SUPERPOSITION OF WAVES

8 Superposition

8.1 Stationary waves

Candidates should be able to:

1. explain and use the principle of superposition
2. show an understanding of experiments that demonstrate stationary waves using microwaves, stretched strings and air columns (it will be assumed that end corrections are negligible; knowledge of the concept of end corrections is not required)
3. explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes
4. understand how wavelength may be determined from the positions of nodes or antinodes of a stationary wave

8.2 Diffraction

Candidates should be able to:

1. explain the meaning of the term diffraction
2. show an understanding of experiments that demonstrate diffraction including the qualitative effect of the gap width relative to the wavelength of the wave; for example diffraction of water waves in a ripple tank

8.3 Interference

Candidates should be able to:

1. understand the terms interference and coherence
2. show an understanding of experiments that demonstrate two-source interference using water waves in a ripple tank, sound, light and microwaves
3. understand the conditions required if two-source interference fringes are to be observed
4. recall and use $\lambda = ax / D$ for double-slit interference using light

8.4 The diffraction grating

Candidates should be able to:

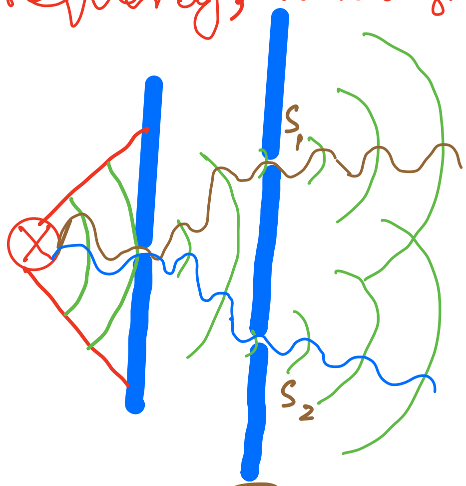
1. recall and use $d \sin \theta = n\lambda$
2. describe the use of a diffraction grating to determine the wavelength of light (the structure and use of the spectrometer are not included)

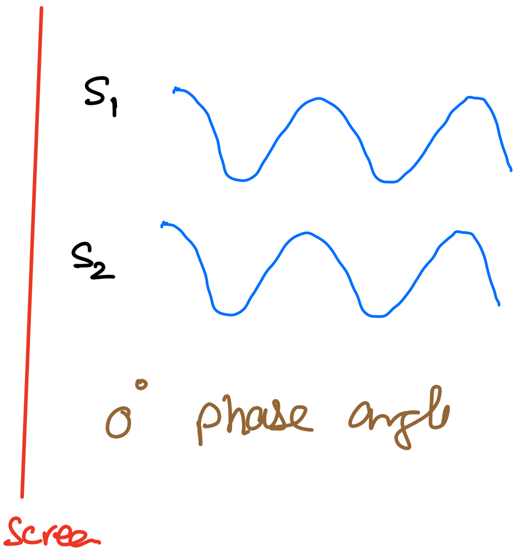
Coherent Sources :-

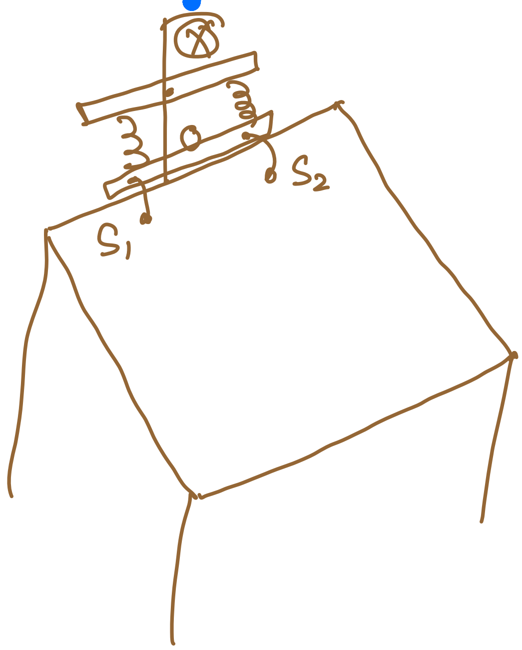
together   Inheritance

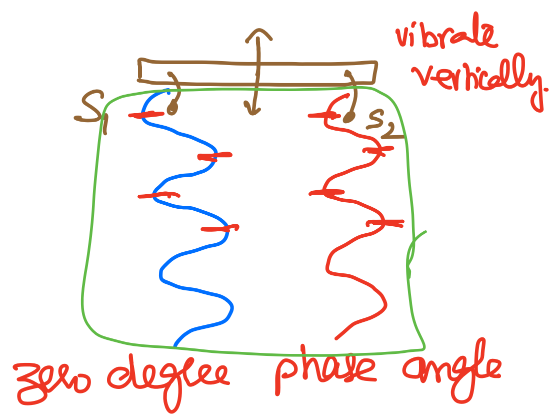
Def Sources which emit waves having constant phase angle in between them.

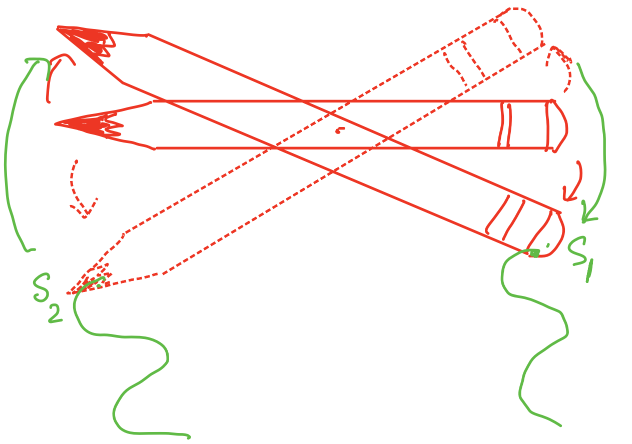
Coherent sources are derived from single source and emit waves having same time period, frequency, wavelength and speed.

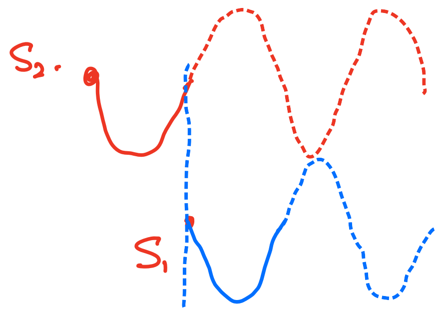










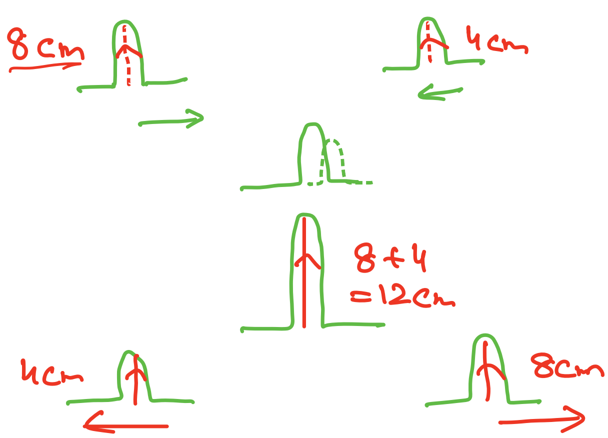


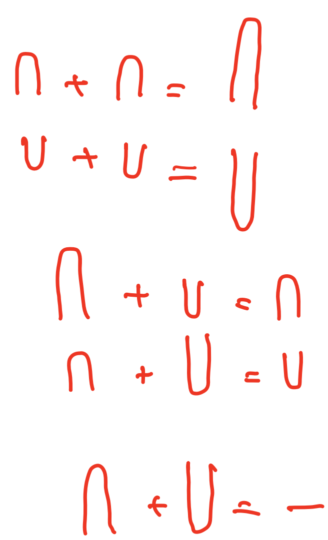
Phase angle of 180°

So both S_1 and S_2 are coherent sources.

Superposition Principle:

Statement: When two or more waves ~~interfere~~ meet/overlap at a point, the resultant wave has ~~amplitude~~ displacement which is equal to the vector sum of their individual ~~amplitudes~~ displacements.





Interference of waves :-

Def. Superposition of waves from coherent sources which support each other at one point and cancel out the effect of each other at another point is interference.

(annotations: "∩+∩ or ∪+∪" above "each other"; "∩+∪" above "cancel")

Conditions:

1. Waves must meet at a point.
2. Waves must be from coherent sources ie they must have constant phase angle inbetween them.
3. Waves must be of same type.
4. If transverse, then both must be polarised in the same plane.

Types of interference:-

- Constructive interference ∩ on ∩ or ∪ on ∪
- Destructive interference ∩ on ∪

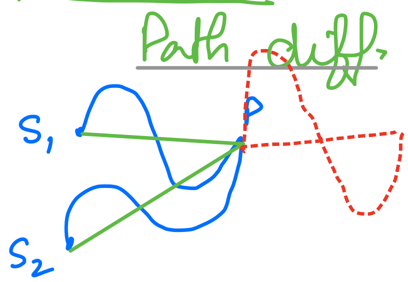
Path difference:- This is the difference of distances travelled by both waves from their sources to their meeting point. path difference is represented in terms of wavelength.

Phase difference (ϕ): $\phi = \left(\frac{\text{Path diff}}{\text{wavelength}}\right) 2\pi$

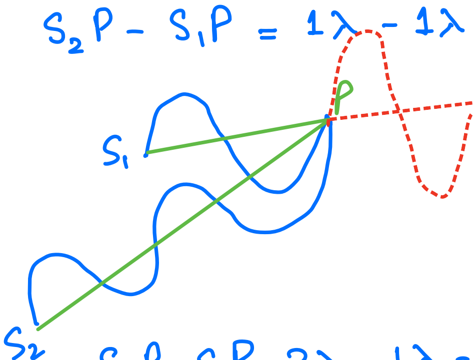
Constructive interference:- Superposition of coherent waves which support each other to provide a resultant wave with maximum displacements. ie $\cap + \cap$ or $\cup + \cup$

Conditions

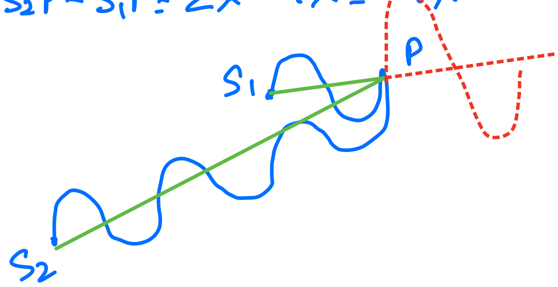
Path diff.



$S_2P - S_1P = 1\lambda - 1\lambda = 0\lambda$



$S_2P - S_1P = 2\lambda - 1\lambda = 1\lambda$



$S_2P - S_1P = 3\lambda - 1\lambda$
$= 2\lambda$

Phase diff.

$\phi = \left(\frac{0\lambda}{\lambda}\right)2\pi$
$= 0\pi$

$\phi = \left(\frac{1\lambda}{\lambda}\right)2\pi$
$= 2\pi$

$\phi = \left(\frac{2\lambda}{\lambda}\right)2\pi$
$= 4\pi$

In general, for constructive interference, (if waves from sources have 0° phase angle)

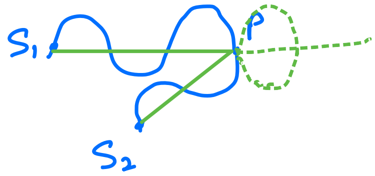
Path difference = $0\lambda, 1\lambda, 2\lambda, \ldots\ldots n\lambda$
= integral multiple of wavelength

Phase difference = $0\pi, 2\pi, 4\pi, \ldots\ldots 2n\pi$
= even multiple of π

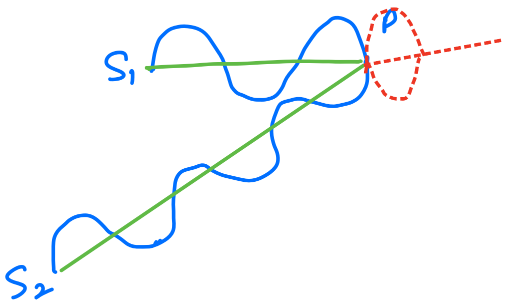
Destructive interference: Superposition of coherent waves which cancel out each others effect to provide a resultant wave with minimum or zero displacement.

Conditions

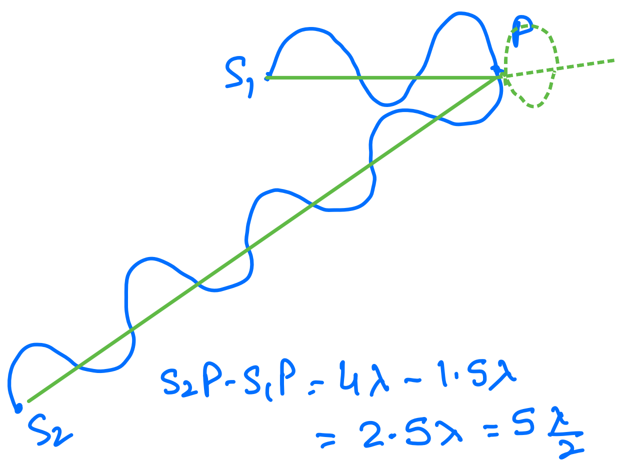
Path difference



$S_1P - S_2P = 1.5\lambda - 1\lambda$
$= 1\frac{\lambda}{2}$



$S_2P - S_1P = 3\lambda - 1.5\lambda = 1.5\lambda$
$= 3\frac{\lambda}{2}$



$S_2P - S_1P = 4\lambda - 1.5\lambda$
$= 2.5\lambda = 5\frac{\lambda}{2}$

Phase difference

$\phi = \left(\frac{1\frac{\lambda}{2}}{\lambda}\right)2\pi$
$= \pi$

$\phi = \left(\frac{3\frac{\lambda}{2}}{\lambda}\right)2\pi$
$= 3\pi$

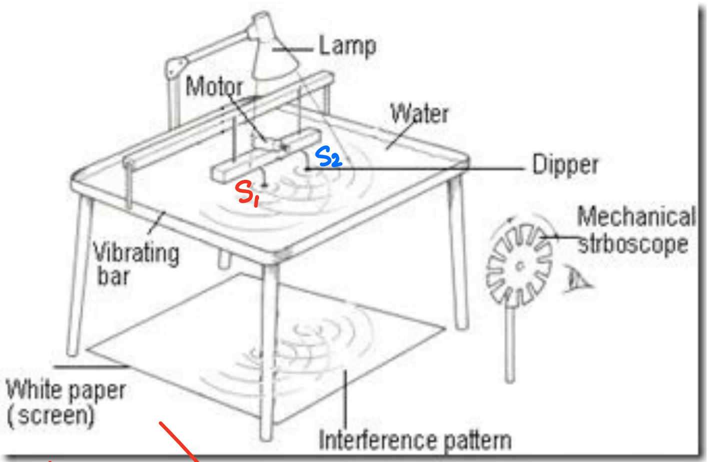
$\phi = \left(\frac{5\frac{\lambda}{2}}{\lambda}\right)2\pi$
$= 5\pi$

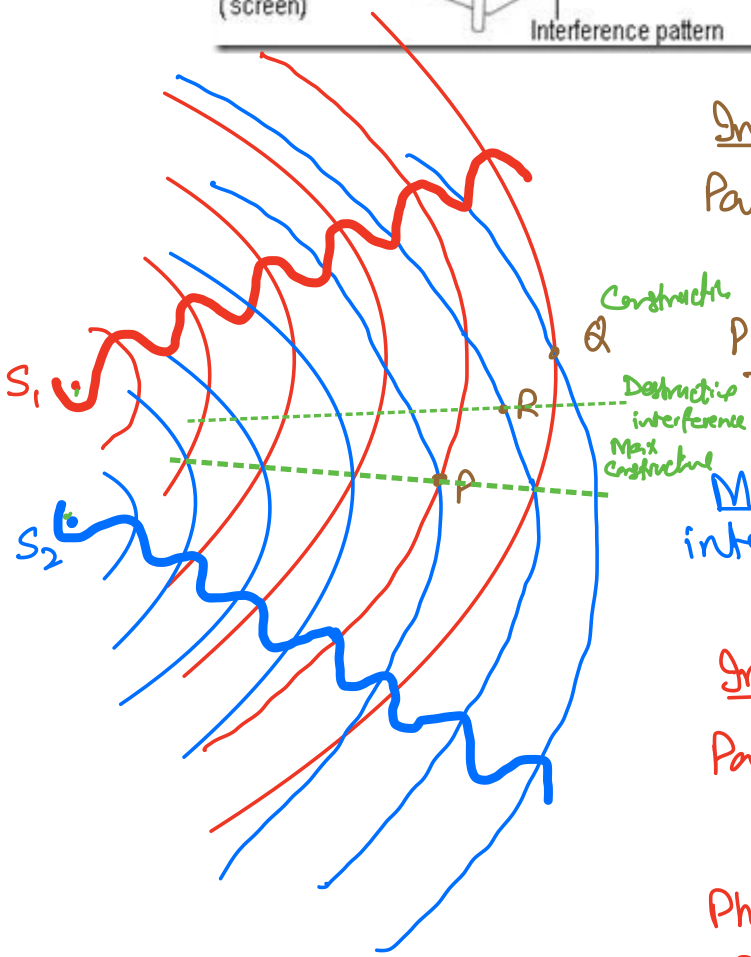
In general, for Destructive interference, (if waves from sources have 0° phase angle)
Path diff $= 1\frac{\lambda}{2}, 3\frac{\lambda}{2}, 5\frac{\lambda}{2}, \ldots\ldots (2n+1)\frac{\lambda}{2}$
= odd multiple of half wavelength

Phase difference = $1\pi, 3\pi, 5\pi, \ldots\ldots (2n+1)\pi$
= odd multiple of π

Note: The conditions of path and phase difference for constructive and Destructive interferences alters if phase angle from sources S_1 and S_2 becomes 180°.

INTERFERENCE OF WATER WAVES:-





Interference at P:

Path diff. = $S_2P - S_1P$
$= 5\lambda - 5\lambda = 0\lambda$

Phase diff.
$\phi = \left(\frac{0\lambda}{\lambda}\right) 2\pi = 0\pi$

Maximum constructive interference occur at P

Interference at Q:

Path diff. = $S_2Q - S_1Q$
$= 7\lambda - 6\lambda = 1\lambda$

Phase diff = $\left(\frac{1\lambda}{\lambda}\right)(2\pi) = 2\pi$

Both waves meet constructively at Q.

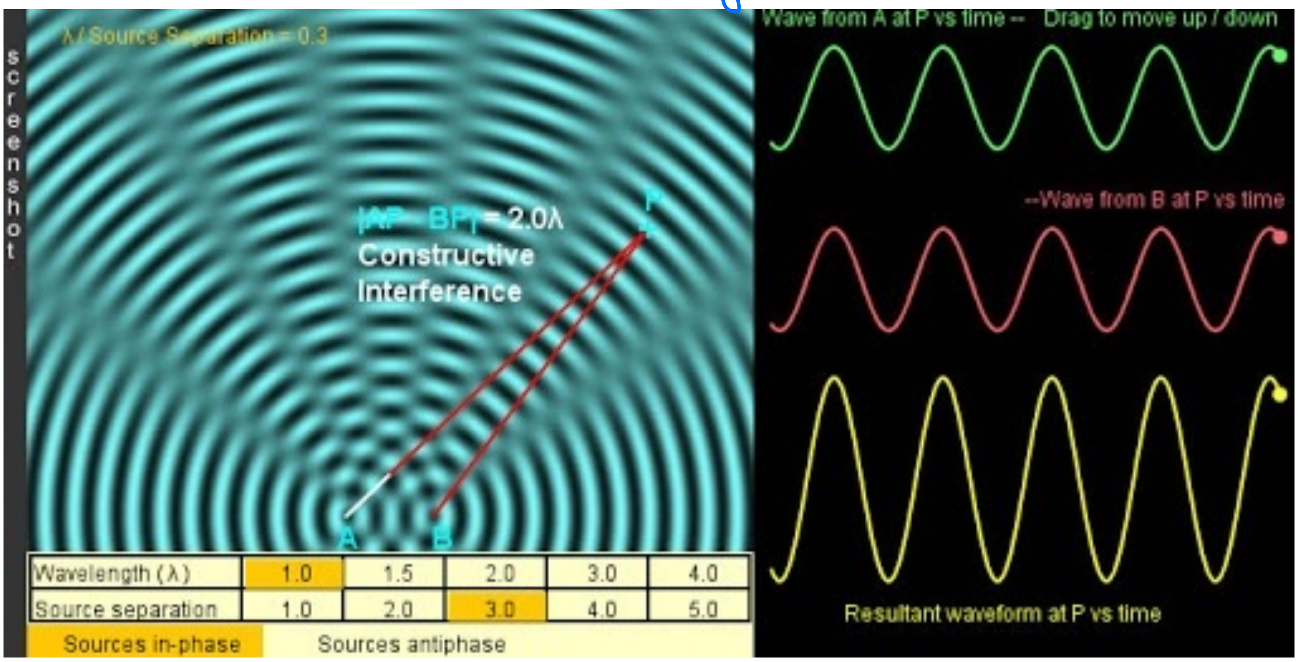
Interference at R:

Path diff $= S_2R - S_1R$

$= 6\lambda - 5.5\lambda = 1\frac{\lambda}{2}$

Phase diff $= \left(\frac{1\frac{\lambda}{2}}{\lambda}\right)2\pi = \pi$

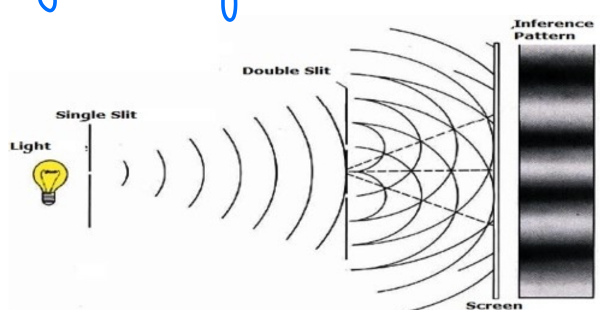
waves meet destructively at R.

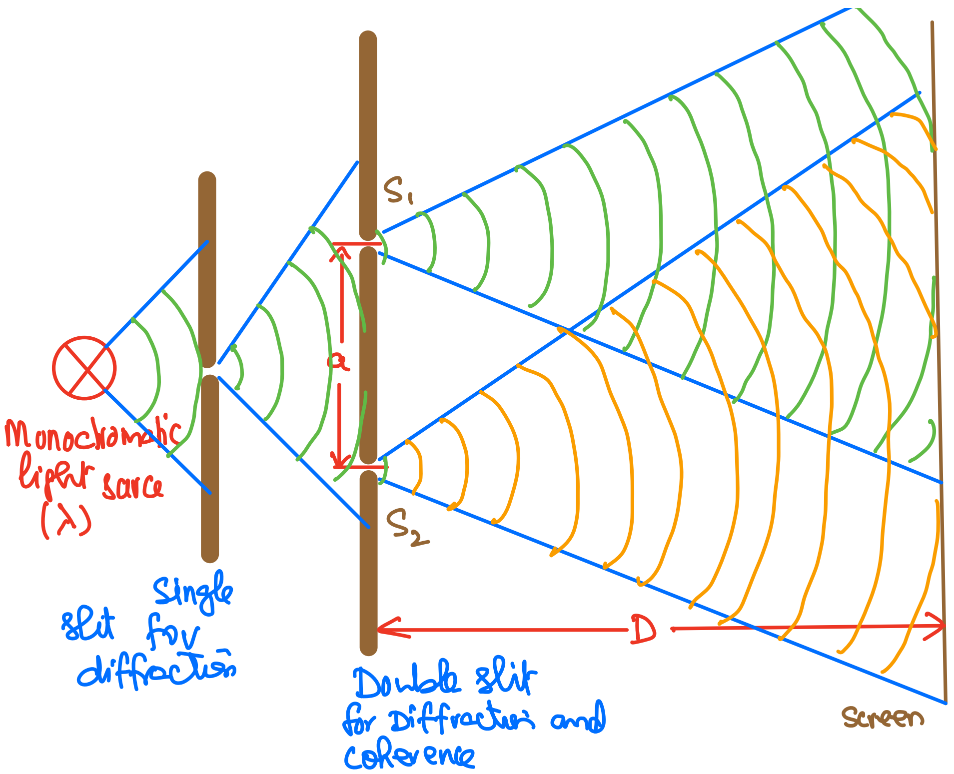


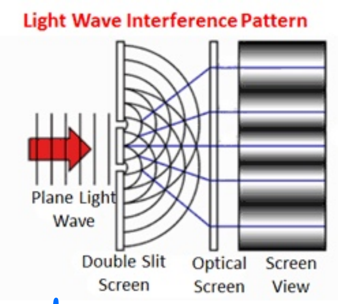
Interference of Light waves (Young Double slits experiment)

Significance:

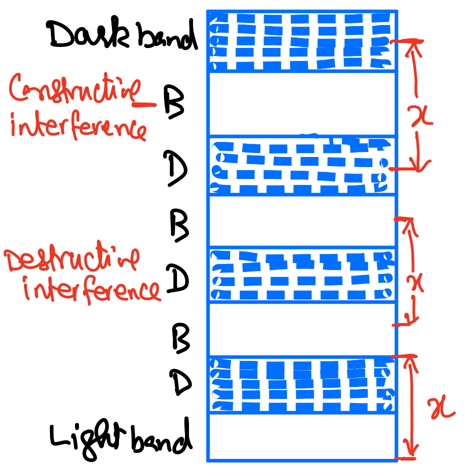
This experiment is used to determine wavelength of monochromatic light source.







Observation: Alternate bright and dark bands also known as interference fringes are observed on the screen due to constructive and destructive interference of light waves.



Fringe spacing: Distance between two adjacent bright or dark fringes.

Symbol: x

Formula: $x = \frac{\lambda D}{a}$

Here λ - Wavelength of Monochromatic light

D - Distance b/w double slits and screen

a - Distance b/w double slits (0.3mm to 3mm)

Appearences of fringes

- Fringe spacing
- Contract

Fringe Spacing : $x\uparrow = \dfrac{\lambda\uparrow D\uparrow}{a\downarrow}$

Fringe spacing incleases if

(i) $\lambda\uparrow$ i.e. replace light source with one having larger wavelength.

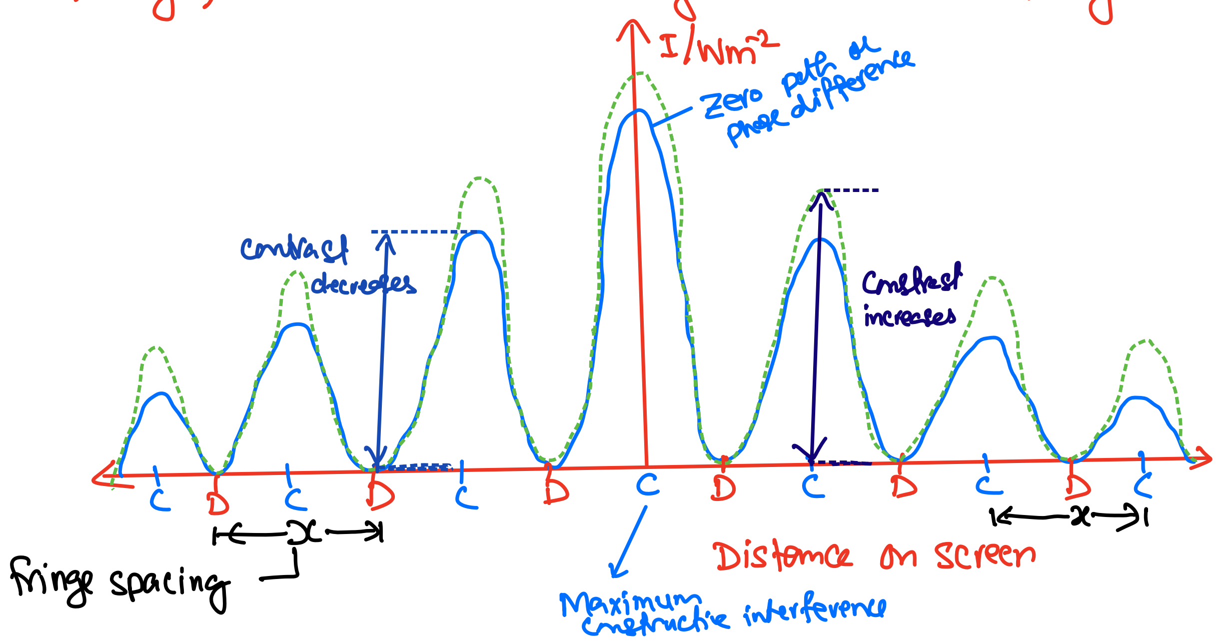
(ii) $D\uparrow$ i.e Distance between double slits and screen is increased.

(iii) $a\downarrow$ i.e. Separation b/w double slits is decreased.

Contrast :

Difference between black and white part of fringes.

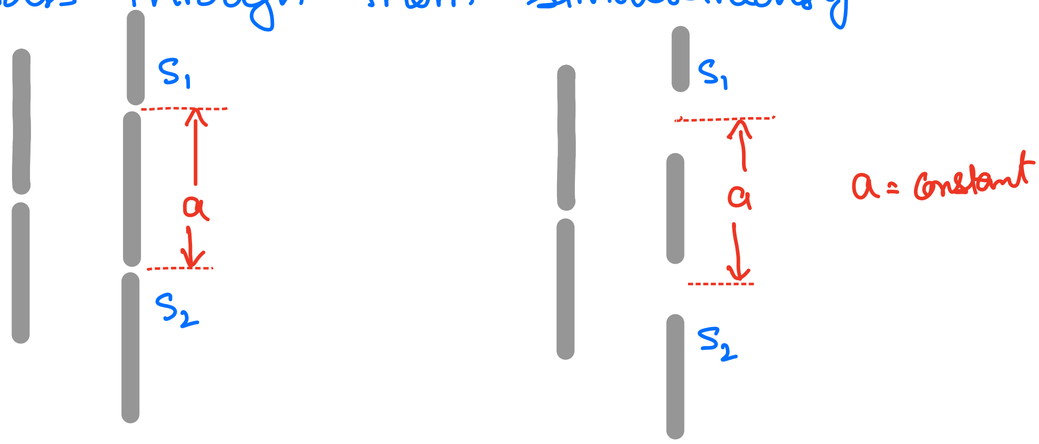
Dependance : (i) (Contrast)$\uparrow$ if (Brightness of bright fringe)$\uparrow$ with no change in dark fringe.



(a) Increase the intensity of light source by increasing its power rating.

(b) Decrease the distance between light source and screen.

(c) Increase the width of each slit of double slits so that greater waves can pass through them simultaneously.



(d) Decrease the wavelength of monochromatic light source. i.e.

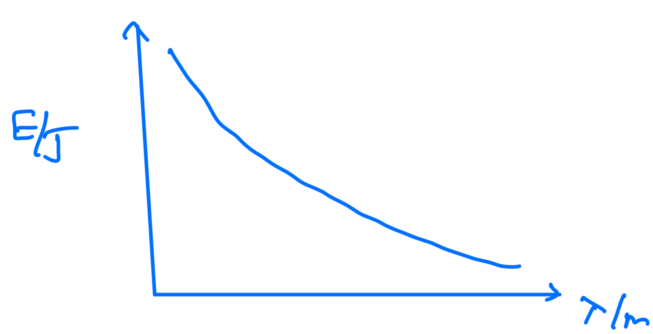
Energy of e.m. wave: $E = hf$

But $v = f\lambda \Rightarrow f = \frac{c}{\lambda}$

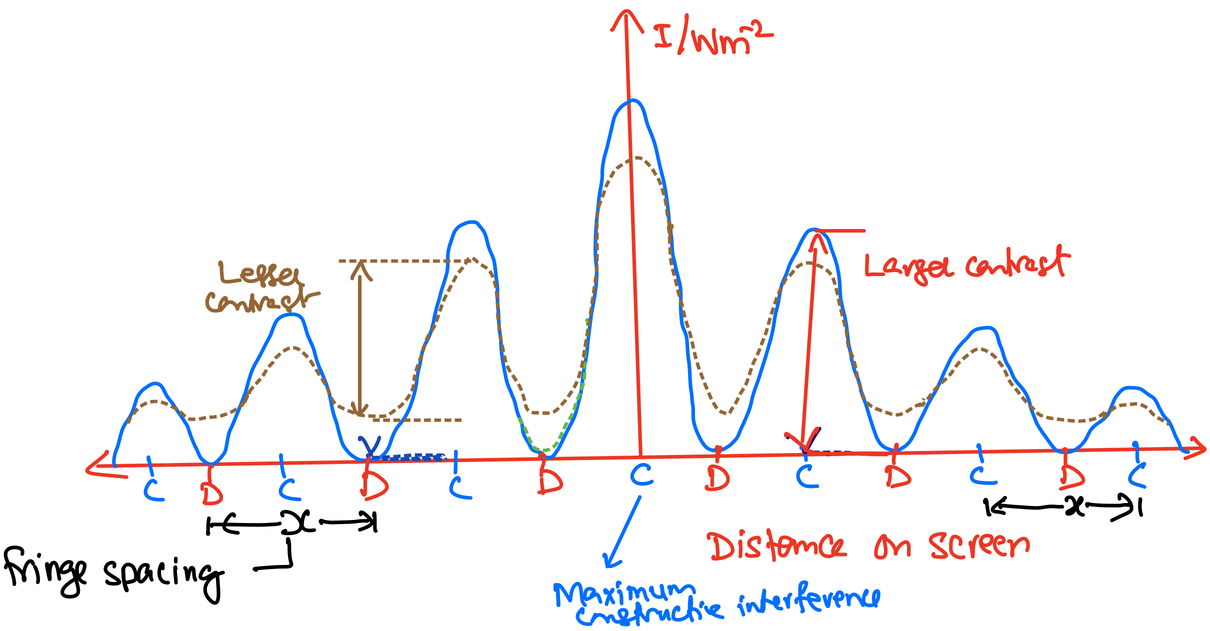
For e.m. waves $v = c = 3.00 \times 10^{8}\,ms^{-1}$

$E = \frac{hc}{\lambda} \Rightarrow E = \frac{Constant}{\lambda} \Rightarrow E \propto \frac{1}{\lambda}$

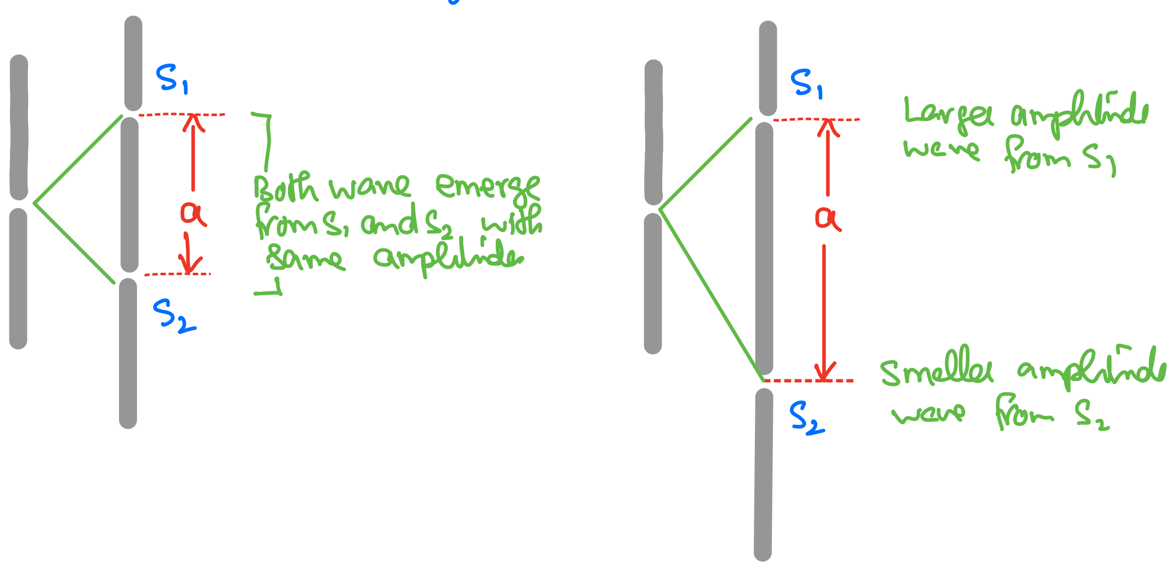
h - Planck's Constant
$= 6.63 \times 10^{-34}\,Js$

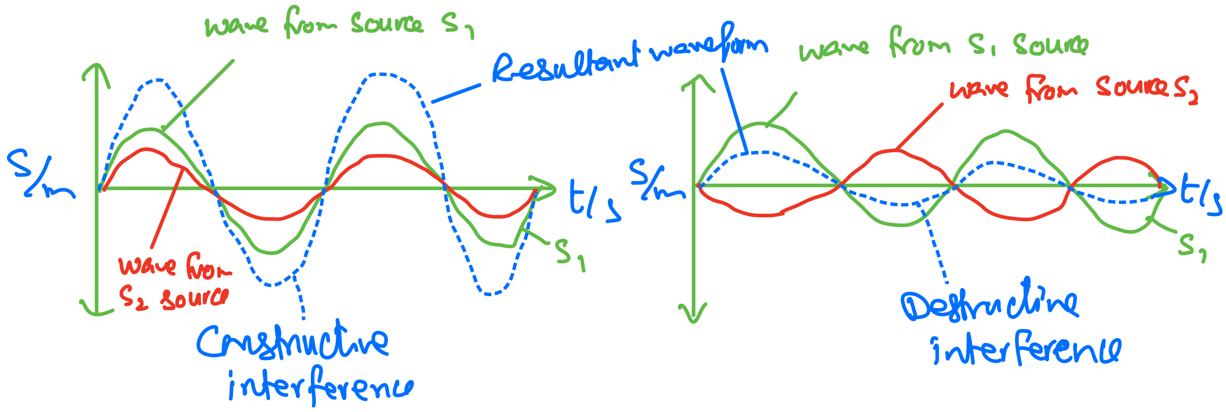


(ii) (Contrast)↓ if (Brightness of bright fringe)↓ and dark fringe becomes a little brighter.



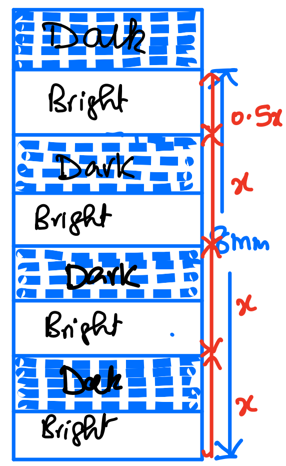
This happens if intensity of light incident on one slit of double slits is decreased.





Q) What should be the wavelength of wave if this pattern is observed at 4 cm away from double slits with separation of 0.6 mm.

D = 4 cm, a = 0.6 mm

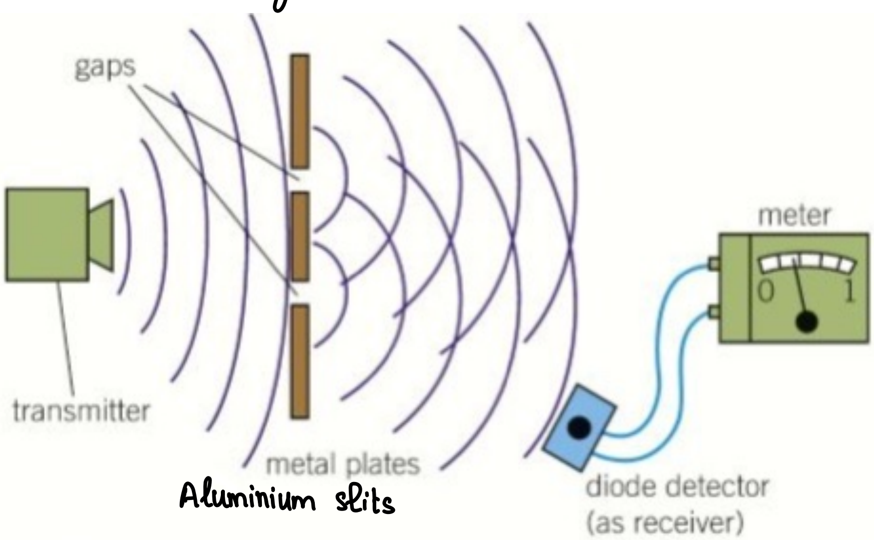


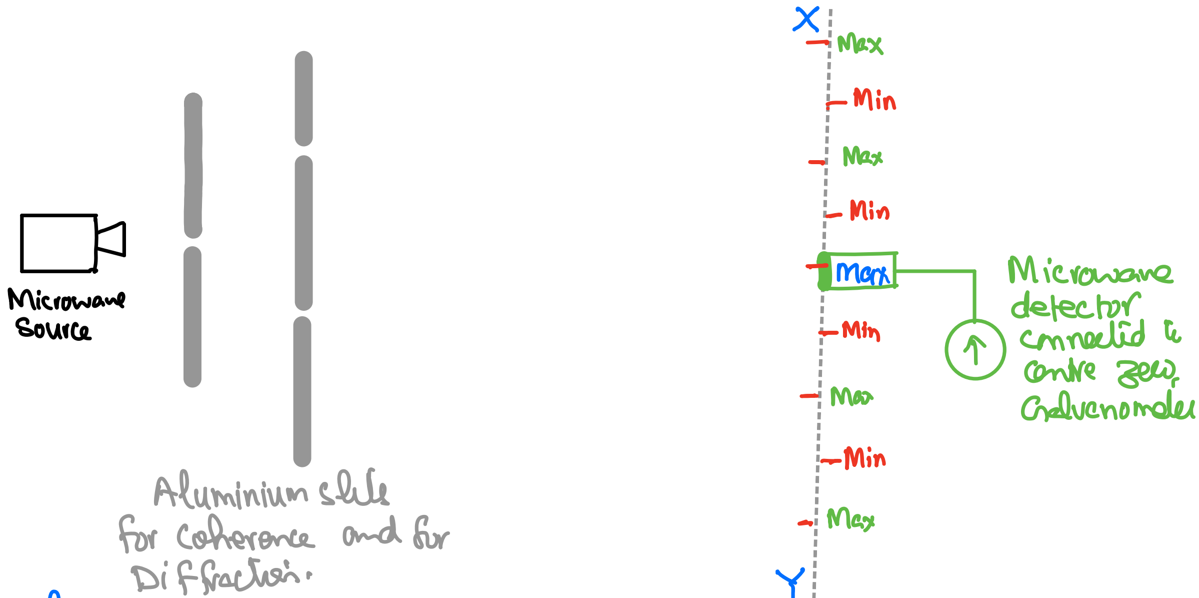
$$3.5x = 8 \Rightarrow x = \frac{8}{3.5} = 2.29\,mm$$

$$x = \frac{\lambda D}{a}$$

$$2.29 \times 10^{-3} = \frac{(\lambda)(4 \times 10^{-2})}{0.6 \times 10^{-3}} \Rightarrow \lambda = \quad \times 10 \quad m$$

Interference of Microwaves:-





Aluminium absorbs microwaves incident on it.

Observation:- When microwave detector connected to centre zero Galvanometer is moved along XY, alternate regions of maxima and minima are obtained due to constructive and Destructive interference of microwaves from coherent Aluminium slits.

Result: Microwaves also exhibit interference property.

Diffraction:

Def.: Spreading of waves after passing through a small aperture/gap or round an obstacle is diffraction.

Note:

1- There is no change in frequency, wavelength or speed of incident or diffracted wave but velocity changes due to change in the direction of motion.

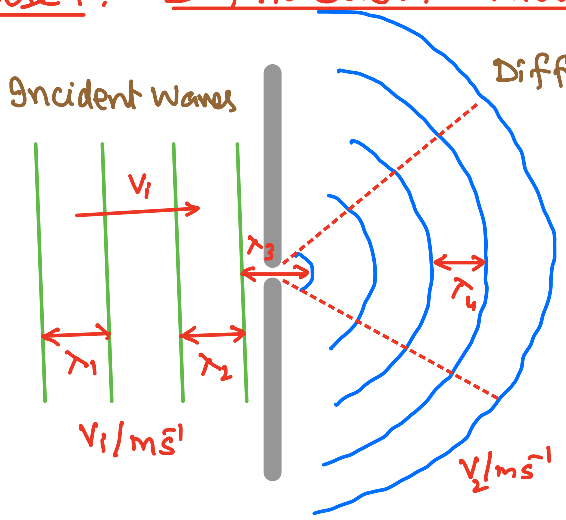
2- (Diffraction) ↑ i.e larger spreading if (wavelength of wave) ≥ (size of aperture)

3- (Diffraction) ↓ i.e smaller spreading if (wavelength of wave) < (size of aperture)

4- Shadow formation is also due to diffraction of waves round an obstacle.

Diffraction patterns:-

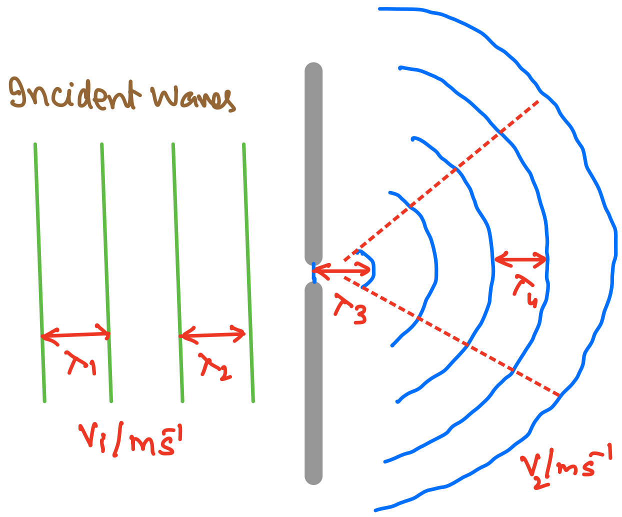
Case 1: Diffraction through a narrow gap/aperture:



(i) $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$

(ii) speed $v_1 = v_2$

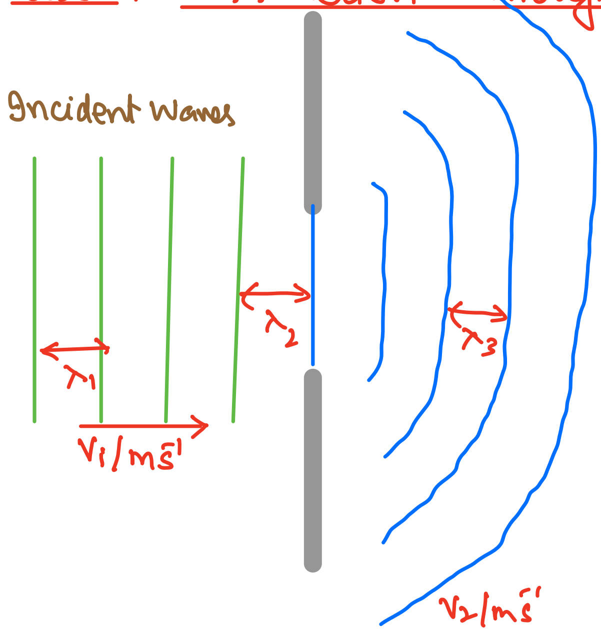
Diffracted waves are circular



$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$

diffracted waves are circular

Case 2: Diffraction through a wide gap:-

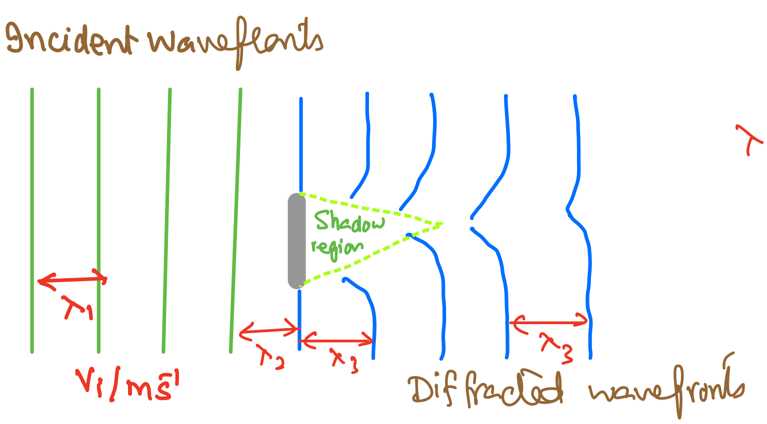


(i) $\lambda_1 = \lambda_2 = \lambda_3$

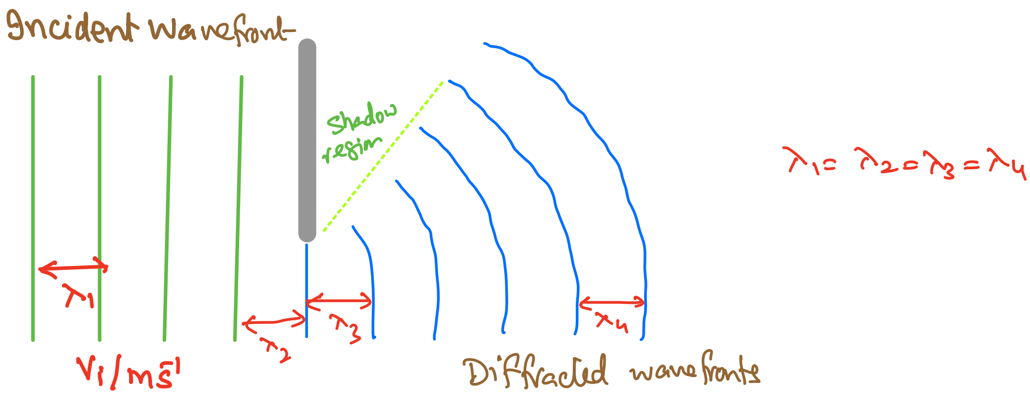
(ii) speed $v_1 = v_2$

(iii) Diffracted waves are straight from the middle and curved from edges.

Case 3: Diffraction round an obstacle



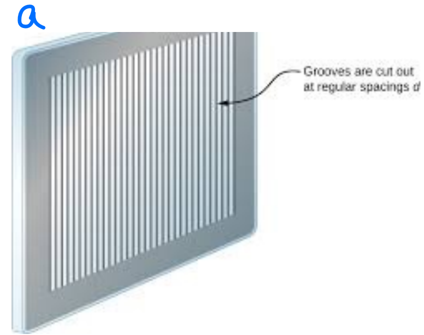
$\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4$

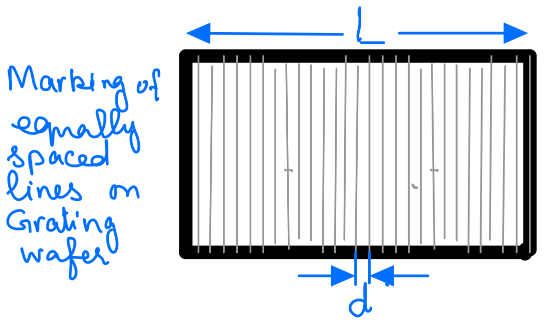


Case 4: Diffraction of microwaves / radio waves round a hill top:-

Diffraction Grating:-

Def. A slice or wafer made of plastic, glass or metal having 5000 to 6000 lines per inch ruled on it. Each line behave like a slit and diffract incident waves to provide a pattern on screen called diffraction pattern.





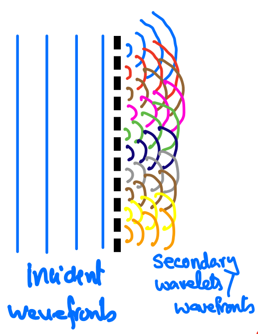
Line spacing = $\frac{\text{Length of grating wafer}}{\text{no. of lines ruled on it}}$

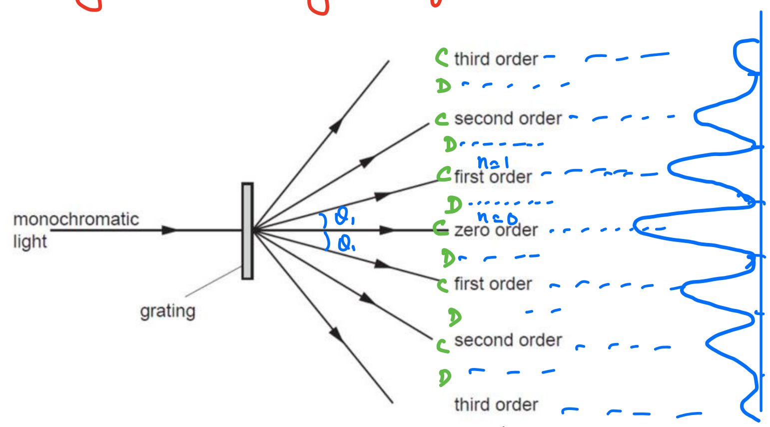
$$d = \frac{L}{N}$$

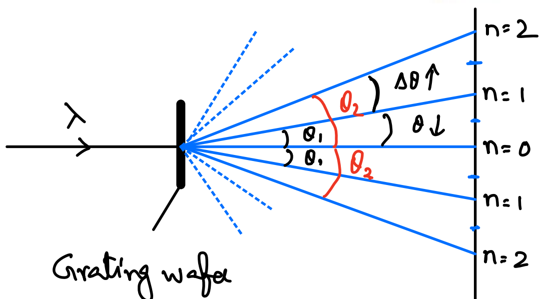
650 lines/mm

$d = \frac{10^{-3}}{650} = \_\_\_\_ m$

Diffraction through Grating wafer:







Monochromatic light is diffracted through lines of wafer and further meet on a screen to provide alternate bright and dark bands with multiple orders of diffraction (n).

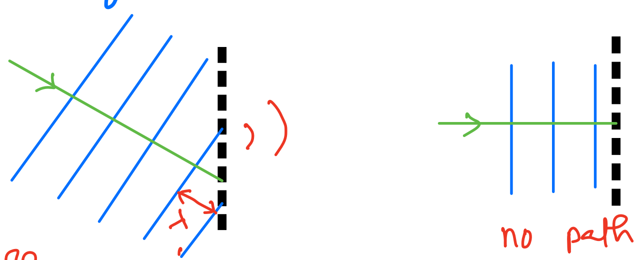
Formula: $n\lambda = d \sin\theta$

Here

- n – order of diffraction
- λ – wavelength of monochromatic light
- $d = \frac{L}{N} = \frac{\text{Length of Grating}}{\text{no. of lines on it}}$
- θ – Angle of diffraction of any order with zero order diffracted beam.

Condition for $n\lambda = d\sin\theta$:-

There must be no path difference b/w waves incident on Grating wafer. i.e incident waves are perpendicular to the plane of Grating wafer



Angle for maximum diffraction:

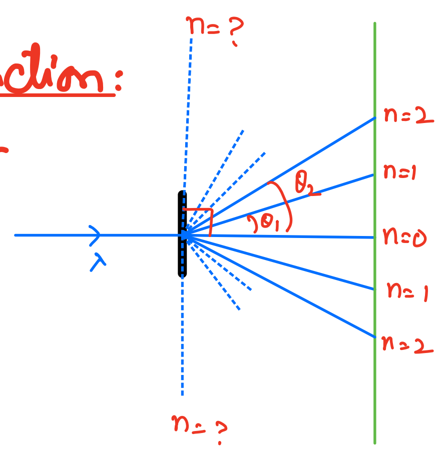
Maximum order of diffraction:-

Diffraction is maximum if $\theta = 90°$

$$n\lambda = d \sin 90°$$

$$n\lambda = d(1)$$

$$\boxed{n = \frac{d}{\lambda}}$$



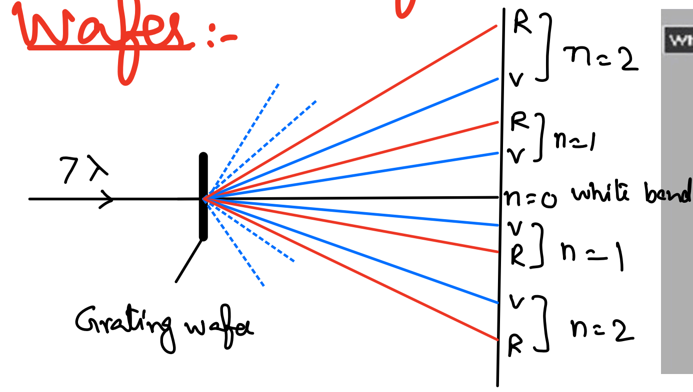
Total orders of diffraction:

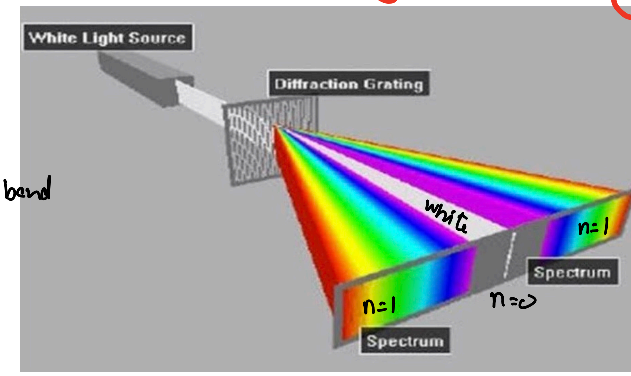
Total orders of diffraction = orders at one side + orders at other side + zero order

$$= n + n + 1$$

$$= (2n+1)$$

Diffraction of visible light through Grating wafer :-





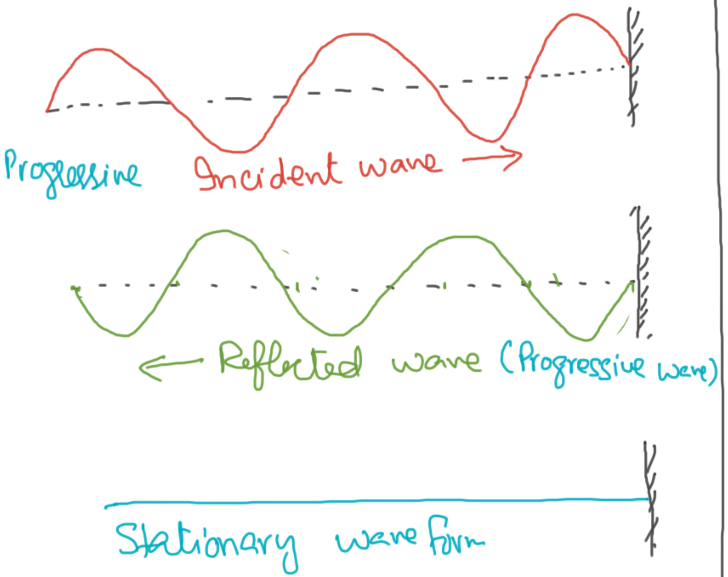
Since white light is composed of seven component colours and each colour has its definite wavelength but the line spacing on Grating wafer is constant. So each colour is diffracted at different angle to provide visible spectrum.

Note :

1. We always get a white band for zero order diffracted beam and a visible spectrum for all other orders of diffraction.
2. The angle of diffraction for red colour is greatest and of violet is least for any order of diffraction.

STANDING OR STATIONARY WAVES :-

Concept :



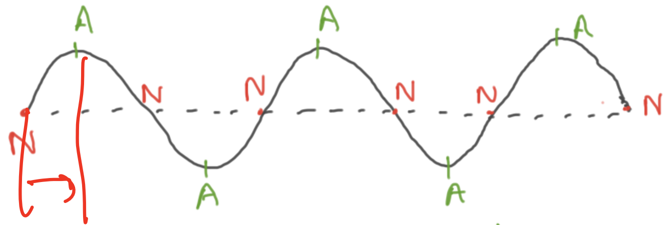
Def. When incident and reflected waves having same speed (frequency and wavelength) but travelling in opposite directions are superposed, stationary waves are formed.

Note:

1) The particle on stationary waves whose ~~displacement~~ amplitude remain zero is called Node.

2) The particle on stationary wave whose amplitude is

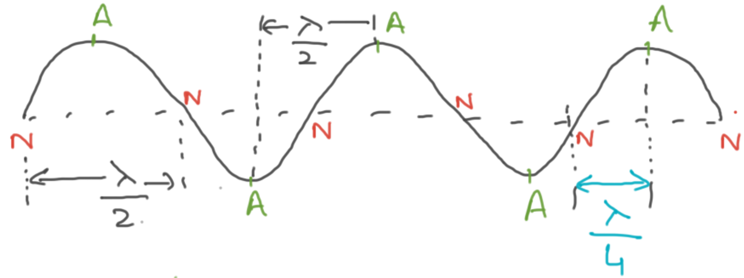
maximum is called 'anti-node'.



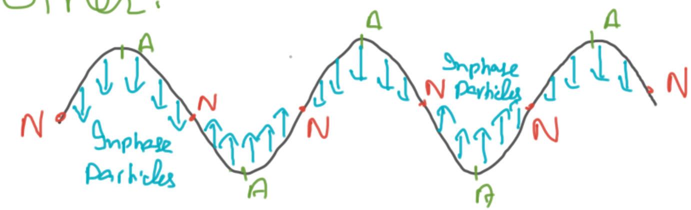
A- Antinode
N- Node

3) The distance between two adjacent nodes or two adjacent anti-nodes is equal to half of wavelength ($\frac{\lambda}{2}$).

4) The distance between a node and its adjacent antinode is quarter of wavelength ($\frac{\lambda}{4}$)



5) All particles between two adjacent nodes move in the same direction and therefore in phase with each other.

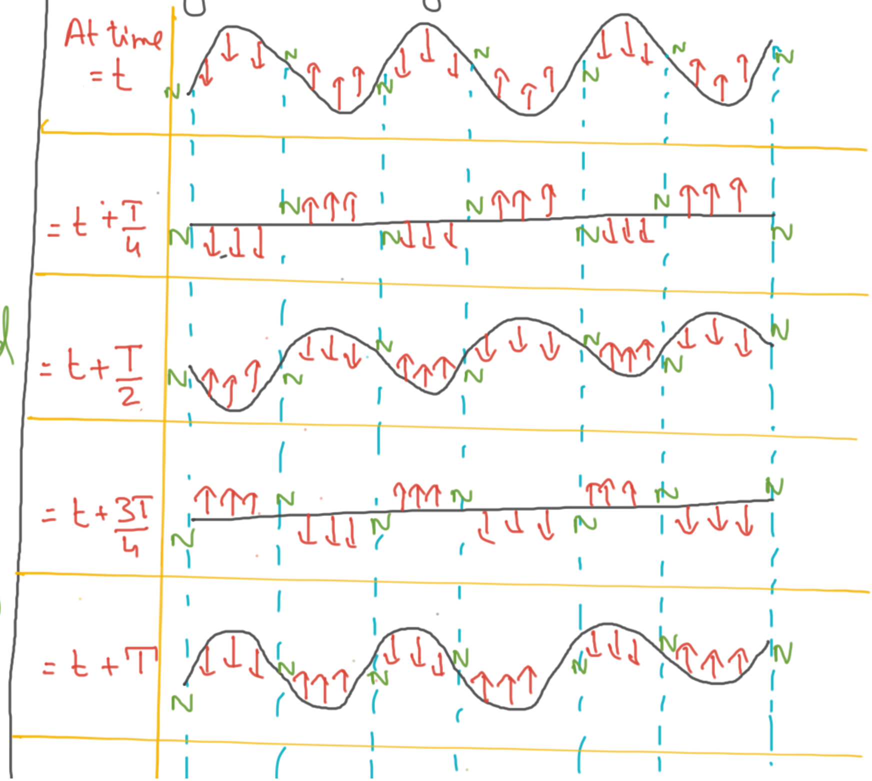


6) Particles on either side of node move in opposite directions and therefore 180° out of phase with each other.

7) No energy is transferred along the wave due to static position of nodes.

8) The speed of stationary wave is same as that of superposing waves

Formation of stationary waves along a string at multiple times



Comparison of stationary and progressive waves

Property	Stationary Wave	Progressive Wave
Energy & Momentum	No net transfer from one point to another	Both move with speed: $c = f \times \lambda$
Amplitude	Varies from zero at NODES to a maximum at ANTINODES	Is the same for all particles within a wave
Frequency	All particles oscillate at the same frequency except those at nodes	All particles oscillate at the same frequency
Wavelength	This is equal to TWICE the distance between adjacent nodes	This is equal to the distance between particles at the same phase
Phase difference between two particles	Between nodes all particles are at the same phase. Any other two particles have phase difference equal to '$m\pi$' where 'm' is the number of nodes between the particles	Any two particles have phase difference equal to '$2\pi d / \lambda$' where 'd' is the distance between the two particles

Stationary waves along a stretched string (Guitar or Sitar string)

S.No.	Mode	Waveform	Relationship b/w wavelength and Length of string	Frequency.
1.	—		—	—
2.	Fundamental mode or First harmonic		$L = \frac{\lambda_0}{2}$ $\lambda_0 = 2L$	$f_0 = \frac{v}{\lambda_0}$ $f_0 = \frac{v}{2L}$
3	2[nd] harmonic Fix String from the middle during motion to make it a node.		$L = \lambda_1$	$f_1 = \frac{v}{\lambda_1} = \frac{v}{L}$ $f_1 = 2f_0$
4	3[rd] harmonic Fix String from two points along its length apart from end points		$L = \frac{3\lambda_2}{2}$ $\lambda_2 = \frac{2L}{3}$	$f_2 = \frac{v}{\lambda_2} = \frac{v}{\frac{2L}{3}}$ $f_2 = 3\left(\frac{v}{2L}\right) = 3f_0$

In general, along a stretched string, stationary transverse waves of frequency f_0, $2f_0$, $3f_0$, $4f_0$, ------- nf_0 are produced. where $f_0 = \frac{v}{2L}$ and n reprents no. of loops along the wave.

Standing Waves for Five Resonant Frequencies

Appearance of the Standing Wave	Mode of Vibration	Number of Loops	Wavelength(λ) in Terms of L ($\lambda = 2L/n$)	Frequency (f)
	Fundamental First Harmonic n=1	1	2L	(f_0) Minimum frequency
	First Overtone Second Harmonic n=2	2	L	$2f_0$
	Second Overtone Third Harmonic n=3	3	2/3L	$3f_0$
	Third Overtone Fourth Harmonic n=4	4	1/2L	$4f_0$
	Fourth Overtone Fifth Harmonic n=5	5	2/5L	$5f_0$

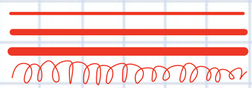
Factors that determine the fundamental frequency of a vibrating string

- The frequency of vibration depends on
  - the mass per unit length of the string,
  - the tension in the string and,
  - the length of the string.
- The fundamental frequency is given by

$$f_o = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

where T = tension
μ = mass per unit length
L = length of string

$$f_0 \propto \sqrt{\frac{T}{\mu}}$$



Stationary waves in a closed and an open pipe:-

Closed pipe i.e Mouth organ

Identify: Pipe which is closed from one end and exposed to external atmosphere from the other end is closed pipe.

Note:

1) If air is blown at or side of open end, stationary longitudinal waves are

Open pipe i.e flute

Identify: Pipe which is exposed to external atmosphere from both ends is open pipe.

Note:

1) When air is blown at one end, stationary longitudinal waves are produced along the pipe.

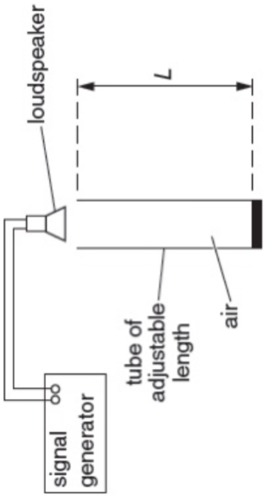
produced along the pipe.

(2) We always get a node at the closed because compression of one waveform meet with the rarefaction of other.

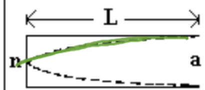
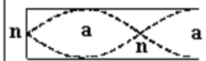
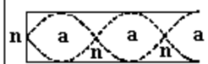
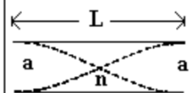
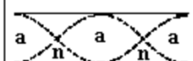
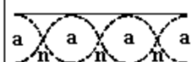
(3) We always get an antinode at the open end because here pressure is either greater than or less than atmospheric pressure.

(2) We alwy get an antinodes at open ends due to diffrence of either of pressure of compression or of rarefaction in comparison to external constant atmospheric pressure.

(3) We always get a node between two adjacent anti-nodes.



26.3.01 Standing waves in air columns

	Harmonic	Frequency	Standing wave	Wavelength
Air column closed at one end	1st harmonic (fundamental)	$f_0 = \frac{v}{\lambda_0}$ $f_0 = v / 4L$ (natural)		$L = \frac{\lambda_0}{4}$ $\lambda_0 = 4L$
	Blow forcefully at open end 2nd harmonic	$f_1 = \frac{v}{\lambda_1}$ $3\ f_0$		$L = \frac{3\lambda_1}{4}$ $\lambda_1 = 4/3\ L$
	Blow more forcefully 3rd harmonic	$5\ f_0$		$L = \frac{5\lambda_2}{4}$ $\lambda_2 = 4/5\ L$
Air column open at both ends	1st harmonic (fundamental)	$f_0 = \frac{v}{\lambda_0}$ $f_0 = v / 2L$ (natural)		$L = \frac{\lambda_0}{2}$ $\lambda_0 = 2\ L$
	Blow forcefully 2nd harmonic	$2f_0$		$\lambda_1 = L$
	Blow more 3rd harmonic forcefully	$3\ f_0$		$L = \frac{3\lambda_2}{2}$ $\lambda_2 = 2/3\ L$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{\frac{4L}{3}} = 3\left(\frac{v}{4L}\right)$

$f_1 = 3f_0$

$f_2 = \frac{v}{\lambda_2} = \frac{v}{\frac{4L}{5}} = 5\left(\frac{v}{4L}\right)$

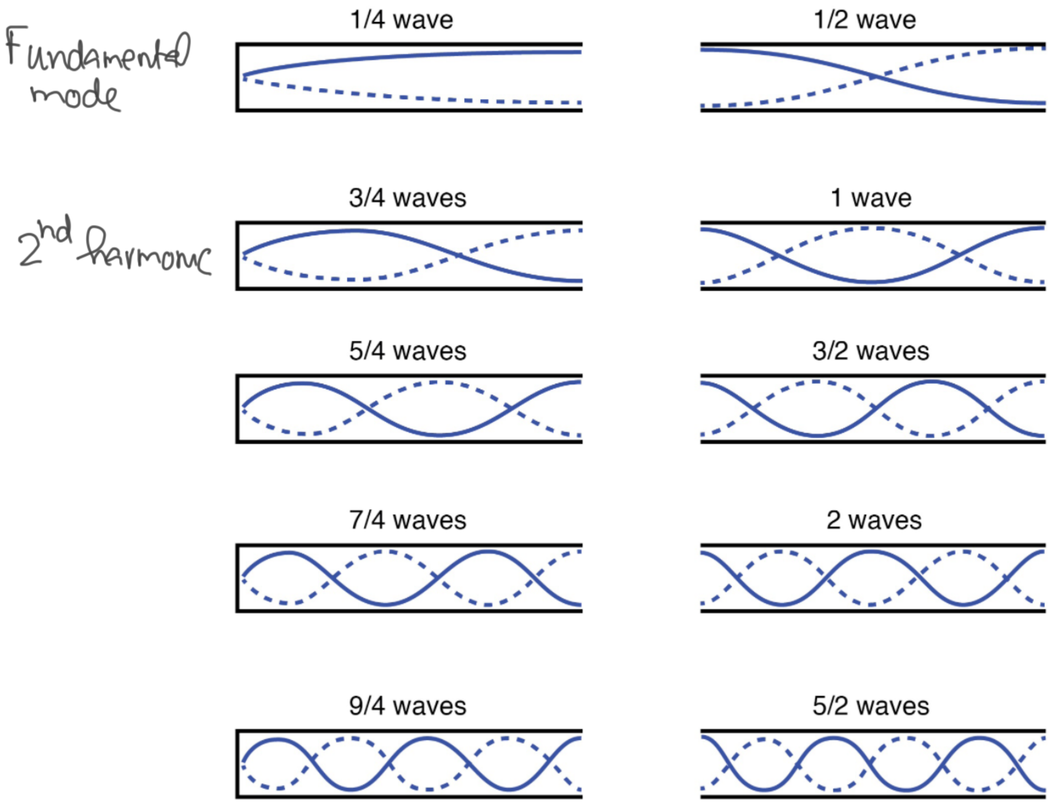
$f_2 = 5f_0$

$f_1 = \frac{v}{\lambda_1} = \frac{v}{L} \Rightarrow f_1 = 2f_0$

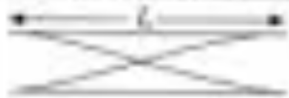
$f_2 = \frac{v}{\lambda_2} = \frac{v}{\frac{2L}{3}} = 3\left(\frac{v}{2L}\right) = 3f_0$

n = node, a = antinode

Standing sound waves in open-ended tubes



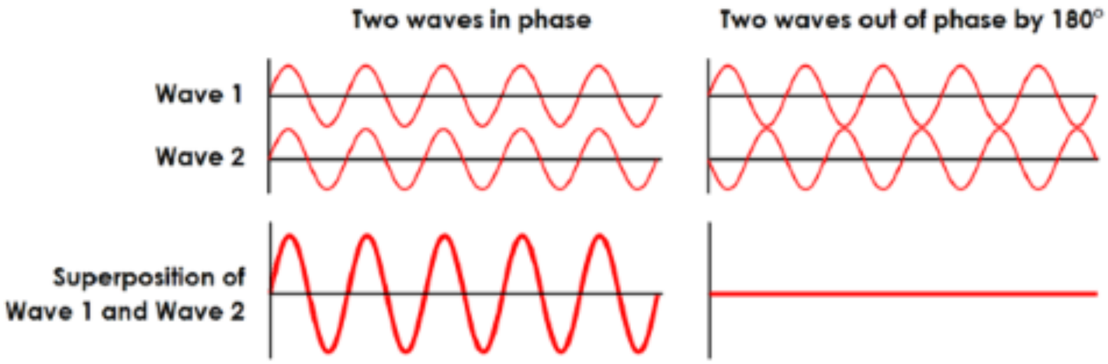
Modes of Vibration of Standing Waves

Mode	String	Closed Pipe	Open Pipe
1st harmonic or fundamental	 $\lambda = 2L$	 $\lambda = 4L$	 $\lambda = 2L$
2nd harmonic or 1st overtone	 $\lambda = \frac{2L}{2}$		 $\lambda = \frac{2L}{2}$
3rd harmonic or 2nd overtone	 $\lambda = \frac{2L}{3}$	 $\lambda = \frac{4L}{3}$	 $\lambda = \frac{2L}{3}$
4th harmonic or 3rd overtone	 $\lambda = \frac{2L}{4}$		 $\lambda = \frac{2L}{4}$
5th harmonic or 4th overtone	 $\lambda = \frac{2L}{5}$	 $\lambda = \frac{4L}{5}$	 $\lambda = \frac{2L}{5}$

Principle of Superposition

- Some headphones offer noise cancellation. This feature relies on the principle of superposition of waves, to remove unwanted sounds from listener's surroundings, allowing them to focus on the voice.
- A microphone on the outside of the headphone detects the background noise. The speaker inside the headphone then produces waves which cancels all the external sounds.
- People use these headphones without music to allow them to sleep in the noisy environments like aircraft cabins
- HOW this happens is explained in this CH

Principle of Superposition



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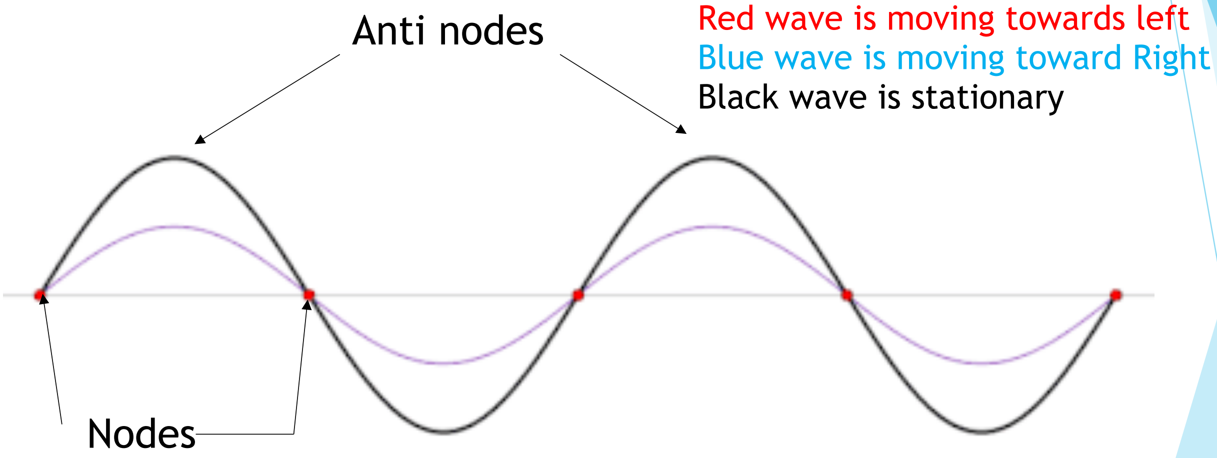
Stationary Waves

- As the wind blows over the mountain’s peak a stationary wave or standing wave is formed.
- If the air contains enough moisture the oscillation in the stationary wave make the water condense into a shape of a standing cloud
- Lets understand that what is a stationary wave

- As the name indicates, a stationary wave does not travel in space but oscillates unlike a progressive wave which travels in space

Formation of Stationary Wave

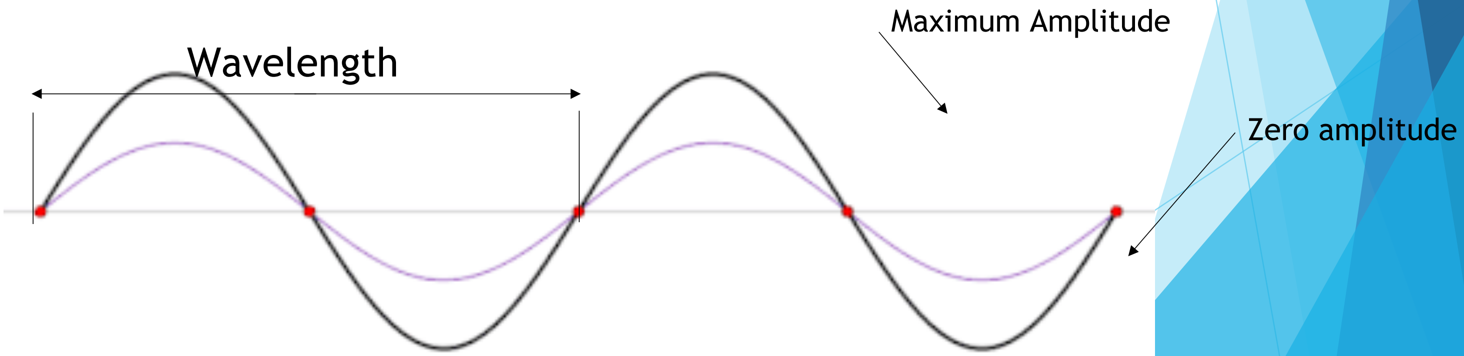
- A stationary wave is formed by superposition of two progressive waves of same type, amplitude and frequency, travelling in opposite direction



- A stationary wave is one in which some points are permanently at rest (zero displacement) and are called nodes,
- other point in between these nodes who vibrate or oscillates about their position are called antinodes

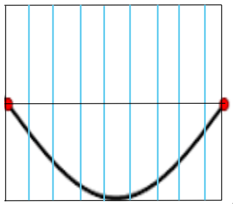
Properties of stationary waves

- Distance between two consecutive nodes/antinodes is half of the wavelength
- Frequency of the stationary waves is the same as the frequency of original progressive waves
- Amplitude of stationary waves is the sum of the amplitudes of the progressive waves, it varies from zero at nodes to maximum at antinodes
- There is not net energy transfer by a stationary wave unlike aa progressive wave



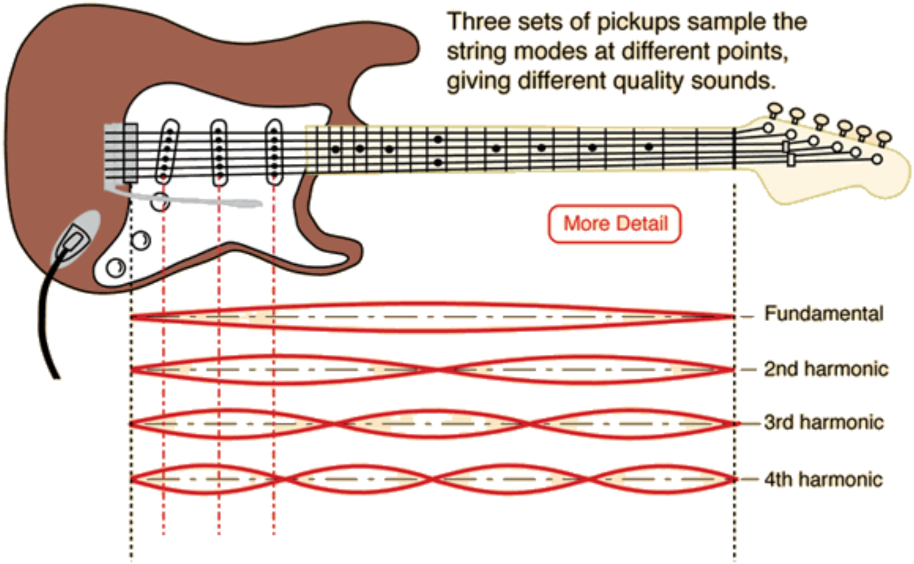
Phase difference along a stationary wave

- In between the adjacent nodes all the particles are oscillating in phase with each other that is they reach their maximum and minimum position at same time
- On different sides of nodes, the particles are at anti-phase with each other that is when the particles of the waves on one side of the node reach their maximum position the particles of the wave on the other side of the node reach their minimum position and vice versa

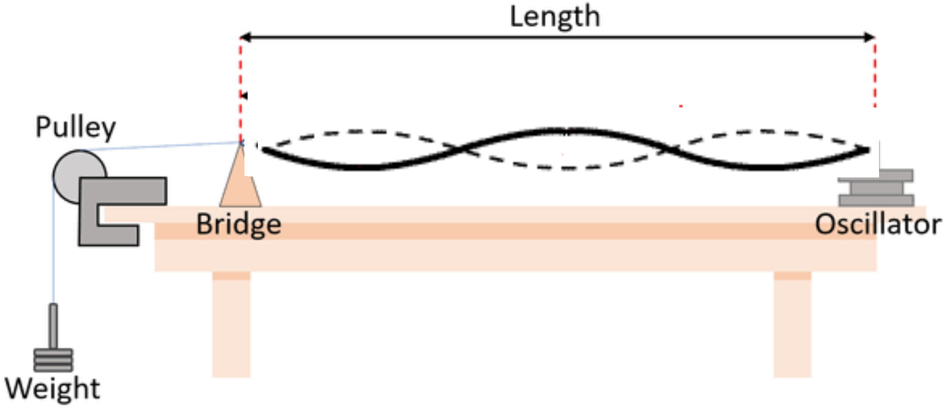


Harmonics

- Different harmonics are produced with several different wavelengths
- If the string is plucked exactly from the center, it vibrates with its fundamental mode of vibration also called fundamental frequency (shown in first case of figure) with the distance between the two nodes equal to half of the wavelength
- String can form higher harmonics depending upon the position from which they are plucked
- When the frequency of vibration increases, wavelength decreases



Melde’s Experiment



- Melde’s experiment is a simple way to investigate stationary wave on a string
- A vibration generator is used to change the frequency until a stable stationary wave is produced
- With this apparatus the length of the string can be varied and tension in the spring can also be varied
- Problem:
- If the fundamental frequency of a string is 300 Hz sketch the shape of the string when the frequency of vibration generator is
  - 300 Hz
  - 600 Hz
  - 450 Hz
  - 900 Hz

For fundamental frequency we have half wave between the fixed point

F	=	W
300 Hz	=	half wave
600 Hz	=	one full wave
900 Hz	=	one complete and one half wave

Formation of stationary waves using microwaves

- Stationary wave can be formed by reflecting microwave of a metal sheet so that two micro waves of same frequency are travelling in opposite directions
- Setup the equipment as shown in figure
- We know that the distance between two consecutive nodes or two consecutive antinodes is $\lambda/2$
- The distance between the transmitter and the metal sheet is adjusted so that it becomes equal to $(n \lambda/2)$ where n is an integer
- When n will be equal to 1 fundamental frequency will be formed and the shape of the string between the transmitter and the metal plate will be

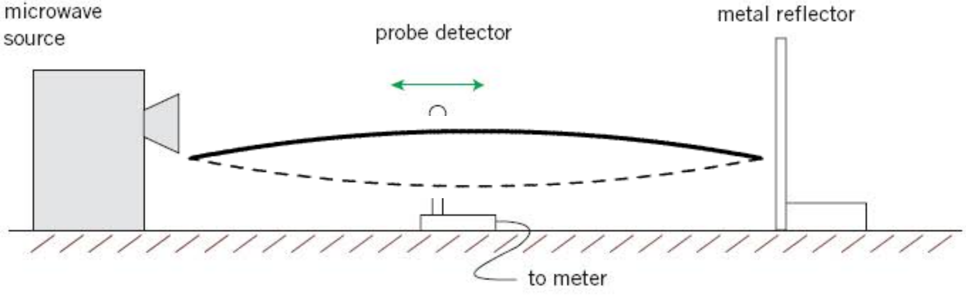


Figure 15.26 Using microwaves to demonstrate stationary waves

Formation of stationary waves in air columns

- Stationary waves are not only formed in transverse waves like electromagnetic waves but they are also formed in longitudinal waves like sound waves
- Most woodwind instruments produces notes from stationary waves in air columns
- Gently blowing over the top of a tube creates a standing wave inside it which produces a sound note at a particular frequency
- The length of the tube determines the wavelength of sound note it produces

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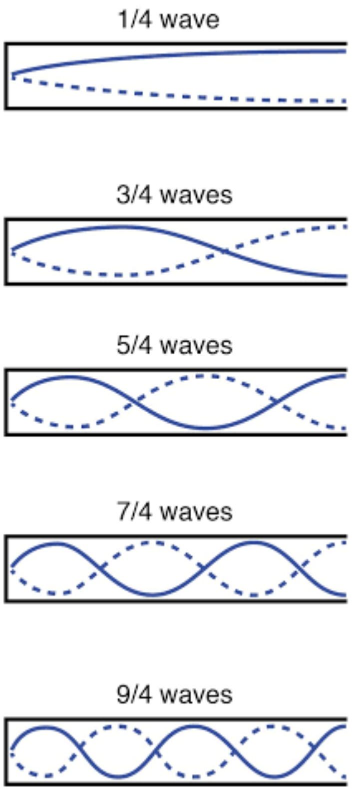
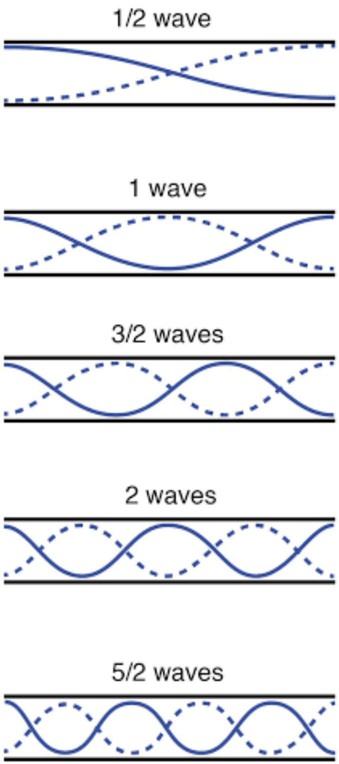
Stationary wave in tube closed at one end

- In order to form a stationary wave in a tube closed at one end there must be a node at the closed end and an antinode at the open end.
- At the closed end the air can not move so there must be a node at closed end
- At the open end the air is free to oscillate so there is an antinode at the open end
- Fundamental mode of vibration will simply have a node at the closed end and antinode at the open end
- Because there can never be a node at the open end and an antinode on close end so a complete wave with nodes at both the ends can not be formed for example for $2f_o$, $4f_o$ ...



Stationary wave in tube open at both ends

- At the open end the air is free to oscillate so there is an antinode at the open end
- So a tube opened at both the end will have antinodes at both the end
- Unlike a closed tube, harmonics of all the forms are possible in open ends tube i.e. f_0, $2f_0$, $3f_0$, $4 f_0$ .....



Fundamental
1^{st} Harmonic

2^{nd} Harmonic

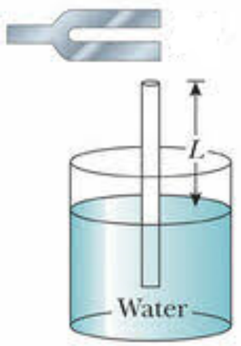
3^{rd} Harmonic

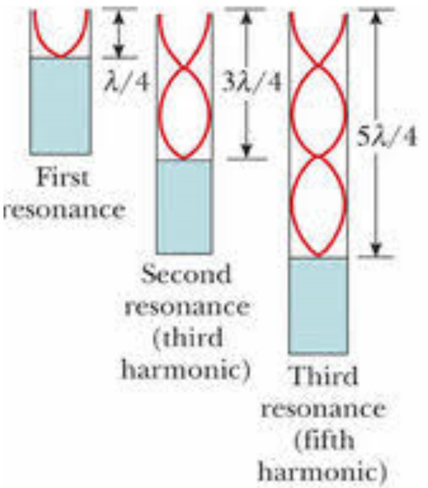
4^{th} Harmonic

5^{th} Harmonic

Speed of sound in resonance tube

- In this experiment the speed of sound in air is to be found by using tuning forks of known frequency.
- The apparatus of the experiment consist of a long cylindrical plastic tube attached to water reservoir and a tuning fork
- The wavelength of the sound will be determined by making use of resonance of air column
- In the apparatus the length of the tube can be changed by raising and lowering it in the water
- A vibrating tuning fork is placed just above the tube, listening for amplification of tone. When the resonance is found, a pronounced reinforcement of the sound will be heard
- When the frequency of tuning fork matches the fundamental frequency the length of the tube above water L= λ so the wave length, $\lambda = 4L$
- The speed of the sound in the air column can be found by using the formula $v=f\lambda$. Where f is the frequency of the tuning fork



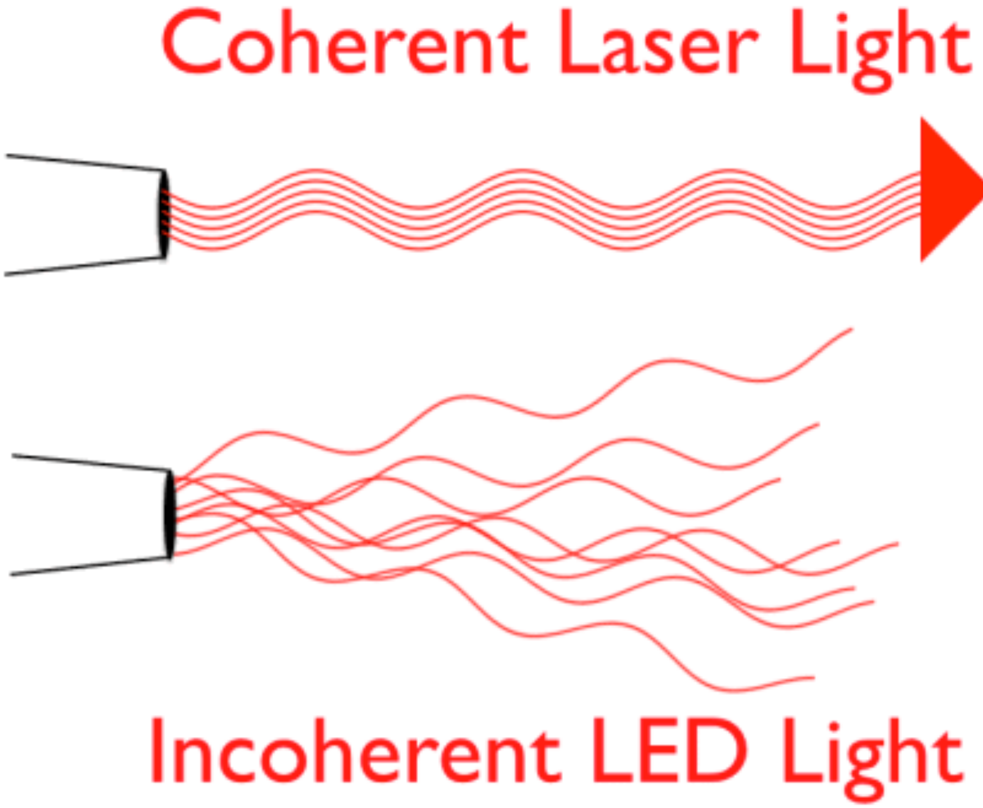


Interference

- If you watch the ripples in pond when rain drops fall, you will see an interference pattern.
- As the wave formed by a rain drop moves outwards, it interferes with waves formed by other rain drops
- Raindrops which fall randomly forms an unstable interference pattern.

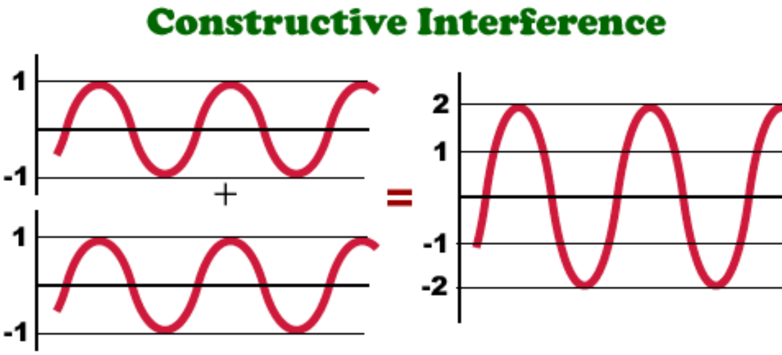
Coherence

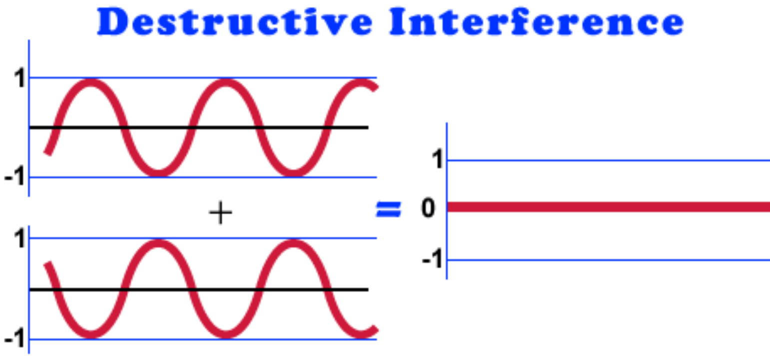
- If the waves emitting from a source have same frequency, wavelength, amplitude and are in phase with each other then they are coherent waves
- For stable pattern of interference, the waves should be coherent
- Filament bulbs do not emit light waves of same frequency and wavelength so they are not coherent
- Whereas a laser light emits the light waves who have same frequency and wavelength and are in phase with each other



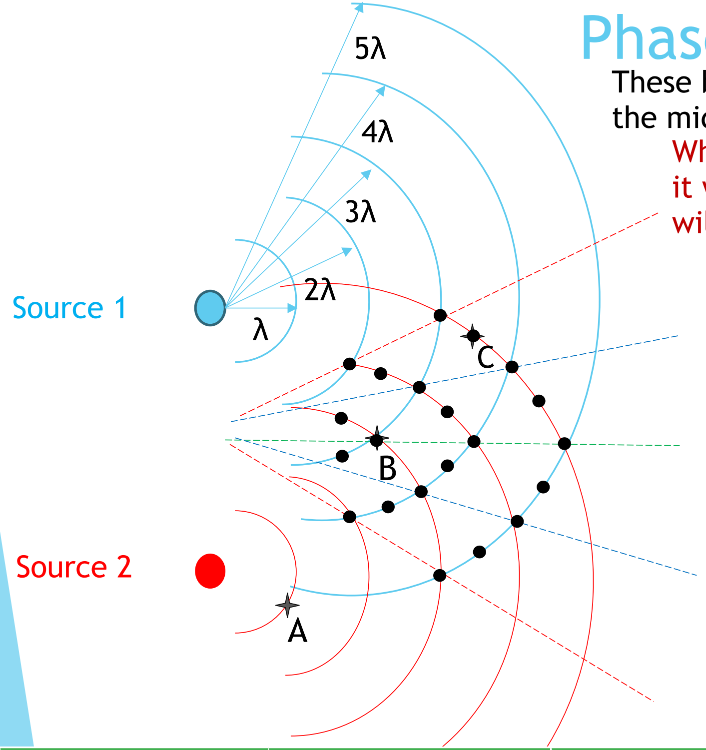
Constructive and destructive interference

- If two waves are in phase with each other i.e. if their crests arrive at a point exactly at same time they will interfere constructively
- A resultant wave will be produced which will have the crest much higher than either of the two individual waves and the trough much deeper.
- For example in the figure if the amplitude of both the waves is 1 at the crest then the amplitude of the resultant wave will be 1 + 1 = 2 at the crest.
- If two waves are out of phase with each other i.e. if their crest of one wave arrive at a point exactly the same time as the trough of the second wave, they will interfere destructively
- A resultant wave will be produced which will have the crest much smaller (or may be zero in some cases) than either of the two individual waves.
- For example in the figure if the amplitude of first wave is 1 and at the same time the amplitude of the second wave is -1 then the resultant wave will be 1 - 1 = 0 at that moment.





Phase Difference



These blue and red lines represent the crest and at the middle of two crests we have trough

When a crest interferers a crest for example at point A & B it will be constructive interference and the resultant wave will have a maxima

When a crest interferers a Trough for example at point C it will be destructive interference and the resultant wave will have a minima

	Point A	Point B	Point C
Red Wavelength	1λ	3λ	5λ
Blue wavelength	5λ	3λ	3.5λ
Path difference	5-1= 4λ	3-3= 0λ	5-3.5= 1.5λ
Phase difference	4(2π) = 8π In Phase Maxima	0(2π) = 0 In Phase Maxima	1.5(2π) = 3π Out of Phase Minima
Resultant			

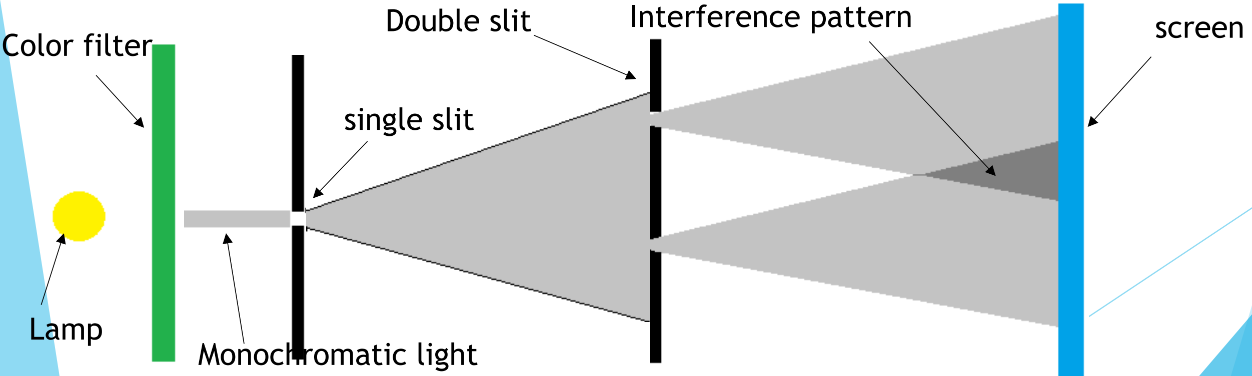
ORDER	2nd Order Maxima	1st Order Maxima	Central maxima	1st Order Minima	2nd Order Maxima
Path Difference	2λ	λ	0	0.5λ	1.5λ
Phase Difference	4π	2π	0	π	3π

Young's double slit experiment

- In 18th century Newton rejected the idea of light being a wave.
- He proposed that light is a stream of tiny particles. The idea remained 'the accepted scientific theory' for nearly 100 years
- Then a scientist named Young performed a wonderful experiment to demonstrate that light streams can interfere with each other, hence concluding that light must be a wave

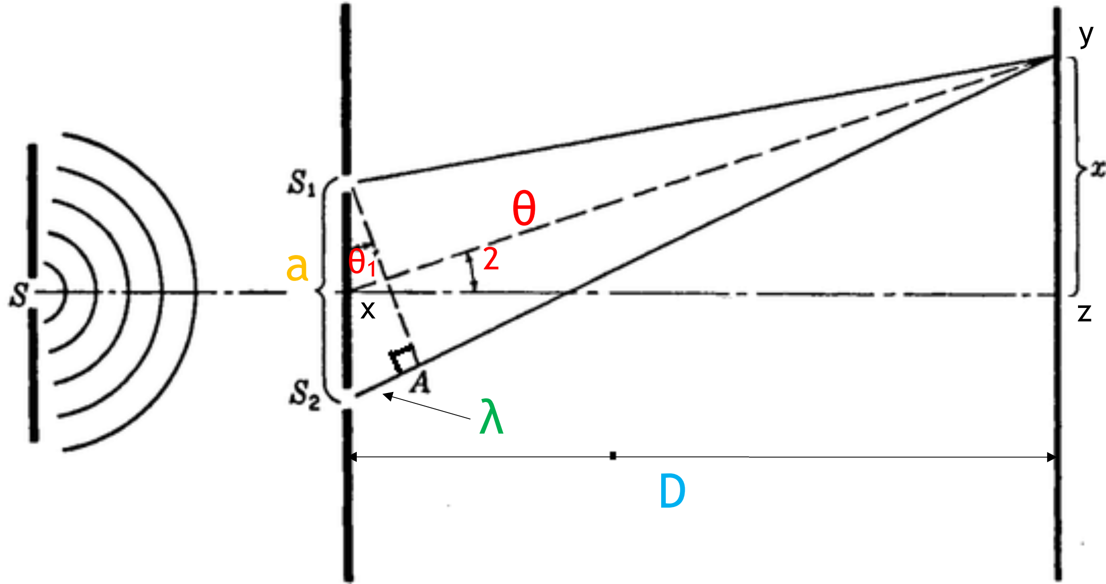
Young's double slit experiment

- Apparatus of this experiment need a filament lamp, a color filter, single slit, double slit and a screen
- Two coherent waves are needed to make a stable interference pattern
- Young achieved this by using a monochromatic light (to produce two waves of same frequency wavelength and amplitude)
- He produced a monochromatic light by placing an color filter in front of a lamp allowing only a specific frequency of light to pass
- He then placed a narrow sheet having single slit in front of this monochromatic light to diffract it
- Light diffracting from the single slit arrives the double slit and it again diffracts from the double slit.
- The coherent waves getting out of double slits make an interference pattern on the screen
- Young's double slit experiment successfully demonstrated the wave nature of light



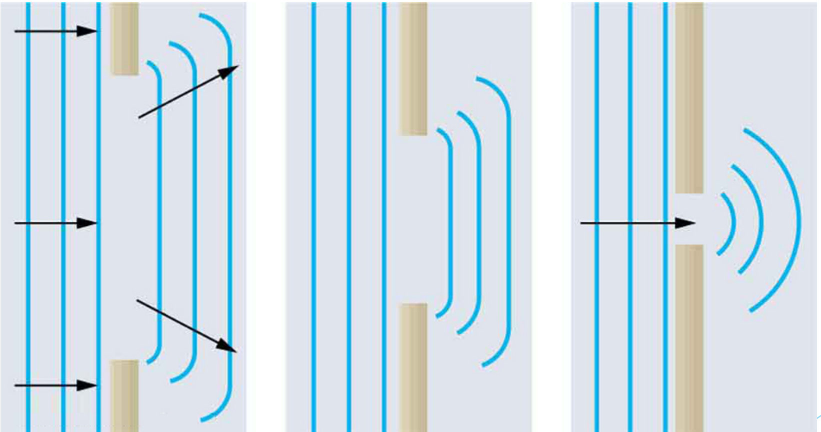
Mathematical treatment

- We can derive the formula to calculate the wavelength of light used to form interference pattern
- In figure the separation between the two slits S_1 and S_2 is **a.** an interference pattern is observed on the screen at a distance **D** from the slits. Where D is much greater than a **D>>a**
- ***x*** on the screen is the distance between two bright fringes
- The path difference S_2A is equal to one wavelength λ. And the angles θ_1 & θ_2 are equal as shown in figure
  - As the angles are very small so we can use the trigonometric approximation that $\sin \theta_1 = \tan \theta_2$
  - So in triangle AS_1S_2
  - $\sin \theta_1 =$
  - And in triangle xyz
  - $\tan \theta_2 = x/D$
  - As $\sin \theta_1 = \tan \theta_2$
  - = ? λ =

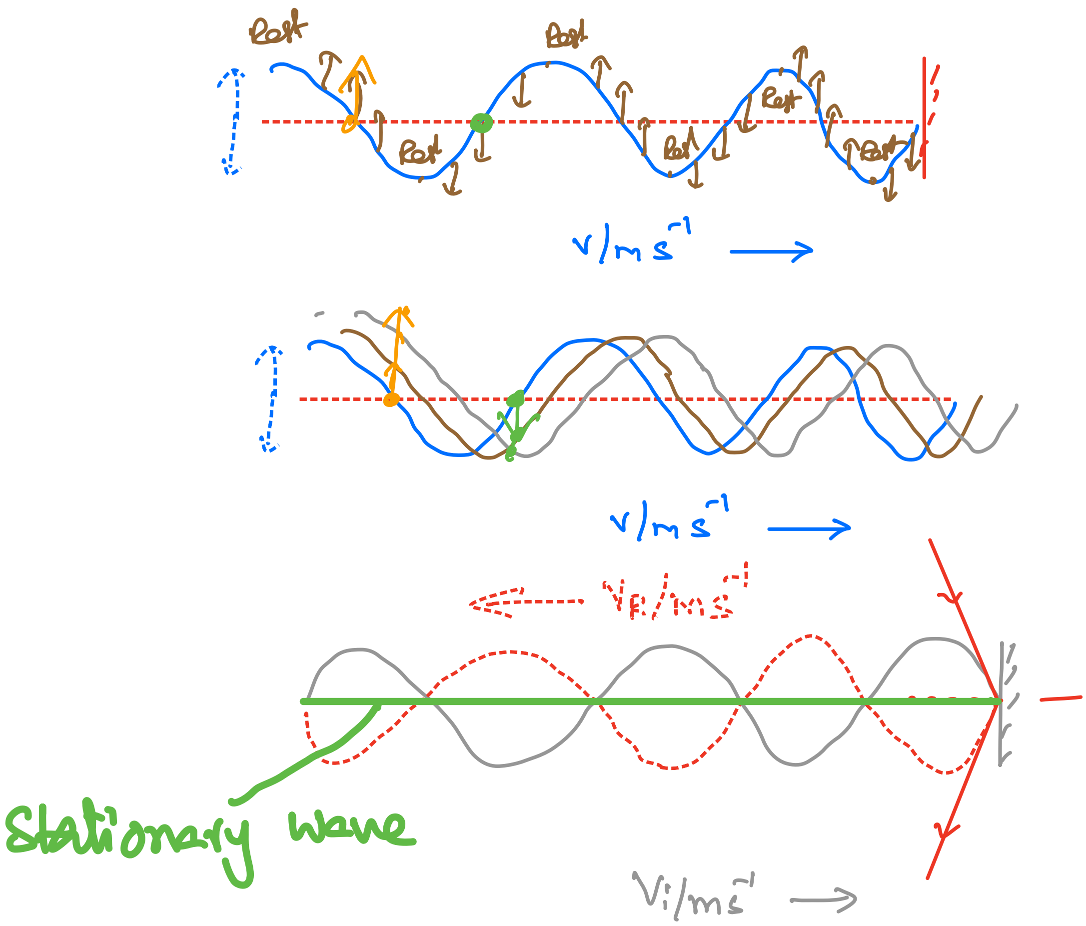


Diffraction

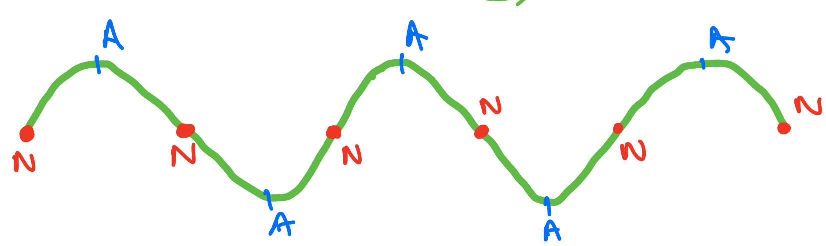
- The bending of a wave around the edges of an opening or an obstacle is called **diffraction**.
- Diffraction is a wave characteristic and occurs for all types of waves
- The amount of bending is more extreme for a small opening.
- Speed wavelength and frequency of a wave does not change after diffraction



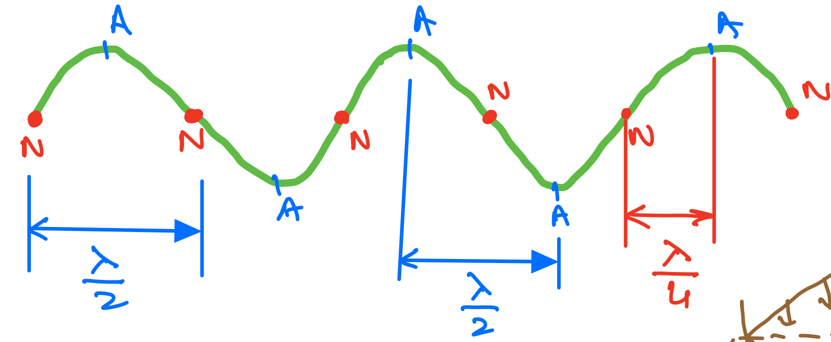
AHMAD MAHMOOD

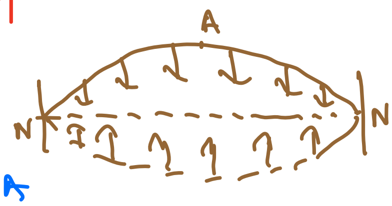


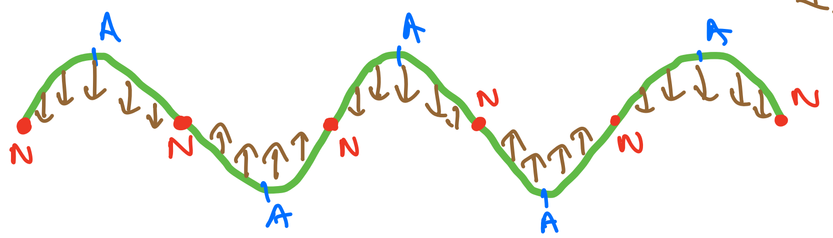
$V_i = -V_R$ (velocity)

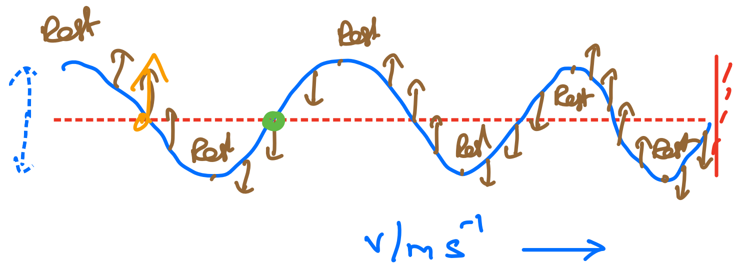


Node – Zero amplitude particles (N)
Antinode – Max amplitude particles (A)

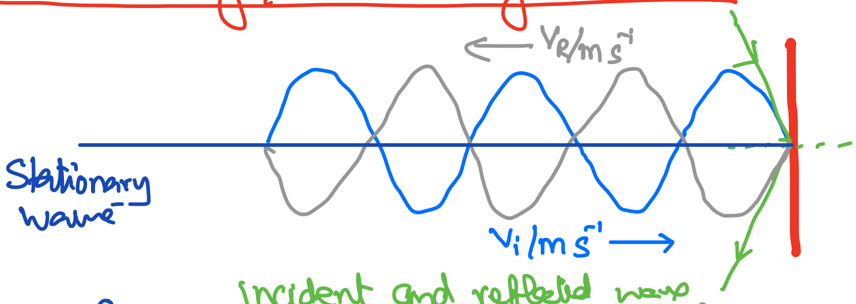

A
N
λ/2
λ/4


A
N


A
N


Rest
v/ms⁻¹

Stationary / Standing waves :-



$f_i = f_R$

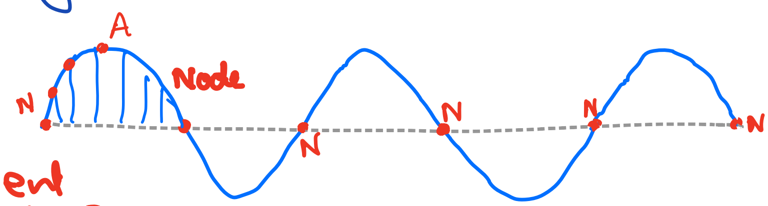
$\lambda_i = \lambda_R$

Speed $V_i = V_R$

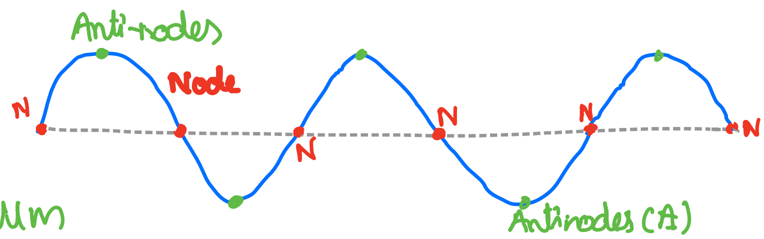
Velocity $V_i = -V_R$

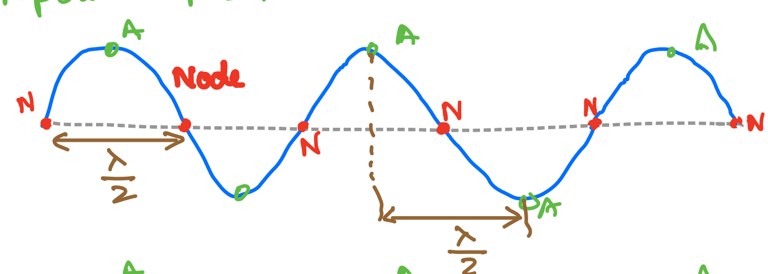
When two waves (incident and reflected wave) having same speed but travelling in opposite direction overlap, stationary waves are formed.

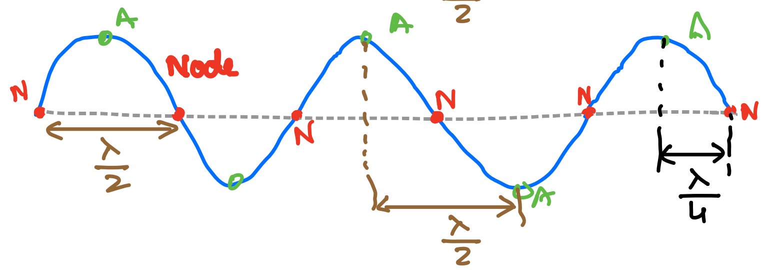
Node: Displacement remain zero.



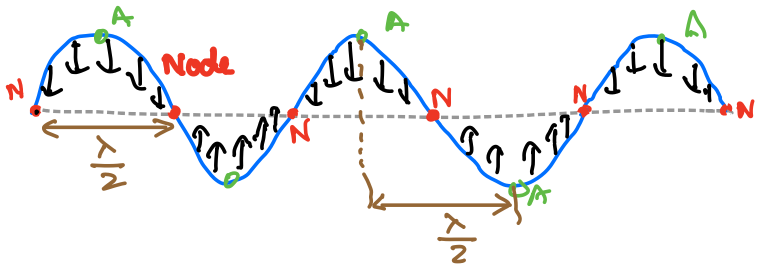
Antinode: Maximum amplitude position

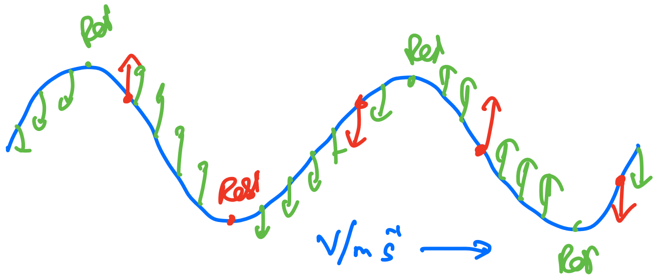


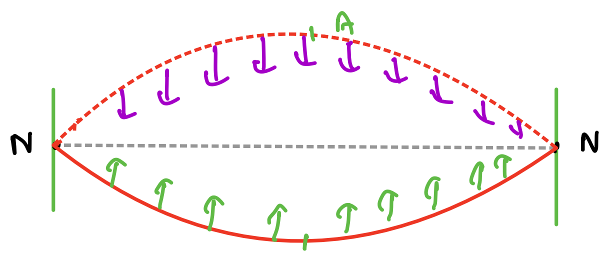




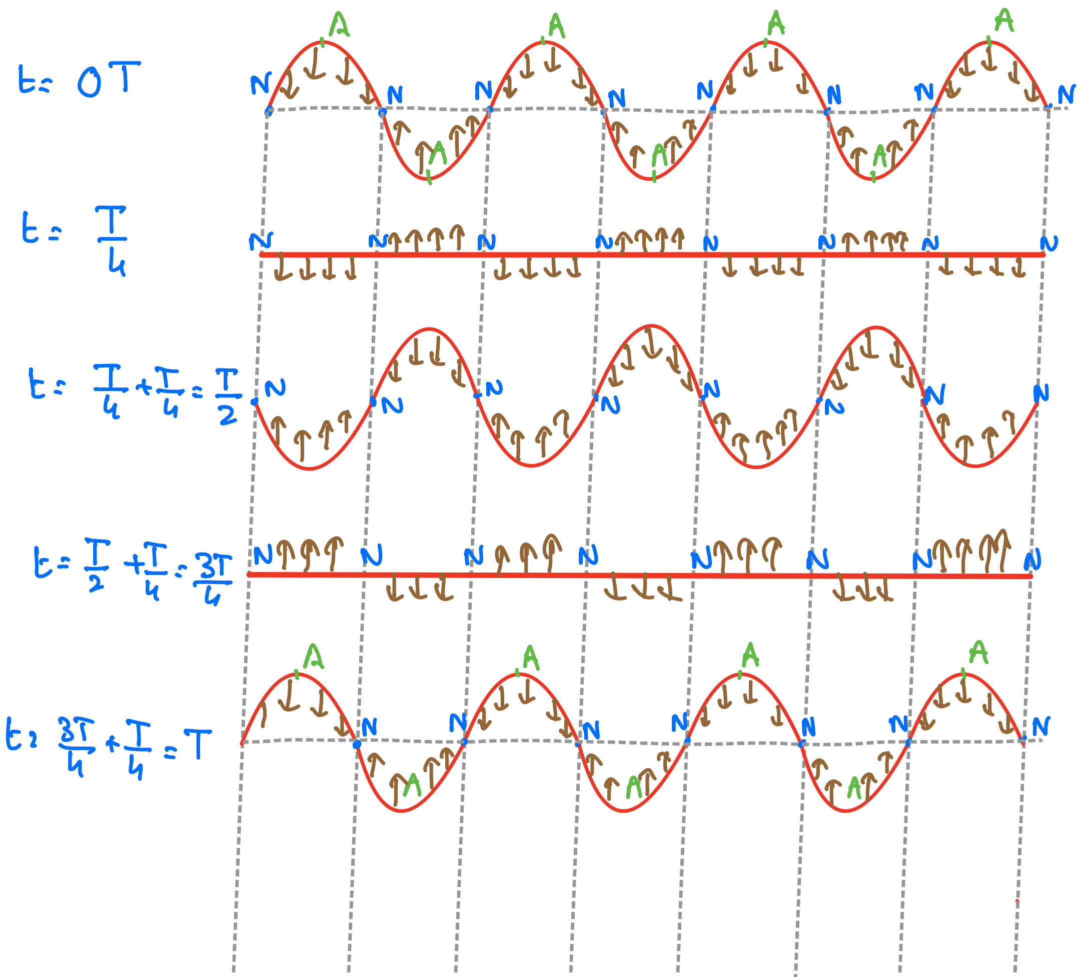
All particles b/w two adjacent nodes move in in same direction and therefore inphase with each other.





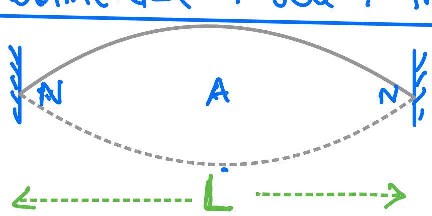


Formation of stationary waves at multiple times:-



Stationary waves along a stretched string (Guitar / Sitaar string):

Fundamental mode / frequency:



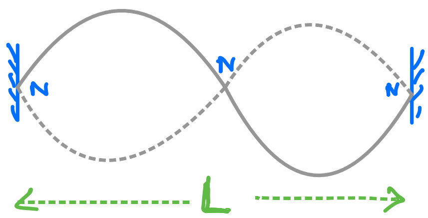
$$\frac{\lambda_0}{2} = L \Rightarrow \lambda_0 = 2L$$

$$v = f_0 \lambda_0 \Rightarrow f_0 = \frac{v}{\lambda_0}$$

$$\boxed{f_0 = \frac{v}{2L}} \quad \text{——— (1)}$$

First overtone frequency: f_1

Fix a point along the string to form a node as shown



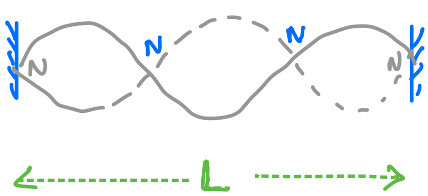
$L = \lambda_1$

$V = f_1 \lambda_1 \Rightarrow f_1 = \frac{V}{\lambda_1}$

$f_1 = \frac{V}{L} \Rightarrow \boxed{f_1 = 2f_0}$

2^{nd} overtone frequency: f_2

Fix the string from two points to form nodes as shown



$L = \frac{3\lambda_2}{2} \Rightarrow \lambda_2 = \frac{2L}{3}$

$V = f_2 \lambda_2 \Rightarrow f_2 = \frac{V}{\lambda_2}$

$f_2 = \frac{V}{\frac{2L}{3}} \Rightarrow f_2 = 3\left(\frac{V}{2L}\right)$

$\boxed{f_2 = 3f_0}$

In general, along a stretched string, stationary transverse wave of frequency f_0, $2f_0$, $3f_0$, $4f_0$, ------ nf_0 are produced where $n = 1, 2, 3, 4$ - - - and represent no. of loops and

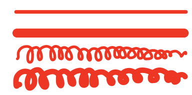
$f_0 = \frac{V}{2L}$

Speed of stationary wave along a stretched string:

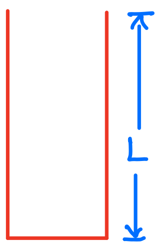
$$v \propto \sqrt{\frac{T}{\mu}}$$, T – Tension in string, μ – mass per unit length of string.

$$f\lambda \propto \sqrt{\frac{T}{\mu}}$$

$$f \propto \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$$

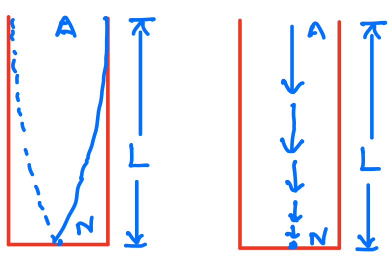


Formation of stationary waves along a closed pipe: (mouth organ)



Length of closed pipe = L

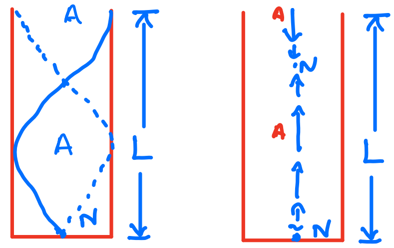
Fundamental mode/ frequency: Blow calmly at top or side of open end.



$$L = \frac{\lambda_0}{4} \Rightarrow \lambda_0 = 4L$$

$$f_0 = \frac{v}{\lambda_0} \Rightarrow \boxed{f_0 = \frac{v}{4L}}$$

First overtone frequency: Blow air forcefully at or side of open end.



$$L = \frac{\lambda_1}{2} + \frac{\lambda_1}{4} = \frac{3\lambda_1}{4}$$

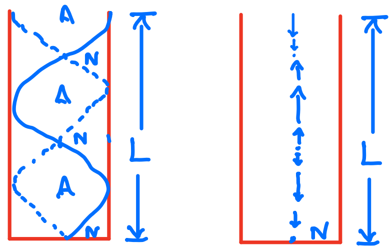
$$\lambda_1 = \frac{4L}{3}$$

$$v = f_1 \lambda_1 \Rightarrow f_1 = \frac{v}{\lambda_1}$$

$$f_1 = \frac{v}{\frac{4L}{3}} \Rightarrow f_1 = 3\left(\frac{v}{4L}\right)$$

$$\boxed{f_1 = 3 f_0}$$

2nd overtone frequency: Blow more force to get another node apart two earlier nodes inside pipe.



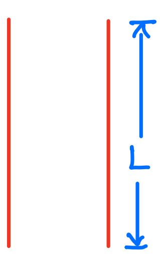
$$L = \frac{5\lambda_2}{4} \Rightarrow \lambda_2 = \frac{4L}{5}$$

$$v = f_2 \lambda_2 \Rightarrow f_2 = \frac{v}{\lambda_2} = \frac{v}{\frac{4L}{5}}$$

$$f_2 = 5\left(\frac{v}{4L}\right) \Rightarrow \boxed{f_2 = 5 f_0}$$

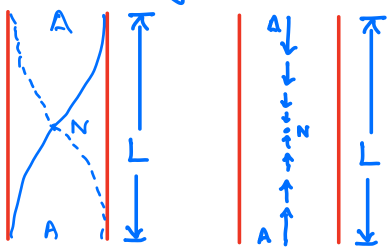
Therefore, in a closed pipe, stationary longitudinal waves of frequency f_0, $3f_0$, $5f_0$, $7f_0$, ----- $(2n+1)f_0$ are produced where $n = 0, 1, 2, 3, ---$ and represent no. of nodes in between open and closed end.

Formation of stationary waves along an open pipe: (flute)



Length of open pipe = L

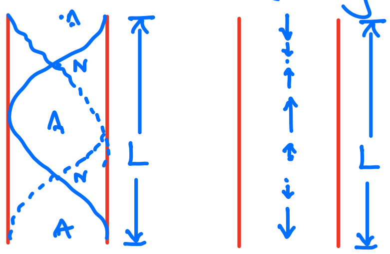
Fundamental mode/ frequency : Blow calmly at top or side of open end.



$$L = \frac{\lambda_0}{2} \Rightarrow \lambda_0 = 2L$$

$$f_0 = \frac{V}{\lambda_0} \Rightarrow \boxed{f_0 = \frac{V}{2L}} \quad \text{—(1)}$$

First overtone frequency : Blow air forcefully at or side of open end.

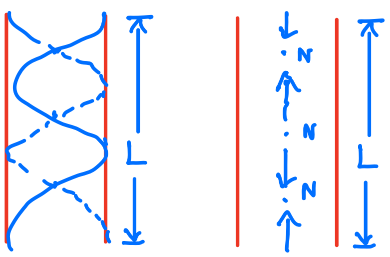


$$L = \lambda_1$$

$$V = f_1 \lambda_1 \Rightarrow f_1 = \frac{V}{\lambda_1}$$

$$f_1 = \frac{V}{L} \Rightarrow \boxed{f_1 = 2f_0} \quad \text{From (1)}$$

2nd overtone frequency: Blow more force to get ano node apart two earlier nodes inside pipe.



$$L = \frac{6\lambda_2}{4} \Rightarrow L = \frac{3\lambda_2}{2}$$

$$\lambda_2 = \frac{2L}{3}$$

$$V = f_2 \lambda_2 \Rightarrow f_2 = \frac{V}{\lambda_2} = \frac{V}{\frac{2L}{3}}$$

$$f_2 = 3\left(\frac{V}{2L}\right) \Rightarrow \boxed{f_2 = 3f_0}$$

In general, in an open pipe, stationary longitudinal waves of frequency $f_0, 2f_0, 3f_0$ ------ nf_0 are produced where $n = 1, 2, 3, --$ and represent no. of nodes inbetween two ends.

5 Light reflected from the surface of smooth water may be described as a polarised transverse wave.

(a) By reference to the direction of propagation of energy, explain what is meant by

(i) a *transverse wave*,

....

....[1]

(ii) *polarisation*.

....

....[1]

(b) A glass tube, closed at one end, has fine dust sprinkled along its length. A sound source is placed near the open end of the tube, as shown in Fig. 5.1.

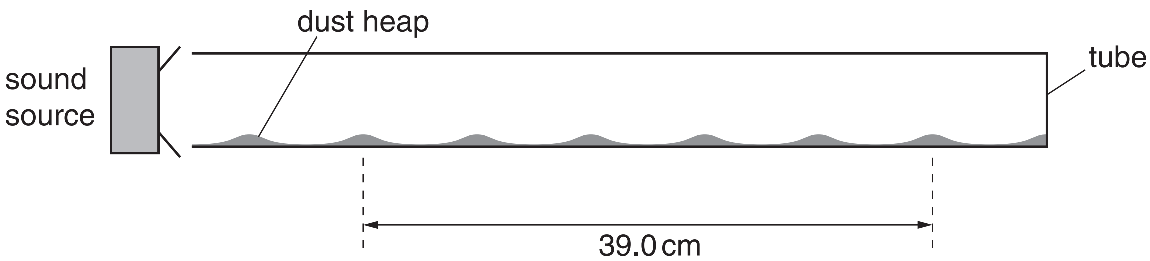


Fig. 5.1

The frequency of the sound emitted by the source is varied and, at one frequency, the dust forms small heaps in the tube.

(i) Explain, by reference to the properties of stationary waves, why the heaps of dust are formed.

....

....

....

....[3]

For Examiner's Use

(ii) One frequency at which heaps are formed is 2.14 kHz.
The distance between six heaps, as shown in Fig. 5.1, is 39.0 cm.
Calculate the speed of sound in the tube.

speed = ... $m\,s^{-1}$ [3]

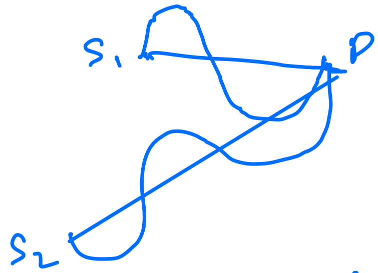
(c) The wave in the tube is a stationary wave. Explain, by reference to the formation of a stationary wave, what is meant by the speed calculated in **(b)(ii)**.

....

....

....

....[3]



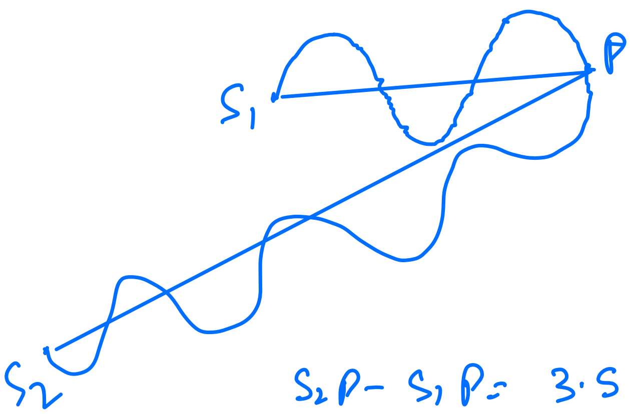
$S_2P - S_1P = 1.5 - 1$

$= 0.5\lambda$

$= 1\frac{\lambda}{2}$

$\phi = \left(\frac{1\frac{\lambda}{2}}{\lambda}\right)2\pi$

$= \pi$



$S_2P - S_1P = 3.5\lambda - 1.5\lambda$

$= 2\lambda$

$\phi = \left(\frac{2\lambda}{\lambda}\right)2\pi$

$= 4\pi$