q22

22 The stress-strain graphs for loading and unloading four different materials are shown.

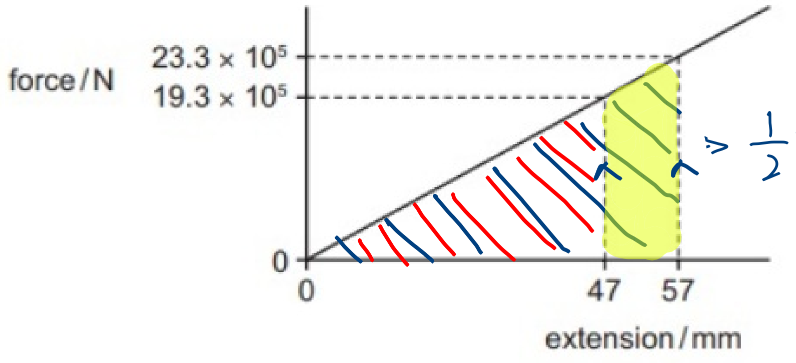
Which material exhibits purely elastic behaviour?



s17 qp12 q19

19 A cable on a suspension bridge supports a weight of 19.3×10^5 N. This weight causes the cable to stretch by 47 mm.

A lorry crossing the bridge then increases the force on the cable to 23.3×10^5 N. The force-extension graph for the cable is shown.



$\Rightarrow \frac{1}{2} \times \left[(19.3 + 23.3) \times 10^5\right] \times (10 \times 10^{-3})$

$= 21300$

What is the **increase** in strain energy in the cable when the lorry is crossing the bridge?

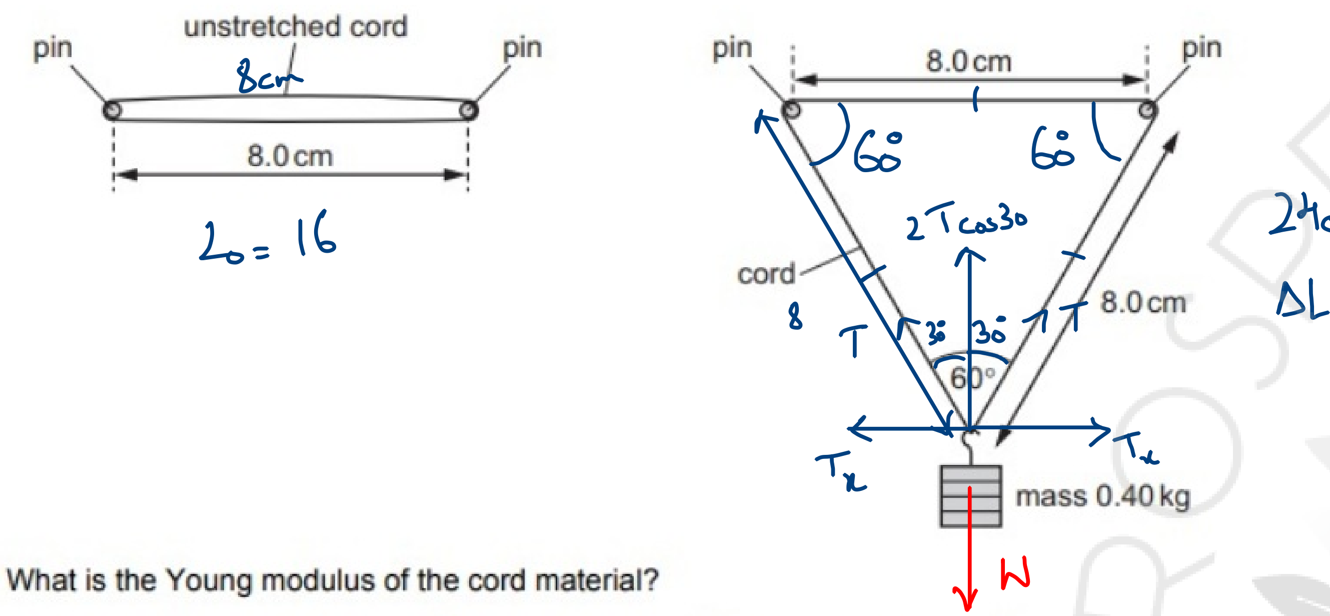
A 21 kJ **B** 23 kJ **C** 45 kJ **D** 66 kJ

s20 qp12 q18

18 An elastic cord of unstretched total length 16.0 cm and cross-sectional area $2.0 \times 10^{-6}\,\text{m}^2$ is held horizontally by two smooth pins a distance 8.0 cm apart.

L_o A

The cord obeys Hooke's law. A load of mass 0.40 kg is suspended centrally on the cord. The angle between the two sides of the cord supporting the load is 60°.



24cm = total

$\Delta L = 24 - 16$

$= 8\text{cm}$

What is the Young modulus of the cord material?

A 5.7×10^5 Pa   **B** 1.1×10^6 Pa   **C** 2.3×10^6 Pa   **D** 3.9×10^6 Pa

$$E = \frac{F \times L_o}{A \times \Delta L}$$

$$= \frac{2.27 \times 16\times10^{-2}}{2\times10^{-6} \times 8\times10^{-2}}$$

$$= 2.3 \times 10^6$$

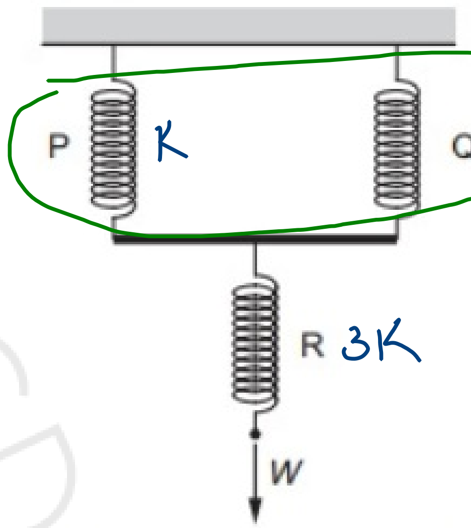
$$W = 2T\cos 30^\circ$$

$$0.40 \times 9.81 = 2 \times T \times \cos 30^\circ$$

$$T = \frac{0.40 \times 9.81}{2 \times \cos 30} = 2.27\,\text{N}$$

w21 qp12 q21

21 Three springs are arranged vertically as shown.



Series

parallel $\Rightarrow K_{net} = K + K = 2K$

$$\frac{1}{K_{net}} = \frac{1^{\times 3}}{2K_{\times 3}} + \frac{1^{\times 2}}{3K_{\times 2}}$$

$$\frac{1}{K_{net}} = \frac{5}{6K} \Rightarrow K_{net} = \frac{6K}{5}$$

Springs P and Q are identical and each has spring constant k. Spring R has spring constant $3k$.

What is the increase in the overall length of the arrangement when a force W is applied as shown?

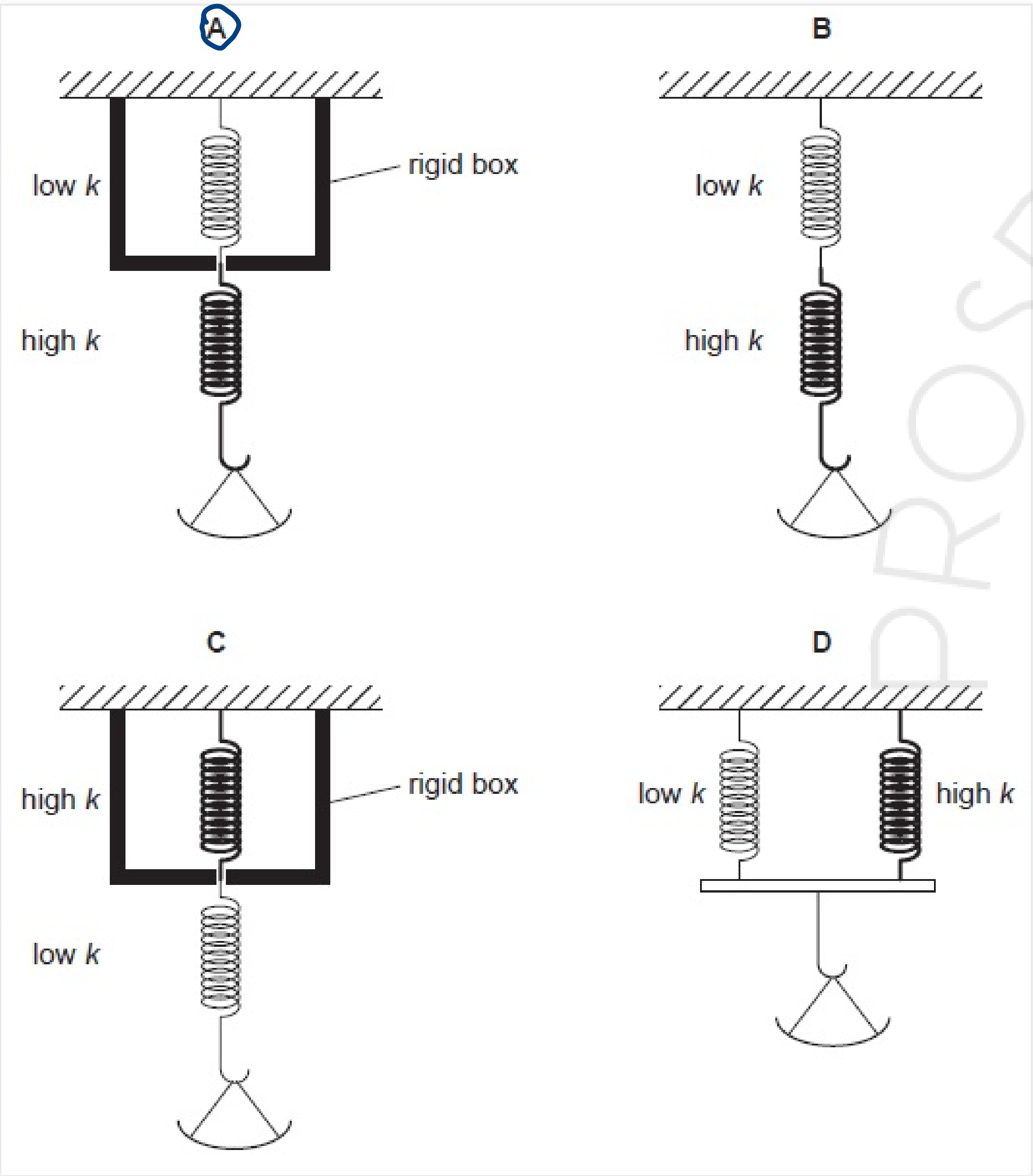
A $\frac{5}{6}\frac{W}{k}$ **B** $\frac{4}{3}\frac{W}{k}$ **C** $\frac{7}{2}kW$ **D** $4kW$

$$W = K_{net} \times \Delta L_{net}$$

$$W = \frac{6K}{5} \times \Delta L_{net}$$

$$\frac{5W}{6K} = \Delta L_{net}$$

To determine mass of food in a pan, a scale is used that has high sensitivity for small masses but low sensitivity for large masses.
To do this, two springs are used, each with different spring constant k. One of the springs has a low spring constant and the other has a high spring constant.
Which arrangement of springs would be suitable?



sensitivity:- the smallest change you can measure

small masses → e↑ → K↓

large masses → e↓ → K↑

4 **(a)** State Hooke's law.

The tensile force on an object under stress is directly proportional to the change in length given limit of proportionality is not exceeded. [1]

(b) A spring is fixed at one end. A compressive force F is applied to the other end. The variation of the force F with the compression x of the spring is shown in Fig. 4.1.

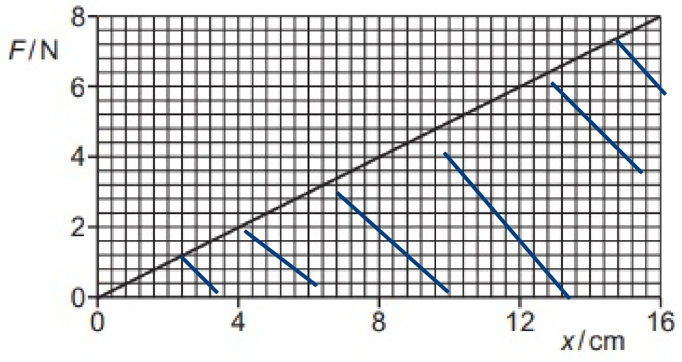


Fig. 4.1

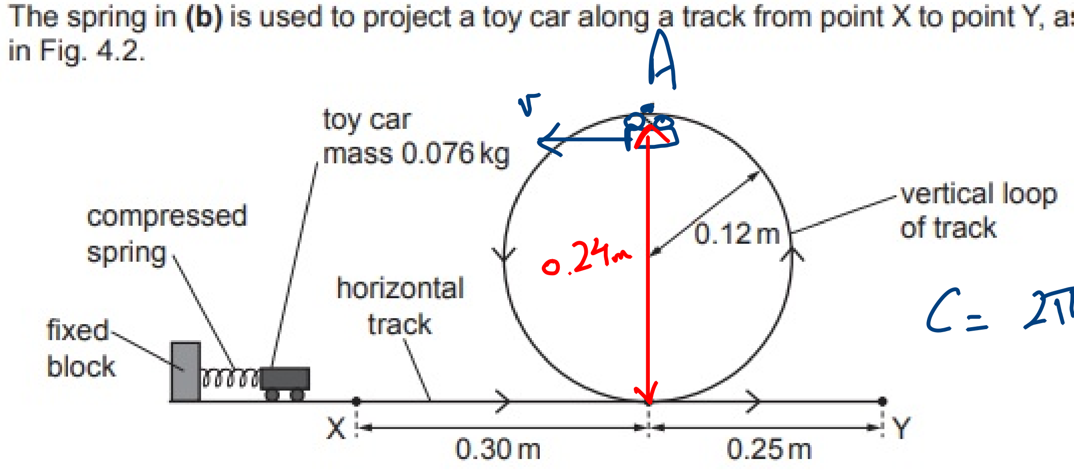
Show that the elastic potential energy of the spring is 0.64 J when its compression is 16.0 cm.

$E = \frac{1}{2} \times x \times F$

$= \frac{1}{\cancel{2}} \times (16 \times 10^{-2}) \times \cancel{8}^{4} = 0.64\,J$

[2]

(c) The spring in **(b)** is used to project a toy car along a track from point X to point Y, as illustrated in Fig. 4.2.



$C = 2\pi r$

Fig. 4.2 (not to scale)

The spring is initially given a compression of 16.0 cm. The car of mass 0.076 kg is held against one end of the compressed spring. When the spring is released it projects the car forward. The car leaves the spring at point X with kinetic energy that is equal to the initial elastic potential energy of the compressed spring.

The car follows the track around a vertical loop of radius 0.12 m and then passes point Y. Assume that friction and air resistance are negligible.

Calculate:

(i) the speed of the car at X

$K.E = E.P.E$

$0.64 = \frac{1}{2} \times 0.076 \times v^2 \Rightarrow v = 4.104$

speed = 4.1 m s^{-1} [2]

(ii) the kinetic energy of the car when it is at the top of the loop

$E_X = E_A$

$K.E_X = G.P.E_A + K.E_A$

$0.64 = (0.076)(9.81)(0.24) + K.E_A$

$K.E_A = 0.46$

kinetic energy = 0.46 J [3]

(iii) the speed of the car at Y.

speed = 4.1 m s^{-1} [1]

(d) In practice, a resistive force due to friction and air resistance acts on the car so that its kinetic energy at Y is 0.23 J less than its kinetic energy at X.

Determine the average resistive force acting on the car for its movement from X to Y.

$K.E_Y = K.E_X \underbrace{-0.23}_{\text{loss}} \rightarrow$ Work done against friction

$0.23 = W_{F_r} = F_r \times s$

$0.23 = F_r \times (0.30 + 2\pi(0.12) + 0.25) = 0.177$

average resistive force = 0.18 N [3]

[Total: 12]

(b) A block of mass 0.40 kg slides in a straight line with a constant speed of $0.30\,\mathrm{m\,s^{-1}}$ along a horizontal surface, as shown in Fig. 3.1.

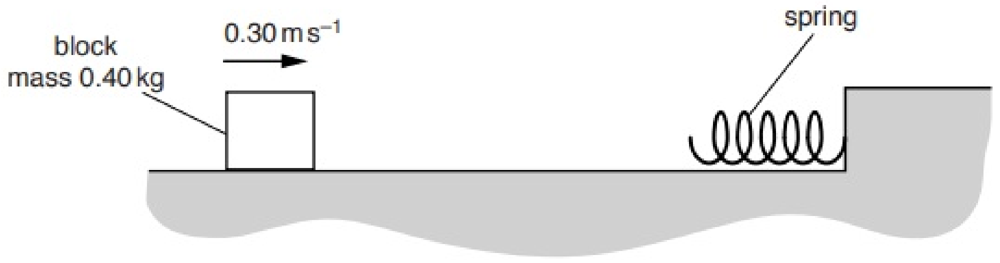


Fig. 3.1

The block hits a spring and decelerates. The speed of the block becomes zero when the spring is compressed by 8.0 cm.

(i) Calculate the initial kinetic energy of the block.

$K.E = \frac{1}{2}mv^2$

$= \frac{1}{2}(0.40)(0.30)^2$

$= 0.018$

kinetic energy = 0.018 J [2]

(ii) The variation of the compression x of the spring with the force F applied to the spring is shown in Fig. 3.2.

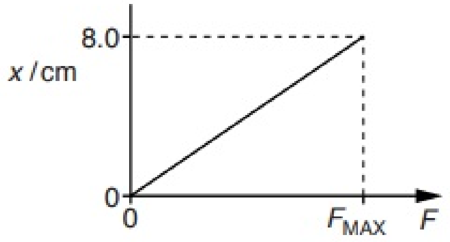


Fig. 3.2

Use your answer in **(b)(i)** to determine the maximum force F_{MAX} exerted on the spring by the block.
Explain your working.

$K.E = E.P.E$

$0.018 = \frac{1}{2} \times F \times \Delta L$

$0.018 = \frac{1}{2} \times F_{max} \times (8 \times 10^{-2})$

$F_{max} = \frac{0.018 \times 2}{8 \times 10^{-2}}$

F_{MAX} = 0.45 N [3]

(iii) Calculate the maximum deceleration of the block.

$F = m \times a$

$0.45 = 0.40 \times a$

$a = 1.1\ \mathrm{ms^{-2}}$

deceleration = 1.1 $\mathrm{m\,s^{-2}}$ [1]

(iv) State and explain whether the block is in equilibrium

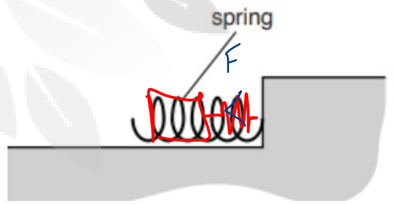
1. before it hits the spring,

yes, as it is travelling with a constant speed

2. when its speed becomes zero.

Not in equilibrium as the tension developed provides a net force horizontally on the block

[2]



m18 qp22 q3

3 (a) For the deformation of a wire under tension, define

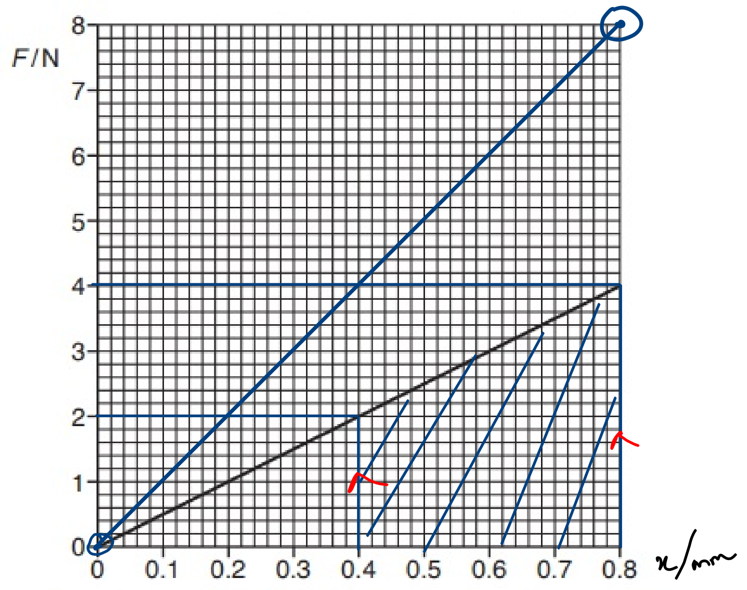
(i) *stress*,

Ratio of tensile force applied to cross sectional area of a deformed solid. [1]

(ii) *strain*.

Ratio of change in length to original length of a deformed solid. [1]

(b) A wire is fixed at one end so that it hangs vertically. The wire is given an extension x by suspending a load F from its free end. The variation of F with x is shown in Fig. 3.1.



The wire has cross-sectional area $9.4 \times 10^{-8}\,m^2$ and original length 2.5 m.

(i) Describe how measurements can be taken to determine accurately the cross-sectional area of the wire.

Measure the diameter of the wire at atleast 3 different points on the wire using a micrometer screw gauge. Average the diameters and then calculate the area using $A = \frac{\pi d^2}{4}$ where d is diameter. [3]

(ii) Determine the Young modulus E of the material of the wire.

$$E = \frac{F \times L_0}{A \times \Delta L} = \frac{4 \times 2.5}{(9.4\times10^{-8}) \times (0.8\times10^{-3})}$$

$$= 1.33 \times 10^{11}$$

E = 1.3×10^{11} Pa [2]

(iii) **Use Fig. 3.1** to calculate the increase in the energy stored in the wire when the load is increased from 2.0 N to 4.0 N.

$$\text{Area} = \frac{1}{2} \times (a+b) \times h$$

$$= \frac{1}{2} \times (2+4) \times (0.4\times10^{-3})$$

$$= 1.2 \times 10^{-3}$$

increase in energy = 1.2×10^{-3} J [2]

(c) The wire in **(b)** is replaced by a new wire of the same material. The new wire has twice the length and twice the diameter of the old wire. The new wire also obeys Hooke's law.

E same

On Fig. 3.1, sketch the variation with extension x of the load F for the new wire from $x = 0$ to $x = 0.80$ mm. [2]

[Total: 11]

Wire 1

E, L_0, d

$A_1 = \frac{\pi d^2}{4}$

$A_1 = \frac{A_2}{4}$

$A_2 = 4A_1$

$$E = \frac{F \times L_0}{A_1 \times \Delta L}$$

Wire 2:-

$E, 2L_0, 2d$

$A_2 = \frac{\pi (2d)^2}{4} \Rightarrow \pi d^2$

$$E = \frac{F_2 \times 2L_0}{4A_1 \times \Delta L}$$

Comparison

$$\frac{F \times \cancel{L_0}}{\cancel{A_1} \times \cancel{\Delta L}} = \frac{F_2 \times \cancel{2}\cancel{L_0}}{\cancel{4}\cancel{A_1} \times \cancel{\Delta L}}$$

$$F = \frac{F_2}{2}$$

$$F_2 = 2F$$

w19 qp21 q4

4 The variation with extension x of the force F applied to a spring is shown in Fig. 4.1.

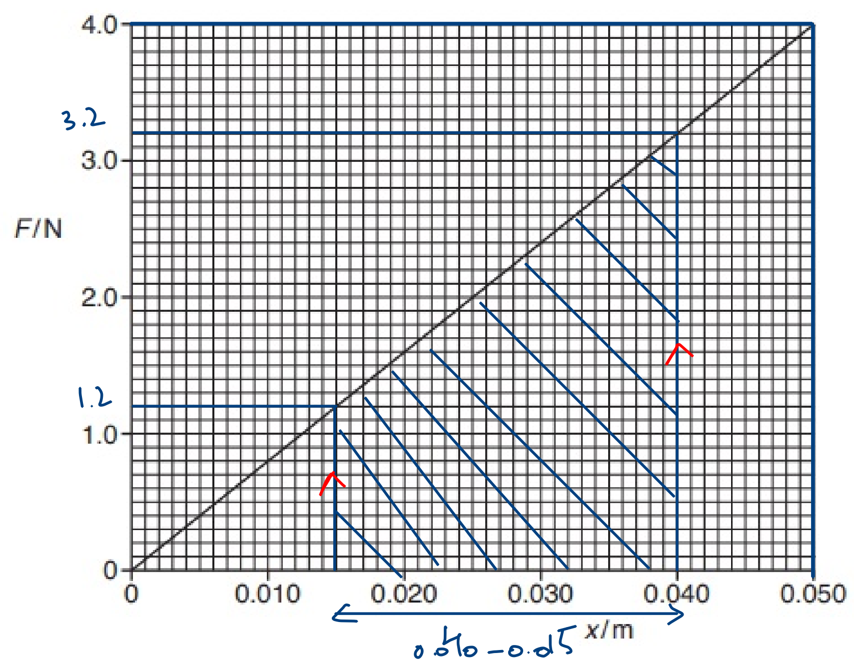


Fig. 4.1

The spring has an unstretched length of 0.080 m and is suspended vertically from a fixed point, as shown in Fig. 4.2.

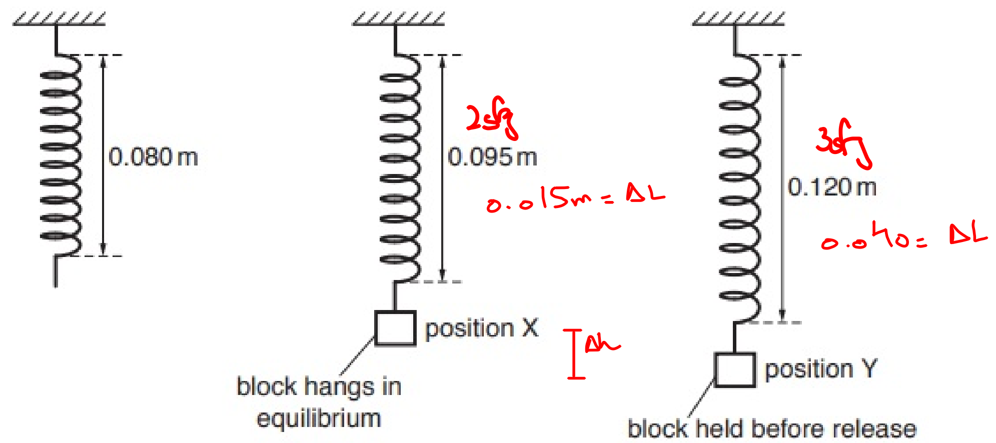


Fig. 4.2 **Fig. 4.3** **Fig. 4.4**

A block is attached to the lower end of the spring. The block hangs in equilibrium at position X when the length of the spring is 0.095 m, as shown in Fig. 4.3.

The block is then pulled vertically downwards and held at position Y so that the length of the spring is 0.120 m, as shown in Fig. 4.4. The block is then released and moves vertically upwards from position Y back towards position X.

(a) Use Fig. 4.1 to determine the spring constant of the spring.

$K = \frac{F}{\Delta L} = \frac{4}{0.050} = 80$

spring constant = ...80...... Nm^{-1} [2]

(b) Use Fig. 4.1 to show that the decrease in elastic potential energy of the spring is 0.055 J when the block moves from position Y to position X.

$A = \frac{1}{2} \times (a+b) \times h$

$= \frac{1}{2} \times (1.2 + 3.2) \times (0.025)$

$= 0.055\ J$ shown

[2]

(c) The block has a mass of 0.122 kg. Calculate the increase in gravitational potential energy of the block for its movement from position Y to position X.

$G.P.E = mg\Delta h = (0.122)(9.81) \times (0.025)$

$= 0.0299$

increase in gravitational potential energy = ...0.030...... J [2]

(d) Use the decrease in elastic potential energy stated in **(b)** and your answer in **(c)** to determine, for the block, as it moves through position X:

(i) its kinetic energy $E_{input} = E_{output}$

$E.P.E = G.P.E + K.E$

$0.055 = 0.030 + K.E$

kinetic energy = ...0.025...... J [1]

(ii) its speed.

$K.E = \frac{1}{2} mv^2$

$0.025 = \frac{1}{2}(0.122)v^2$

speed = ...0.64...... ms^{-1} [2]

[Total: 9]