PHYSICAL QUANTITIES AND MEASUREMENT TECHNIQUES

Physical Quantity

Any quantity that can be measured.

7 Basic Physical Quantities

	SI Base Units!
Length	meter (m)
Mass	kilogram (kg)
Time	seconds (s)
Electric Current	Ampere (A)
Luminous Intensity	candela (cd)
Thermodynamic Temperature	Kelvin (K)
Amount of Substance	moles (mol)

Derived Quantities

Quantities that are made up of two or more basic quantities, e.g. force, acceleration, density, moment.

Representing Derived Units in terms of SI base Units

i, Area

$$A = \pi r^2 \qquad\qquad A = L \times b \qquad\qquad A = 2\pi r h$$

$$= (m)^2 \qquad\qquad\quad = m \times m \qquad\qquad = m \times m$$

$$= m^2 \qquad\qquad\qquad = m^2 \qquad\qquad\quad = m^2$$

ii, Volume

$$V = L \times b \times h \qquad\qquad V = \frac{4}{3} \pi r^3 \qquad\qquad V = \pi r^2 h$$

$$= m \times m \times m$$

$$= m^3 \qquad\qquad\qquad\quad = (m)^3 \qquad\qquad = (m)^2 (m)$$

$$\qquad\qquad\qquad\qquad\qquad = m^3 \qquad\qquad\quad = m^3$$

iii, Density

$$\rho = \frac{m}{V} \qquad\qquad\qquad \rho = \frac{kg}{m^3} \quad or \quad kg m^{-3}$$

iv, Velocity

$$v = \frac{s}{t}$$

$$v = \frac{m}{s}$$

$$v = ms^{-1}$$

v, Acceleration

$$a = \frac{\Delta v}{\Delta t}$$

$$a = \frac{v-u}{t}$$

$$= \frac{ms^{-1} - ms^{-1}}{s}$$

$$= \frac{ms^{-1}}{s}$$

$$= ms^{-1-1}$$

$$= ms^{-2}$$

vi, Force

$$F = ma$$

$$= kg \cdot ms^{-2}$$

$$N = kg\,ms^{-2}$$

vii, Work

$$W = F \times s$$

$$= ma \times s$$

$$= kg \cdot ms^{-2} \cdot m$$

$$J = kg \cdot m^2 s^{-2}$$

viii, Energy

$$E = mgh$$

$$= kg \cdot ms^{-2} \cdot m$$

$$J = kg\,m^2 s^{-2}$$

$$E = mc^2$$

$$= kg \cdot (ms^{-1})^2$$

$$J = kg\,m^2 s^{-2}$$

ix, Pressure

$$P = \frac{F}{A}$$

$$= \frac{ma}{A}$$

$$= \frac{kg \cdot ms^{-2}}{m^2}$$

$$= kgm^{1-2}s^{-2}$$

$$Pa = kgm^{-1}s^{-2}$$

$$P = \rho gh$$

$$= kgm^{-3} \cdot ms^{-2} \cdot m$$

$$= kgm^{-3+1+1} \cdot s^{-2}$$

$$Pa = kgm^{-1}s^{-2}$$

x, Power

$$P = \frac{E}{t}$$

$$= \frac{mgh}{t}$$

$$= \frac{kg \cdot ms^{-2} \cdot m}{s}$$

$$= kgm^2 s^{-2-1}$$

$$W = kgm^2 s^{-3}$$

xi, Voltage

$$V = \frac{W}{q}$$

$$= \frac{F \times s}{I \times t}$$

$$= \frac{kgms^{-2} \cdot m}{A \cdot s}$$

$$= kgm^2 s^{-3} \cdot A^{-1}$$

xii, Resistance

$$R = \frac{V}{I}$$

$$= \frac{kg\,m^2 s^{-3} A^{-1}}{A}$$

$$\Omega = kg\,m^2 s^{-3} A^{-2}$$

Find the unit of an unknown quantity

i, $Q = mc\Delta T$ find the units of c in SI base units.

Q: heat energy
m: mass
T: Temperature

$$E = mgh$$
$$= kg\,m s^{-2} \cdot m$$

$$c = \frac{Q}{m\Delta T}$$ $$E = kg\,m^2 s^{-2}$$

$$c = \frac{kg\,m^2 s^{-2}}{kg \cdot K}$$

$$c = m^2 s^{-2} K^{-1}$$

ii, $F = 6\pi \eta r v$, find the units of η in terms of SI base units

F: force   r: radius   v: velocity η: coefficient of viscosity

$$\frac{F}{6\pi r v} = \eta$$

$$\eta = \frac{kg\, m s^{-2}}{m \cdot m s^{-1}}$$

$$= kg\, m^{-1} \cdot s^{-2-(-1)}$$

Poise $= kg\, m^{-1} s^{-1}$

Homogeneity of Equations

The units of quantities on one side of equation should be equal to the units on the other side.

Check of homogeneity

① $T^2 = \dfrac{4\pi^2 \ell}{g}$

$(s)^2 = \dfrac{m}{ms^{-2}}$

$s^2 = s^2$

The equation is homogenous.

② $s^2 = ut^2 + \dfrac{1}{2}at^3$

$m^2 = (ms^{-1})(s)^2 + (ms^{-2})(s)^3$

$m^2 = ms^{-1+2} + ms^{-2+3}$

$m^2 \neq ms + ms$

Not homogenous!

③ $\dfrac{P}{\rho} = \dfrac{v^2}{2g} + h$

$P = \rho g h$
$= kg\,m^{-3} \cdot m\,s^{-2} \cdot m$
$= kg\,m^{-1}s^{-2}$

P: pressure ρ: density
v: velocity   g: grav. acc.
h: height

$\dfrac{kg\,m^{-1}s^{-2}}{kg\,m^{-3}} = \dfrac{(m s^{-1})^2}{(m s^{-2})} + m$

$m^{-1+3}\,s^{-2} = \dfrac{m^2 s^{-2}}{m s^{-2}} + m$

$m^2 s^{-2} \neq m + m$

Not homogenous!

4 A cylindrical tube rolling down a slope of inclination θ moves a <u>distance L</u> in <u>time T</u>. The equation relating these quantities is

$$L\left(3 + \frac{a^2}{P}\right) = QT^2 \sin\theta$$

$$3L + \frac{La^2}{P} = QT^2\sin\theta$$

$$m$$

Where <u>a is the internal radius</u> of the tube and P and Q are constants.

Which line gives the correct units for P and Q?

$$\frac{La^2}{P}$$

$$QT^2\sin\theta \quad \rightarrow \text{unit less !}$$

$$m \cdot m^2 = m$$

$$Q \cdot (s)^2 = m$$

$$\frac{m^2}{P} = m \qquad Q = \frac{m}{s^2} \quad \text{or } ms^{-2}$$

$$m^2 = P$$

	P	Q
A	m^2	$m^2 s^{-2}$
B	m^2	ms^{-2}
C	m^2	$m^3 s^{-2}$
D	m^3	ms^{-2}

Finding unknown power over variables using SI base Units

Example #1

$$T = 2\pi \left(\frac{L}{g}\right)^x$$

$$s = \left(\frac{m}{ms^{-2}}\right)^x$$

$$s = (s^2)^x$$

$$s = s^{2x}$$

as eqt is homogenous. so comparing powers.

$$1 = 2x$$

$$\boxed{x = 0.5}$$

Example #2

$$P = \frac{1}{3}\rho c^y$$

P: pressure
ρ: density
c: speed

$$kg\,m^{-1}s^{-2} = kg\,m^{-3}\cdot(ms^{-1})^y$$

$$kg\,m^{-1}s^{-2} = kg\,m^{-3}\cdot m^y\cdot s^{-y}$$

$$kg\,m^{-1}s^{-2} = kg\,m^{y-3}\cdot s^{-y}$$

comparing powers.

<u>m.</u>
$$-1 = y-3$$
$$-1+3 = y$$
$$\boxed{y = 2}$$

<u>s.</u>
$$+2 = +y$$
$$\boxed{y = 2}$$

1 The drag force F acting on a moving sphere obeys an equation of the form $F = kAv^2$, where A represents the sphere's frontal area and v represents its speed.

What are the base units of the constant k?

$$k = \frac{F}{Av^2} = \frac{kg\,m\,s^{-2}}{m^2 \cdot (m\,s^{-1})^2}$$

A $kg\,m^5\,s^{-4}$ **B** $kg\,m^{-2}\,s^{-1}$ **C** $kg\,m^{-3}$ **D** $kg\,m^{-4}\,s^2$

$$= \frac{kg\,m\,s^{-2}}{m^2 \cdot m^2\,s^{-2}}$$

$$= kg\,m^{1-2-2}$$

$$= kg\,m^{-3}$$

2 The table contains some quantities, together with their symbols and units.

quantity	symbol	unit
gravitational field strength	g	$N\,kg^{-1}$
density of liquid	ρ	$kg\,m^{-3}$
vertical height	h	m
volume of part of liquid	V	m^3

$E = mgh$

$= kg \cdot m\,s^{-2} \cdot m$

$= kg\,m^2\,s^{-2}$

Which expression has the units of energy?

A $g\rho hV$ **B** $\dfrac{\rho hV}{g}$ **C** $\dfrac{\rho g}{hV}$ **D** $\rho g^2 h$

$m\,s^{-2} \cdot kg\,m^{-3} \cdot m \cdot m^3$

$kg\,m^{1-3+1+3}\,s^{-2} \Rightarrow kg\,m^2\,s^{-2}$

3 The drag coefficient C_d is a number with no units. It is used to compare the drag on different cars at different speeds. It is given by the equation

$$C_d = \frac{2F}{\rho v^n A}$$

$$1 = \frac{kgm\,s^{-2}}{kgm^{-3}.(ms^{-1})^n.m^2}$$

$$1 = \frac{ms^{-2}}{m^{-3}.m^n.s^{-n}.m^2}$$

where F is the drag force on the car, ρ is the density of the air, A is the cross-sectional area of the car and v is the speed of the car.

What is the value of n?

$$1 = \frac{ms^{-2}}{m^{-3+n+2}s^{-n}}$$

$$1 = m^{1-n+1}s^{-2-(-n)}$$

$$1 = m^{2-n}s^{n-2}$$

A 1

B 2

C 3

D 4

$$1 = \frac{ms^{-2}}{m^{n-1}s^{-n}}$$
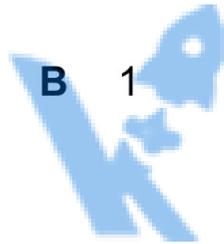
$m.$

$0 = 2-n$

$n = 2$

$s.$

$0 = n-2$

$n = 2$
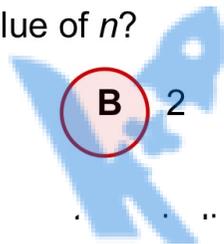
3 The speed v of a liquid leaving a tube depends on the change in pressure ΔP and the density ρ of the liquid. The speed is given by the equation

$$v = k\left(\frac{\Delta P}{\rho}\right)^n$$

$$ms^{-1} = \left(\frac{kg\,m^{-1}s^{-2}}{kg\,m^{-3}}\right)^n$$

$$P = \rho g h$$
$$= kg\,m^{-3}.\,ms^{-2}.\,m$$
$$= kg\,m^{-1}s^{-2}$$

where k is a constant that has no units.

$$ms^{-1} = (m^2 s^{-2})^n$$

What is the value of n?

$$ms^{-1} = m^{2n} s^{-2n}$$

A $\dfrac{1}{2}$ B 1   C $\dfrac{3}{2}$ D 2

m. s.
$1 = 2n$ $-1 = -2n$
$n = \dfrac{1}{2}$ $n = \dfrac{1}{2}$

4 A cylindrical tube rolling down a slope of inclination θ moves a distance L in time T. The equation relating these quantities is

$$L\left(3 + \frac{a^2}{P}\right) = QT^2 \sin\theta$$

Where a is the internal radius of the tube and P and Q are constants.

Which line gives the correct units for P and Q?

	P	Q
A	m^2	$m^2\,s^{-2}$
B	m^2	$m\,s^{-2}$
C	m^2	$m^3\,s^{-2}$
D	m^3	$m\,s^{-2}$

Handwritten working:

$$3L + \frac{a^2 L}{P} = QT^2 \sin\theta$$

$$m + \frac{a^2 L}{P} = QT^2$$

$$\frac{a^2 L}{P} = m$$

$$\frac{(m)^2(m)}{P} = m$$

$$P = m^2$$

$$QT^2 = m$$

$$Q(s)^2 = m$$

$$Q = \frac{m}{s^2}$$

$$= m\,s^{-2}$$

4

Answer **all** the questions in the spaces provided.

1 **(a)** Show that the SI base units of power are $\text{kg}\,\text{m}^2\,\text{s}^{-3}$.

$$P = \frac{E}{t}$$

$$P = \frac{mgh}{t}$$

$$P = \frac{kg \cdot ms^{-2} \cdot m}{s}$$

$$P = kg\,m^2\,s^{-3}$$

[3]

(b) The rate of flow of thermal energy $\frac{Q}{t}$ in a material is given by

$$\frac{Q}{t} = \frac{CAT}{x}$$

where A is the cross-sectional area of the material,
T is the temperature difference across the thickness of the material,
x is the thickness of the material,
C is a constant.

Determine the SI base units of C.

$$kg\,m^2\,s^{-3} = C\,\frac{m^2 \cdot K}{m}$$

$$kg\,m\,s^{-3}\,K^{-1} = C$$

base units .... [4]

Measurement Techniques

1. Length

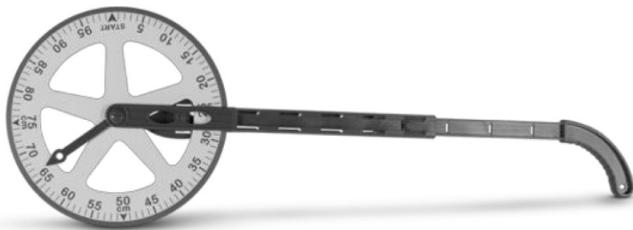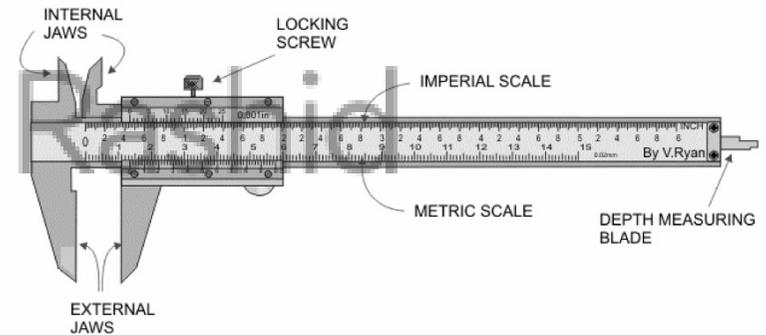|	Range	Precision	
Trundle Wheel	several meters	0.1 cm	
Measuring Tape	several meters	0.1 cm	
Meter rule	100 cm	0.1 cm	
Vernier Caliper	20~25 cm	0.01 cm	} Small but
Micrometer Screw Gauge	4~5 cm	0.001 cm	} precise lengths

tape

meter rule
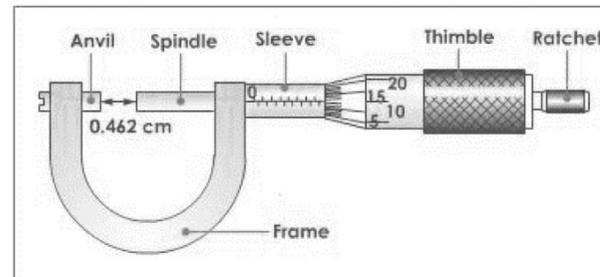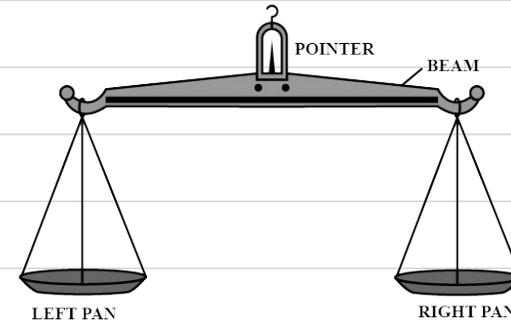
Figure 2 Vernier Caliper
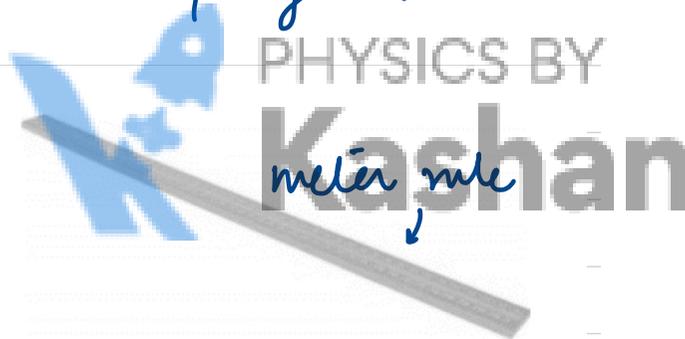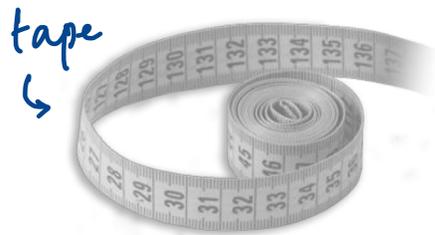
Figure 1 Trundle Wheel

Figure 3 Micrometer Screw Gauge

2. Mass

- Electronic Balance
- Beam Balance

Electronic Balance

Beam balance

3. Weight

- Newton meter/Spring Balance
- Compression balance
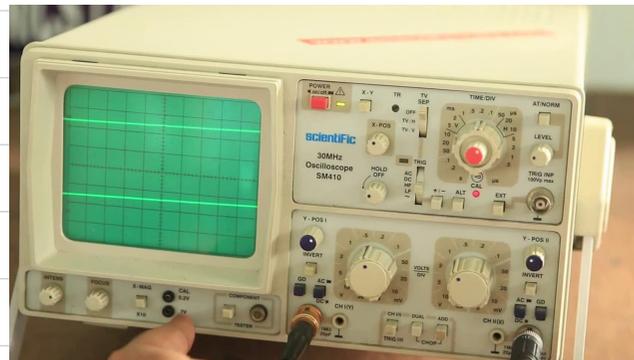

Spring Balance

Compression Balance

4. Time

- Stopwatch
- Clock
- CRO

Cathode Ray Oscilloscope

5. Temperature

- Liquid in glass thermometer
- Thermocouple Thermometers
- Temperature Sensors

Thermocouple

6. Current

- Ammeter
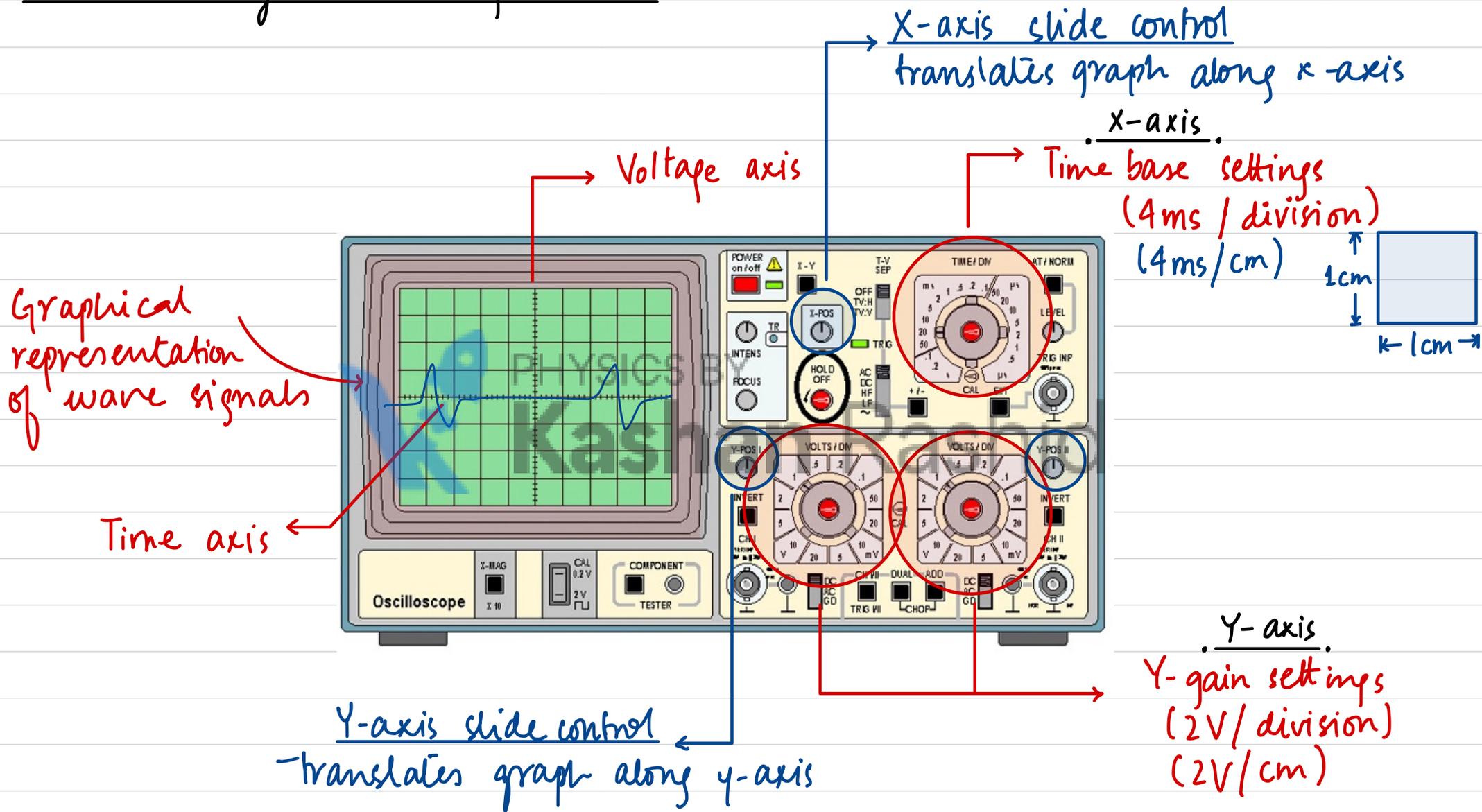- Galvanometer
- Multimeter

Ammeter

Galvanometer

Multimeter

## 7. Voltage

- Voltmeter
- Multimeter

Cathode Ray Oscilloscope (CRO)

X-axis slide control translates graph along x-axis

Voltage axis

X-axis
Time base settings
(4ms / division)
(4ms/cm)

1cm

← 1cm →

Graphical representation of wave signals

Time axis

Y-axis slide control translates graph along y-axis

Y-axis
Y-gain settings
(2V/division)
(2V/cm)

Oscilloscope

Type #1: Drawing a graph using Input Signal and Settings

Information Signal
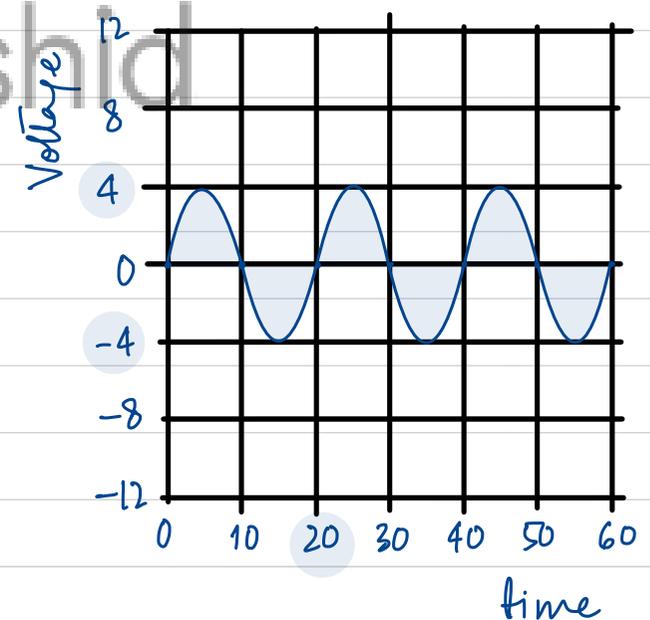
- Sine wave
- $V_0 = 4.0$ V
- $f = 50$ Hz

Setting A

Y-gain: 2.0 V/div
Time-base: 5 ms/div

$f = \dfrac{1}{T}$ so $50 = \dfrac{1}{T}$
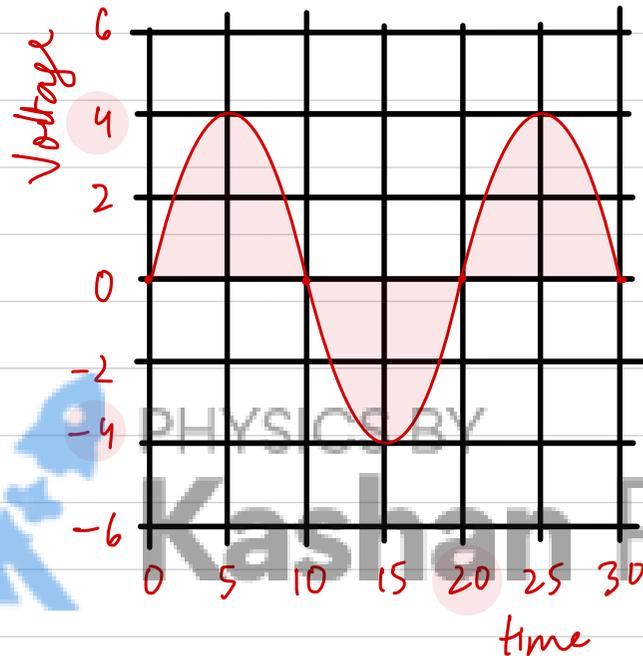
$T = 0.02$ s or 20 ms

Setting B

Y-gain: 4.0 V/div
Time base: 10 ms/div

👉 When the **settings** for the CRO is **increased**, the **graph shrinks** along that axis

5 The diagram shows a cathode-ray oscilloscope (c.r.o.) being used to measure the rate of rotation of a flywheel.

$1 ms = 1000 \mu s$

$3000 \text{ rev} - 1 \text{min}$

$3000 \text{ rev} - 60 s$

$1 \text{ rev} \swarrow$ flywheel

$T = \dfrac{60 \times 1}{3000}$

$T = 0.02 s \quad 1s = 1000 ms$

$20 ms$

20 ms

10 cm

0 20 40 60 80 100

M coil

PHYSICS BY Kashan Rashid
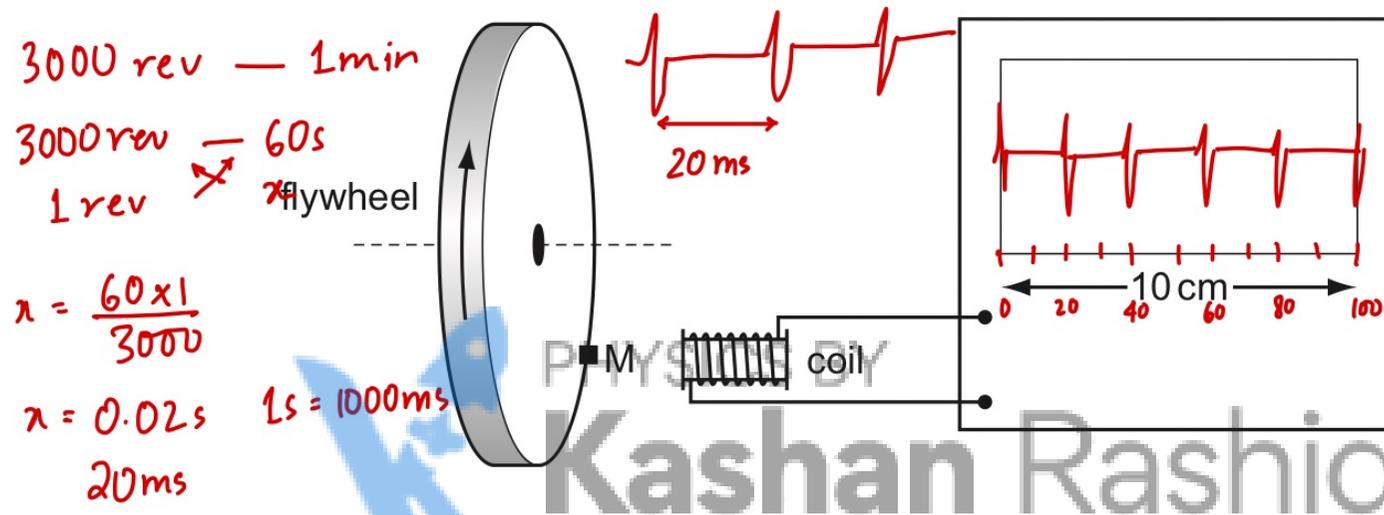
The flywheel has a small magnet M mounted on it. Each time the magnet passes the coil, a voltage pulse is generated, which is passed to the c.r.o. The display of the c.r.o. is 10 cm wide. The flywheel is rotating at a rate of about 3000 revolutions per minute.

Which time-base setting will display clearly separate pulses on the screen?

A 1 s cm^{-1} **B** 10 ms cm^{-1} **C** $100 \mu \text{s cm}^{-1}$ **D** $1 \mu \text{s cm}^{-1}$

$0.1 ms$

4 A signal that repeats periodically is displayed on the screen of a cathode-ray oscilloscope.

10 ms
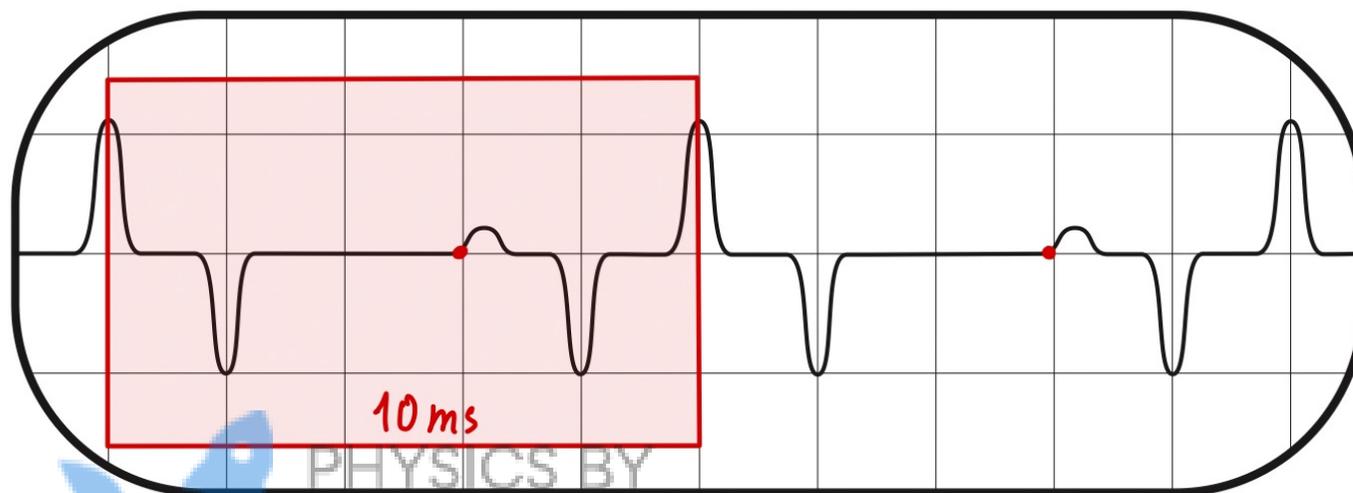
The screen has 1 cm squares and the time base is set at 2.00 ms cm⁻¹.

$$f = \frac{1}{10 \times 10^{-3}}$$

What is the frequency of this periodic signal?

$$f = 100\,Hz$$

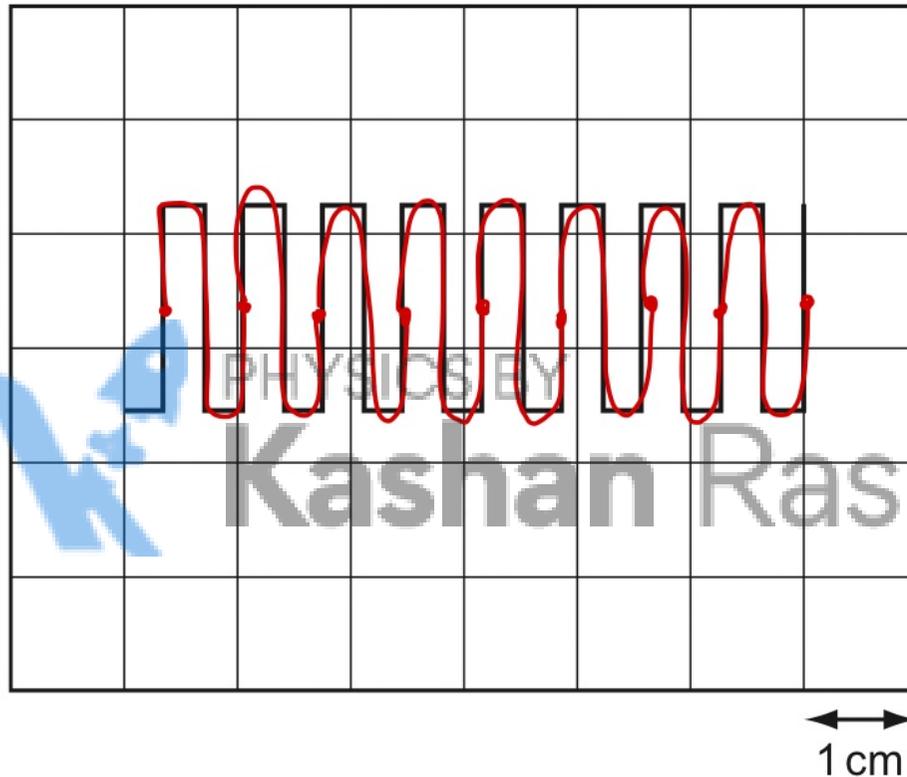**A** 50 Hz   **B** 100 Hz   **C** 125 Hz   **D** 200 Hz

5 The diagram shows a square-wave trace on the screen of a cathode-ray oscilloscope. A grid of 1 cm squares covers the screen. The time-base setting is $10\,\text{ms}\,\text{cm}^{-1}$.

8.5 waves — 60 ms

x — 1 s

$x = \dfrac{8.5 \times 1}{60 \times 10^{-3}}$

$x = 141\,\text{Hz}$

1 cm

1 cm

8.5 waves — 60 ms

1 wave   — x

$x = \dfrac{60 \times 1}{8.5}$

$x = 7.06\,\text{ms}$ (time period)

$f = \dfrac{1}{T}$

$f = \dfrac{1}{7.06 \times 10^{-3}}$

$f = 141\,\text{Hz}$

What is the approximate frequency of the square wave?

A   70 Hz   **B**   140 Hz   **C**   280 Hz   **D**   1400 Hz

Unit Conversion

$$\text{Tera (T)} \qquad \times 10^{12}$$

Prefix	Exponents
Giga (G)	$\times 10^9$
Mega (M)	$\times 10^6$
kilo (k)	$\times 10^3$
deci (d)	$\times 10^{-1}$
centi (c)	$\times 10^{-2}$
mili (m)	$\times 10^{-3}$
micro (µ)	$\times 10^{-6}$
nano (n)	$\times 10^{-9}$

$$\text{pico (p)} \qquad 10^{-12}$$

Note: you need to remember the prefix and their exponent to solve the conversion

1. _If a prefix is to be <mark>inserted, divide</mark> the value by the exponent of that prefix._

800m to km $k: 10^3$ $$\frac{800}{10^3} = 0.8 \text{ km}$$	48000g to Mg $M: 10^6$ $$\frac{48000}{10^6} = 0.048 \text{ Mg}$$
2.8 x 10^{-12} s to μs $\mu: 10^{-6}$ $$\frac{2.8 \times 10^{-12}}{10^{-6}} = 2.8 \times 10^{-6} \mu s$$	490 x 10^{-6} N to mN $m: 10^{-3}$ $$\frac{490 \times 10^{-6}}{10^{-3}} = 0.49 \text{ mN}$$

2. _If a prefix is to removed, multiply the value by the exponent of that prefix._

6700 μs to s $\mu: 10^{-6}$ 6700×10^{-6} $6.7 \times 10^{-3} s$	30 MN to N $M: 10^6$ 30×10^6 $3.0 \times 10^7 N$
6.05 x 10^{-6} nJ to J $n: 10^{-9}$ $6.05 \times 10^{-6} \times 10^{-9}$ $6.05 \times 10^{-15} J$	48000 mV to V $m: 10^{-3}$ 48000×10^{-3} $48 V.$

3. _If a prefix is to be replaced with the other, combine both rule._

5000 μs to ms $\mu: 10^{-6}$ $m: 10^{-3}$	65 kN to dN $k: 10^3$ $d: 10^{-1}$
$\dfrac{5000 \times 10^{-6}}{10^{-3}}$ 5 ms	$\dfrac{65 \times 10^3}{10^{-1}}$ $6.5 \times 10^5 \, dN$
8.85 x 10⁻¹² GJ to μJ $G: 10^9$ $\mu: 10^{-6}$	6.7 x 10⁻⁵ mC to MC $m: 10^{-3}$ $M: 10^6$
$\dfrac{8.85 \times 10^{-12} \times 10^9}{10^{-6}}$ $8850 \, \mu J$	$\dfrac{6.7 \times 10^{-5} \times 10^{-3}}{10^6}$ $6.7 \times 10^{-14} \, MC$

3 When a constant braking force is applied to a vehicle moving at speed v, the distance d moved by the vehicle in coming to rest is given by the expression

$$d = kv^2$$

where k is a constant.

When d is measured in metres and v is measured in metres per second, the constant has a value of k_1.

What is the value of the constant when the distance is measured in metres, and the speed is measured in kilometres per hour?

A $0.0772\,k_1$ **B** $0.278\,k_1$ **C** $3.60\,k_1$ **D** $13.0\,k_1$

$d = k_1 v^2$

$m \swarrow$ $\searrow ms^{-1}$

$d = k_1 \left(v \times \dfrac{10^3}{3600} \right)^2$

$v \dfrac{km}{h} \xrightarrow[\times 3600]{\times 10^3} \dfrac{m}{s}$

$d = k_2 v^2$

$m \swarrow$ $\searrow kmh^{-1}$

$k_2 = \boxed{}\, k_1$

$d = k_1 \cdot v^2 \cdot 0.0772$

$d = \boxed{0.00772\, k_1}\, v^2$

$m \swarrow \quad k_2$ $\searrow kmh^{-1}$

PRECISION VS ACCURACY

Precision: It tells how close the values are to one another.
No. of decimal places of a value helps improve precision.

e.g.   4.2 , 3.9 , 4.1   (precise)
       4.2 , 2.1, 6.8   (not precise) "Scattered data"

Accuracy: It tells how close the values are to the <u>true value</u>.
Accuracy depends upon the quality of experiment performed to determine values.

e.g.   g (true value) = 9.8 ms⁻²

Student A : g = 9.6 , 10.2 , 9.5 (Accurate)
Student B : g = 7.1 , 5.5 , 9.3 (Not Accurate)

Darts

Dart Board

no. of values

g_{true}
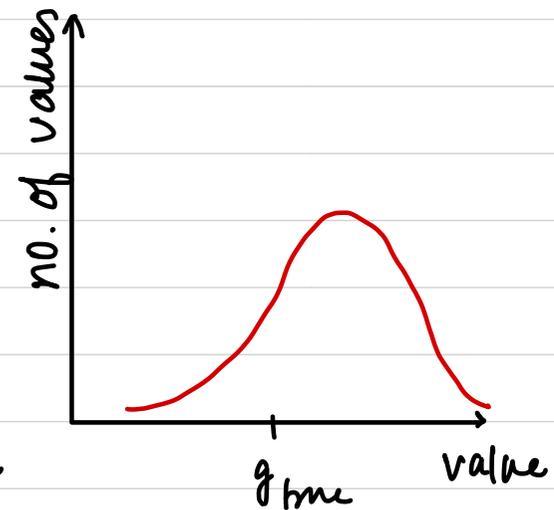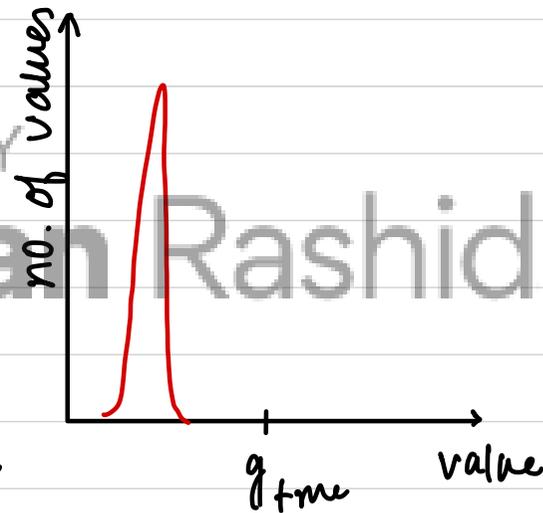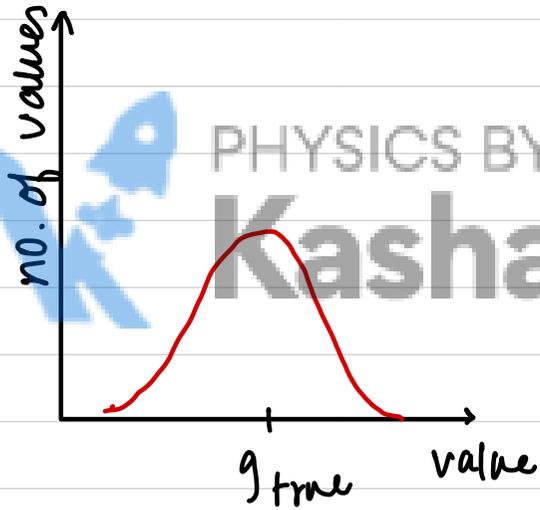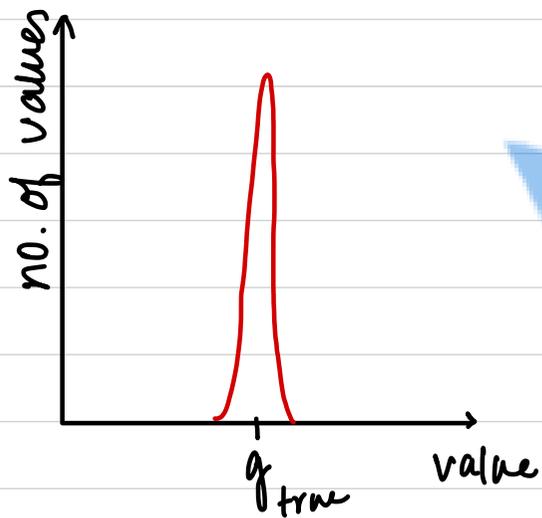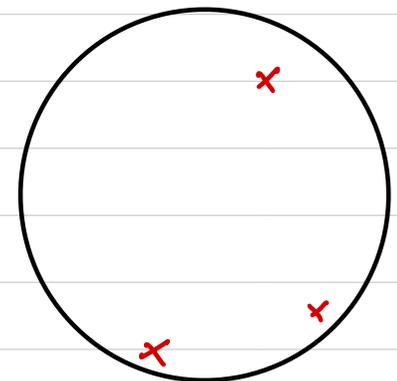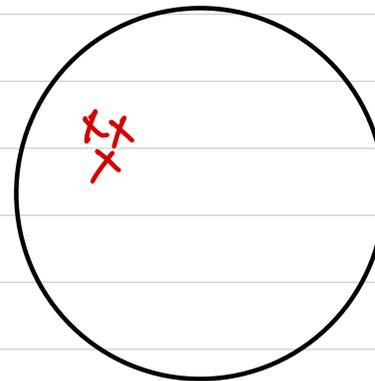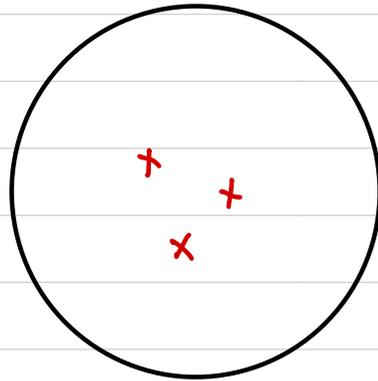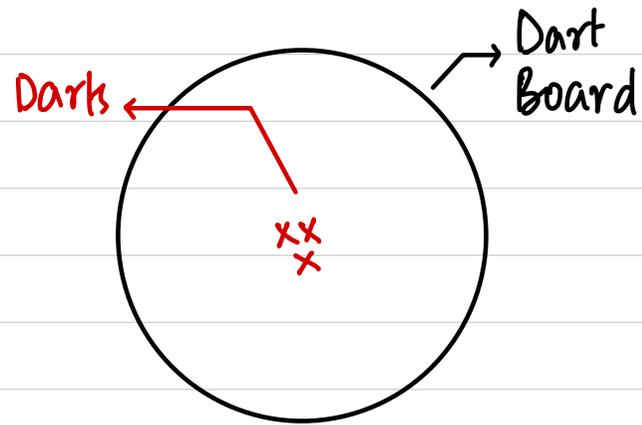
value

Both Accurate and Precise

Accurate but not precise

Not Accurate but precise

Neither Accurate Nor precise

Error & Uncertainty

Error

The difference in the true value and the obtained value during experiment.

$$\text{Error} = \left| \text{true value} - \text{obtained value} \right|$$

Types of Error

i. Systematic Error: Error introduced due to the fault in method of experiment, instrument used or equation adopted while performance.

They cannot be removed by repeat & average.

To eliminate systematic error, find out the problem in experiment and correct it
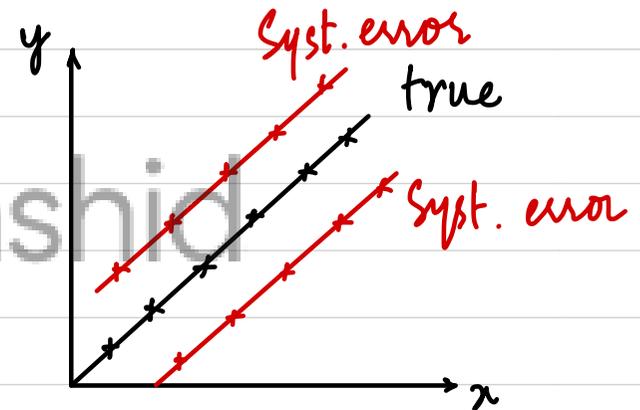
- Systematic Error causes offset of data on either side of the true value.

- Reduces Accuracy.

ji, Random Error

- Error introduced due to the fluctuations going on it surrounding e.g. temperature, wind speed, humidity etc. It may also occur due to human reaction time error.

- They can be removed / reduced by repeat and average.
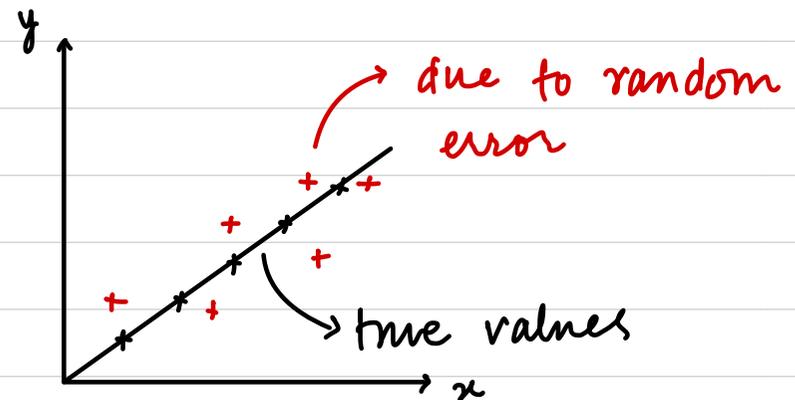
- Random errors create scatter on both sides of the true value.

- Reduce precision.

Uncertainity

- It is the margin of doubt present in the results obtained during experiment.

- Example:
  values of g calculated are 10.9, 10.5, 10.2, 9.8, 9.4, 9.6, 9.0

  average value of $g = \frac{\Sigma g}{n} = \frac{10.9 + 10.5 + 10.2 + 9.8 + 9.4 + 9.6 + 9.0}{7}$

  $g_{avg} = 9.9 \text{ N kg}^{-1}$

Uncertainity $= \dfrac{\text{max value} - \text{min value}}{2}$

so $\Delta g = \dfrac{10.9 - 9.0}{2}$

$\Delta g = 1.0$

so the value of g is quoted as $g = 9.9 \pm 1.0$

→ Uncertainity reflects the precision of your experiment or results.
  • less precision ⟶ more scattered data ⟶ high uncertainity

→ Instruments that have more precision i.e. give values to more no. of d.p. incur less uncertainity in your results.

Rules for writing uncertainity with principle value

$$9.9 \pm 0.1$$

Principle value   Uncertainity

1. No. of d.p in your uncertainity should be either "equal to" or "less than" the no. of d.p in principle value.

9.9 ± 0.95 ✗
9.9 ± 1.0 ✓
9.9 ± 1 ✓

2. In case of exponents i.e. 10^x, both principle value and uncertainity should have the same power of exponent.

$5.96 \times 10^{12} \pm 1.23 \times 10^{11}$ ✗
$5.96 \times 10^{12} \pm 0.123 \times 10^{12}$ ✗
$5.96 \times 10^{12} \pm 0.12 \times 10^{12}$ ✓
$(5.96 \pm 0.12) \times 10^{12}$ ✓

$$* \ \begin{cases} y = a + b \\ y = a - b \end{cases}$$

$$* \ y = a^n$$

$$\Delta x$$

$$11.5 \ \pm 0.1$$

Absolute

Uncertainity

Fractional

$$\frac{\Delta x}{x} \qquad \frac{0.1}{11.5}$$

$$* \ y = na$$

Percentage

$$* \ \begin{cases} y = a \times b \\ y = \dfrac{a}{b} \end{cases}$$

$$\Delta x\% = \frac{\Delta x}{x} \times 100$$

$$\Delta x\% = \frac{0.1}{11.5} \times 100$$

$$= 0.87 \ \%$$
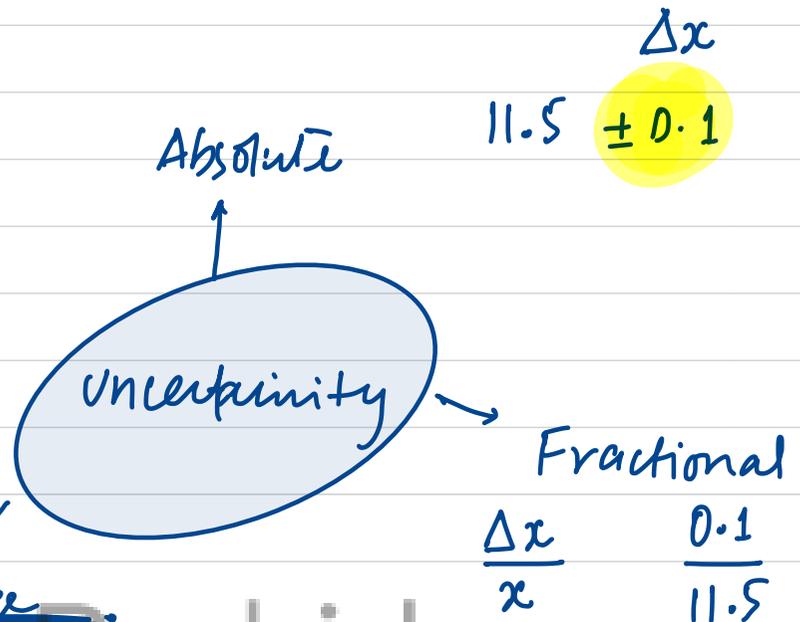
$$21.6 \ cm \ \Big| \ \pm 5\%$$

$$5\% \ of \ 21.6$$

$$\frac{5}{100} \times 21.6 = \pm 1.1$$

Representation of Uncertainity

Absolute Uncertainity
$$\Delta x$$

$$12.0 \pm 0.1$$

Absolute Uncertainity

Fractional Uncertainity
$$\frac{\Delta x}{x}$$

$$\frac{0.1}{12.0} = 0.0083$$

Percentage Uncertainity
$$\Delta x \% = \frac{\Delta x}{x} \times 100$$

$$\frac{0.1}{12.0} \times 100 = 0.83\%$$

Rules of Uncertainity

1. Addition & Subtraction

$$y = a + b$$
$$y = a - b$$

$$\boxed{\Delta y = \Delta a + \Delta b}$$

e.g. $a = 12.0 \pm 0.1$
$b = 4.0 \pm 0.2$

if $y = a + b$
$$y = 12.0 + 4.0$$
$$y = 16.0$$

$$\Delta y = \Delta a + \Delta b$$
$$= 0.1 + 0.2$$

$$\Delta y = \pm 0.3$$

if $y = a - b$
$$y = 12.0 - 4.0$$
$$y = 8.0$$

$$\Delta y = \Delta a + \Delta b$$
$$= 0.1 + 0.2$$

$$\Delta y = 0.3$$

$$y = 8.0 \pm 0.3$$

higher $\Delta y \%$

$$y = 16.0 \pm 0.3$$

Lower $\Delta y \%$

2. If a coefficient is multiplied with a variable having power of "1".

$y = na$ where n is a coefficient

$$\boxed{\Delta y = n \Delta a}$$

e.g. $P = 2l + 2b$

$b = 2.0 \pm 0.1$ cm

$l = 4.0 \pm 0.1$ cm

$P = 2(4) + 2(2)$
$P = 12.0$ cm

$\Delta P = 2\Delta l + 2\Delta b$
$\Delta P = 2(0.1) + 2(0.1)$
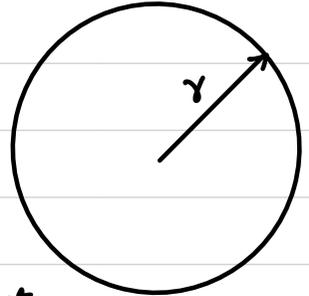$\quad = \pm 0.4$ cm

$$\boxed{P = (12.0 \pm 0.4) \text{ cm}}$$

$y = 3a$
$y = a + a + a$

$\Delta y = \Delta a + \Delta a + \Delta a$

$\Delta y = 3 \Delta a$

let $r = 2.53 \pm 0.02$

find the value of circumference along with its uncertainty.

$C = 2\pi r$

$C = 2\pi r$

$C = 2\pi(2.53)$
$C = 15.9$ cm

$\Delta C = 2\pi \Delta r$
$\quad = 2\pi(0.02)$
$\Delta C = 0.13$ cm

$$\boxed{C = (15.9 \pm 0.1) \text{ cm}}$$

3. If variables are being multiplied or divided

$$y = a \times b \quad \text{or} \quad y = \frac{a}{b}$$

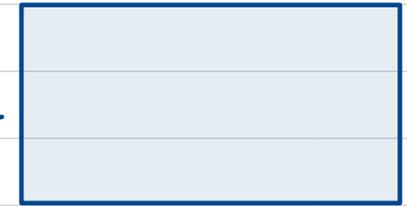$$\boxed{\Delta y\% = \Delta a\% + \Delta b\%}$$

OR

$$\frac{\Delta y}{y} \times 100 = \frac{\Delta a}{a} \times 100 + \frac{\Delta b}{b} \times 100$$

$$\boxed{\frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b}}$$

Example #1

$$b = 3.0 \pm 0.2$$

Find the area along with its uncertainty.

$$\ell = 12.0 \pm 0.1$$

$$A = \ell \times b$$
$$= 12.0 \times 3.0$$
$$= 36.0$$

$$\frac{\Delta A}{A} = \frac{\Delta L}{L} + \frac{\Delta b}{b}$$

$$\frac{\Delta A}{36.0} = \frac{0.1}{12.0} + \frac{0.2}{3.0}$$
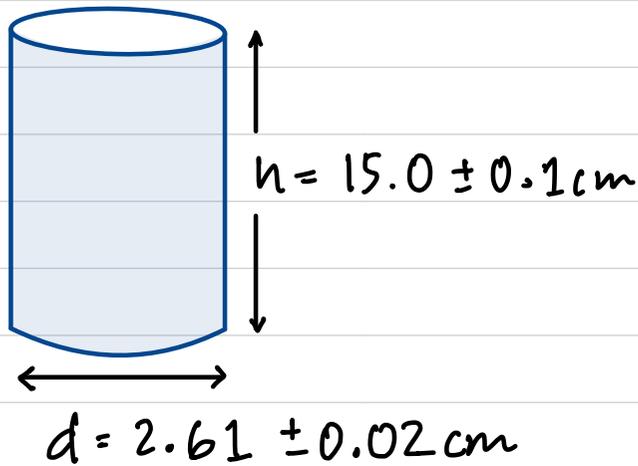
$$\Delta A = 2.7$$

$$\boxed{A = 36.0 \pm 2.7}$$

OR

$$\boxed{A = 36 \pm 3}$$

1st : S.f of principle value

2nd : d.p of uncertainty

Example # 2

Find the surface Area along with its uncertainty

$h = 15.0 \pm 0.1$ cm

$d = 2.61 \pm 0.02$ cm

$A = 2\pi r h$

$r = 0.5 d$
$r = 0.5 (2.61)$
$r = 1.305$

$\Delta r = 0.5 \Delta d$
$\Delta r = 0.5 (0.02)$
$\Delta r = 0.01$

$A = 2\pi (1.305)(15.0)$
$A = 122.99$ or 123 cm^2

$\dfrac{\Delta A}{A} = \dfrac{\Delta r}{r} + \dfrac{\Delta h}{h}$

$\dfrac{\Delta A}{123} = \dfrac{0.01}{1.305} + \dfrac{0.1}{15.0}$

$\Delta A = \pm 1.76$ cm^2

$A = (123 \pm 2)$ cm^2

Example #3

$v = \dfrac{s}{t}$, where v: velocity s: displ. t: time

Given that s: 53.6 ± 0.1 cm

$\qquad\qquad\quad t$: 2.53 ± 0.02 s

Calculate the value of "v" along with its uncertainity.

$v = \dfrac{s}{t}$ so $v = \dfrac{53.6}{2.53}$; $v = 21.2$ cms⁻¹

$\dfrac{\Delta v}{v} = \dfrac{\Delta s}{s} + \dfrac{\Delta t}{t}$ hence $v = 21.2 \pm 0.2$

$\dfrac{\Delta v}{21.2} = \dfrac{0.1}{53.6} + \dfrac{0.02}{2.53}$

$\Delta v = \pm 0.21$

4. If a variable has a power other than 1

$$y = a^n \quad \text{so} \quad \boxed{\Delta y\% = n\,\Delta a\%}$$

$$y = a^3$$

OR

$$\boxed{\frac{\Delta y}{y} = n\,\frac{\Delta a}{a}}$$

- $\Delta y\% = 3\,\Delta a\%$
- $\dfrac{\Delta y}{y} = 3\,\dfrac{\Delta a}{a}$

$$y = a \cdot a \cdot a$$
$$\Delta y\% = \Delta a\% + \Delta a\% + \Delta a\%$$
$$\Delta y\% = 3\,\Delta a\%$$
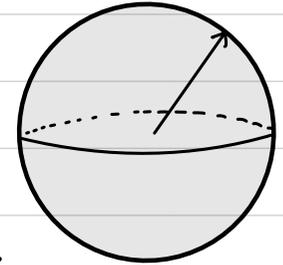
Example #1

$$r = 2.61 \pm 0.01 \text{ mm}$$

Calculate the volume of sphere along with its uncertainty.

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi (2.61)^3$$

$$\boxed{V = 74.5 \text{ mm}^3}$$

$$\frac{\Delta V}{V} = 3\,\frac{\Delta r}{r}$$

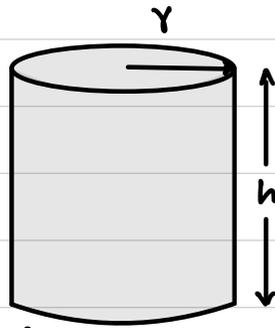$$\frac{\Delta V}{74.5} = 3\left(\frac{0.01}{2.61}\right)$$

$$\Delta V = 0.86 \text{ mm}^3$$

$$\boxed{V = 74.5 \pm 0.9 \text{ mm}^3}$$

Example #2 (Combining Rules)

$r = 4.63 \pm 0.01$ cm

$h = 25.6 \pm 0.2$ cm

Calculate the volume of the cylinder along with its

i. absolute uncertainty

ii. percentage uncertainty

$$V = \pi r^2 h$$

$$V = \pi (4.63)^2 (25.6)$$

$$V = 1724.06 \text{ cm}^3$$

i. $\dfrac{\Delta V}{V} = 2\dfrac{\Delta r}{r} + \dfrac{\Delta h}{h}$

$$\frac{\Delta V}{1724.06} = 2\left(\frac{0.01}{4.63}\right) + \left(\frac{0.2}{25.6}\right)$$

$$\Delta V = 20.91 \text{ cm}^3$$

$$\boxed{V = 1720 \pm 21} \quad \text{or} \quad \boxed{1720 \pm 20}$$

ii. $\Delta V\% = 2\Delta r\% + \Delta h\%$

$$\Delta V\% = \left\{ 2\left(\frac{0.01}{4.63}\right) + \left(\frac{0.2}{25.}\right) \right\} \times 100$$

$$\Delta V\% = 1.21\% \quad \text{or} \quad \boxed{1.2\%}$$