Stationary Waves

Formation of a Stationary wave

→ When the incident wave is reflected at one end, the incident and reflected wave interfere, producing a stationary wave.

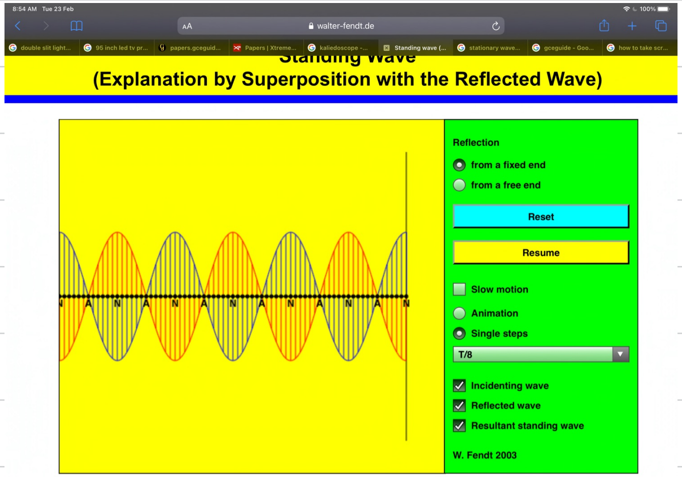
→ The regions of constructive and destructive interference are produced.

Progressive Wave	Stationary Wave
→ Waves which have a net transfer of energy from one point to other.	→ No net transfer of energy observed as energy is confined between two points.
→ The waveform appears to translate from one point to another.	→ The waveform appears to be standing in one region b/w points.
→ The amplitude of each point is same and equal to amplitude of wave.	→ Every point has a different amplitude.

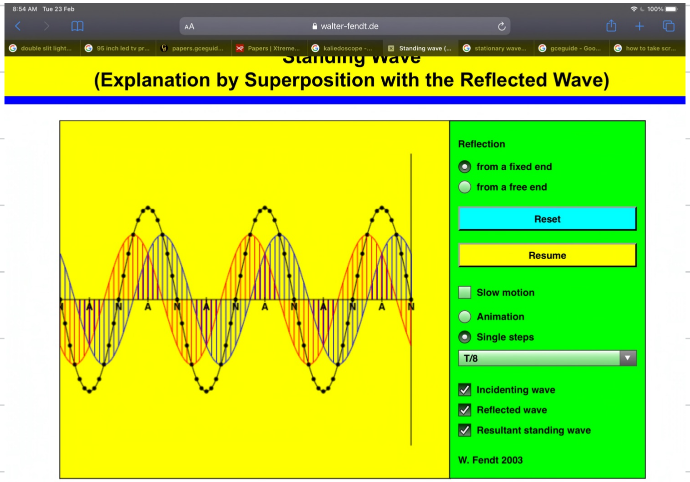
Progressive Waves	Stationary Waves
→ Every point on a wave has a different phase angle based on its location.	→ All points between nodes are in phase and all point adjecent to nodes are out of phase.
→ It comprises of crests, troughs, compressions and varefactions.	→ It comprises of nodes and antinodes.

NODE: Points of zero displacement on a stationary wave. There is always a destructive interference here.

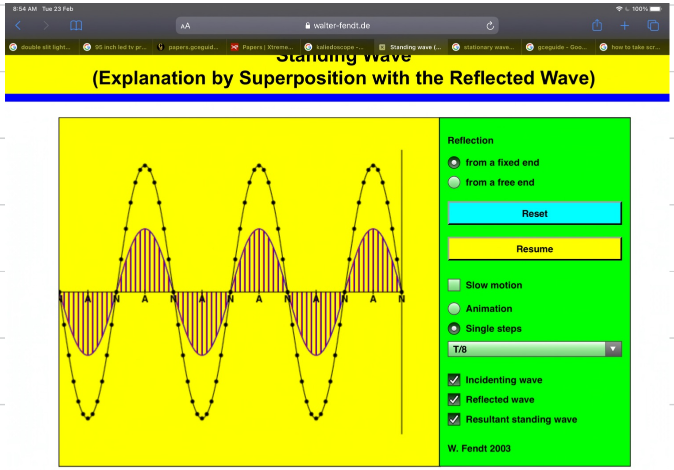
ANTINODE: Point of maximum amplitude/displacement on a stationary wave.



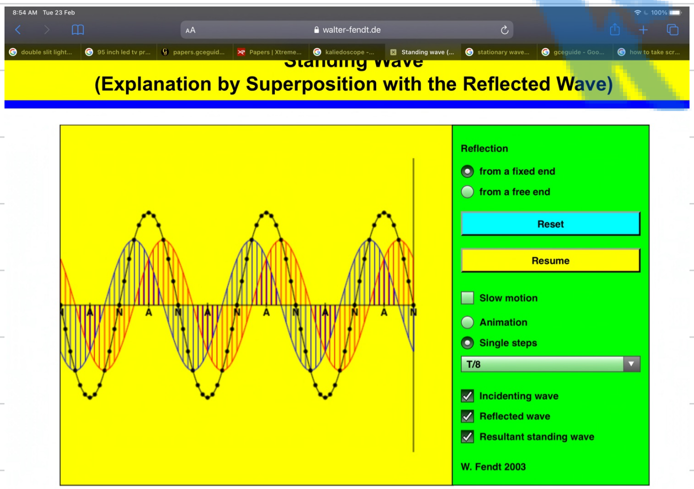
$t = 0$



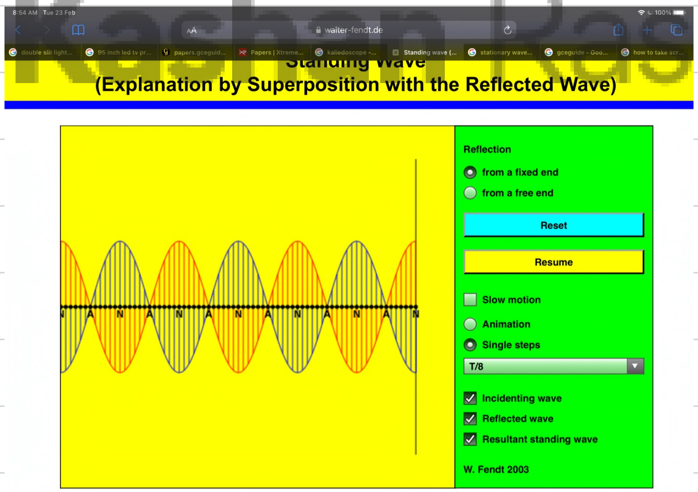
$t = T/8$



$t = T/4$

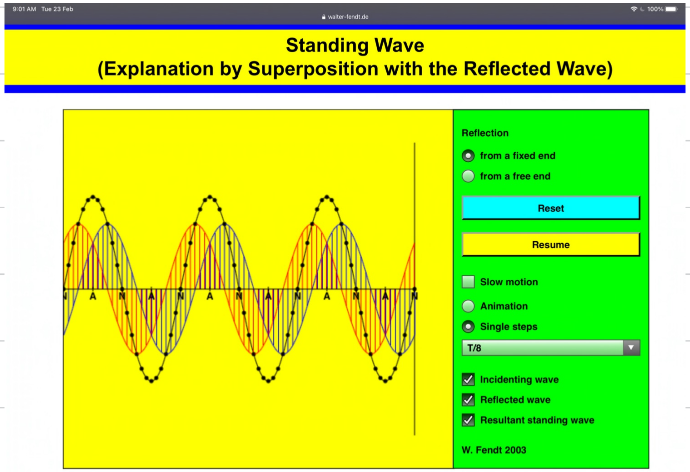


$t = \frac{3T}{8}$

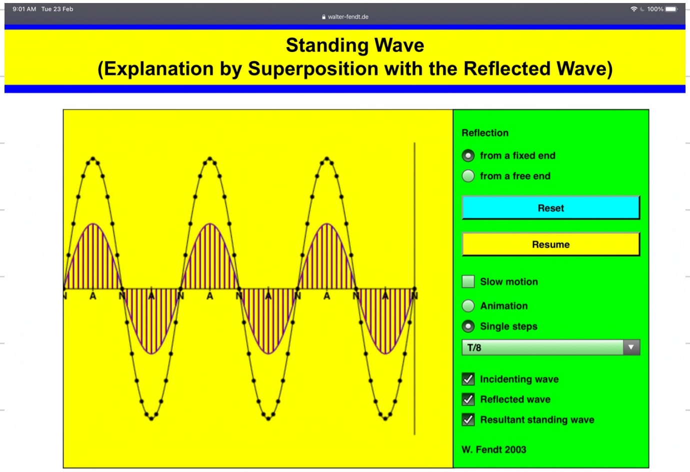


$t = \frac{T}{2}$

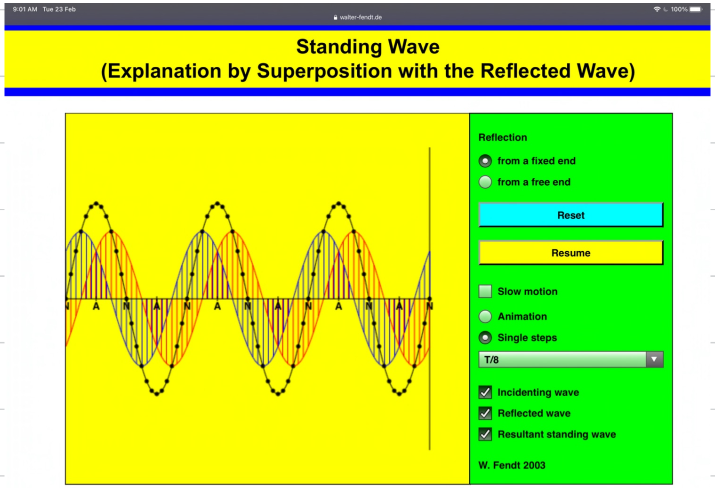
When a stationary wave starts from zero amplitude to max and then returns to zero, it is half cycle completed and time elapsed is $t = \frac{T}{2}$.



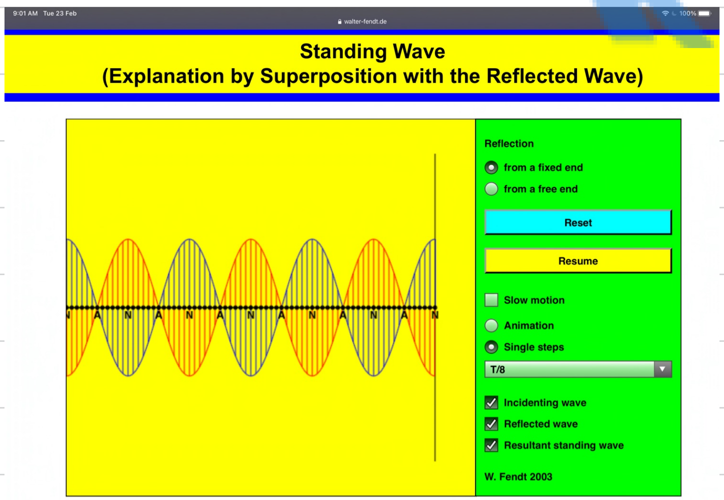
$t = \frac{5T}{8}$



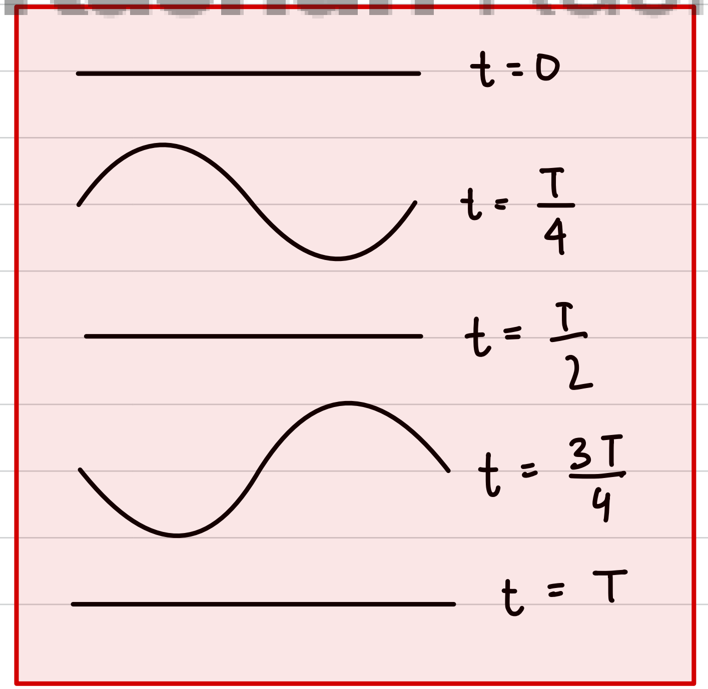
$t = \frac{3T}{4}$



$t = \frac{7T}{8}$

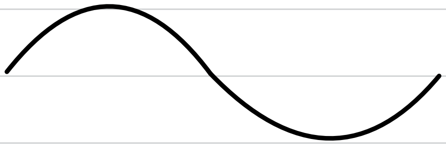


$t = T$

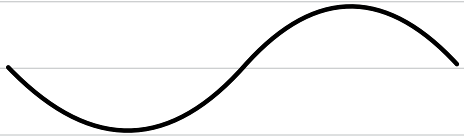


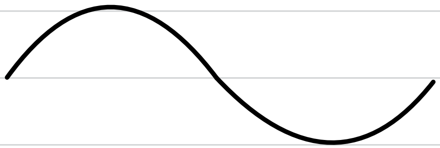
ONE COMPLETE CYCLE OF A STATIONARY WAVE

$T = 2s$

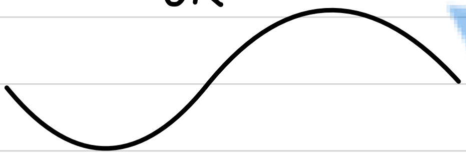


after $t = \frac{T}{2}$





OR



after $t = \frac{T}{4}$

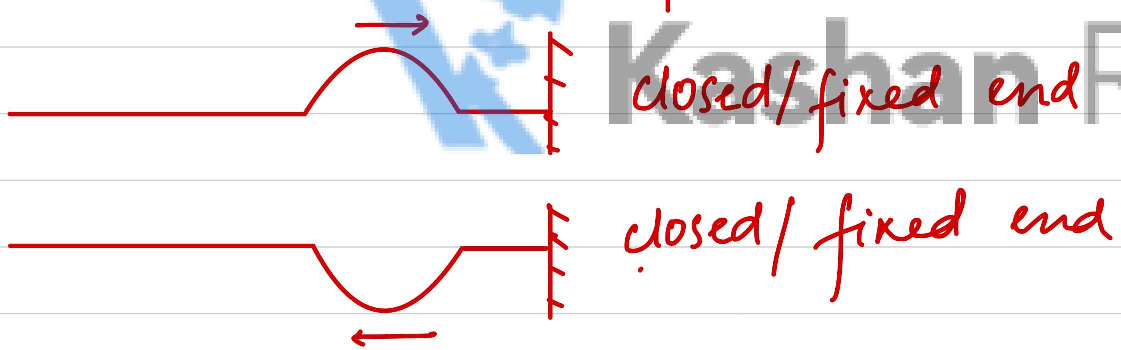
after $t = \frac{T}{2}$

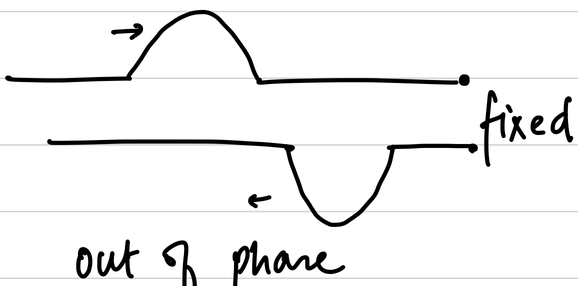
Reflection of wave from fixed end

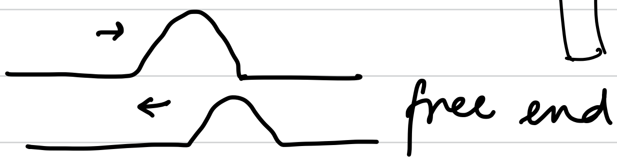
- Reflection of a wave from a fixed end is out of phase with the incident wave.

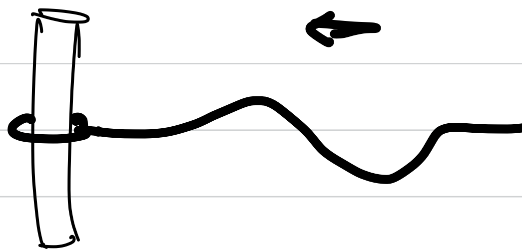
- Crest goes, trough returns.

- As a result, at the fixed end, destructive interference is observed. The point is hence called a NODE!









fixed end: node   free end: antinode.

Reflection of wave from an open end

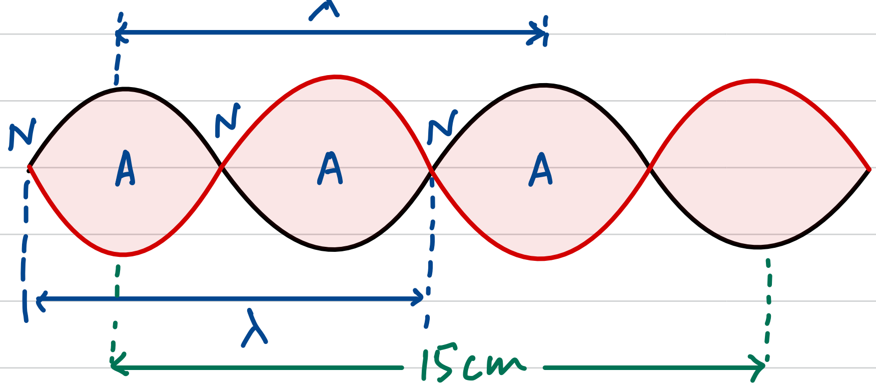
- Reflected wave from an open end is always in phase with the incident wave.

- Crest goes, crest returns
- Trough goes, trough returns

- The waves are hence in phase with one another and hence a constructive interference is observed at the open end. An antinode is always formed at the open end.

open end

open end



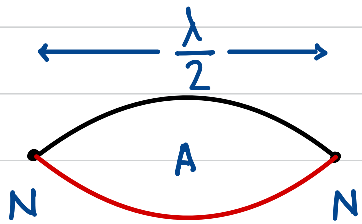
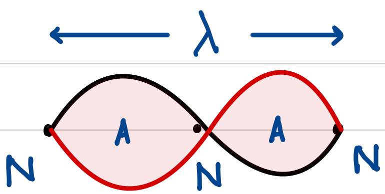
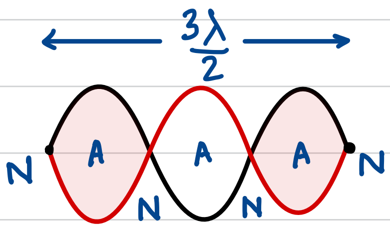
find wavelength

$1.5\lambda = 15\text{ cm}$

$\lambda = 10\text{ cm}$

FORMATION OF A STATIONARY WAVE

- **BOTH ENDS FIXED (CLOSED)**

Shape	Harmonic	Tone	Wavelength	frequency
	–	–	–	–
	1^{st} Harmonic	Tone	$L=\frac{\lambda}{2}$ $\lambda=2L$	$v=f\lambda$ so $v=f(2L)$ $f=\frac{v}{2L}$
	2^{nd} Harmonic	1^{st} overtone	$L=\lambda$ $\lambda=L$	$v=f\lambda$ so $v=f(L)$ $f=\frac{v}{L}$
	3^{rd} Harmonic	2^{nd} overtone	$L=3\frac{\lambda}{2}$ $\lambda=\frac{2L}{3}$	$v=f\lambda$ so $v=f\left(\frac{2L}{3}\right)$ $f=\frac{3v}{2L}$

Fundamental frequency (f_0): The smallest (minimum) frequency of vibration required to form a stationary wave.

	1st Harmonic	2nd Harmonic	3rd Harmonic	nth Harmonic
	$f_0 = \frac{(1)v}{2L}$ fundamental frequency	$f = \frac{(2)v}{2L} = 2f_0$	$f = \frac{3v}{2L} = 3f_0$	$f = \frac{nv}{2L} = nf_0$
	50 Hz	100 Hz	150 Hz	200 Hz - - -

where $n = 1, 2, 3, 4 \ldots$ (integral multiples)

Example.

A stationary wave of speed $v = 50\,ms^{-1}$ is formed on a stretched spring fixed at both ends, of length 0.5m.

Calculate

i, fundamental frequency

ii, 11^{th} Harmonic

iii, 20^{th} overtone

i, $f_0 = \frac{1v}{2L}$

$f_0 = \frac{(50)}{2(0.5)}$

$f_0 = 50$ Hz

ii, $f = 11 f_0$

$= 11 \times 50$

$f = 550$ Hz

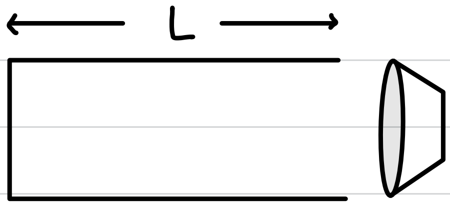
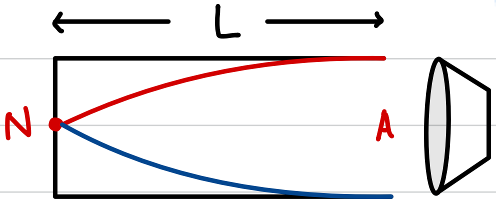
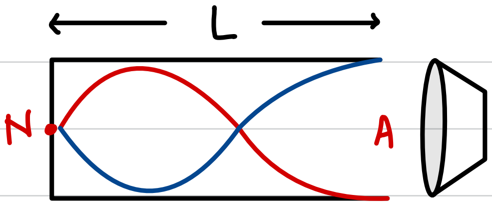
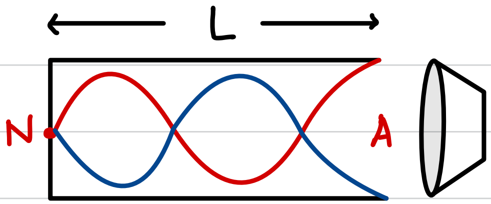
iii, 20^{th} overtone → 21^{st} Harmonic

$f = 21 f_0$

$f = 21 \times 50$

$f = 1050$ Hz

- **ONE END FIXED AND ONE END OPEN**

Shape	Harmonic	Tone	Wavelength	Frequency
	–	–	–	–
	1st Harmonic	Tone	$L = \frac{\lambda}{4}$ $\lambda = 4L$	$v = f\lambda$ so $v = f(4L)$ $f = \frac{v}{4L}$
	2nd Harmonic	1st overtone	$L = \frac{3\lambda}{4}$ $\lambda = \frac{4L}{3}$	$v = f\lambda$ so $v = f\left(\frac{4L}{3}\right)$ $f = \frac{3v}{4L}$
	3rd Harmonic	2nd overtone	$L = \frac{5\lambda}{4}$ $\lambda = \frac{4L}{5}$	$v = f\lambda$ so $v = f\left(\frac{4L}{3}\right)$ $f = \frac{5v}{4L}$

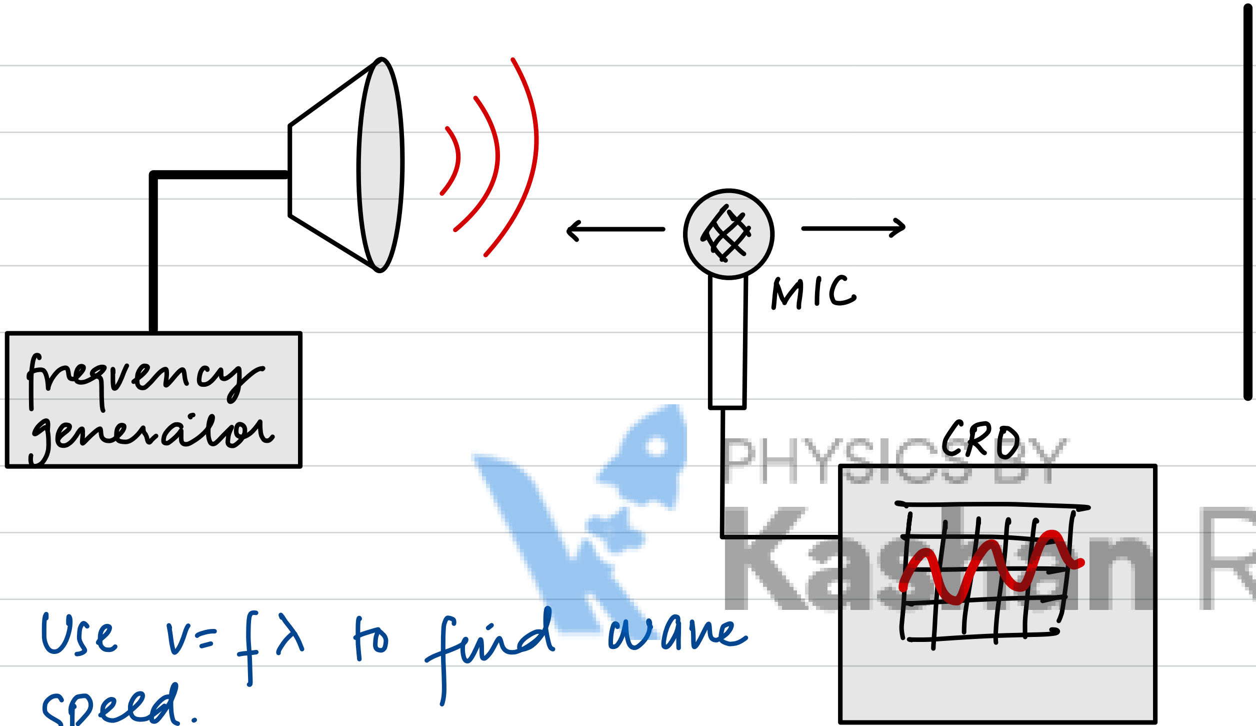
	1st Harmonic	2nd Harmonic	3rd Harmonic	nth Harmonic
	$f_0 = \frac{v}{4L}$	$f = \frac{3v}{4L}$	$f = \frac{5v}{4L}$	$f = \frac{(2n-1)v}{4L}$
	f_0	$f = 3f_0$	$f = 5f_0$	$f = (2n-1)f_0$

where $n = 1, 3, 5, 7 \ldots$ (odd numbers)

Speed of a Stationary Wave

When the incident wave is reflected at one end, the incident and reflected waves interfere forming a stationary wave. The speed of the stationary wave the speed of either the incident or reflected wave.

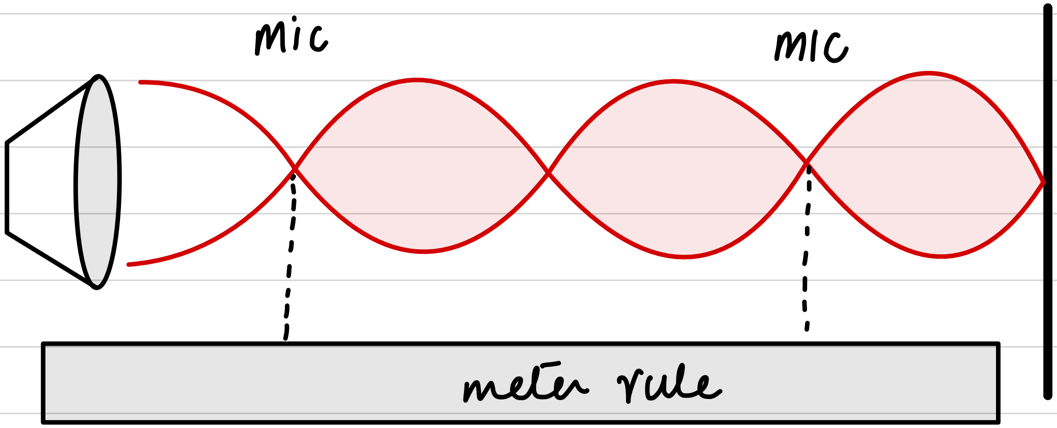
Experiment #1 : Loudspeaker + CRO



- The frequency of sound is adjusted so that a stationary wave is produced b/w speaker and barrier.

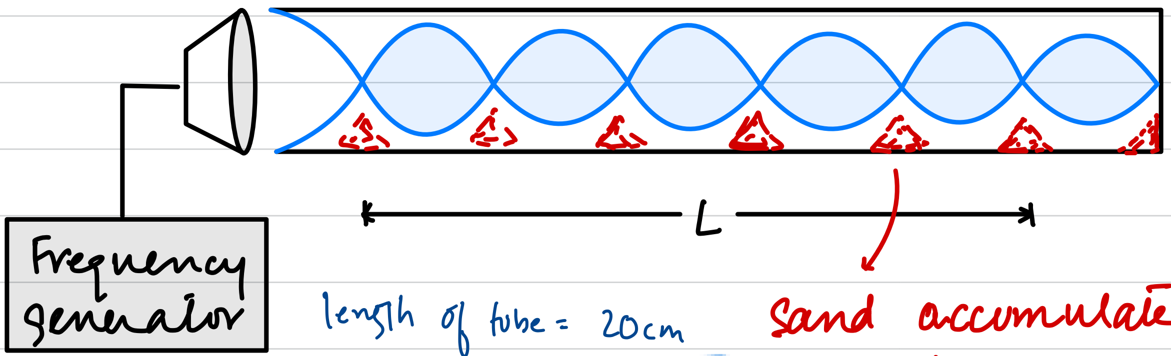
- A mic connected to CRO is used to determine the position of Nodes and Antinodes.

- Use $v = f\lambda$ to find wave speed.



- The length and nodes are used to determine wavelength. Frequency is determined using generator.

Experiment #2 : Sand + Tube experiment



length of tube = 20cm

$\lambda = ??$

$3.25\lambda = 20\ cm$

$\lambda = 6.15\ cm$

Sand accumulated at points.

- Due to the formation of stationary wave, grains of sand are accumulated at regions of Nodes and displaced from regions of Antinode.

- Nodes have minimum vibration
- Antinodes have maximum vibration

- Determine the frequency of sound from frequency generator
- Measure wavelength by determining the length b/w heaps of sand.
- Use $v = f\lambda$ to determine wavespeed.

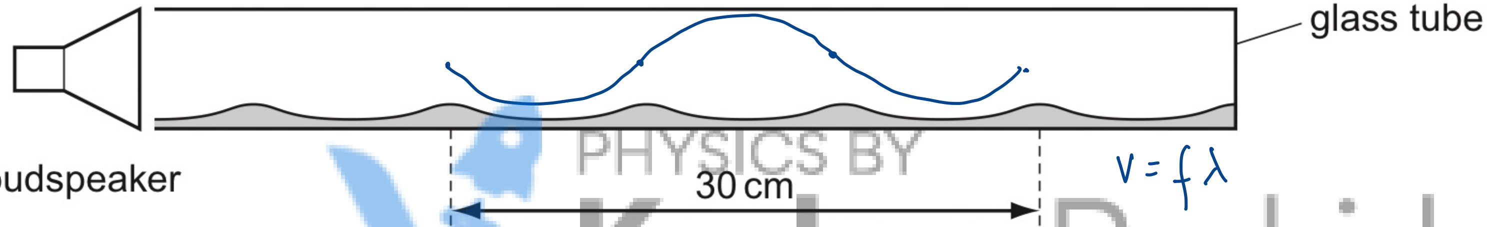
29 A horizontal glass tube, closed at one end, has a layer of dust laid inside it on its lower side. Sound is emitted from a loudspeaker that is placed near the open end of the tube.

The frequency of the sound is varied and, at one frequency, a stationary wave is formed inside the tube so that the dust forms small heaps.

$1.5\lambda = 30$

The distance between four heaps of dust is 30 cm.

$\lambda = 20\text{cm}$



$v = f\lambda$

$330 = f(0.2)$

$\boxed{f = 1650\text{ Hz}}$

The speed of sound in the tube is $330\,\text{m}\,\text{s}^{-1}$.

What is the frequency of the sound emitted by the loudspeaker?

A 1650 Hz **B** 2200 Hz **C** 3300 Hz **D** 6600 Hz

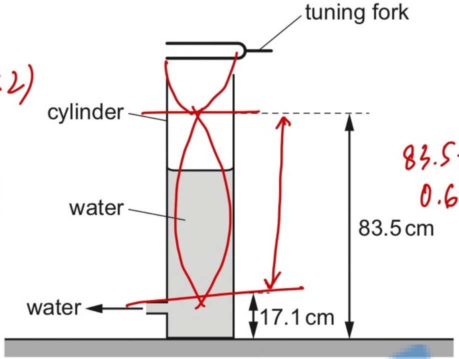
24 A vibrating tuning fork is held above a glass cylinder filled to the top with water. The water level is steadily lowered. A loud sound is first heard when the water level is 83.5 cm above the bench. The next loud sound is heard when the water level is 17.1 cm above the bench.

$v = f\lambda$

$340 = f(0.66 \times 2)$

$f = \frac{340}{(0.66 \times 2)}$

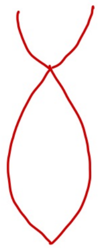
$f = 256\,Hz$



NOT TO SCALE

$83.5 - 17.1$

$0.66\,m$



The speed of sound in air is 340 m s^{-1}.

What is the frequency of the tuning fork?

A 128 Hz **B** 256 Hz **C** 384 Hz **D** 512 Hz

PHYSICS **9702/11**

Paper 1 Multiple Choice **October/November 2012**

1 hour

Additional Materials:

$3(8) = 24\,Hz$

$5(8) = 40\,Hz$

$7(8) = 56\,Hz$

$v = f\lambda$

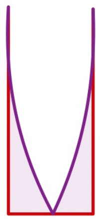
$320 = f(40)$

$f = 8\,Hz$

28 A musical organ produces notes by blowing air into a set of pipes that are open at one end and closed at the other.

What is the lowest frequency of sound produced by a pipe of length 10 m? (The speed of sound in the pipe is 320 m s^{-1}.)

A 4 Hz **B** 8 Hz **C** 16 Hz **D** 32 Hz



$10 = \frac{\lambda}{4}$

$\lambda = 40\,m$

4 (a) State two features of a stationary wave that distinguish it from a progressive wave.

1. Stationary comprises of nodes and antinodes but progressives have crest or troughs.

2. All points between Nodes are in phase in stationary. In progressive, points have different phase angles. [2]

For Examiner's Use

(b) A long tube is open at one end. It is closed at the other end by means of a piston that can be moved along the tube, as shown in Fig. 4.1.

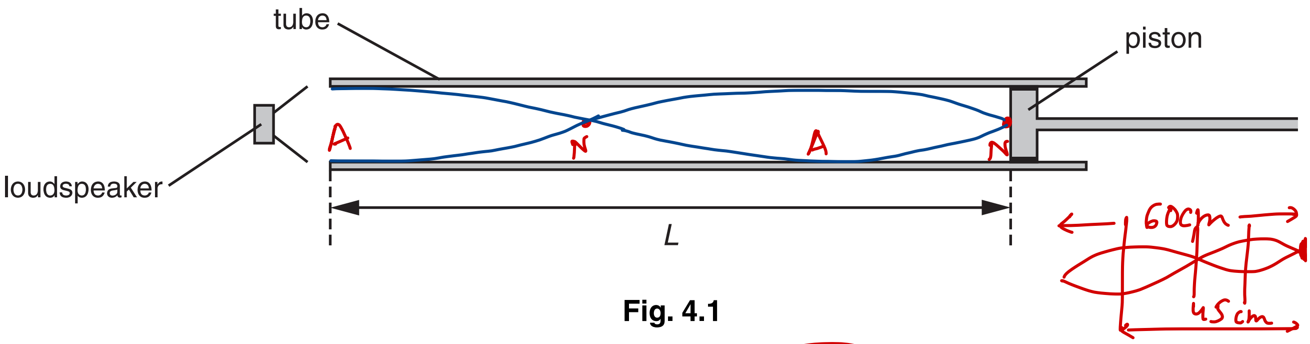


Fig. 4.1

A loudspeaker producing sound of frequency 550 Hz is held near the open end of the tube.
The piston is moved along the tube and a loud sound is heard when the distance *L* between the piston and the open end of the tube is 45 cm.
The speed of sound in the tube is 330 m s^{-1}.

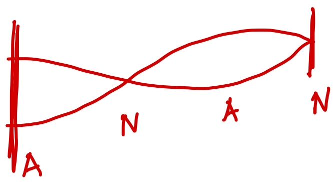
(i) Show that the wavelength of the sound in the tube is 60 cm.

$v = f\lambda$

$330 = 550 \times \lambda$

$\lambda = 0.6\,m$

60 cm



[1]

(ii) On Fig. 4.1, mark all the positions along the tube of

1. the displacement nodes (label these with the letter N),

2. the displacement antinodes (label these with the letter A).

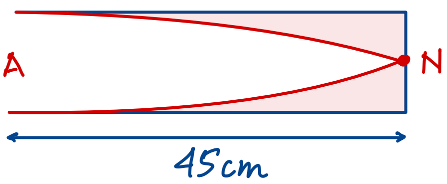
[3]

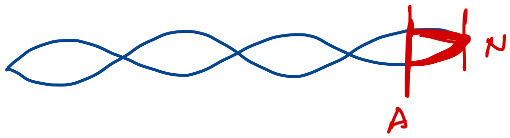
freq. ↓ λ ↑

(c) The frequency of the sound produced by the loudspeaker in **(b)** is gradually reduced.

Determine the lowest frequency at which a loud sound will be produced in the tube of length L = 45 cm.

For Examiner's Use





$45 = \frac{\lambda}{4}$ so $\lambda = 180\text{cm}$ (1.8 m)

$v = f\lambda$

$330 = f(1.8)$

$f = 183\text{ Hz}$

frequency = ... Hz [3]

5 Fig. 5.1 shows a string stretched between two fixed points P and Q.

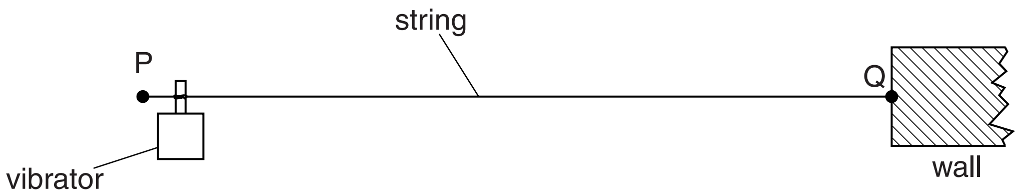


Fig. 5.1

A vibrator is attached near end P of the string. End Q is fixed to a wall. The vibrator has a frequency of 50 Hz and causes a transverse wave to travel along the string at a speed of 40 m s^{-1}.

(a) **(i)** Calculate the wavelength of the transverse wave on the string.

$v = f\lambda$

$40 = 50 \times \lambda$

$\lambda = 0.8\,m$

wavelength = 0.8 m [2]

(ii) Explain how this arrangement may produce a stationary wave on the string.

Incident wave coming from P will reflect at Q and both of them will interfere.

[2]

(b) The stationary wave produced on PQ at one instant of time *t* is shown on Fig. 5.2. Each point on the string is at its maximum displacement.

$1.5\lambda = \text{length}$

$1.5(0.8) = \text{length}$

$\text{length} = 1.2\,m$

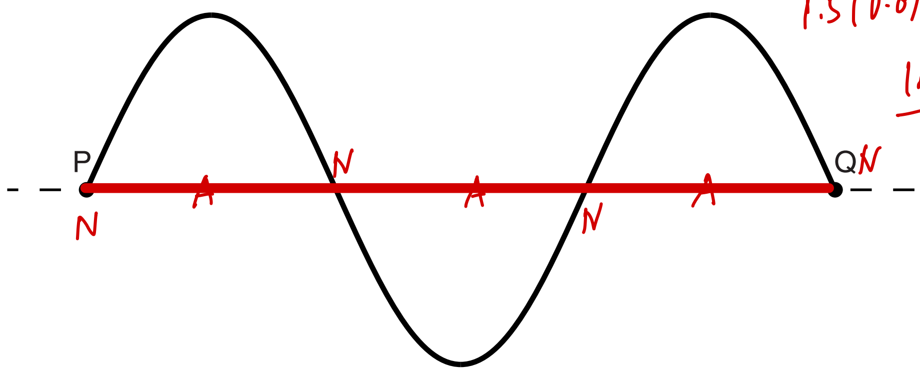


Fig. 5.2 (not to scale)

(i) On Fig. 5.2, label all the nodes with the letter **N** and all the antinodes with the letter **A**. [2]

(ii) Use your answer in **(a)(i)** to calculate the length of string PQ.

For Examiner's Use

length = ... m [1]

+10 ms

(iii) On Fig. 5.2, draw the stationary wave at time (t + 5.0 ms). Explain your answer.

$T = \frac{1}{f}$ $T = \frac{1}{50}$ $T = 0.02s$ $\boxed{T = 20ms}$ $5ms \rightarrow \frac{T}{4}$

.... [3]