

**SECTION I**

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**MEASUREMENT**

**Chapter 1: Measurement**

- SI Units
- Errors and Uncertainties
- Scalars and Vectors

a. Recall the following base quantities and their units; mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol).

Base Quantities	SI Units	
	Name	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Amount of substance	mole	mol
Temperature	Kelvin	K
Current	ampere	A
Luminous intensity	candela	cd

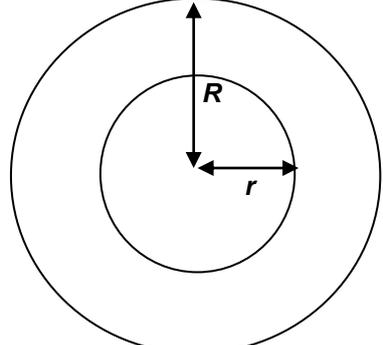
b. Express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate.

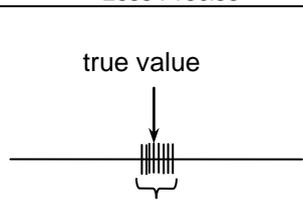
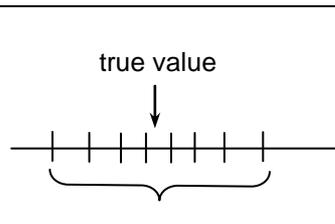
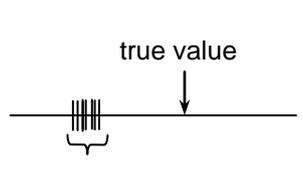
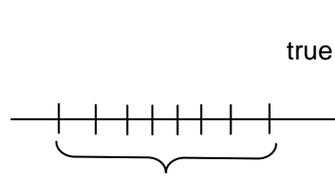
A derived unit can be expressed in terms of products or quotients of base units.

Derived Quantities	Equation	Derived Units
Area (A)	$A = L^2$	$m^2$
Volume (V)	$V = L^3$	$m^3$
Density ( $\rho$ )	$\rho = \frac{m}{V}$	$\frac{kg}{m^3} = kg\ m^{-3}$
Velocity (v)	$v = \frac{L}{t}$	$\frac{m}{s} = m\ s^{-1}$
Acceleration (a)	$a = \frac{\Delta v}{t}$	$\frac{m\ s^{-1}}{s} = m\ s^{-2}$
Momentum (p)	$p = m \times v$	$(kg)(m\ s^{-1}) = kg\ m\ s^{-1}$

Derived Quantities	Equation	Derived Unit		Derived Units
		Special Name	Symbol	
Force (F)	$F = \frac{\Delta p}{t}$	Newton	N	$\frac{kg\ m\ s^{-1}}{s} = kg\ m\ s^{-2}$
Pressure (p)	$p = \frac{F}{A}$	Pascal	Pa	$\frac{kg\ m\ s^{-2}}{m^2} = kg\ m^{-1}\ s^{-2}$
Energy (E)	$E = F \times d$	joule	J	$(kg\ m\ s^{-2})(m) = kg\ m^2\ s^{-2}$
Power (P)	$P = \frac{E}{t}$	watt	W	$\frac{kg\ m^2\ s^{-2}}{s} = kg\ m^2\ s^{-3}$
Frequency (f)	$f = \frac{1}{t}$	hertz	Hz	$\frac{1}{s} = s^{-1}$
Charge (Q)	$Q = I \times t$	coulomb	C	A s
Potential Difference (V)	$V = \frac{E}{Q}$	volt	V	$\frac{kg\ m^2\ s^{-2}}{A\ s} = kg\ m^2\ s^{-3}\ A^{-1}$
Resistance (R)	$R = \frac{V}{I}$	ohm	$\Omega$	$\frac{kg\ m^2\ s^{-3}\ A^{-1}}{A} = kg\ m^2\ s^{-3}\ A^{-2}$

c.	<p>Show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication <i>SI Units, Signs, Symbols and Systematics (The ASE Companion to 5-16 Science, 1995)</i>.</p> <p>Self-explanatory</p>																																	
d.	<p>Use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (<math>\mu</math>), milli (m), centi (c), deci (d), kilo (K), mega (M), giga (G), tera (T).</p> <table border="1" data-bbox="248 501 1393 824"> <thead> <tr> <th>Multiplying Factor</th> <th>Prefix</th> <th>Symbol</th> </tr> </thead> <tbody> <tr> <td><math>10^{-12}</math></td> <td>pico</td> <td>p</td> </tr> <tr> <td><math>10^{-9}</math></td> <td>nano</td> <td>n</td> </tr> <tr> <td><math>10^{-6}</math></td> <td>micro</td> <td><math>\mu</math></td> </tr> <tr> <td><math>10^{-3}</math></td> <td>milli</td> <td>m</td> </tr> <tr> <td><math>10^{-2}</math></td> <td>centi</td> <td>c</td> </tr> <tr> <td><math>10^{-1}</math></td> <td>deci</td> <td>d</td> </tr> <tr> <td><math>10^3</math></td> <td>kilo</td> <td>k</td> </tr> <tr> <td><math>10^6</math></td> <td>mega</td> <td>M</td> </tr> <tr> <td><math>10^9</math></td> <td>giga</td> <td>G</td> </tr> <tr> <td><math>10^{12}</math></td> <td>tera</td> <td>T</td> </tr> </tbody> </table>	Multiplying Factor	Prefix	Symbol	$10^{-12}$	pico	p	$10^{-9}$	nano	n	$10^{-6}$	micro	$\mu$	$10^{-3}$	milli	m	$10^{-2}$	centi	c	$10^{-1}$	deci	d	$10^3$	kilo	k	$10^6$	mega	M	$10^9$	giga	G	$10^{12}$	tera	T
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e.	<p><b>Make reasonable estimates of physical quantities included within the syllabus.</b></p> <p>When making an estimate, it is only reasonable to give the figure to <u>1 or at most 2 significant figures</u> since an estimate is not very precise.</p> <table border="1" data-bbox="248 987 1393 1137"> <thead> <tr> <th>Physical Quantity</th> <th>Reasonable Estimate</th> </tr> </thead> <tbody> <tr> <td>Mass of 3 cans (330 ml) of Coke</td> <td>1 kg</td> </tr> <tr> <td>Mass of a medium-sized car</td> <td>1000 kg</td> </tr> <tr> <td>Length of a football field</td> <td>100 m</td> </tr> <tr> <td>Reaction time of a young man</td> <td>0.2 s</td> </tr> </tbody> </table> <ul style="list-style-type: none"> <li>- Occasionally, students are asked to estimate the area under a graph. The usual method of counting squares within the enclosed area is used. (eg. Topic 3 (Dynamics), N94P2Q1c)</li> <li>- Often, when making an estimate, a formula and a simple calculation may be involved.</li> </ul> <p><b>EXAMPLE 1E1</b> Estimate the average running speed of a typical 17-year-old's 2.4-km run.</p> $\text{velocity} = \frac{\text{distance}}{\text{time}}$ $= \frac{2400}{12.5 \times 60} = 3.2$ $\approx 3 \text{ m s}^{-1}$ <p><b>EXAMPLE 1E2 (N08/ I/ 2)</b> Which estimate is realistic?</p> <table border="1" data-bbox="248 1693 1393 1998"> <thead> <tr> <th>Option</th> <th>Explanation</th> </tr> </thead> <tbody> <tr> <td><b>A</b> The kinetic energy of a bus travelling on an expressway is 30 000 J</td> <td>A bus of mass <math>m</math> travelling on an expressway will travel between 50 to 80 km h<sup>-1</sup>, which is 13.8 to 22.2 m s<sup>-1</sup>. Thus, its KE will be approximately <math>\frac{1}{2} m(18^2) = 162m</math>. Thus, for its KE to be 30 000J: <math>162m = 30\ 000</math>. Thus, <math>m = 185\text{kg}</math>, which is an absurd weight for a bus; ie. This is not a realistic estimate.</td> </tr> <tr> <td><b>B</b> The power of a domestic light is 300 W.</td> <td>A single light bulb in the house usually runs at about 20 W to 60 W. Thus, a <i>domestic</i> light is unlikely to run at more than 200W; this estimate is rather high.</td> </tr> <tr> <td><b>C</b> The temperature of a hot oven is 300 K.</td> <td>300K = 27 °C. Not very hot.</td> </tr> </tbody> </table>	Physical Quantity	Reasonable Estimate	Mass of 3 cans (330 ml) of Coke	1 kg	Mass of a medium-sized car	1000 kg	Length of a football field	100 m	Reaction time of a young man	0.2 s	Option	Explanation	<b>A</b> The kinetic energy of a bus travelling on an expressway is 30 000 J	A bus of mass $m$ travelling on an expressway will travel between 50 to 80 km h <sup>-1</sup> , which is 13.8 to 22.2 m s <sup>-1</sup> . Thus, its KE will be approximately $\frac{1}{2} m(18^2) = 162m$ . Thus, for its KE to be 30 000J: $162m = 30\ 000$ . Thus, $m = 185\text{kg}$ , which is an absurd weight for a bus; ie. This is not a realistic estimate.	<b>B</b> The power of a domestic light is 300 W.	A single light bulb in the house usually runs at about 20 W to 60 W. Thus, a <i>domestic</i> light is unlikely to run at more than 200W; this estimate is rather high.	<b>C</b> The temperature of a hot oven is 300 K.	300K = 27 °C. Not very hot.															
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<b>D</b>	The volume of air in a car tyre is $0.03 \text{ m}^3$ .		Estimating the width of a tyre, $t$ , is 15 cm or 0.15 m, and estimating $R$ to be 40 cm and $r$ to be 30 cm,  volume of air in a car tyre is $= \pi(R^2 - r^2)t$ $= \pi(0.4^2 - 0.3^2)(0.15)$ $= 0.033 \text{ m}^3$ $\approx 0.03 \text{ m}^3$ (to one sig. fig.)
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<b>f.</b>	<b>Show an understanding of the distinction between systematic errors (including zero errors) and random errors.</b>		
<b>g.</b>	<b>Show an understanding of the distinction between precision and accuracy.</b>		
	<p><b>Random error</b> is the type of error which causes readings to scatter about the true value.</p> <p><b>Systematic error</b> is the type of error which causes readings to deviate in one direction from the true value.</p> <p><b>Precision:</b> refers to the <u>degree of agreement (scatter, spread) of repeated measurements</u> of the same quantity. {NB: regardless of whether or not they are correct.}</p> <p><b>Accuracy</b> refers to the <u>degree of agreement between the result of a measurement and the true value</u> of the quantity.</p>		
	→ → R Error Higher → → → → → → Less Precise → → →		
	↓ ↓ S Error Higher ↓ ↓ ↓ ↓ ↓ ↓ Less Accurate ↓ ↓ ↓	true value 	true value 
		true value 	true value 

<b>h.</b>	<b>Assess the uncertainty in a derived quantity by simple addition of actual, fractional or percentage uncertainties (a rigorous statistical treatment is not required).</b>		
	For a quantity $x = (2.0 \pm 0.1) \text{ mm}$ ,		
	<b>Actual/ Absolute uncertainty,</b>	$\Delta x$	$= \pm 0.1 \text{ mm}$
	<b>Fractional uncertainty,</b>	$\frac{\Delta x}{x}$	$= 0.05$
	<b>Percentage uncertainty,</b>	$\frac{\Delta x}{x} \times 100\%$	$= 5\%$
	If $p = \frac{2x + y}{3}$ or $p = \frac{2x - y}{3}$ ,	$\Delta p$	$= \frac{2\Delta x + \Delta y}{3}$
	If $r = 2xy^3$ or $r = \frac{2x}{y^3}$ ,	$\frac{\Delta r}{r}$	$= \frac{\Delta x}{x} + \frac{3\Delta y}{y}$
	Actual error must be recorded to only <b>1 significant figure</b> , & The <b>number of decimal places</b> a calculated quantity should have is determined by its <u>actual error</u> .		

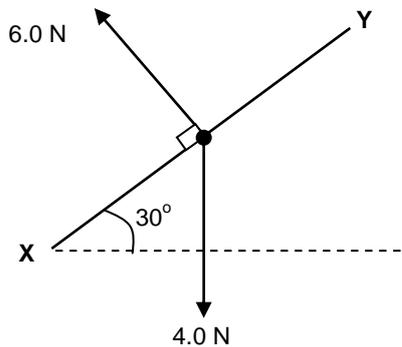
For eg, suppose  $g$  has been initially calculated to be  $9.80645 \text{ m s}^{-2}$  &  $\Delta g$  has been initially calculated to be  $0.04848 \text{ m s}^{-2}$ . The final value of  $\Delta g$  must be recorded as  $0.05 \text{ m s}^{-2}$  {1 sf}, and the appropriate recording of  $g$  is  $(9.81 \pm 0.05) \text{ m s}^{-2}$ .

**i. Distinguish between scalar and vector quantities, and give examples of each.**

Type	Scalar	Vector
<b>Definition</b>	A <b>scalar</b> quantity has a <b>magnitude only</b> . It is completely described by a certain number and a unit.	A <b>vector</b> quantity has both <b>magnitude and direction</b> . It can be described by an arrow whose length represents the magnitude of the vector and the arrow-head represents the direction of the vector.
<b>Examples</b>	Distance, speed, mass, time, temperature, <b>work done, kinetic energy, pressure, power</b> , electric charge etc.  <b>Common Error:</b> Students tend to associate kinetic energy and pressure with vectors because of the vector components involved. However, such considerations have no bearings on whether the quantity is a vector or scalar.	Displacement, velocity, moments (or torque), momentum, force, electric field etc.

**j. Add and subtract coplanar vectors.**  
**k. Represent a vector as two perpendicular components.**

In the diagram below, XY represents a flat kite of weight  $4.0 \text{ N}$ . At a certain instant, XY is inclined at  $30^\circ$  to the horizontal and the wind exerts a steady force of  $6.0 \text{ N}$  at right angles to XY so that the kite flies freely.



By accurate scale drawing	By calculations using sine and cosine rules, or Pythagoras' theorem
<p>Draw a scale diagram to find the magnitude and direction of the resultant force acting on the kite.</p> <p>Scale: <math>1 \text{ cm} \equiv 1.0 \text{ N}</math></p> <p>resultant, R</p> <p><math>6.0 \text{ N}</math></p> <p><math>4.0 \text{ N}</math></p> <p><math>30^\circ</math></p> <p><math>\theta</math></p>	<p>resultant, R</p> <p><math>6.0 \text{ N}</math></p> <p><math>4.0 \text{ N}</math></p> <p><math>30^\circ</math></p> <p><math>\alpha</math></p> <p>Using cosine rule,</p> $a^2 = b^2 + c^2 - 2bc \cos A$ $R^2 = 4^2 + 6^2 - 2(4)(6)(\cos 30^\circ)$ $R = 3.23 \text{ N}$

$R = 3.2 \text{ N} (\approx 3.2 \text{ cm})$   
 at  $\theta = 112^\circ$  to the 4 N vector.

Using sine rule,

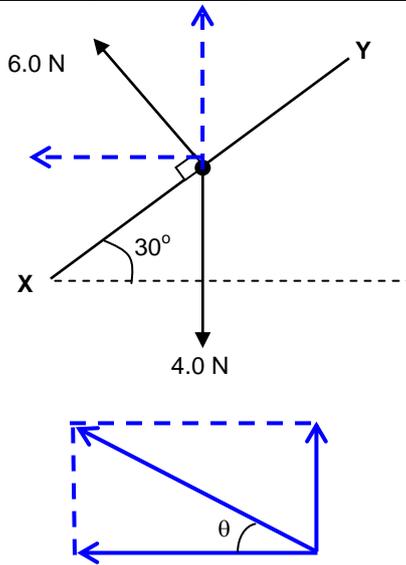
$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{6}{\sin \alpha} = \frac{3.23}{\sin 30^\circ}$$

$$\alpha = 68^\circ \text{ or } 112^\circ$$

$$= 112^\circ \text{ to the 4 N vector}$$

**Summing Vector Components**



$$F_x = -6 \sin 30^\circ$$

$$= -3 \text{ N}$$

$$F_y = 6 \cos 30^\circ - 4$$

$$= 1.2 \text{ N}$$

$$R = \sqrt{(-3)^2 + (1.2)^2}$$

$$= 3.23 \text{ N}$$

$$\tan \theta = \frac{1.2}{3}$$

$$\theta = 22^\circ$$

R is at an angle  $112^\circ$  to the 4 N vector. ( $90^\circ + 22^\circ$ )