WORK ENERGY AND POWER

Work

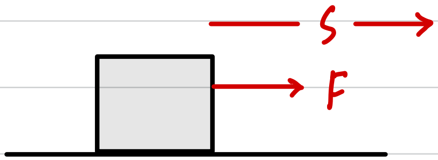
The product of force and displacement parallel to force.

$W = F \times s$

SI Unit: Joules (J)

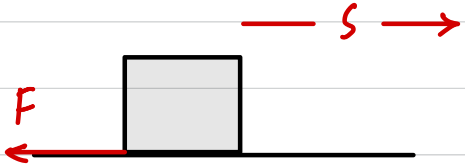
Scalar Quantity

Workdone by the force

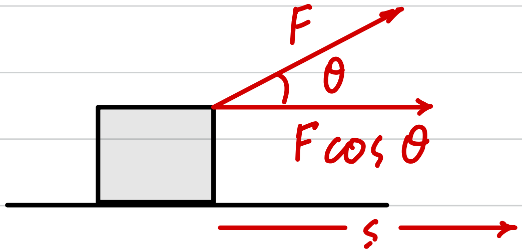


same direction

Workdone against force

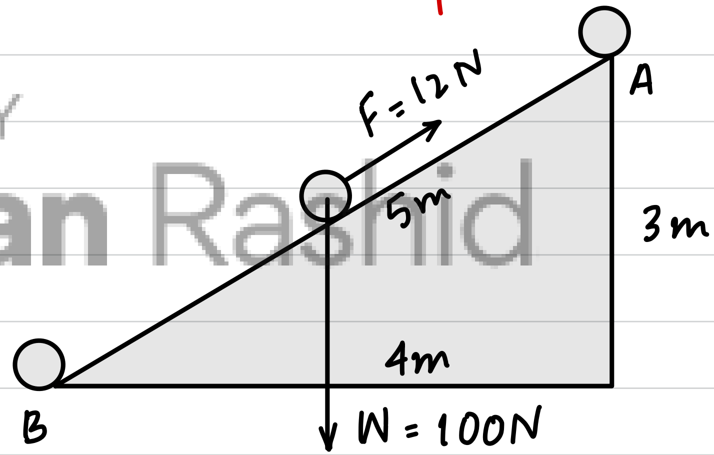


opposite direction



$W = F\cos\theta \times s$

If force is at an angle, determine the component of that force in the direction of motion.



Ball rolls from A to B.

a) Workdone by gravity (Weight)

$W = F \times s$

$W = 100 \times 3$

$W = 300\,J$

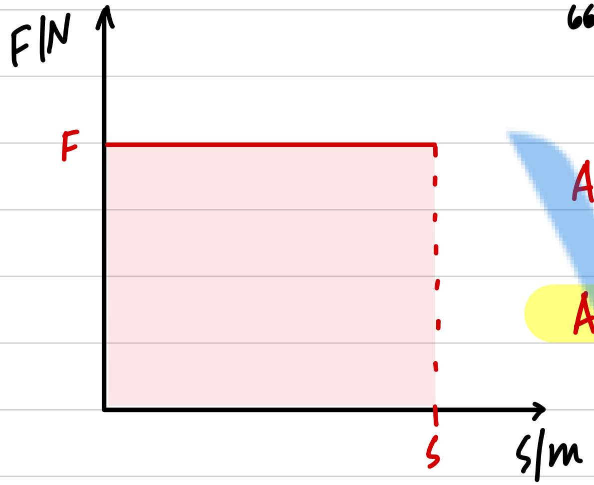
b) Workdone against friction

$W = F \times s$

$= 12 \times 5$

$W = 60\,J$

Graphs



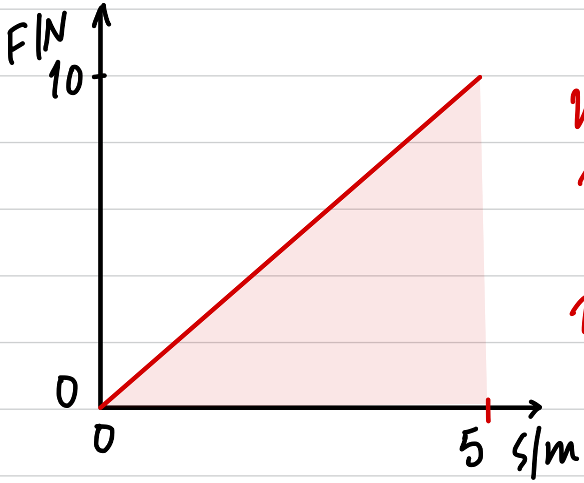
"Constant Force"

Area = $L \times b$

$= F \times s$

Area = Workdone

☑ Area should be between graph and displacement axis !

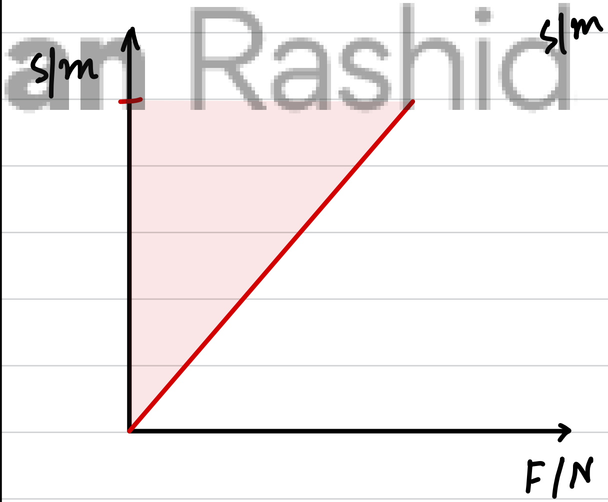


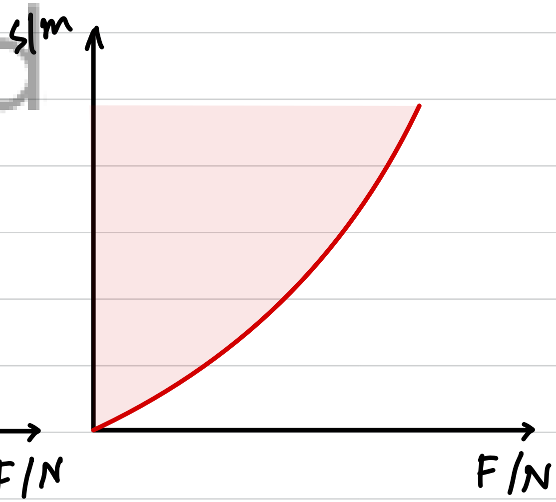
Workdone = Area

$W = \frac{1}{2} bh$

$W = \frac{1}{2}(5)(10)$

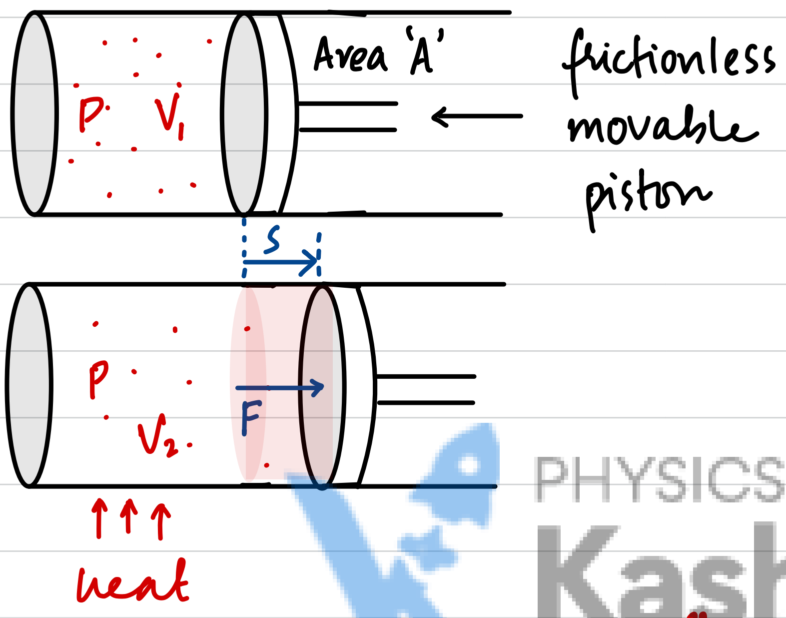
$= 25\,J$





$W = \int F \cdot ds$

Workdone by gas



"Piston moves at constant pressure"

$W = F \times s$

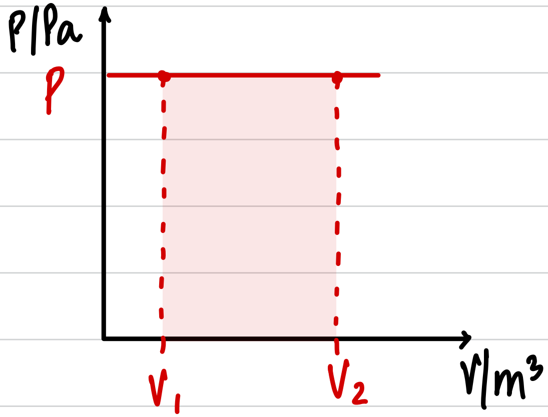
as $P = \frac{F}{A}$ so $F = P \times A$

$W = P \times A \times s$

as $V = A \times l$ so $A \times s = \Delta V$

$W = P \times \Delta V$ or $W = P(V_2 - V_1)$

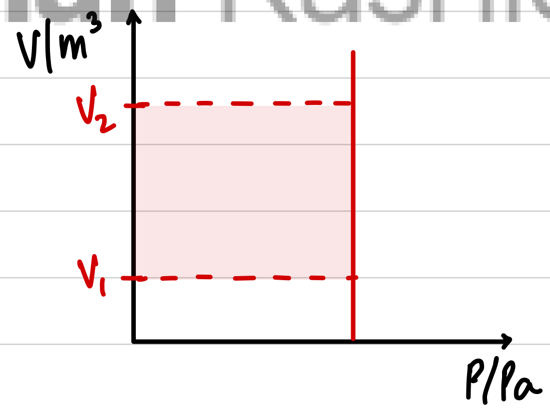
Area should be between graph and Volume axis!

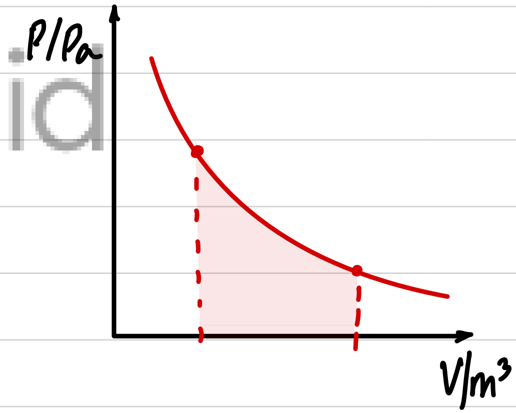


Area = Workdone

$L \times b$

$P \times (V_2 - V_1)$





Energy

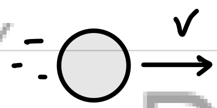
1. Kinetic Energy
2. Potential Energy
   a) Gravitational
   b) Elastic Potential
   c) Electric Potential
3. Heat Energy
4. Light Energy
5. Sound Energy
6. Chemical Energy
7. Internal Energy
   The sum of microscopic K.E and P.E of molecules due to their random motion.

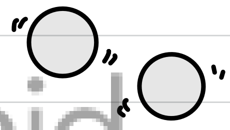
Kinetic Energy

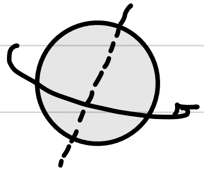
Energy possessed by a body due to its motion.

Kinetic Energy

Translational | Vibrational | Rotational



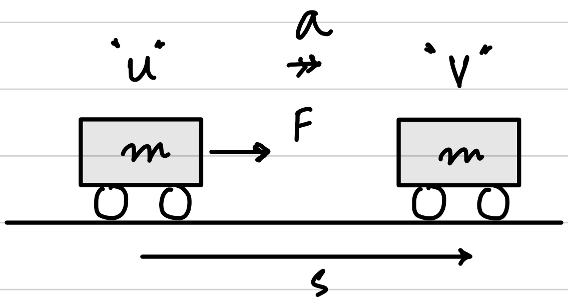




$$E_K = \frac{1}{2}mv^2$$

- SI Unit : Joules (J)
- Scalar

Deriving $K.E = \frac{1}{2}mv^2$



Workdone on body = Gain in Kinetic Energy

$$F \times s = \Delta K.E$$

$$\Delta K.E = ma \times s$$

as $v^2 - u^2 = 2as$ so $s = \frac{v^2 - u^2}{2a}$

$$\Delta K.E = m\cancel{a} \times \left(\frac{v^2 - u^2}{2\cancel{a}}\right)$$

$$\boxed{\Delta K.E = \frac{1}{2}m(v^2 - u^2)}$$

change in K.E !!

if $u = 0$, then $K.E_{initial} = 0$

$$\boxed{K.E = \frac{1}{2}mv^2}$$

K.E in terms of momentum as mass

as $p = mv$ so $v = \frac{p}{m}$

$$K.E = \frac{1}{2}m.v^2$$

$$K.E = \frac{1}{2}m.\left(\frac{p}{m}\right)^2$$

$$K.E = \frac{1}{2}\cancel{m}.\frac{p^2}{m^{\cancel{2}}}$$

$$\boxed{K.E = \frac{p^2}{2m}}$$

$$K.E = \frac{1}{2}mv^2$$

$K.E \propto m$

$m - E$

$2 \times m - 2E$

$4 \times m - 4E$

$\frac{m}{3} - \frac{E}{3}$

$K.E \propto v^2$

$v - E$

$2 \times v - (2)^2 \times E = 4E$

$3 \times v - (3)^2 \times E = 9E$

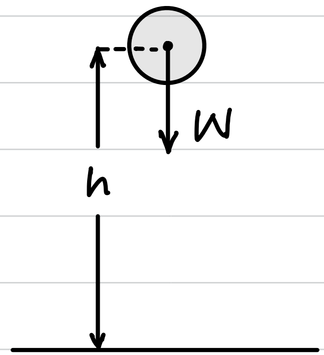
$\frac{v}{3} - \frac{E}{(3)^2} = \frac{E}{9}$

m	v	E
2 m	2v	$E \times 2 \times (2)^2 = 8E$
$\frac{m}{2}$	2v	$\frac{E}{2} \times (2)^2 = 2E$
3m	$\frac{v}{2}$	$\frac{E \times 3}{(2)^2} = \frac{3E}{4}$
$\frac{m}{3}$	$\frac{v}{2}$	$\frac{E}{3 \times (2)^2} = \frac{E}{12}$

Gravitational Potential Energy

Energy possessed by a body due to its position in a gravitational field.

$P.E = mgh$



ball raised to a height 'h'

Workdone against gravity = Gain in G.P.E

$G.P.E = F \times s$

$= W \times h$

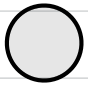
$\boxed{G.P.E = mgh}$

> Workdone against gravity raises the center of Gravity

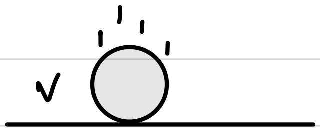
"HEIGHT IS ALWAYS MEASURED TILL THE CENTER OF GRAVITY".

Work- Energy Equation

1\. released from rest

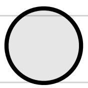


negligible air resistance

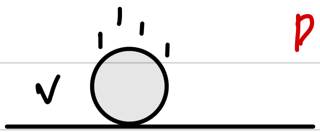


P.E $\longrightarrow$ K.E

2\. released from rest

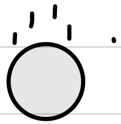


air resistance exists

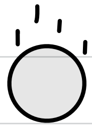


P.E $\rightarrow$ K.E + heat

3\.



falling at terminal velocity (speed constant)

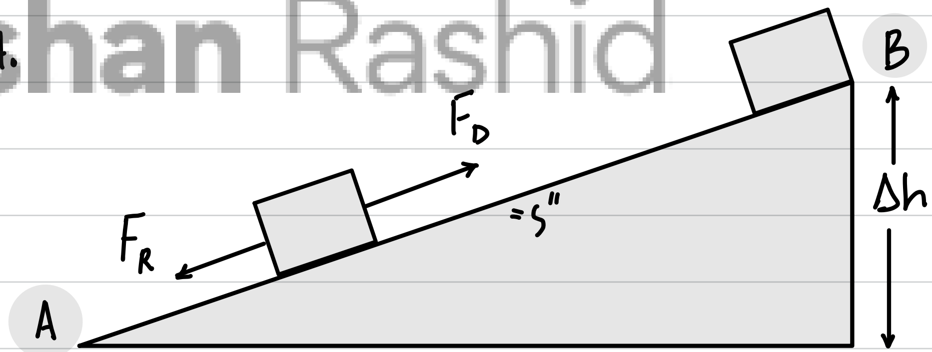


Air resistance exists

P.E $\rightarrow$ heat

Its K.E is constant and no more energy gets converted to K.E.

4\.



final velocity = v

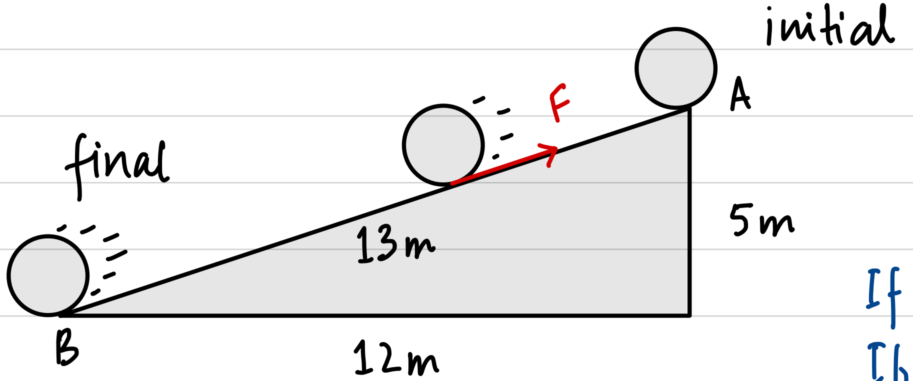
initial velocity = u

$$\boxed{\begin{array}{c}\text{NET WORKDONE = CHANGE IN MECHANICAL ENERGY} \\ W.D_{DF} - W.D_{RF} = \Delta P.E + \Delta K.E\end{array}}$$

F_D : driving force (Engine, tension, pull, push)

F_R : resistive force (Friction, Air resistance)

Example



A ball of mass 5 kg rolls downhill from A to B. It starts from rest and experiences a constant friction force of 5N. Calculate the speed of ball at B.

If height decreases, P.E decreases $\Delta P.E = -ve$ | $\Delta h = -ve$

If height increases, P.E increases $\Delta P.E = +ve$ $\Delta h = +ve$

Method #1

$$W.D_{DF} - W.D_{RF} = \Delta P.E + \Delta K.E$$

$$0 - (F_R \times s) = mg\Delta h + \frac{1}{2}m(v^2 - u^2)$$

$$-(5 \times 13) = (5 \times 9.8)(-5)^2 + \frac{1}{2}(5)(v^2 - 0)$$

$$-65 = -245 + 2.5v^2$$

$$v = 8.485 \simeq 8.5\,ms^{-1}$$

Method # 2

• Loss in $P.E = mg\Delta h$

$= (5)(9.8)(5)$

$= 245J$

• W.D against friction

$W.D = F \times s$

$= 5 \times 13$

$= 65J$

lost as heat!

gain in K.E

$245 - 65 = 180J$

$$\Delta K.E = \frac{1}{2}m(v^2 - u^2)$$

$$180 = \frac{1}{2}(5)(v^2 - 0)$$

$$v = 8.5\,ms^{-1}$$

Power

It is the rate at which work is done.

It is the rate at which energy is converted.

$P = \frac{W}{t}$ $\qquad P = \frac{E}{t}$

$$\boxed{P = \frac{\Delta E}{\Delta t}}$$

- SI Unit : Watt (W)
  J/s
- Scalar Quantity

$P = \frac{W}{t}$ and $W = F \times s$

so $P = \frac{F \times s}{t}$

$$\boxed{P = F \times v}$$

if F : driving force
P : input power from source

if F : resistive force
P : rate at which energy is lost

if F : net force
P : net power output

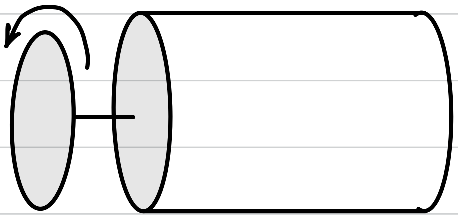
$P = \frac{E}{t}$ and $E_K = \frac{1}{2}mv^2$

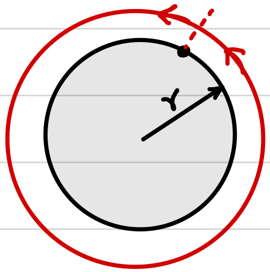
for a moving fluid at constant speed

$P = \frac{1 mv^2}{2t}$

as $\dot{m} = \frac{m}{t}$ (mass flowrate)

$$\boxed{P = \frac{1}{2}\dot{m}v^2}$$





$$v = \frac{d}{t}$$

$$v = \frac{2\pi r \times N}{t}$$

here "N" are the no. of rotations performed in time "t".

✓ $P = F \times v$

Efficiency

The ratio of output to input

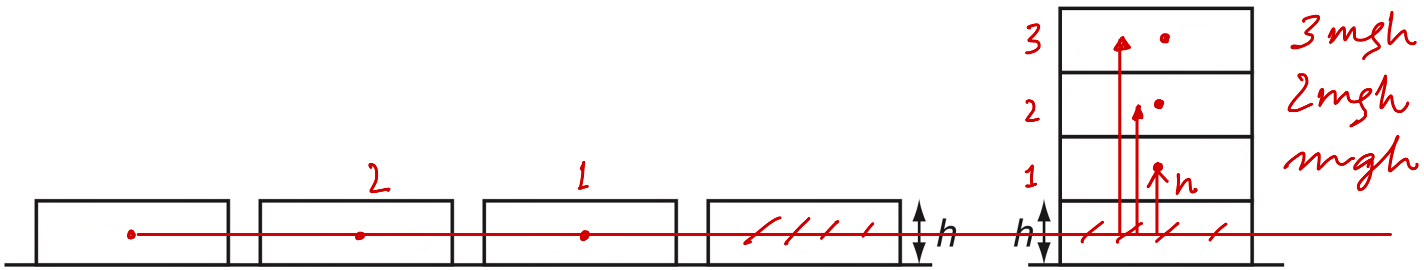
$$\boxed{eff = \frac{output}{input} \times 100}$$

✓ $eff = \frac{P_{out}}{P_{in}}$ ✓ $eff = \frac{W_{out}}{W_{in}}$ ✓ $eff = \frac{E_{out}}{E_{in}}$

✗ $eff = \frac{P_{out}}{W_{in}}$ ✓ $eff = \frac{W_{out}}{E_{in}}$ ✓ $eff = \frac{E_{out}}{W_{in}}$

- It is a unitless quantity.

17 Initially, four identical uniform blocks, each of mass m and thickness h, are spread on a table.



How much work is done on the blocks in stacking them on top of one another?

A 3 mgh **B** 6 mgh **C** 8 mgh **D** 10 mgh

$mgh + 2mgh + 3mgh$

Additional Materials:

$W = mg$

15 A uniform solid block has weight 500 N, width 0.4 m and height 0.6 m. The block rests on the edge of a step of depth 0.8 m, as shown.

$h_f = 0.2\,m$

$h_i = 1.1\,m$

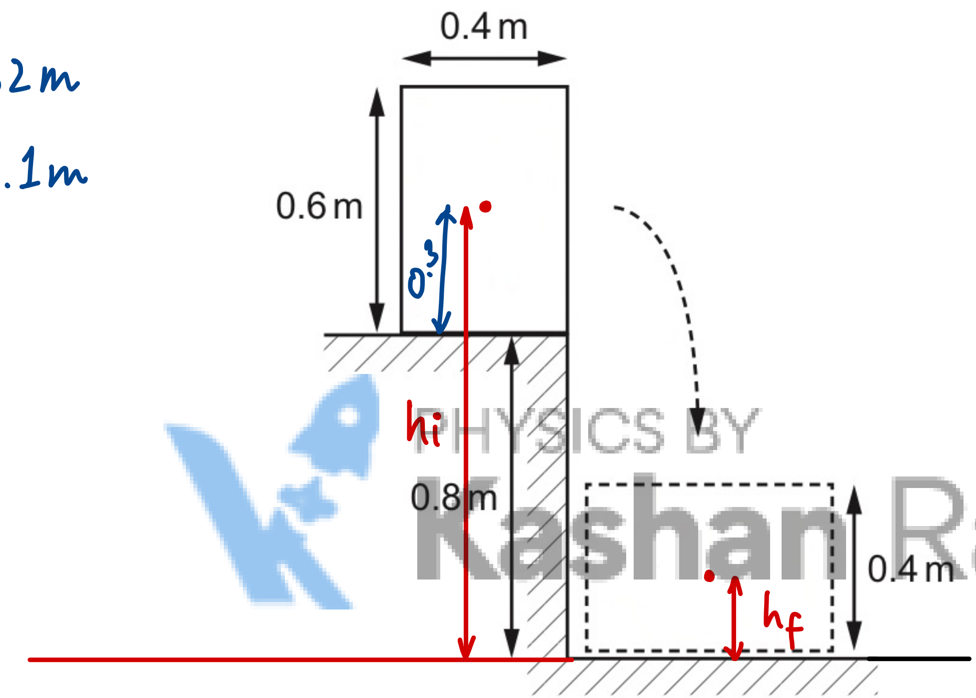
$\Delta E_p = mg\Delta h$

$\Delta E_p = mg(h_f - h_i)$

$= 500(0.2 - 1.1)$

$= 500(0.9)$

$= 450\,J$



The block is knocked over the edge of the step and rotates through 90° before coming to rest with the 0.6 m edge horizontal.

What is the change in gravitational potential energy of the block?

A 300 J **B** 400 J **C** 450 J **D** 550 J

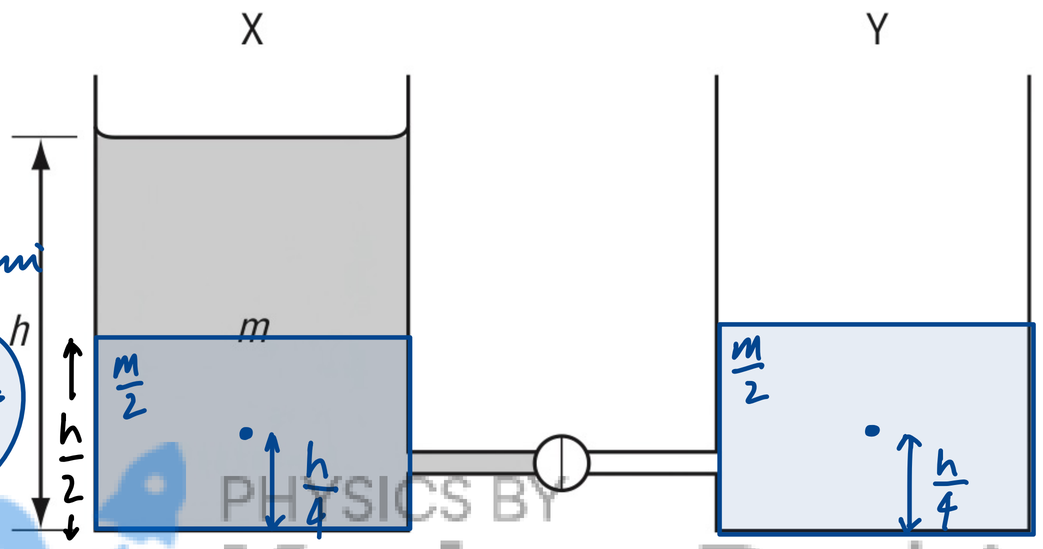
height is taken from base to the center of gravity of body.

15 The diagram shows two identical vessels X and Y connected by a short pipe with a tap.

$$P.E_{initial} = mg\left(\frac{h}{2}\right) = \frac{mgh}{2}$$

change in P.E = final - initial

$$\frac{mgh}{4} - \frac{mgh}{2} \Rightarrow -\frac{mgh}{4}$$



$$P.E_{final} = \left(\frac{m}{2}\right)g\left(\frac{h}{4}\right) \rightarrow X + \left(\frac{m}{2}\right)g\left(\frac{h}{4}\right) \rightarrow Y$$

$$P.E_{final} = \frac{mgh}{8} + \frac{mgh}{8} = \frac{mgh}{4}$$

Initially, X is filled with water of mass m to a depth h, and Y is empty.

When the tap is opened, water flows from X to Y until the depths of water in both vessels are equal.

How much potential energy is lost by the water during this process? (g = acceleration of free fall)

A 0 **B** $\frac{mgh}{4}$ **C** $\frac{mgh}{2}$ **D** mgh

14 A uniform solid cuboid of concrete of dimensions 0.50 m × 1.20 m × 0.40 m and weight 4000 N rests on a flat surface with the 1.20 m edge vertical as shown in diagram 1.

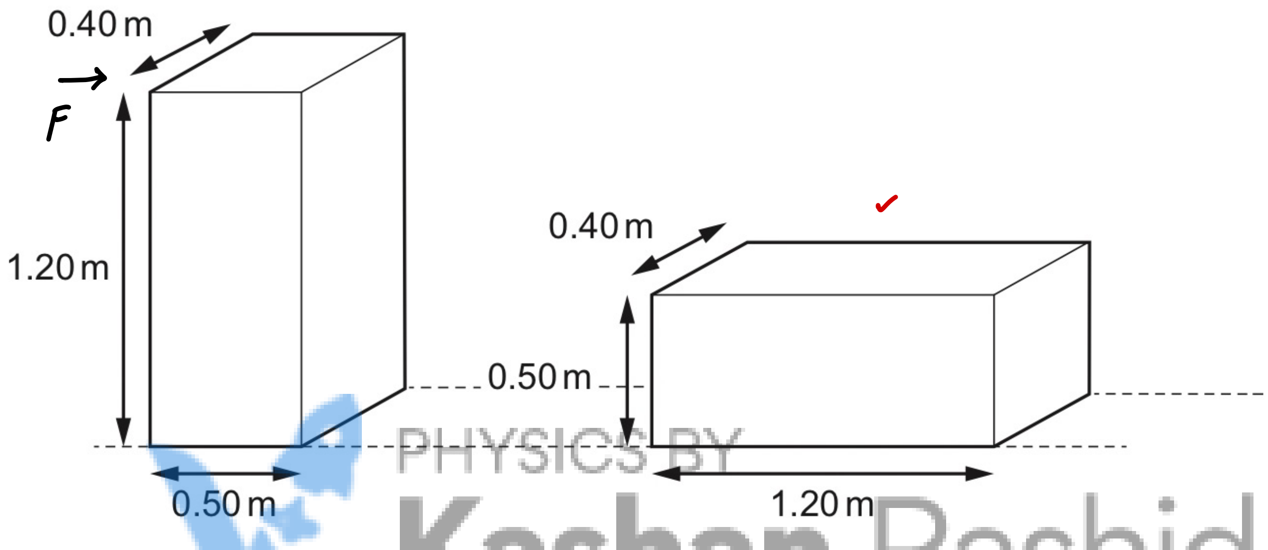


diagram 1 diagram 2

What is the minimum energy required to roll the cuboid through 90° to the position shown in diagram 2 with the 0.50 m edge vertical?

A 200 J **B** 400 J **C** 1400 J **D** 2600 J

$h^2 = p^2 + b^2$

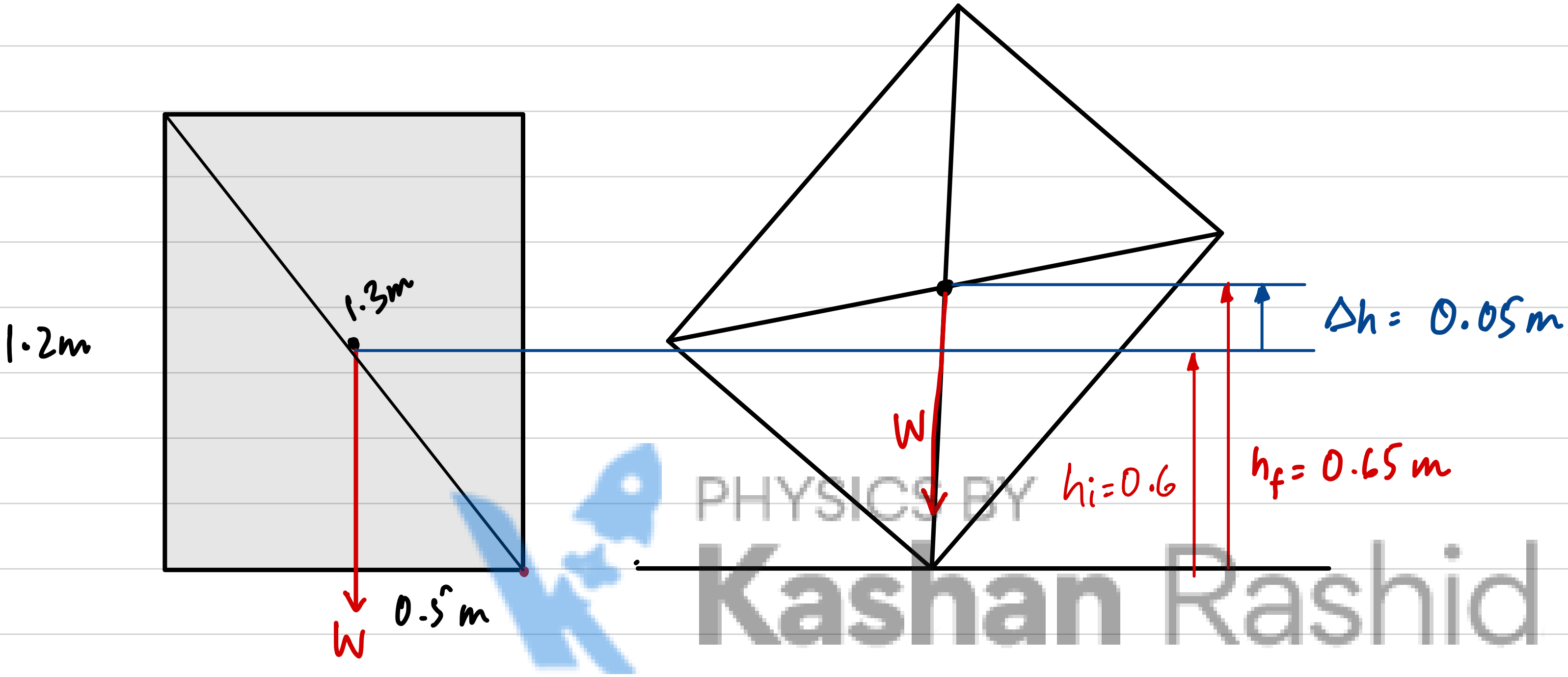
$h^2 = (1.2)^2 + (0.5)^2$

$\boxed{h = 1.3m}$

$\Delta h = h_f - h_i$

$= 0.65 - 0.6$

$= 0.05$



Minimum workdone = Gain in G.P.E before toppling over.

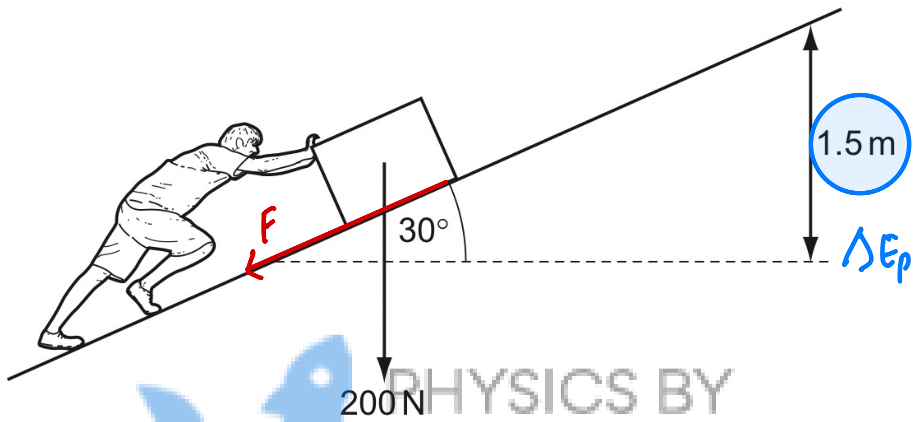
$= mg\Delta h$

$= 4000 \times 0.05$

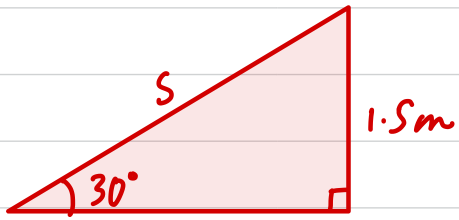
$= 200\ J$

no change in K.E

14 A box of weight 200 N is pushed so that it moves at a steady speed along a ramp, through a height of 1.5 m. The ramp makes an angle of 30° with the ground. The frictional force on the box is 150 N while the box is moving.



$\Delta E_p = mg\Delta h$



$\sin\theta = \frac{p}{h}$

$\sin 30° = \frac{1.5}{s}$

$s = 3\,m$

What is the work done by the person?

A 150 J **B** 300 J **C** 450 J **D** 750 J

$W.D_{DF} - W.D_{RF} = \Delta E_p + \Delta E_K$

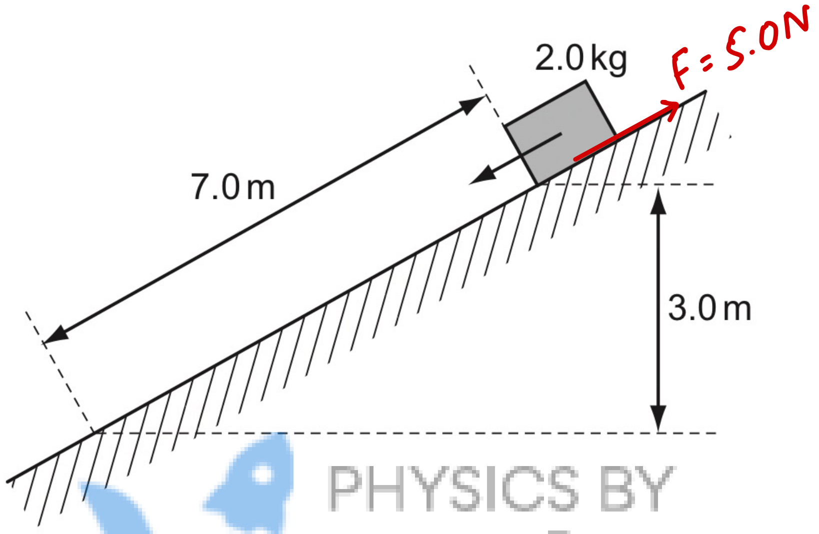
$W.D_{DF} - (150 \times s) = mg\Delta h + 0$

$W.D_{DF} - (150 \times s) = (200)(1.5)$

$W.D_{DF} = (200 \times 1.5) + (150 \times 3)$

$W.D_{DF} = 750\,J$

15 A block of mass 2.0 kg is released from rest on a slope. It travels 7.0 m down the slope and falls a vertical distance of 3.0 m. The block experiences a frictional force parallel to the slope of 5.0 N.



What is the speed of the block after falling this distance?

A 4.9 m s^{-1} **B** 6.6 m s^{-1} **C** 8.6 m s^{-1} **D** 10.1 m s^{-1}

$$\cancel{W.D_{DF}}^{\,0} - W.D_{RF} = \Delta E_p + \Delta E_K$$

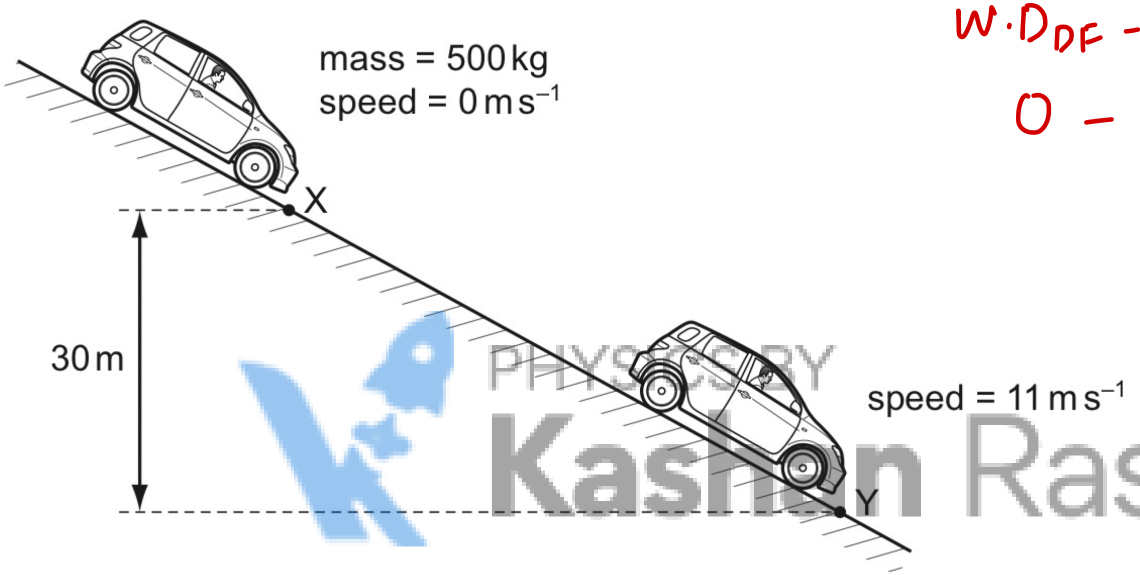
$$0 - (5 \times 7) = -mgh + \frac{1}{2}m(v^2 - u^2)$$

$$-35 = -(2)(9.8)(3) + \frac{1}{2}(2)(v^2 - 0)$$

$$v = 4.9\ ms^{-1}$$

16 A car of mass 500 kg is at rest at point X on a slope, as shown.

The car's brakes are released and the car rolls down the slope with its engine switched off. At point Y the car has moved through a vertical height of 30 m and has a speed of 11 m s^{-1}.



$W.D_{DF} - W.D_{RF} = \Delta E_p + \Delta E_k$

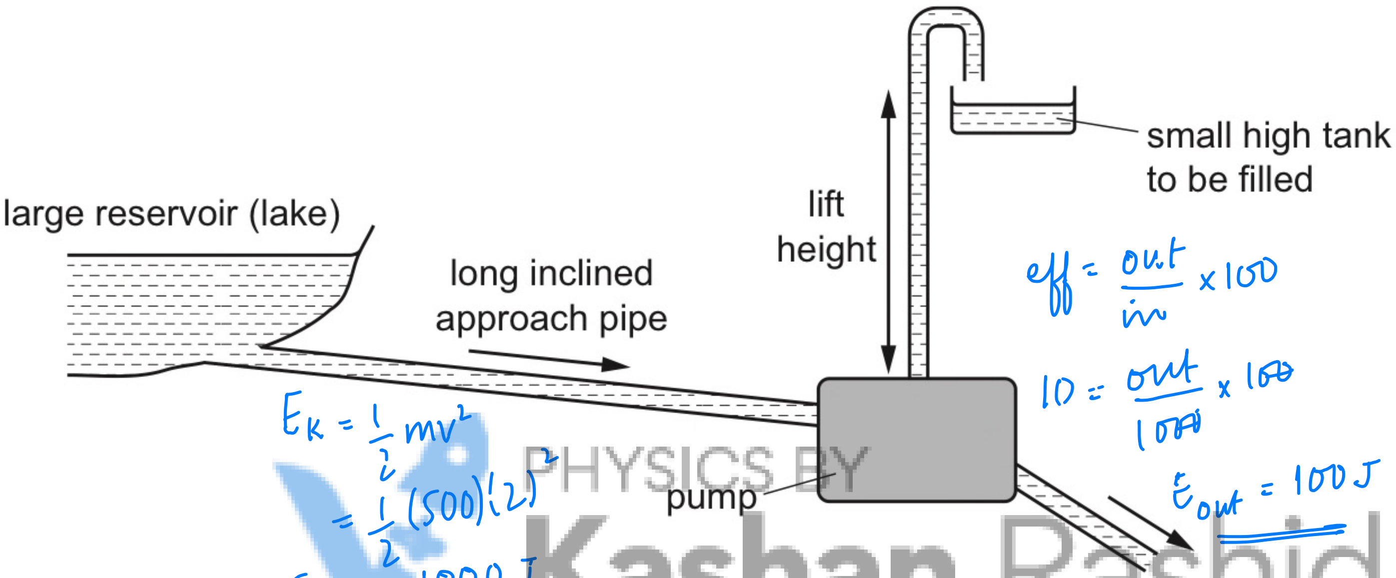
$0 - W.D_{RF} = -(500)(9.8)(30) + \frac{1}{2}(500)(11^2 - 0^2)$

$W.D_{RF} = 116750$ J

What is the energy dissipated by frictional forces when the car moves from X to Y?

A 3.0×10^4 J **B** 1.2×10^5 J **C** 1.5×10^5 J **D** 1.8×10^5 J

18 The diagram shows a pump called a hydraulic ram.



$E_K = \frac{1}{2}mv^2$

$= \frac{1}{2}(500)(2)^2$

$E_K = 1000\,J$

$eff = \frac{out}{in} \times 100$

$10 = \frac{out}{1000} \times 100$

$E_{out} = 100\,J$

$E_p = mgh$

$100 = m \times 9.8 \times 15$

$m = 0.68\,kg$

In one such pump the long approach pipe holds 500 kg of water. A valve shuts when the speed of this water reaches 2.0 m s^{-1} and the kinetic energy of this water is used to lift a small quantity of water by a height of 15 m.

The efficiency of the pump is 10%.

Which mass of water could be lifted 15 m?

A 0.15 kg **B** 0.68 kg **C** 1.5 kg **D** 6.8 kg

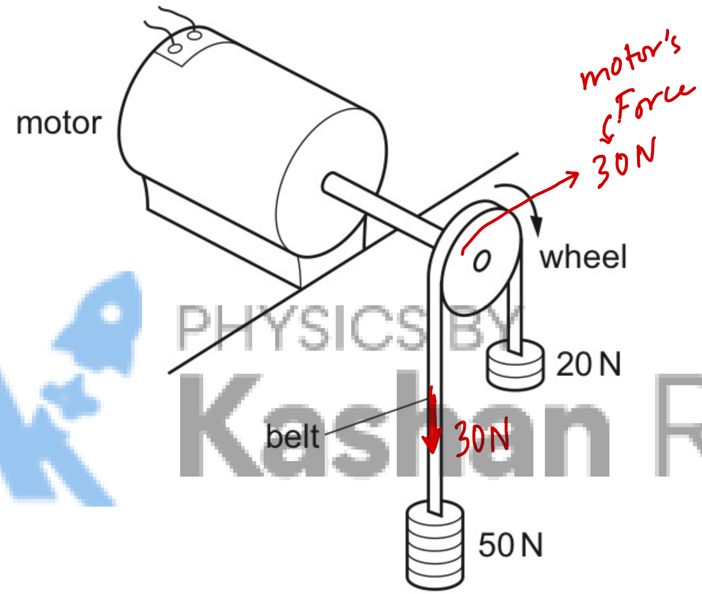
19 The diagram shows an arrangement used to find the output power of an electric motor.

The wheel attached to the motor's axle has a circumference of 0.5 m and the belt which passes over it is stationary when the weights have the values shown.

$P = F \times V$

$= 30 \times 10$

$P = 300\ W$



$V = \frac{d}{t} = \frac{0.5 \times 20}{1}$

$V = 10\ ms^{-1}$

When the wheel is making 20 revolutions per second, what is the output power of the motor?

A 300 W **B** 500 W **C** 600 W **D** 700 W

Oct Nov 2010/P11/Q.18

18 The diagram shows a lift system in which the elevator (mass m_1) is partly counterbalanced by a heavy weight (mass m_2).

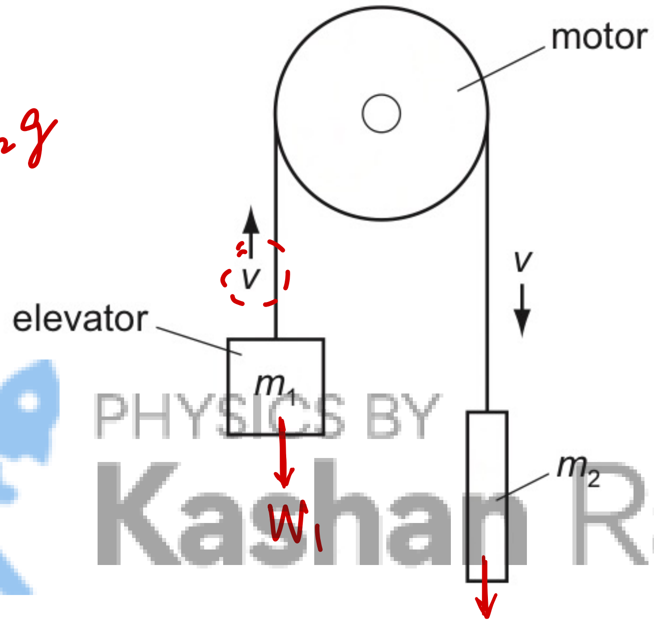
$F = W_1 - W_2$

$F = m_1 g - m_2 g$

$P = F \times v$

$= (m_1 g - m_2 g) \times v$

$= (m_1 - m_2) g v$



Equilibrium exists

↳ no acceleration

↳ force by motor and tension in belt is same.

At what rate does the motor provide energy to the system when the elevator is rising at a steady speed v? (g = acceleration of free fall)

A $\frac{1}{2} m_1 v^2$

B $\frac{1}{2}(m_1 - m_2)v^2$

C $m_1 g v$

D $(m_1 - m_2) g v$

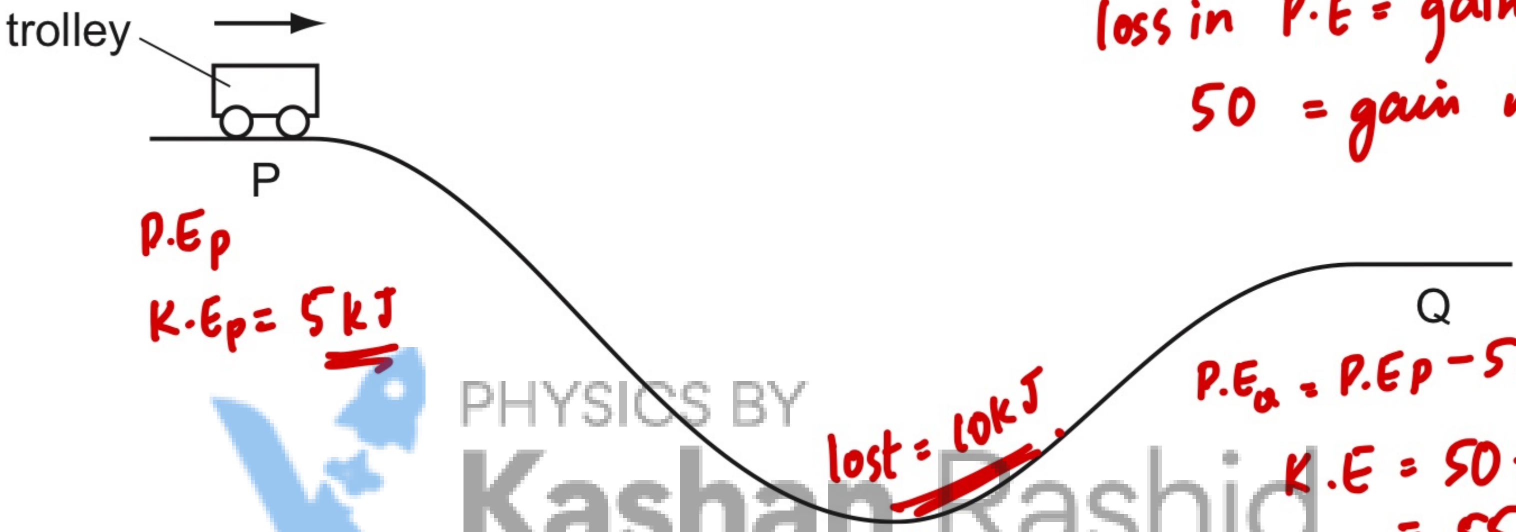
$P = \frac{E}{t}$ → P.E gained by system

$P = \frac{+m_1 g h - m_2 g h}{t}$

$P = \frac{(m_1 - m_2) g h}{t}$

$P = (m_1 - m_2) g v$ ←

18 A trolley runs from P to Q along a track. At Q its potential energy is 50 kJ less than at P.



loss in P.E = gain in K.E
50 = gain in K.E

$P.E_P$
$K.E_P = 5\,kJ$

lost = 10 kJ

$P.E_Q = P.E_P - 50$
already
K.E = 50 + 5
= 55 kJ

At P, the kinetic energy of the trolley is 5 kJ. Between P and Q, the work the trolley does against friction is 10 kJ.

10 kJ lost so now
45 kJ

What is the kinetic energy of the trolley at Q?

A 35 kJ **B** 45 kJ **C** 55 kJ **D** 65 kJ