# DENSITY AND PRESSURE

PHYSICS BY
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## Mass

~~Amount of matter present in a body (O level)~~

The measurement of inertia is called mass (A level and beyond) ✓

**Inertia**: It is the tendency of body to resist sudden changes in its state of rest or motion.

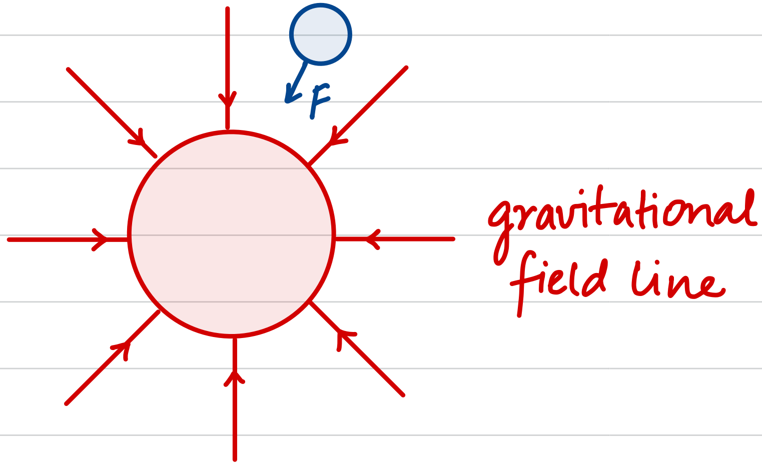


- The greater the mass, the greater is the inertia. Its harder to start or stop a heavier object due to more inertia.
- SI Unit: kg
- Scalar Quantity

Lecture 2 by Professor Julius Sumner Miller | Youtube Video

# Weight and Gravitational Field.

Gravitational field is a region in space around a mass where another mass experiences gravitational force.




Weight is the effect of gravitational field on mass.



$F_{net} = ma$

$W = mg$

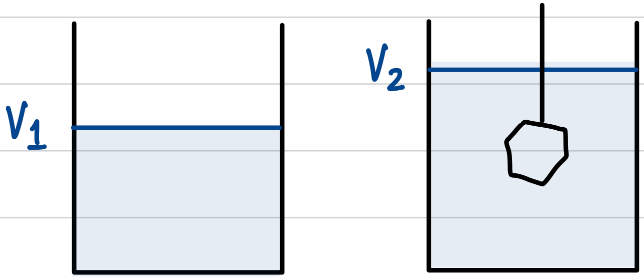
# Volume

Space occupied by an object is called volume.

- Regular shaped object : Formula OR Liquid displacement method
- Irregular shaped object : Liquid displacement method

## Liquid displacement method

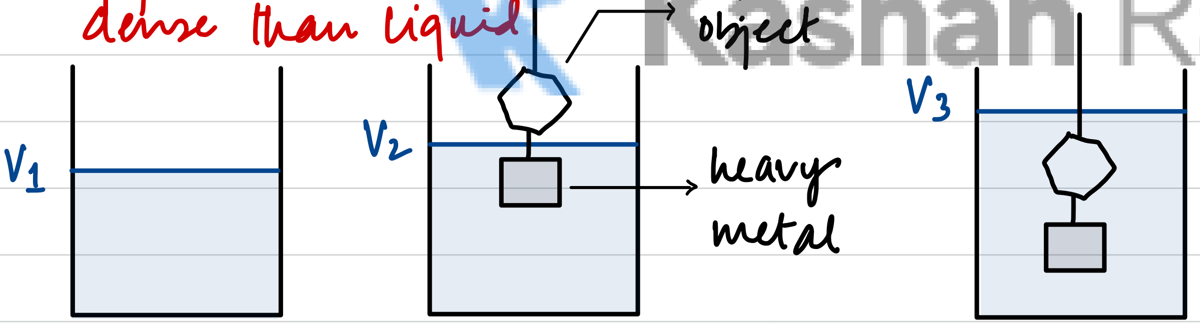
- Object is more dense than liquid




$V_{object} = V_2 - V_1$



- Object is less dense than liquid




$\text{Vol. of object} = V_3 - V_2$

$V_2 = \text{Vol. of liquid} + \text{Vol. of metal}$

$V_3 = \text{Vol. of liquid} + \text{metal} + \text{light object}$

# Density

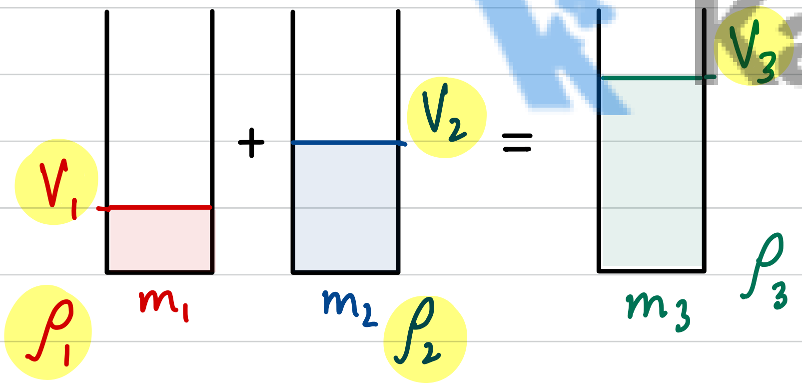
Mass per unit volume

$$\rho = \frac{m}{V}$$

SI Unit: $kgm^{-3}$

**Understanding:** It tells of how much of mass is stuffed/concentrated in a given volume.

## Density of mixture of liquids




$$\text{density of mixture} = \frac{m_{mix}}{V_{mix}} = \frac{m_1 + m_2}{V_{mix}}$$

**10** The density of water is 1.0 g cm$^{-3}$ and the density of glycerine is 1.3 g cm$^{-3}$.

Water is added to a measuring cylinder containing 40 cm$^3$ of glycerine so that the density of the mixture is 1.1 g cm$^{-3}$. Assume that the mixing process does not change the total volume of the liquid.

What is the volume of water added?

**A** 40 cm$^3$ **B** 44 cm$^3$ **C** 52 cm$^3$ **D** 80 cm$^3$

water

$\rho = 1\ gcm^{-3}$

$V_1' = ??$

$m_1 = ??$

$m_1 = \rho V$

glycerine

$\rho_2 = 1.3\ gcm^{-3}$

$V_2 = 40\ cm^3$

$m_2 = ?$

mixture

$\rho_{mix} = 1.1\ gcm^{-3}$

$V_{mix} =$

$m_{mix} = ?$



$$\text{density} = \frac{\text{mass mix}}{V\ \text{mix}}$$

$$1.1 = \frac{m_1 + m_2}{V_1 + V_2}$$

$$1.1 = \frac{(1)V_1 + (1.3)(40)}{V_1 + 40}$$

$$V_1 = 80\ cm^3$$

$$1.1 = \frac{V_1 + 52}{V_1 + 40}$$

$$V_1 = 80\ cm^3$$

# PRESSURE

Pressure

Force per unit surface area.

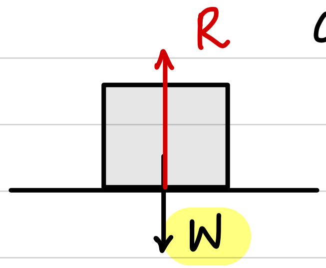
$$P = \frac{F}{A}$$

SI Unit : Pascal (Pa)

Scalar quantity

Things to know about force

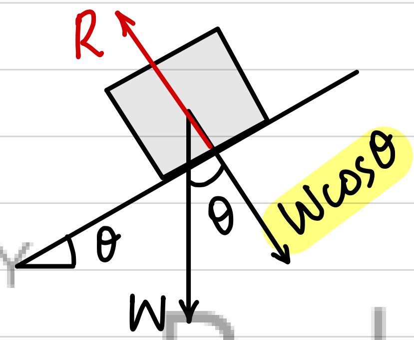
- ☑ It is the contact force exerted by the body on the surface
- ☑ It must make an angle of 90° with the surface.




contact force = Weight

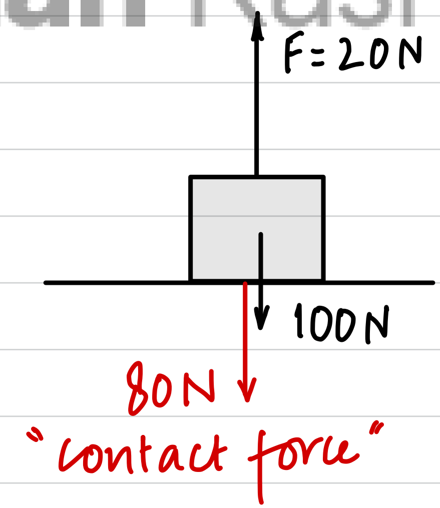
$R = W$

$P = \frac{F}{A}$ so $P = \frac{W}{A}$




$R = W\cos\theta$

$P = \frac{F}{A}$ so $P = \frac{W\cos\theta}{A}$




$P = \frac{F}{A} = \frac{80}{A}$

## Atmospheric Pressure

Pressure exerted due to air molecules present in the surrounding.

**How atmosphere exerts pressure?**

- ☑ Air molecules are in a continuous random motion
- ☑ They collide with one another and with the walls of the body.
- ☑ This results in change of momentum
- ☑ As rate of change of momentum is force, that force per unit area is called pressure of gas.

Values of Atmospheric Pressure

| | |
|---|---|
| 101325 Pa | $1.01 \times 10^5$ Pa |
| 760 mm Hg | 760 torr |
| 1 atm | 14.7 psi |

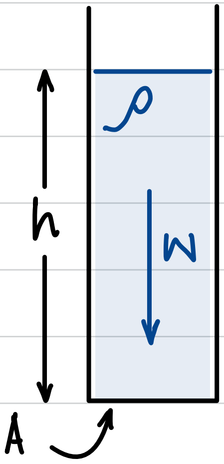


## Atmospheric Pressure decreases with height

Reason 1: Less density of air causes less no. of collisions

Reason 2: Lower temperature results in molecules moving slower so less force and hence less pressure.

# Pressure in Liquid ($P = \rho g h$)




$$P = \frac{F}{A}$$

$$\rightarrow P = \frac{W}{A}$$

$$\rightarrow P = \frac{mg}{A}$$

as $\rho = \frac{m}{V}$ so $m = \rho V$

$$\rightarrow P = \frac{\rho V g}{A}$$

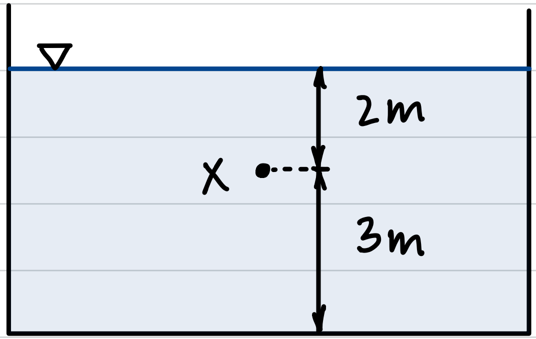
as $V = A \times h$ so

$$\rightarrow P = \frac{\rho (A \times h) g}{A}$$

$$\boxed{P = \rho g h}$$

P: pressure   $\rho$: density
g: gravitational acc   h: depth

Example




$\rho$ : 1000 $kgm^{-3}$
g : 9.8 $ms^{-2}$

Find the value of i, Pressure due to liquid at X
ii, total pressure at X

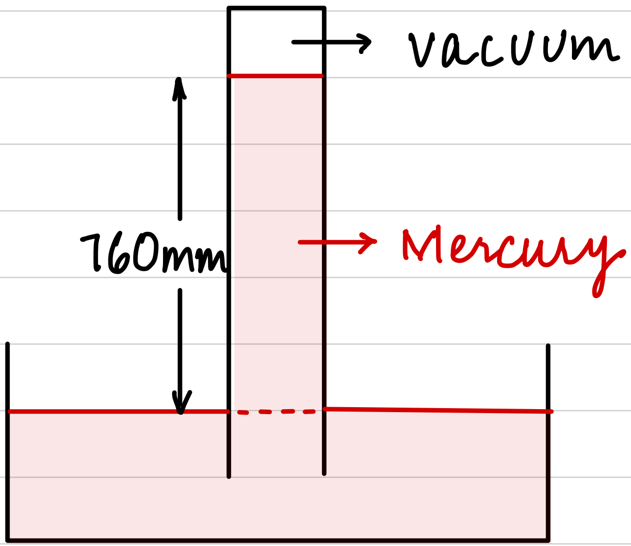
i, $P_L = \rho g h$
$= (1000)(9.8)(2)$
$P_L = 19{,}600$ Pa

ii, $P_T = P_L + P_{atm}$
$= 19600 + 1.01 \times 10^5$
$P_T = 120{,}600$ Pa

$P = \rho g h$ is applicable on solids, liquids and gases provided that they have uniform density.

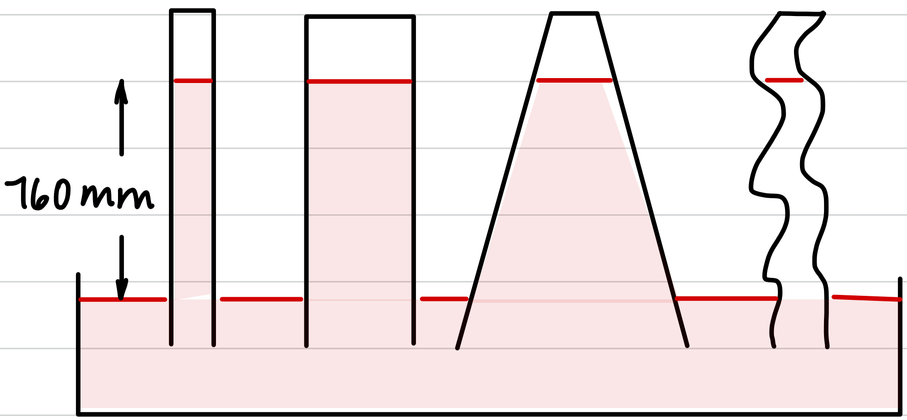
# Barometer

An instrument used to measure the atmospheric pressure.




- At sea level, 760 mm of mercury was able to balance atmospheric pressure.

- At higher pressure, liquid column increases in length and vice versa.




$\rho_{Hg} = 13600\ kgm^{-3}$

$P = \rho g h$

$\frac{101325}{13600 \times 9.8} = h$

$h_{Hg} = 0.76m$ (760mm)

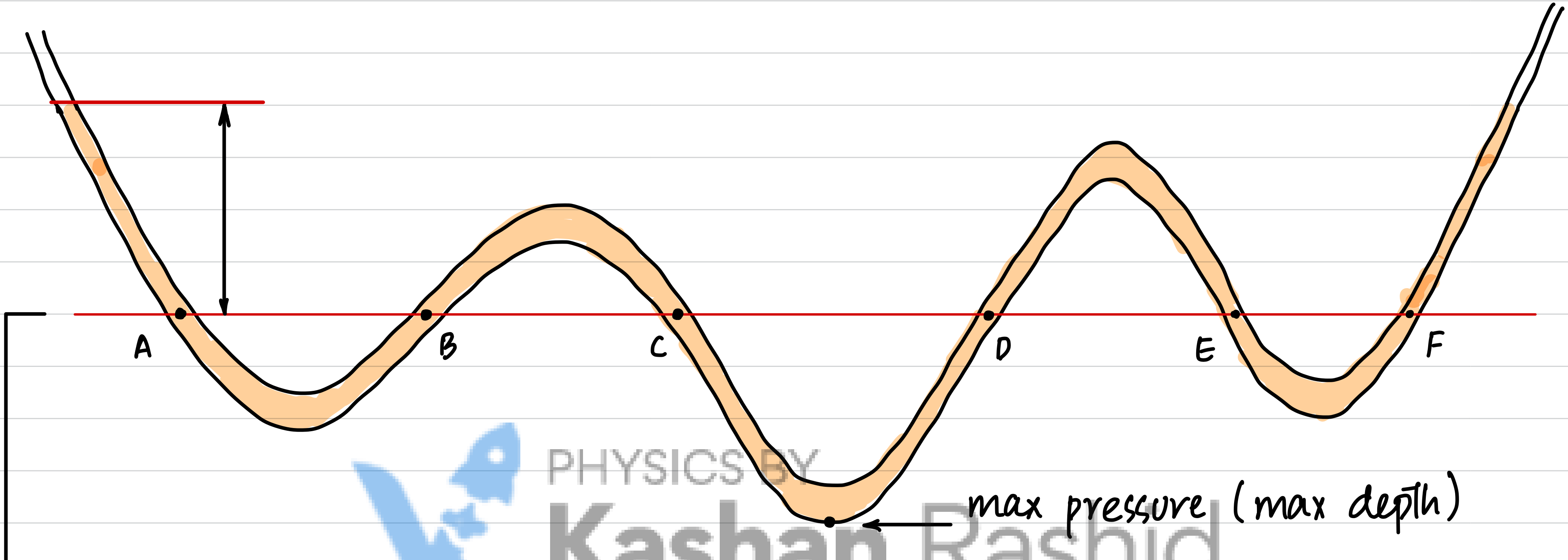
$\rho_{H_2O} = 1000\ kg/m^3$

$P = \rho g h$

$\frac{101325}{1000 \times 9.8} = h$

$h_{H_2O} = 10.3\ m$

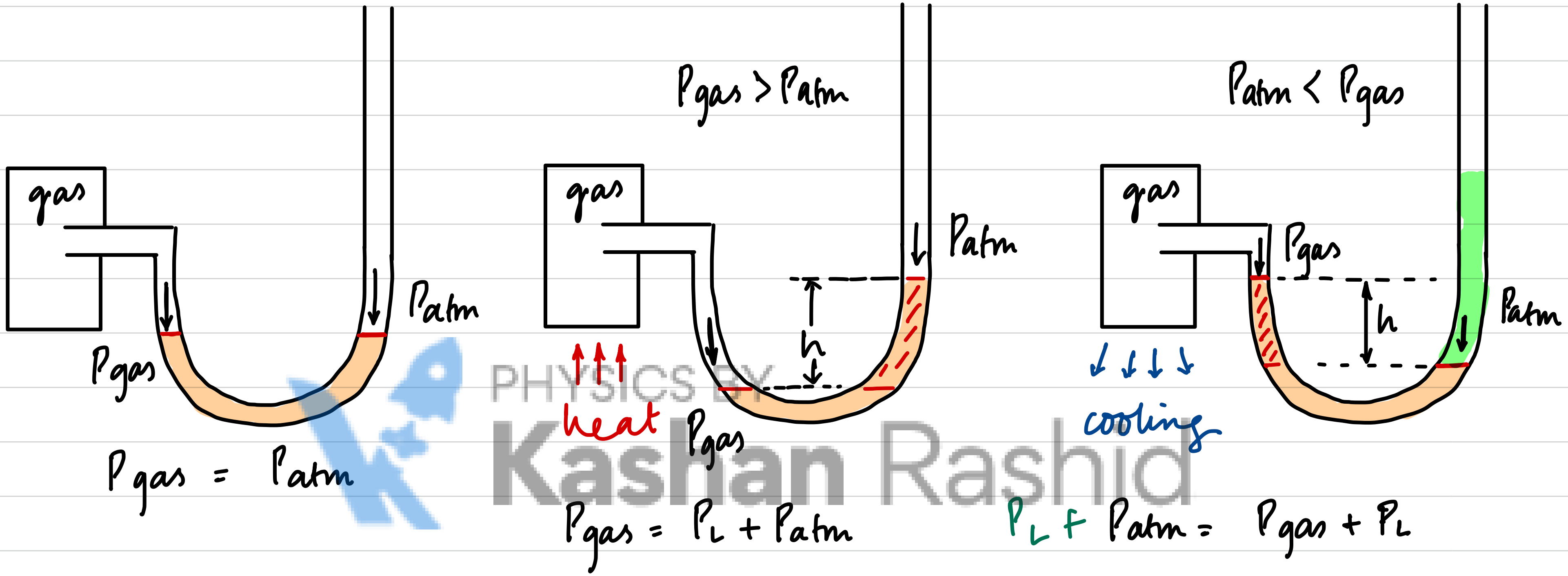
Mercury being heavier required less column to balance atmospheric pressure.

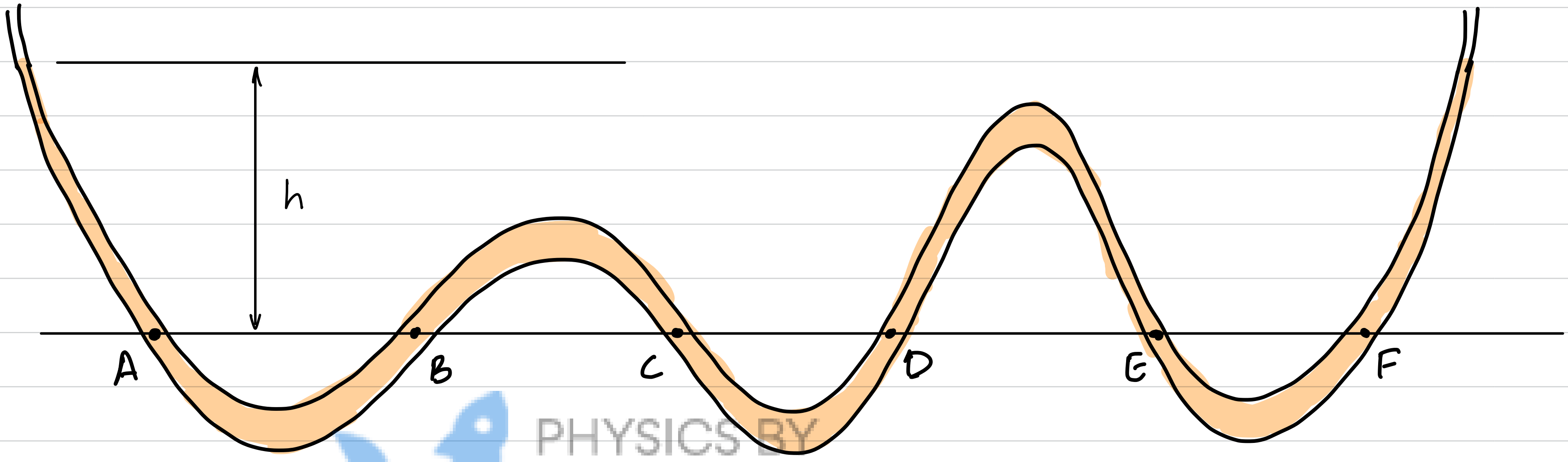





→ Liquid exerts equal pressure at all points at a specific depth.

# Manometer


gas
Pgas
Patm
Pgas = Patm
Pgas > Patm
gas
Patm
h
heat
Pgas
Pgas = PL + Patm
Patm < Pgas
gas
Pgas
h
Patm
cooling
PL + Patm = Pgas + PL



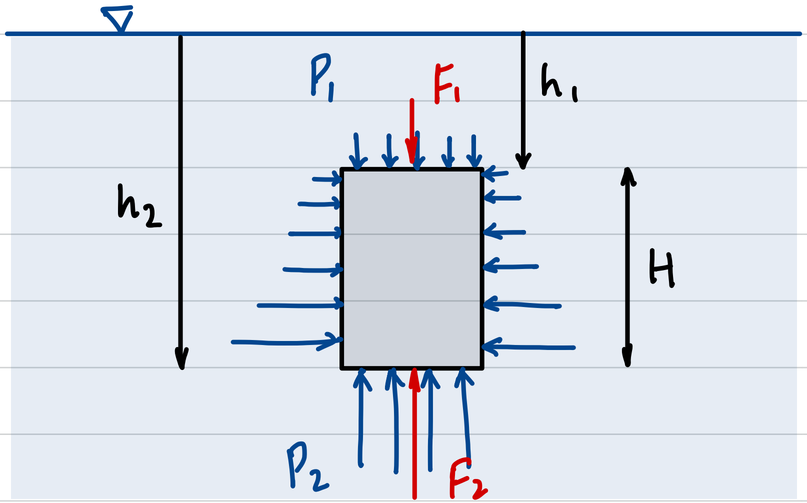


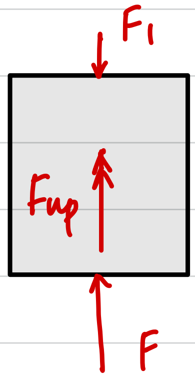

P at A, B, C, D, E and F is exactly same as all points are at the same depth!

# Upthrust

- It is an upward force acting on the body submerged in a fluid.

- It is due to the difference in pressure at the top and bottom surface of the object in the fluid.




$$F_{up} = F_2 - F_1$$

as $P = \frac{F}{A}$ so $F = P \times A$

$$F_{up} = P_2 A - P_1 A$$

$$F_{up} = (P_2 - P_1)\,A$$

$P_1 : P_{L1} + P_{atm}$ $\qquad$ $P_2 : P_{L2} + P_{atm}$

$$P_2 - P_1 = P_{L2} + P_{atm} - (P_{L1} + P_{atm})$$

$$= P_{L2} + \cancel{P_{atm}} - P_{L1} - \cancel{P_{atm}}$$

$$P_2 - P_1 = P_{L2} - P_{L1}$$

$$F_{up} = (P_{L2} - P_{L1})\,A$$

$$F_{up} = (\rho g h_2 - \rho g h_1)\,A$$

$$F_{up} = \rho g (h_2 - h_1)\,A$$

$$F_{up} = \rho g H A$$

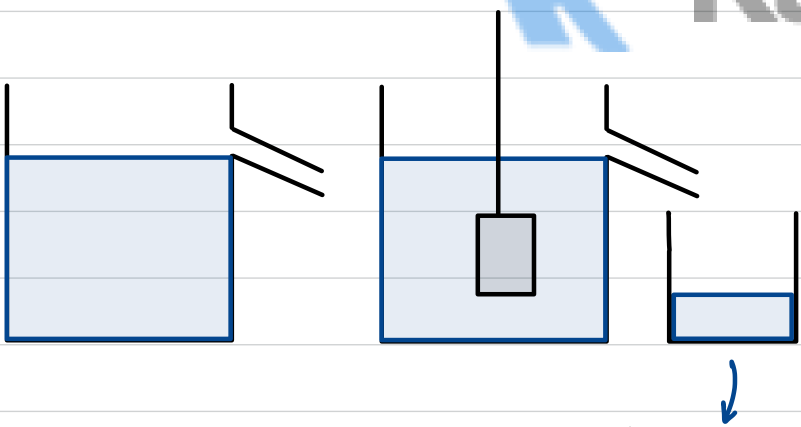
$F_{up} = \rho g H A$

as $A \times H = V_{obj}$ so

$F_{up} = \rho g V_{obj}$

$F_{up}$: Upthrust force
$\rho$: density of liquid
$g$: grav. field strength
$V_{obj}$: volume of object





Vol. of object = Vol. of liquid displaced
$V_{obj} = V$

$F_{up} = \rho g V_{obj}$ – OR

$F_{up} = \rho g V$

$\rho$ → density of liquid

$V$ → volume of liquid "displaced"

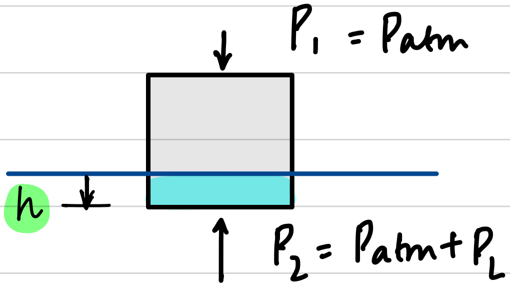
as $\rho = \frac{m}{V}$ so $m = \rho V$

$F_{up} = (\rho V) g$

$F_{up} = mg$

$F_{up} = W$

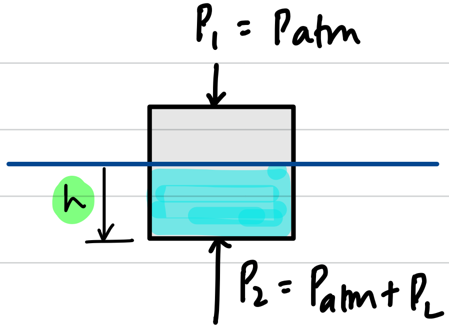
The UPTHRUST force is equal to the weight of the liquid displaced.




$$F_{up} = (P_2 - P_1)A$$
$$= (\cancel{P_a} + P_L - \cancel{P_a})A$$
$$= (P_L)A$$
$$F_{up} = \rho g h A$$

- Less depth, hence less upthrust $\Delta P$ is less.

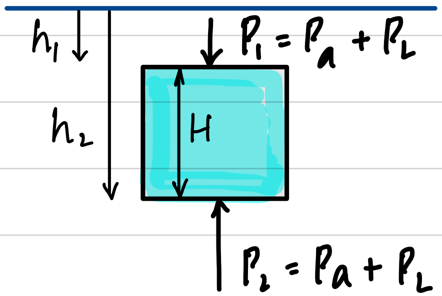
- Less liquid displaced, so less weight and hence less upthrust.




$$F_{up} = (P_2 - P_1)A$$
$$= (\cancel{P_a} + P_L - \cancel{P_a})A$$
$$= (P_L)A$$
$$F_{up} = \rho g h A$$

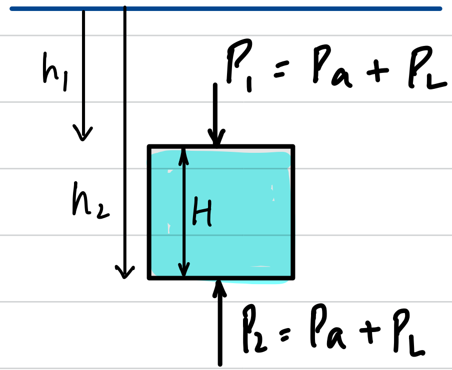
- More depth, hence more upthrust. $\Delta P$ increased.

- Liquid displaced increases so more weight and hence more upthrust.




$$F_{up} = (P_2 - P_1)A$$
$$= (\cancel{P_a} + P_{L_2} - \cancel{P_a} - P_{L_1})A$$
$$= (P_{L_2} - P_{L_1})A$$
$$= \rho g (h_2 - h_1)A$$
$$F_{up} = \rho g H A$$

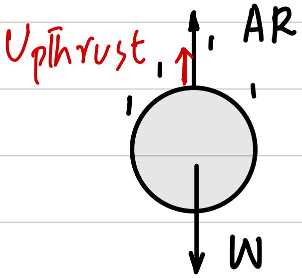
- Upthrust is max as $\Delta P$ is max. Liquid displaced is equal to size of object.




$$F_{up} = (P_2 - P_1)A$$
$$= (\cancel{P_a} + P_{L_2} - \cancel{P_a} - P_{L_1})A$$
$$= (P_{L_2} - P_{L_1})A$$
$$= \rho g (h_2 - h_1)A$$
$$F_{up} = \rho g H A$$

- Upthrust stays same as $\Delta P$ stays same. No more liquid displaces upon further submerging.

# Object moving in a fluid

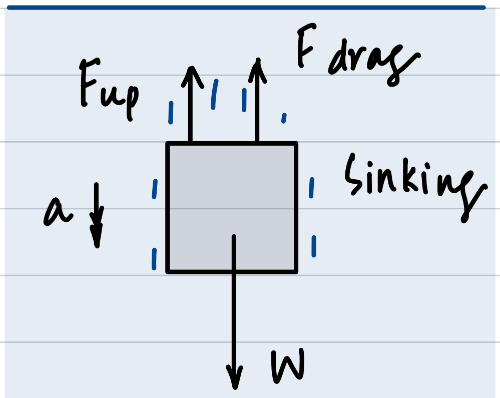



air

In air, for small size objects, upthrust force is negligible!
Reason: Density of air is so less that weight of air displaced because of object is negligible compared to the weight of object itself.

In case of a liquid, upthrust force is significant due to large density.

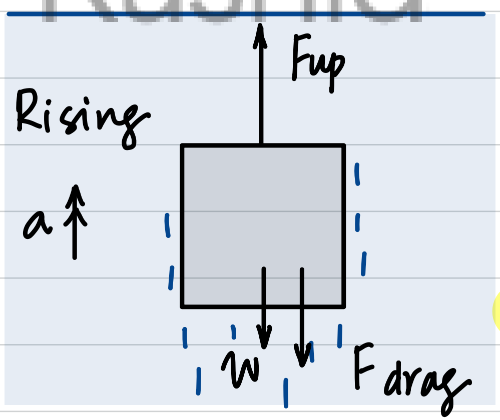





- W > Upthrust
- $\rho_{obj} > \rho_{liq}$

$F_{net} = ma$

$W - F_{up} - F_D = ma$




- W < Upthrust
- $\rho_{obj} < \rho_{liquid}$

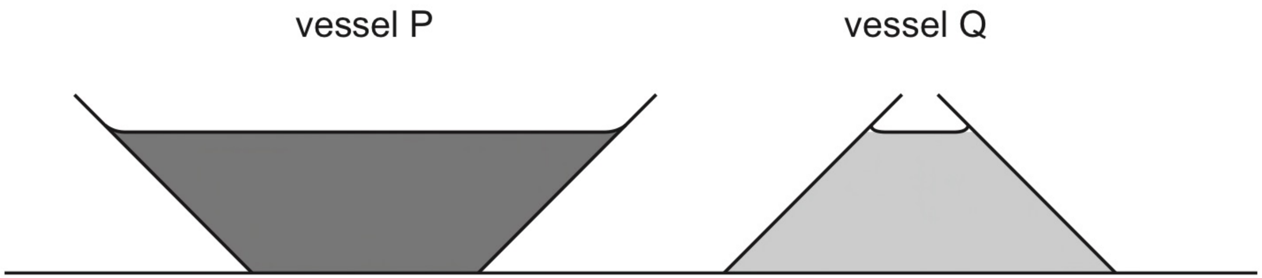
$F_{net} = ma$

$F_{up} - W - F_D = ma$

- Drag force ∝ velocity of object
- As object accelerates in a liquid, drag force increases until resultant force acting on body is zero. Now body moves at constant speed.

MJ/2011/P12

**20** The diagram shows two vessels, P and Q, both with sides inclined at 45°.




Vessel P tapers outwards and vessel Q tapers inwards, as shown.

Both vessels contain a liquid. The depth of the liquid in the vessels is the same. The liquid in vessel P is twice as dense as the liquid in vessel Q.



What is the ratio $\dfrac{\text{pressure due to the liquid on the base of P}}{\text{pressure due to the liquid on the base of Q}}$?

**A** $\dfrac{2}{1}$ **B** $\dfrac{\sqrt{2}}{1}$ **C** $\dfrac{1}{\sqrt{2}}$ **D** $\dfrac{1}{2}$

$$\frac{P_P}{P_Q} = \frac{\rho_P g h_P}{\rho_Q g h_Q} \Rightarrow \frac{P_P}{P_Q} = \frac{2\rho_Q}{\rho_Q}$$

$$\boxed{\frac{P_P}{P_Q} = \frac{2}{1}}$$

ON/2011/P12

**20** A horizontal plate of area $0.036\ m^2$ is beneath the surface of a liquid of density $930\ kg\ m^{-3}$. The force on the plate due to the pressure of the liquid is 290 N.

What is the depth of the plate beneath the surface of the liquid?

**A** 0.88 m **B** 1.13 m **C** 8.7 m **D** 9.1 m

$\frac{F}{A}$ $P = \rho g h$

$F$ $\frac{290}{0.036}$ $8056 = 930 \times g \times h$

$h = 0.88\ m$

$P = 8056\ Pa$

MJ/2016/P12



**15** The density of air on the Earth decreases almost linearly with height from $1.22\ kg\ m^{-3}$ at sea level to $0.74\ kg\ m^{-3}$ at an altitude of 5000 m.

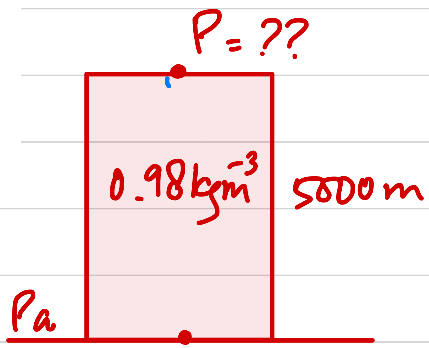
Atmospheric pressure at the Earth's surface on a particular day is 100 000 Pa. The value of *g* between the Earth's surface and an altitude of 5000 m can be considered to have a constant value of $9.7\ m\ s^{-2}$.

What will be the atmospheric pressure at an altitude of 5000 m?

**A** 36 000 Pa **B** 48 000 Pa **(C)** 52 000 Pa **D** 59 000 Pa

$\rho = \frac{1.22 + 0.74}{2}$

$\rho = 0.98\ kgm^{-3}$

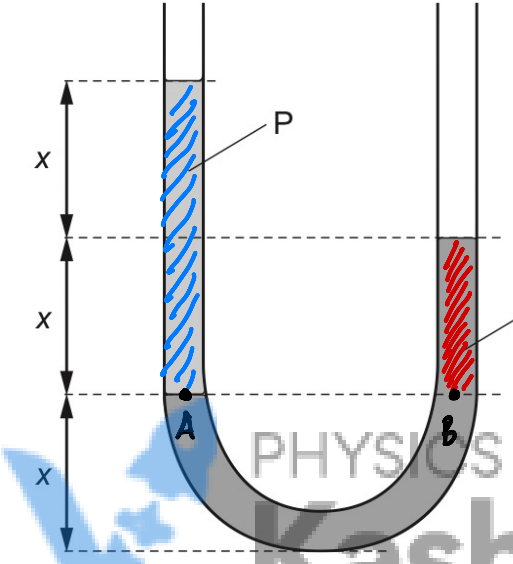



$P = \rho g h$
$= (0.98)(9.7)(5000)$
$= 47530\ Pa$ ✓

$100,000 - 47530 = 52470\ Pa$

ON/2014/P13

**22** The diagram shows two liquids, labelled P and Q, which do **not** mix. The liquids are in equilibrium in an open U-tube.




$P_A = P_B$

$\rho_P g h_P = \rho_Q g h_Q$

$\rho_P (2x) = \rho_Q (x)$

$2\rho_P = \rho_Q$

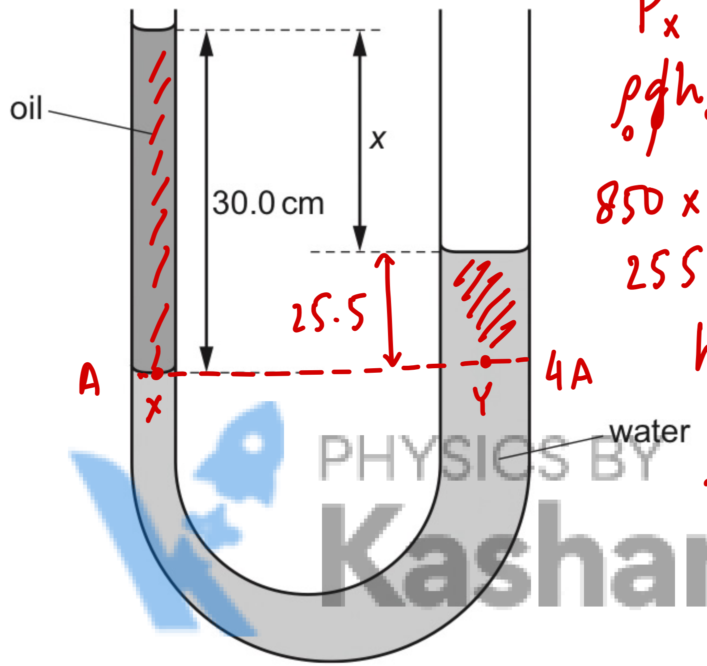
$\frac{\rho_P}{\rho_Q} = \frac{1}{2}$

What is the ratio $\frac{\text{density of P}}{\text{density of Q}}$ ?

**A** $\frac{1}{2}$ **B** $\frac{2}{3}$ **C** $\frac{3}{2}$ **D** 2

ON/2015/P13

**19** A U-tube has one arm of area of cross-section $A$ and the other of cross-section $4A$. The tube contains water of density $1000\,kg\,m^{-3}$ and oil of density $850\,kg\,m^{-3}$, as shown.




$P_X = P_Y$

$\rho_o g h_o = \rho_w g h_w$

$850 \times 30 = 1000 \times h_w$

$25500 = 1000 \times h_w$

$h_w = 25.5\,cm$

$x = 30 - 25.5$

$x = 4.5\,cm$



The column of oil on top of the water in the left-hand arm is of length 30.0 cm.

What is the difference in height $x$ between the levels in the two arms of the tube?

**A** 4.5 cm **B** 6.2 cm **C** 23.8 cm **D** 25.5 cm

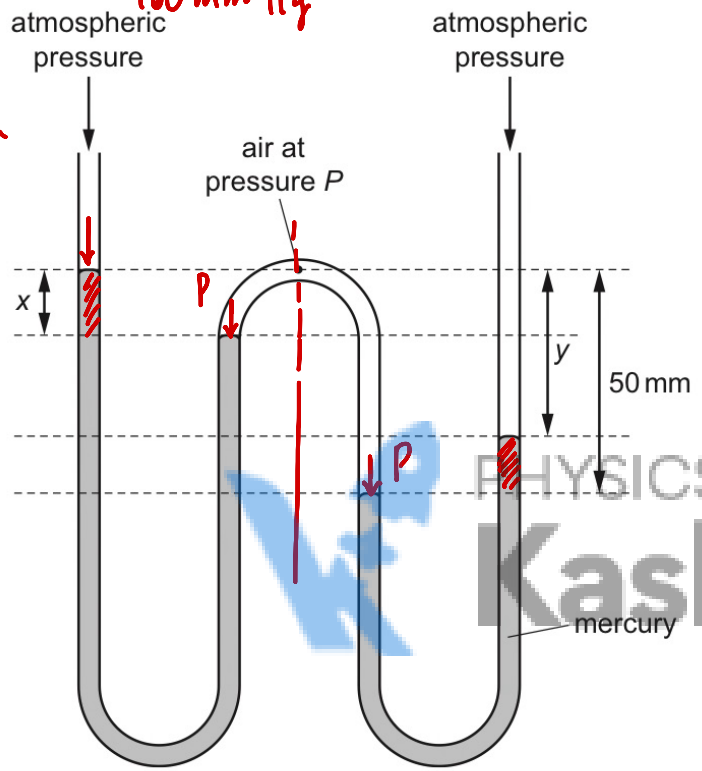
MJ/2015/P13

**21** A W-shaped tube contains two amounts of mercury, each open to the atmosphere. Air at pressure $P$ is trapped in between them. The diagram shows two vertical distances $x$ and $y$.

760 mm Hg

$P_p = P_L + P_{atm}$

$P = x + 760$




NOT TO SCALE

$P_p = P_L + P_{atm}$

$P = (50 - y) + 760$

$P = 810 - y$



Atmospheric pressure is equal to the pressure that would be exerted by a column of mercury of height 760 mm. The pressure $P$ is expressed in this way.

Which values of $x$, $y$ and $P$ are possible?

| | $x$/mm | $y$/mm | $P$/mm of mercury |
|---|---|---|---|
| A ✗ | 20 | 20 | 780 |
| **B** | 20 | 30 | 780 |
| C | 30 | 20 | 810 |
| D ✗ | 30 | 30 | 790 |

$x + 760 = 810 - y$

$x + y = 50$

as $y > x$

$P = x + 760$

$= 20 + 760$

$P = 780$ ✓

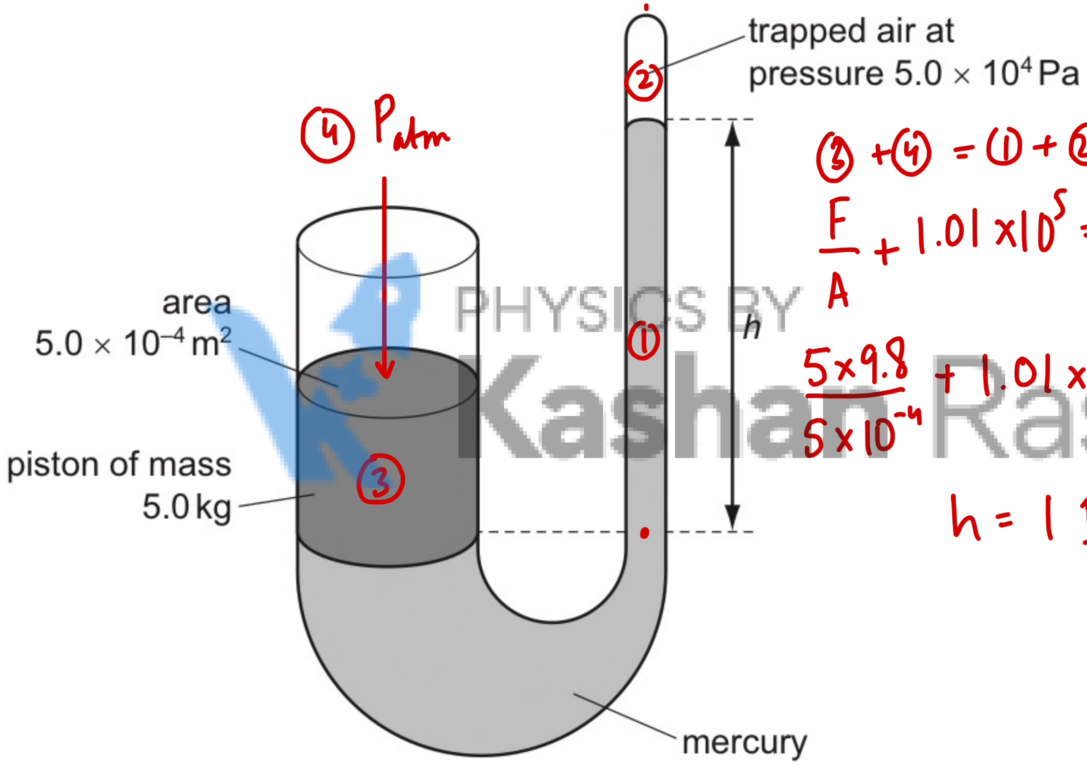
$P = 810 - y$

$P = 810 - 30$

$P = 780$ ✓

**19** A U-tube closed at one end contains mercury. Air at a pressure of $5.0 \times 10^4$ Pa is trapped at the closed end. The other end is open to the atmosphere and is fitted with a piston of mass 5.0 kg and cross-sectional area $5.0 \times 10^{-4}\,\text{m}^2$.

The density of mercury is $13\,600\,\text{kg}\,\text{m}^{-3}$ and atmospheric pressure is $1.01 \times 10^5$ Pa. ✓




③ + ④ = ① + ②

$$\frac{F}{A} + 1.01 \times 10^5 = \rho g h + 5.0 \times 10^4$$

$$\frac{5 \times 9.8}{5 \times 10^{-4}} + 1.01 \times 10^5 = (13600)(9.8)h + 5 \times 10^4$$

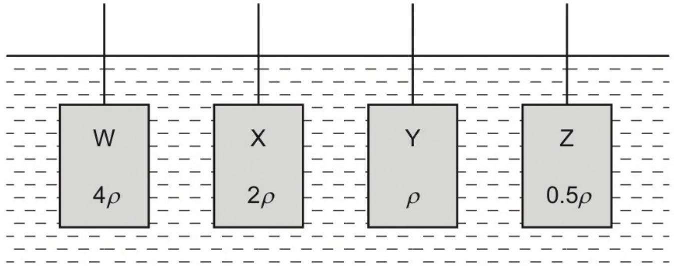
$h = 1.11$ m



What is the height $h$ of the mercury column?

**A** 37 cm **B** 44 cm **C** 74 cm **D** 110 cm

**14** Four cuboids with identical length, breadth and height are immersed in water. The cuboids are held at the same depth and in identical orientations by vertical rods, as shown.




Water has density $\rho$.
Cuboid W is made of material of density $4\rho$.
Cuboid X is made of material of density $2\rho$.
Cuboid Y is made of material of density $\rho$.
Cuboid Z is made of material of density $0.5\rho$.

Which statement is correct?

**A** The upthrust of the water on each of the cuboids is the same.

**B** The upthrust of the water on W is twice the upthrust of the water on X.

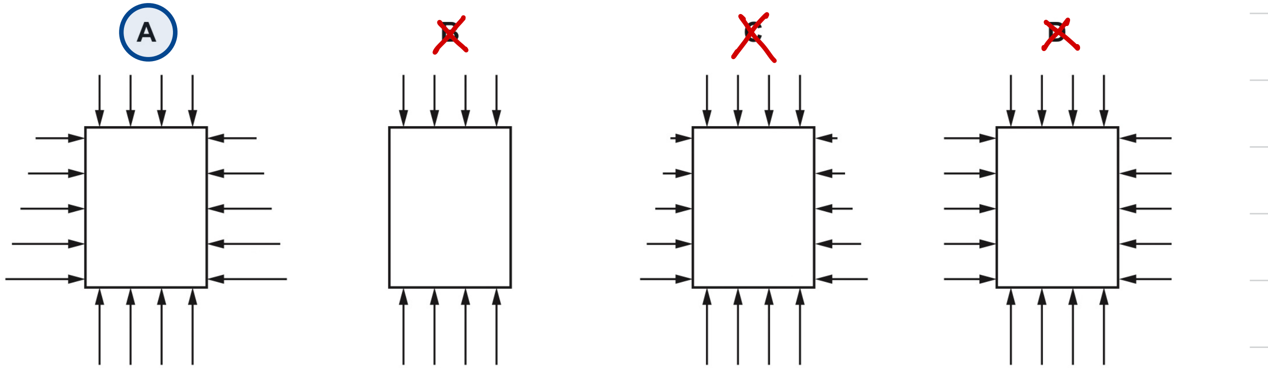
**C** The upthrust of the water on X is twice the upthrust of the water on W.

**D** The upthrust of the water on Y is zero.



**12** A block is submerged vertically in a liquid. The four diagrams show, to scale, the forces exerted by the liquid on the block.

Which diagram correctly shows a possible situation as viewed from the side?




• Same pressure at a particular depth on the top and sides in A. →

• Pressure increases with depth.