Transformation

27-03-23

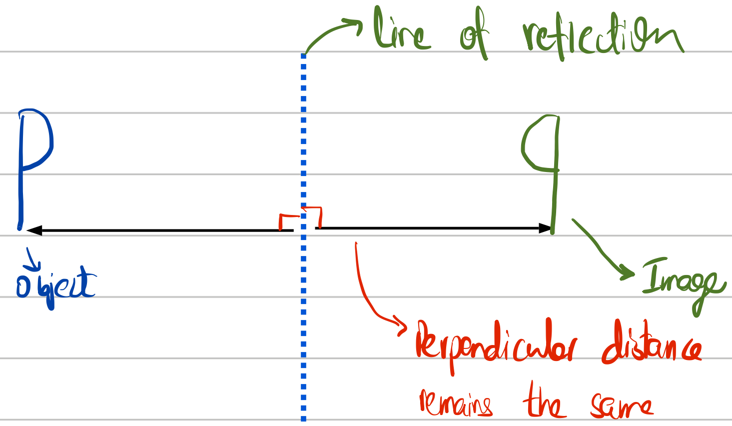
- → Reflection (Easy)
- → Rotation (Tricky)
- → Translation (Easiest)
- → Enlargement (Medium)
- → Shear ✗ (2017 & Before)
- → Stretch ✗ (2017 & Before)

→ Tranformation Matrices (O Level Only)

① Reflection

① Shape & size remains the same (inverted)

② Image & the object are equidistant from the line of reflection



Defined by:

① Line of Reflection

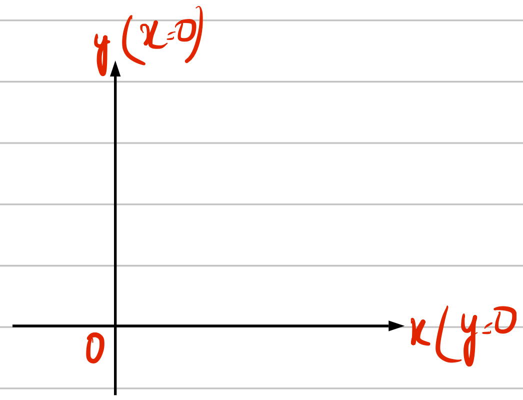
Standard Cases

① Along the x-axis / $y=0$

② Along the y-axis / $x=0$

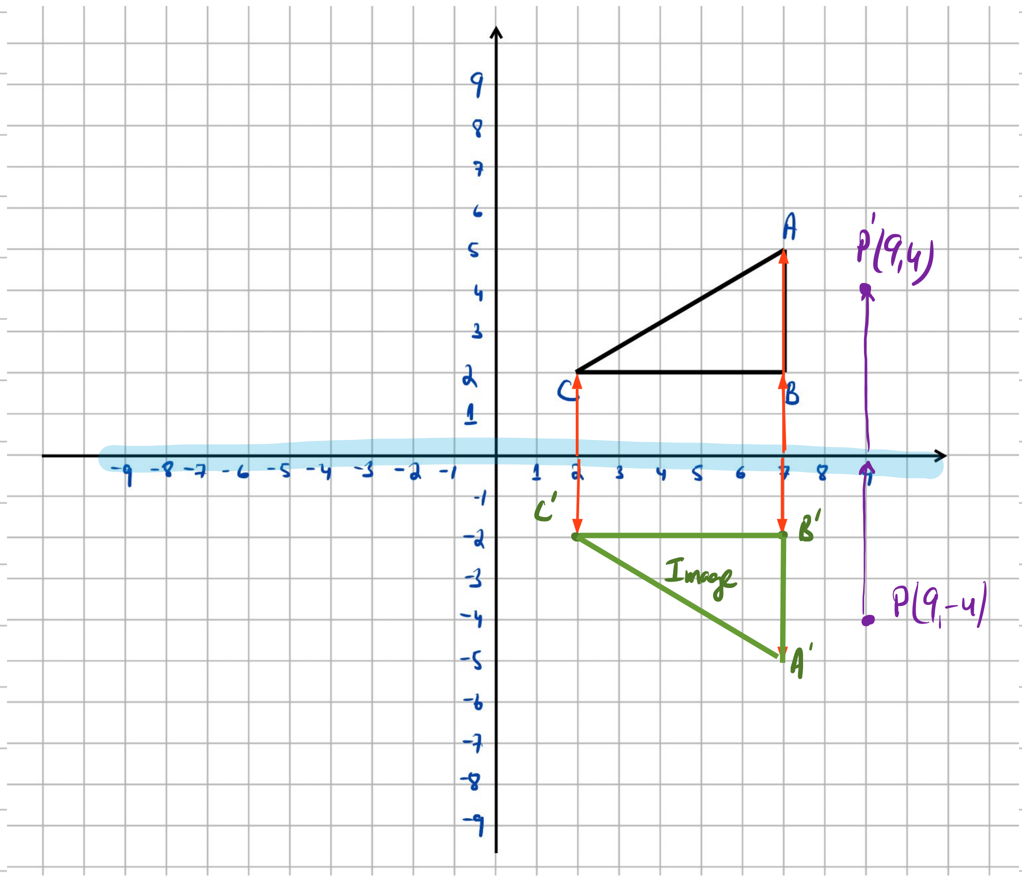
③ Along the line $y=x$

④ Along the line $y=-x$



① Along the x-axis or y=0

Reflect ΔABC along the line $y=0$



$A(7,5) \rightarrow A'(7,-5)$

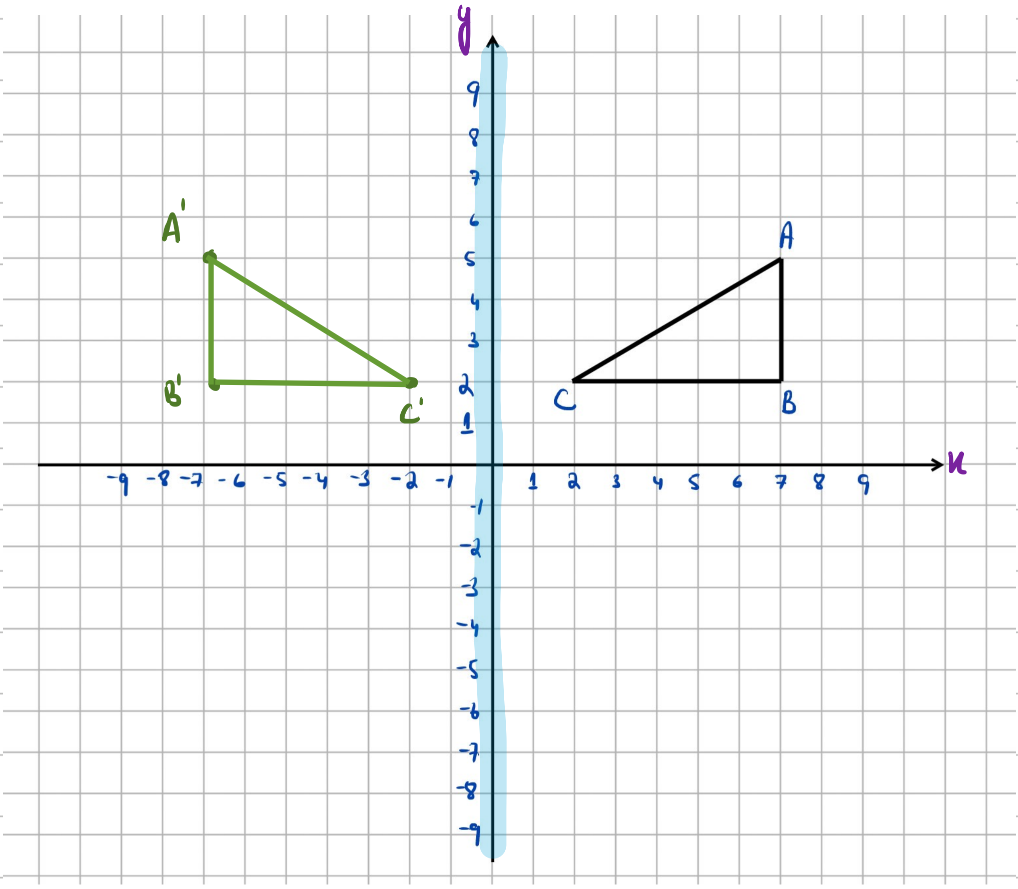
$B(7,2) \rightarrow B'(7,-2)$

$C(2,2) \rightarrow C'(2,-2)$

$P(x,y) \rightarrow P'(x,-y)$

② Along the y-axis

Reflect ΔABC along the line $x=0$



$A(7,5) \longrightarrow A'(-7,5)$

$B(7,2) \longrightarrow B'(-7,2)$

$C(2,2) \longrightarrow C'(-2,2)$

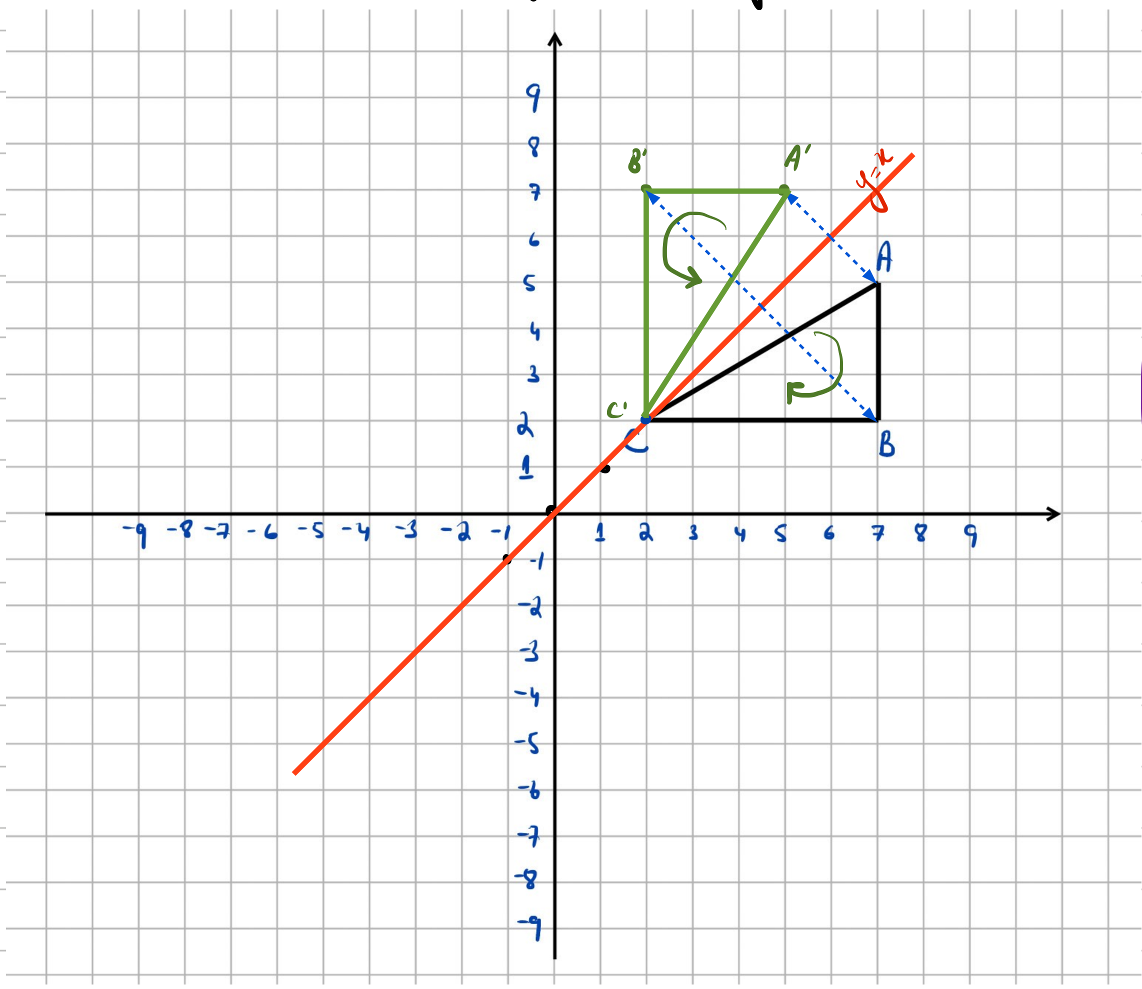
$P(x,y) \rightarrow P'(-x,y)$

③ $y = x$

Reflect ΔABC along the line $y = x$

$y = x$

x	-1	0	1
y	-1	0	1



$A(7, 5) \longrightarrow A'(5, 7)$

$B(7, 2) \longrightarrow B'(2, 7)$

$C(2, 2) \longrightarrow C'(2, 2)$

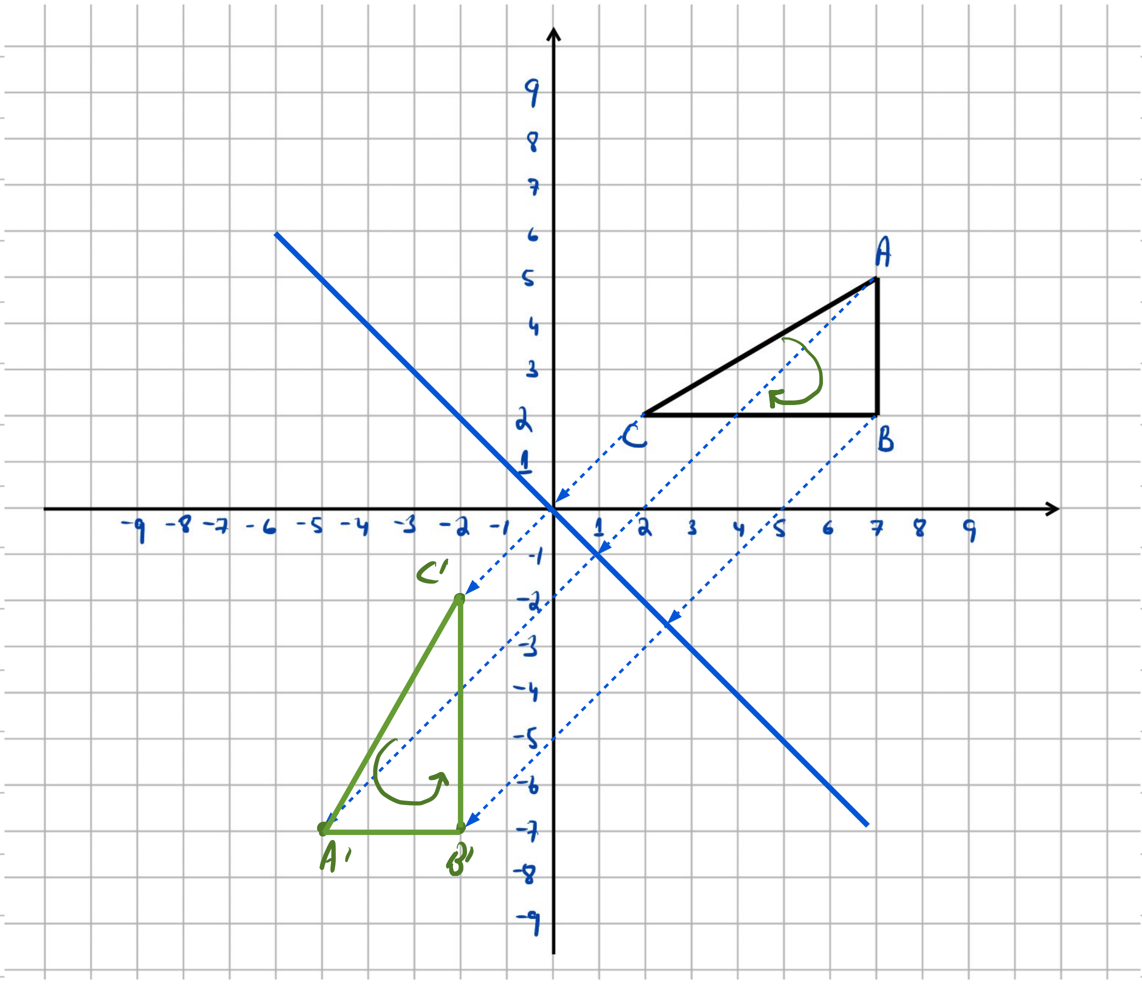
$P(x, y) \longrightarrow P(y, x)$

Invariant Point

Point(s) that remain unchanged. In reflection all the points lying on the line of reflection are invariant.

④ $y = -x$

Reflect ΔABC along the line $y = -x$



$A(7, 5) \longrightarrow A'(-5, -7)$

$B(7, 2) \longrightarrow B'(-2, -7)$

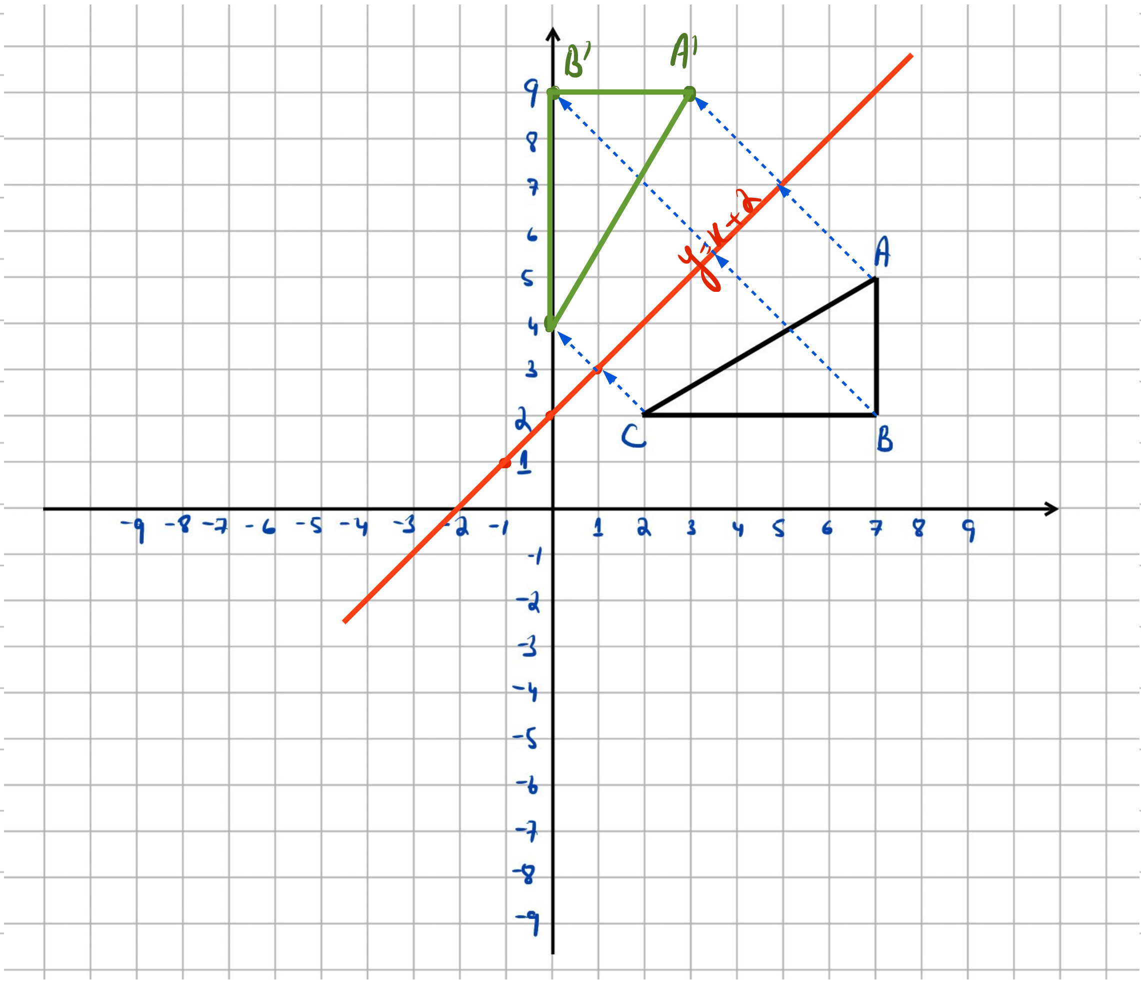
$C(2, 2) \longrightarrow C'(-2, -2)$

$P(x, y) \rightarrow P(-y, -x)$

→ Reflection along any other line

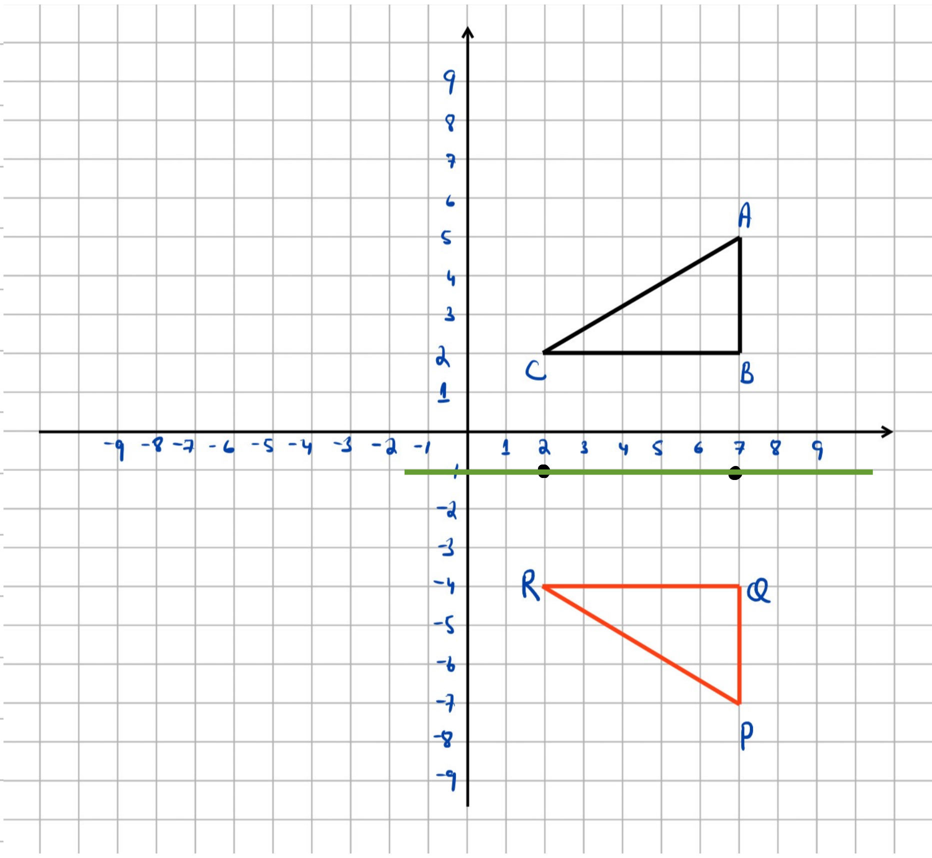
Reflect ΔABC along the line $y = x + 2$

x	-1	0	1
y	1	2	3



→ How to find the line of Reflection ?

Example 1



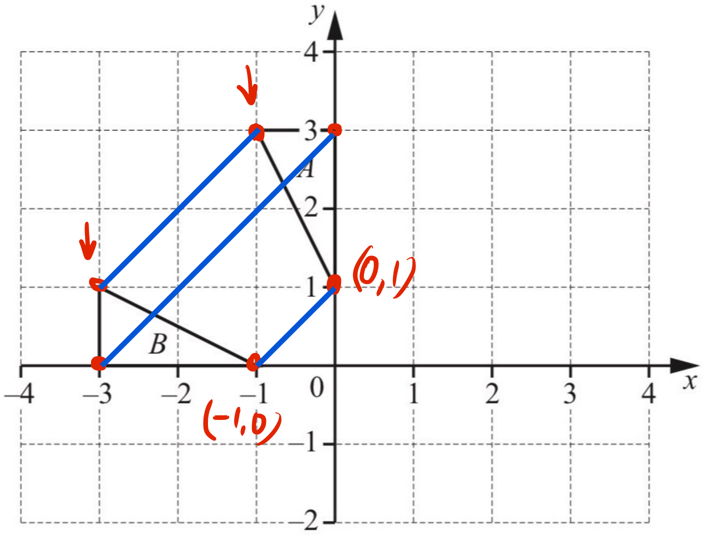
Step 1: Find the Midpoint of A & A'

Step 2: Find the Midpoint of B & B'

Step 3: Join the two points found in S1 & S2

$y = -1$

Example 2



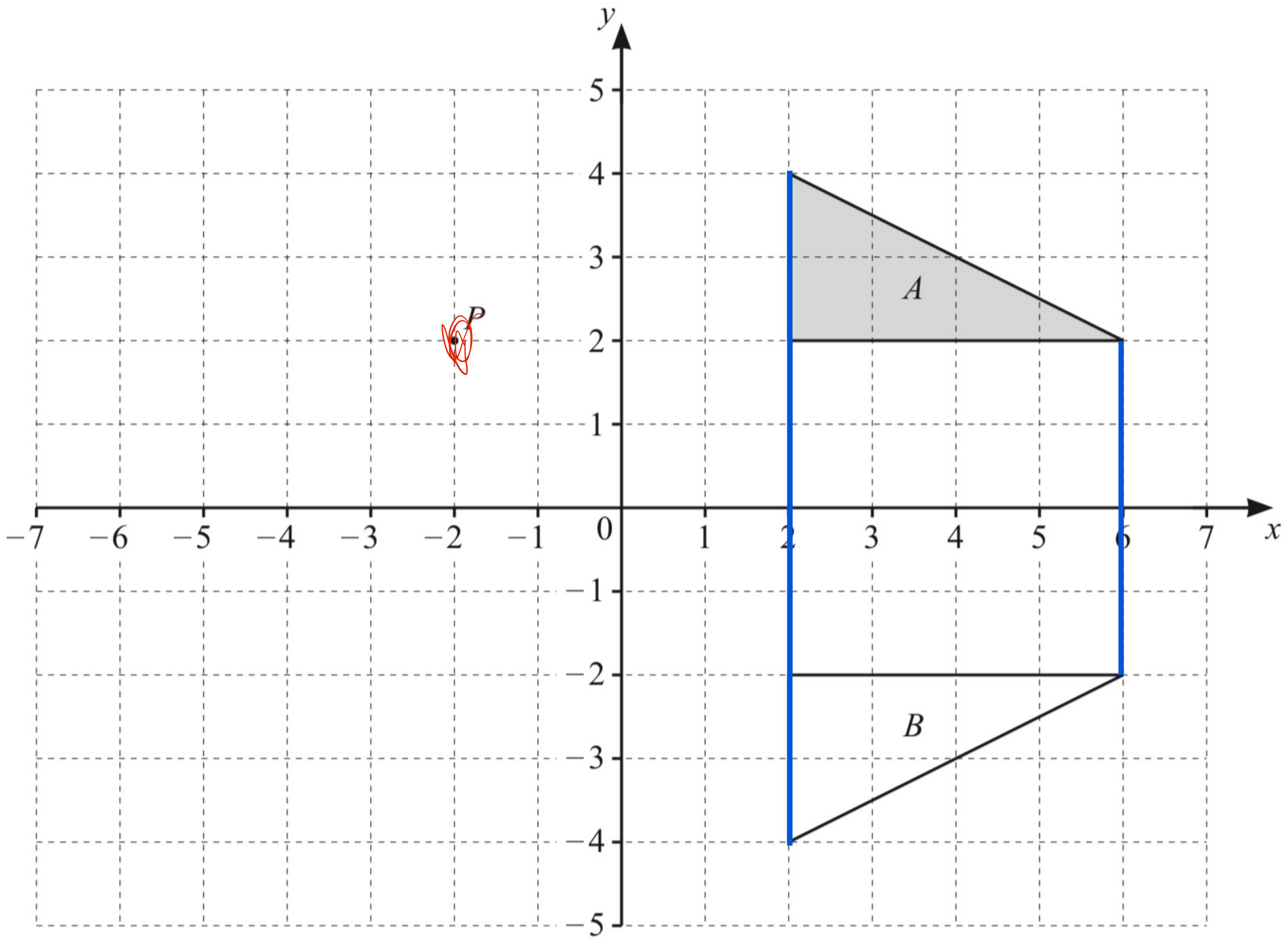
(0,1) (-1,0)
(0,3) (-3,0)
(-1,3) (-3,1)
$(x,y) \to (-y,-x)$

The diagram shows triangles *A* and *B*.

(a) Describe fully the **single** transformation that maps triangle *A* onto triangle *B*.

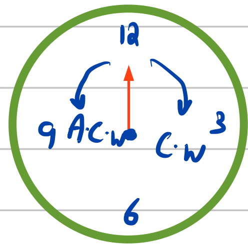
Reflection, along the line $y = -x$

[1] .... [2]



$y = 0$ Equation : $y = 0$

Rotation



Defined by:

① Centre of Rotation
② Angle: 90°, 180°, 270°
③ Direction: Clockwise or Anticlockwise

Note: Centre of rotation is the invariant point.

In Rotation, the size & shape remains the same.

→ 90° C.W / 270° A.C.W
→ 180°
→ 90° A.C.W / 270° C.W

Level 1: Centre of Rotation is origin & object is in line with the centre

Level 2: " " " " " & object is not in line with the centre

Level 3: Centre of Rotation is NOT the origin

Level 4: Rotating entire shapes

Level 1

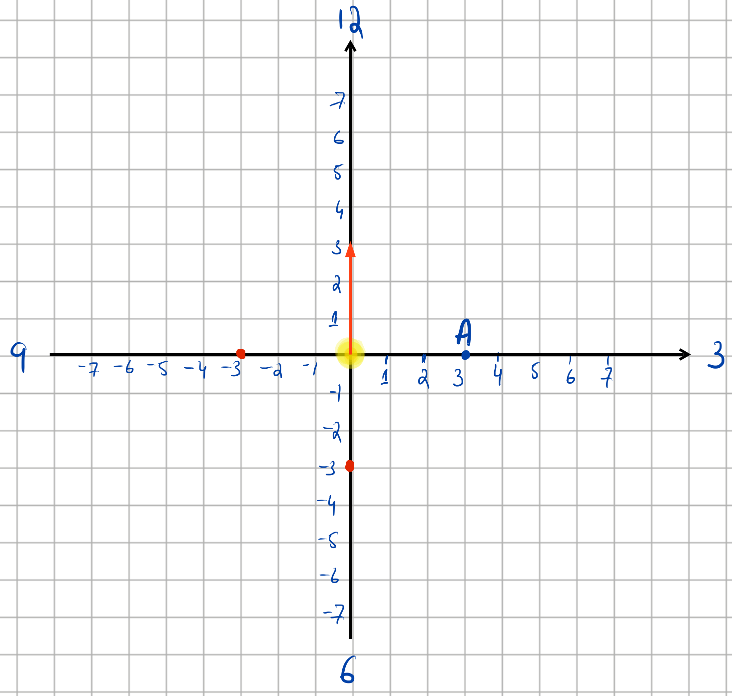
Example 1

Rotate the point A(3,0) about origin

(i) 90° Clockwise (0,-3)

(ii) 180° (-3,0)

(iii) 90° Anti-clockwise (0,3)



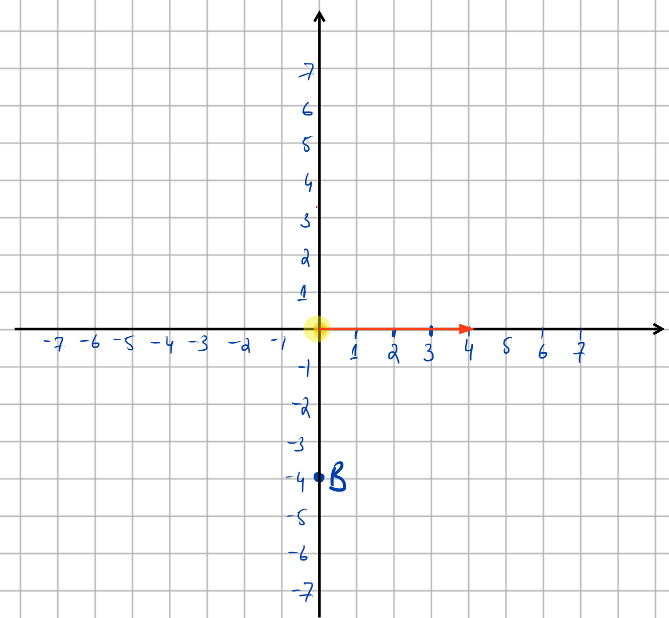
Example 2

Rotate the point B(0,-4) about origin

(i) 90° C.W (-4,0)

(ii) 180° (0,4)

(iii) 90° A.C.W (4,0)



Level 2

Example 1

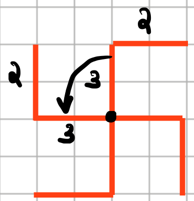
Rotate the point A(2,3) about origin

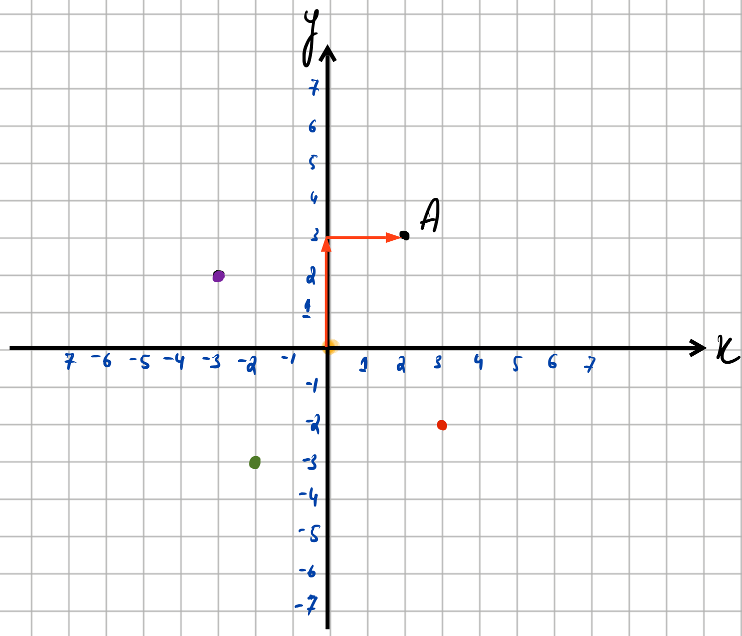
(i) 90° Clockwise (3,−2)

(ii) 180° (−2,−3)

(iii) 90° Anti-Clockwise

(−3,2)





Example 2

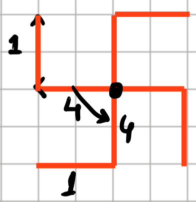
Rotate the point B(−4,1) about origin

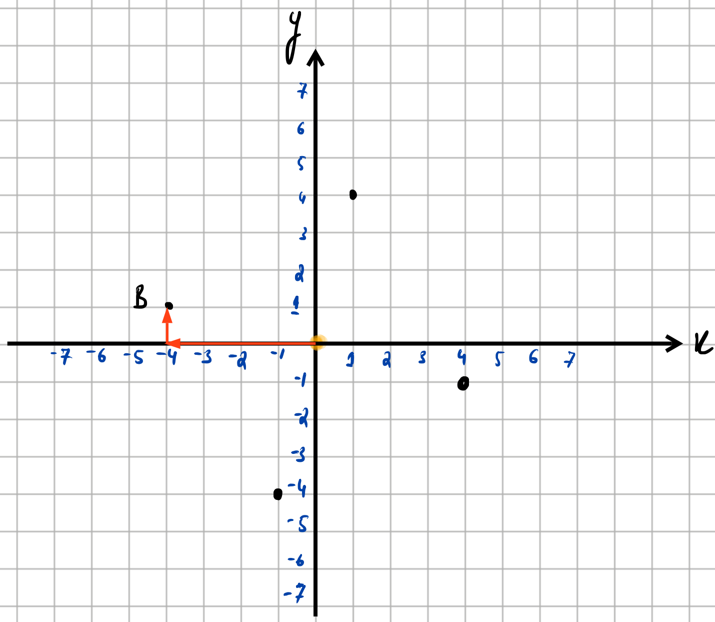
i) 90° Clockwise (1,4)

ii) 180° (4,−1)

iii) 90° Anti-Clockwise

(−1,−4)





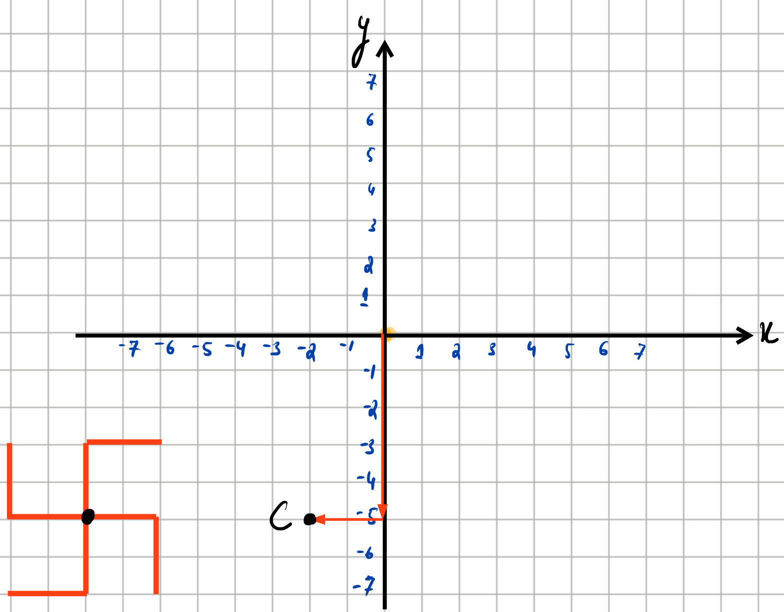
Example 3

Rotate the point C(−2,−5) about origin

i 90° Clockwise (−5,2)

ii 180° (2,5)

(iii) 90° Anti-Clockwise (5,−2)



Level 3

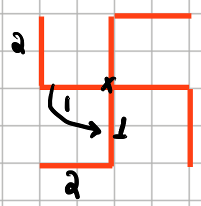
Example 1

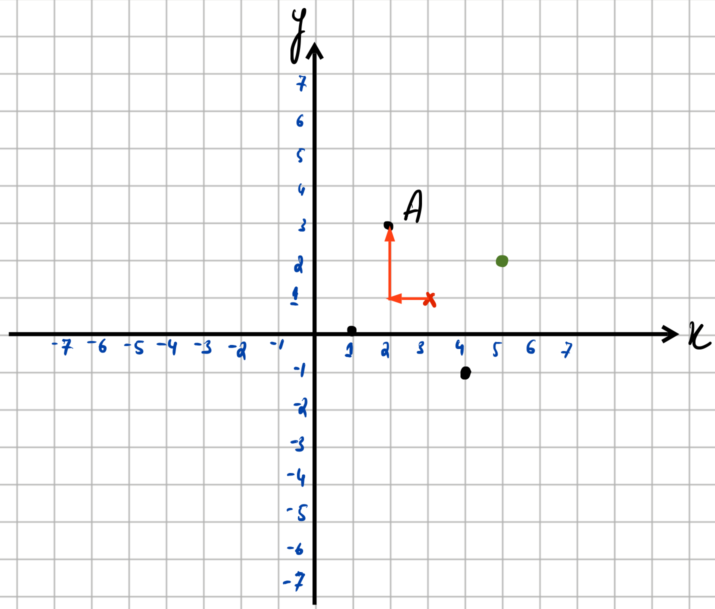
Rotate the point $A(2,3)$ about $X(3,1)$

(i) 90° Clockwise $(5,2)$

(ii) 180° $(4,-1)$

iii) 90° Anti-Clockwise $(1,0)$





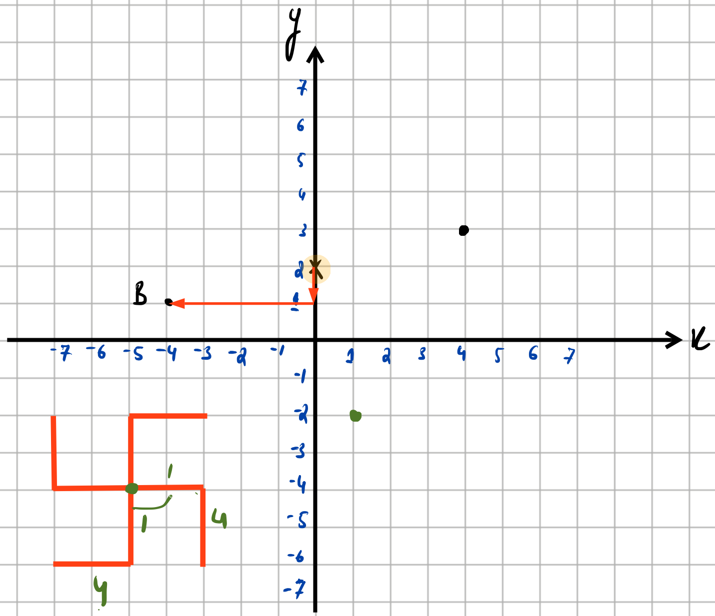
Example 2

Rotate the point $B(-4,1)$ about $X(0,2)$

i) 90° Clockwise $(-1,6)$

ii) 180° $(4,3)$

iii) 90° Anti-Clockwise $(1,-2)$

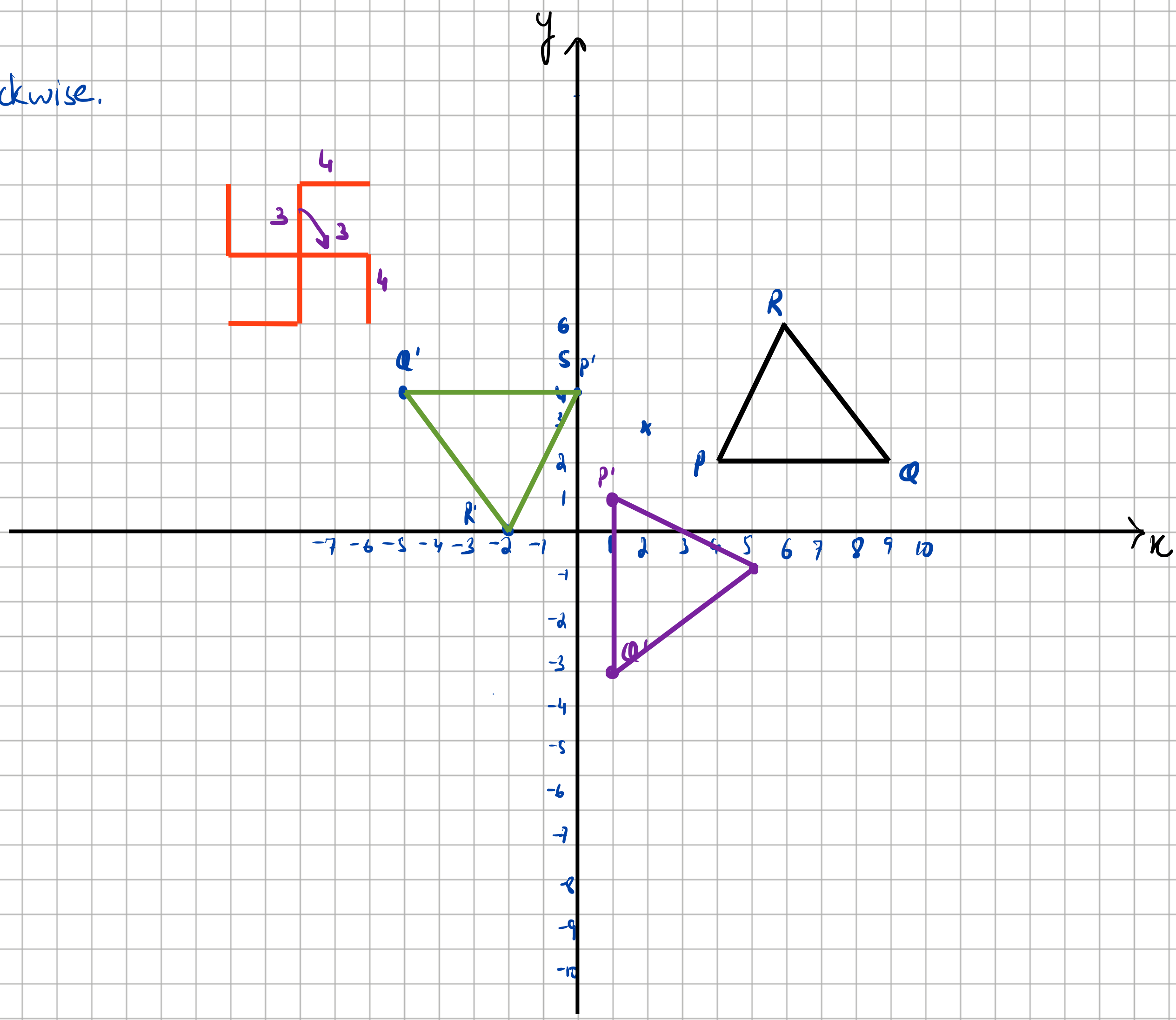


Rotation (Level 4)

Q. Rotate ΔPQR about the centre $x(2,3)$

(a) 180°

(b) 90° clockwise.



→ How to find the centre of Rotation ?

① 180° (Easy)

② 90° (Tricky)

① 180°

Step 1: Join A & A' (object & its image)

Step 2: Join B & B' (Object & its image)

Step 3: The point of intersection is the centre of Rotation ?

② 90° (Drawing a P.b without the help of a compass)

Step 1: Find the Midpoint of A & A'

Step 2: Rotate A or A' 90° C.W or A.C.W using Midpoint as the centre.

Step 3: Find the Midpoint of B & B'

Step 4: Repeat of Step 2 with (B & B')

Step 5: The point of intersection of the two p.b's is the centre of rotation.

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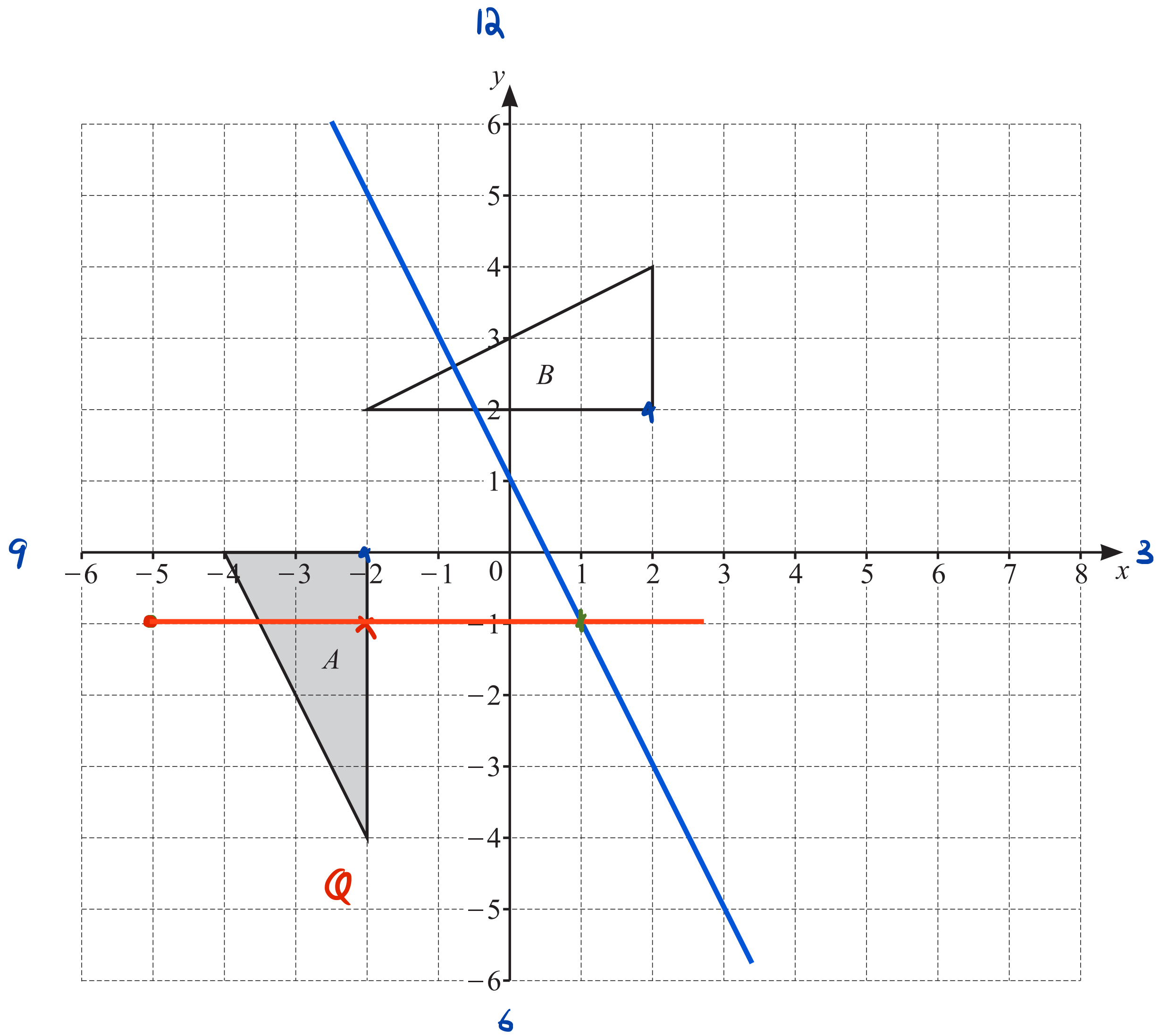


Triangle A and triangle B are drawn on the grid.

(a) Describe fully the **single** transformation that maps triangle A onto triangle B.

Rotation 90° Clockwise Centre (1, −1) ....

.... [3]

(b) Transformation P is represented by the matrix $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$.

P maps triangle A onto triangle C.

On the grid, draw triangle C.

[2]

10 (a)

(2,6) (-4,2)

$\frac{2+-4}{2}, \frac{6+2}{2}$

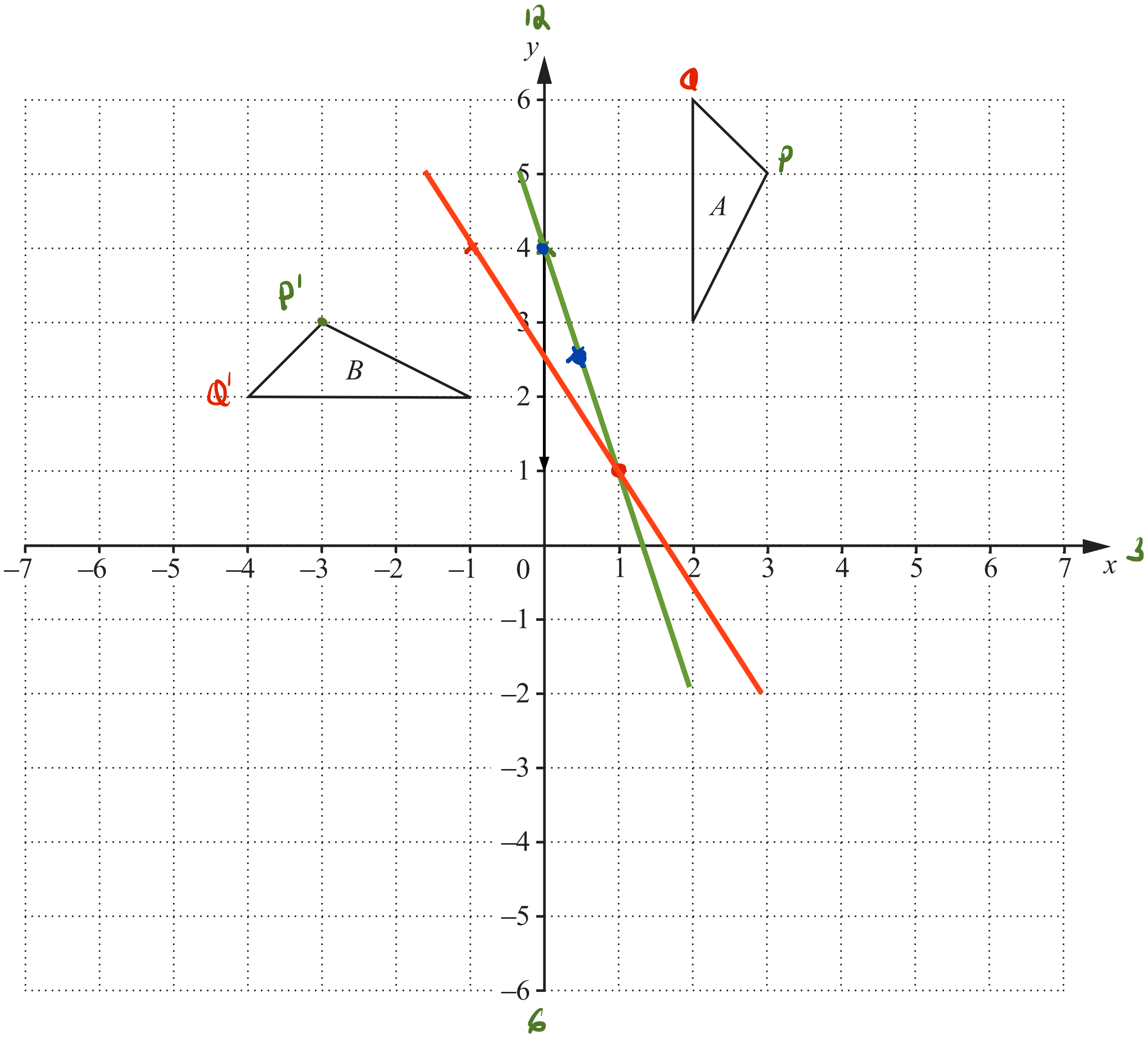
(-1, 4)

(-1,2) (2,3)

$\frac{-1+2}{2}, \frac{2+3}{2}$

$\frac{1}{2}, \frac{5}{2}$

0.5, 2.5

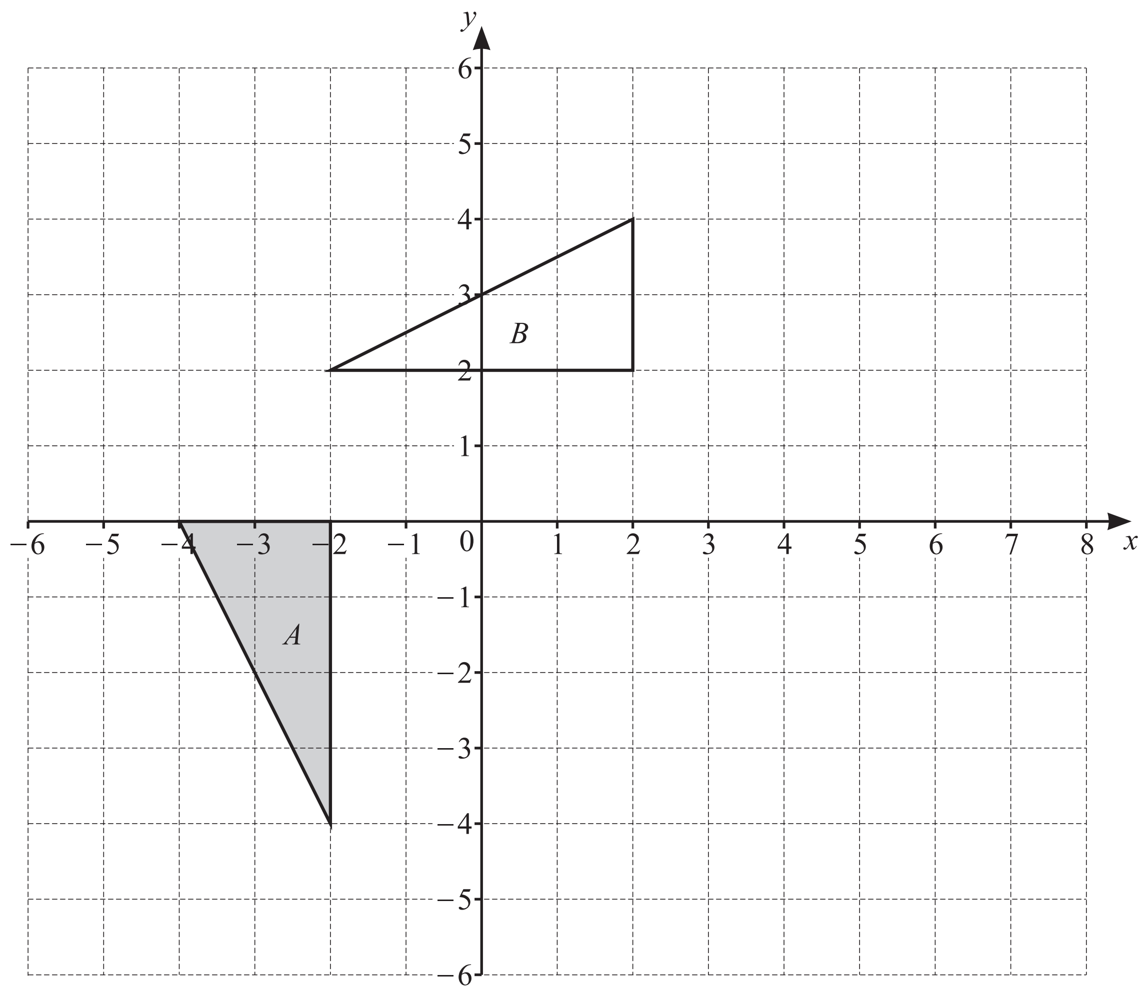


(i) Describe fully the **single** transformation that maps triangle *A* onto triangle *B*.

Answer Rotation 90° A.C.W Centre (1, 1)

.... [2]

23



Triangle *A* and triangle *B* are drawn on the grid.

(a) Describe fully the **single** transformation that maps triangle *A* onto triangle *B*.

....

.... [3]

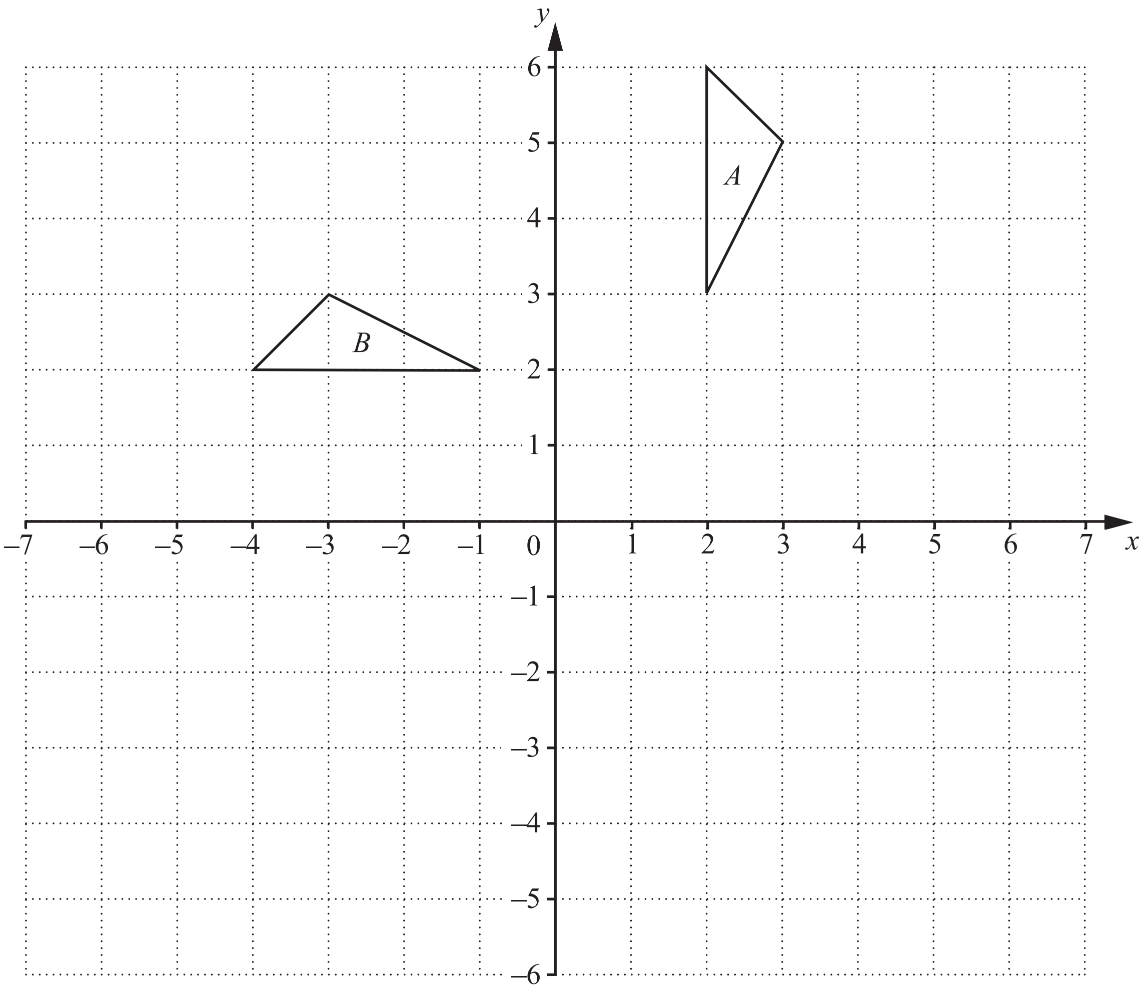
(b) Transformation P is represented by the matrix $\begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$.

P maps triangle *A* onto triangle *C*.

On the grid, draw triangle *C*.

[2]

10 (a)



(i) Describe fully the **single** transformation that maps triangle A onto triangle B.

Answer ....

.... [2]

(ii) Triangle B is mapped onto triangle C by a translation, vector $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$.

Draw and label triangle C. [2]

(iii) Triangle A is mapped onto triangle D by a reflection in the line $y = x$.

Draw and label triangle D. [2]

(iv) Triangle E is geometrically similar to triangle A and its longest side is 12 cm.

Calculate the area of triangle E.

Answer .... cm^2 [2]

Example 1

90°

Describe fully the single transformation that maps ΔA onto ΔB

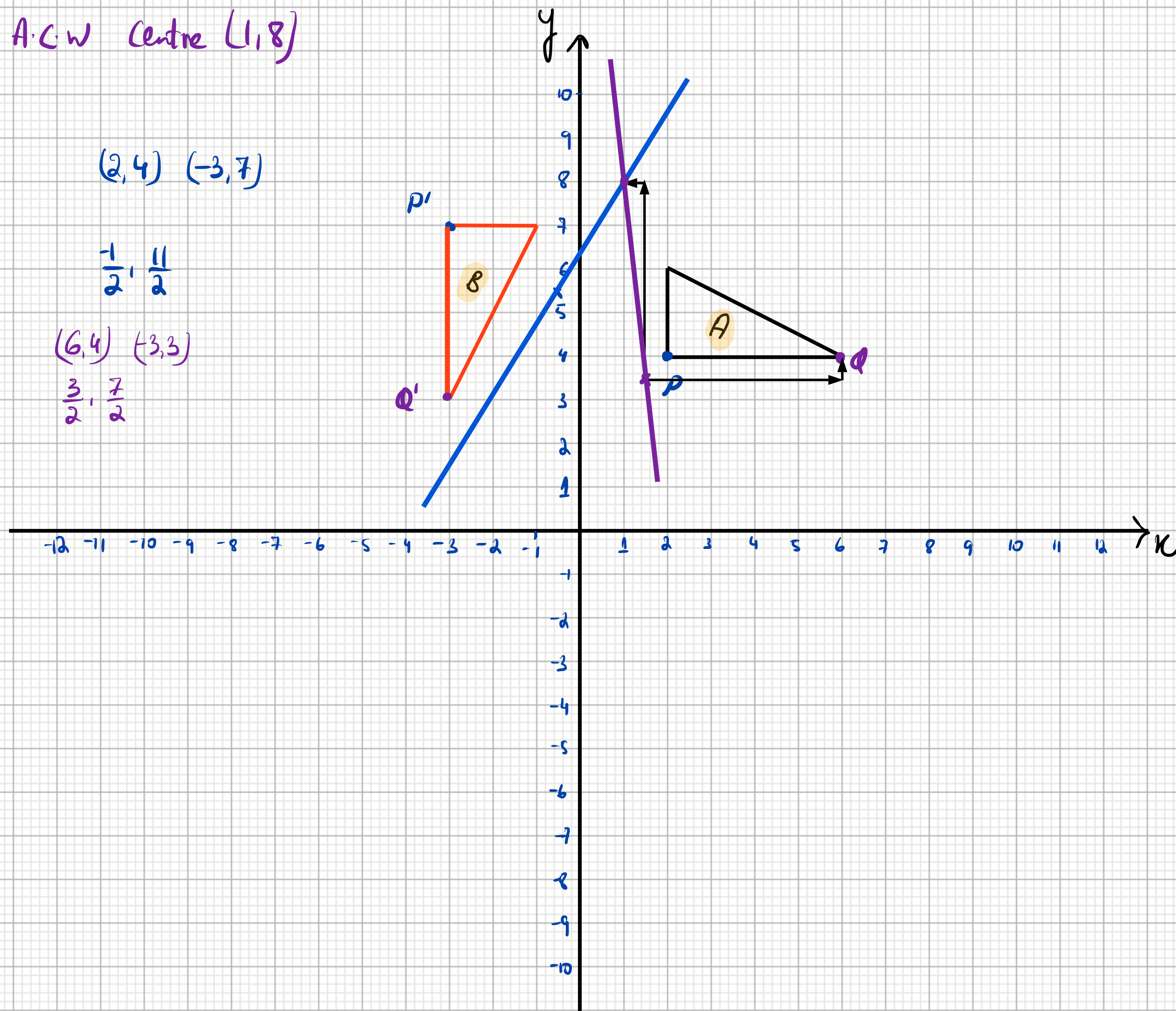
Rotation 90° A.C.W Centre (1,8)

(2,4) (−3,7)

$\frac{-1}{2}, \frac{11}{2}$

(6,4) (−3,3)

$\frac{3}{2}, \frac{7}{2}$



Example 1

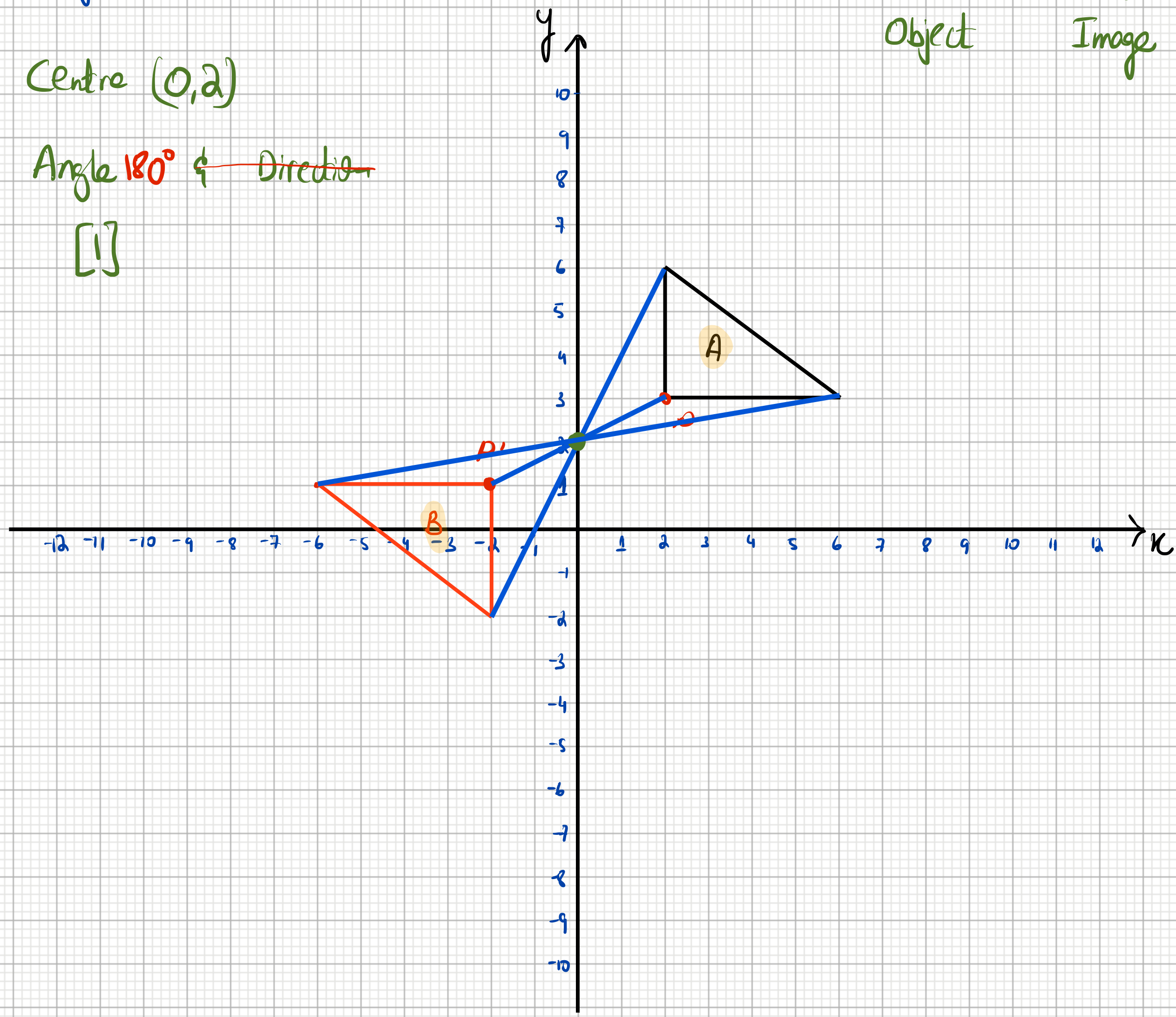
180°

Q. Describe fully the single transformation that maps $\underbrace{\Delta A}_{\text{Object}}$ onto $\underbrace{\Delta B}_{\text{Image}}$

Rotation [1]

Centre (0,2)

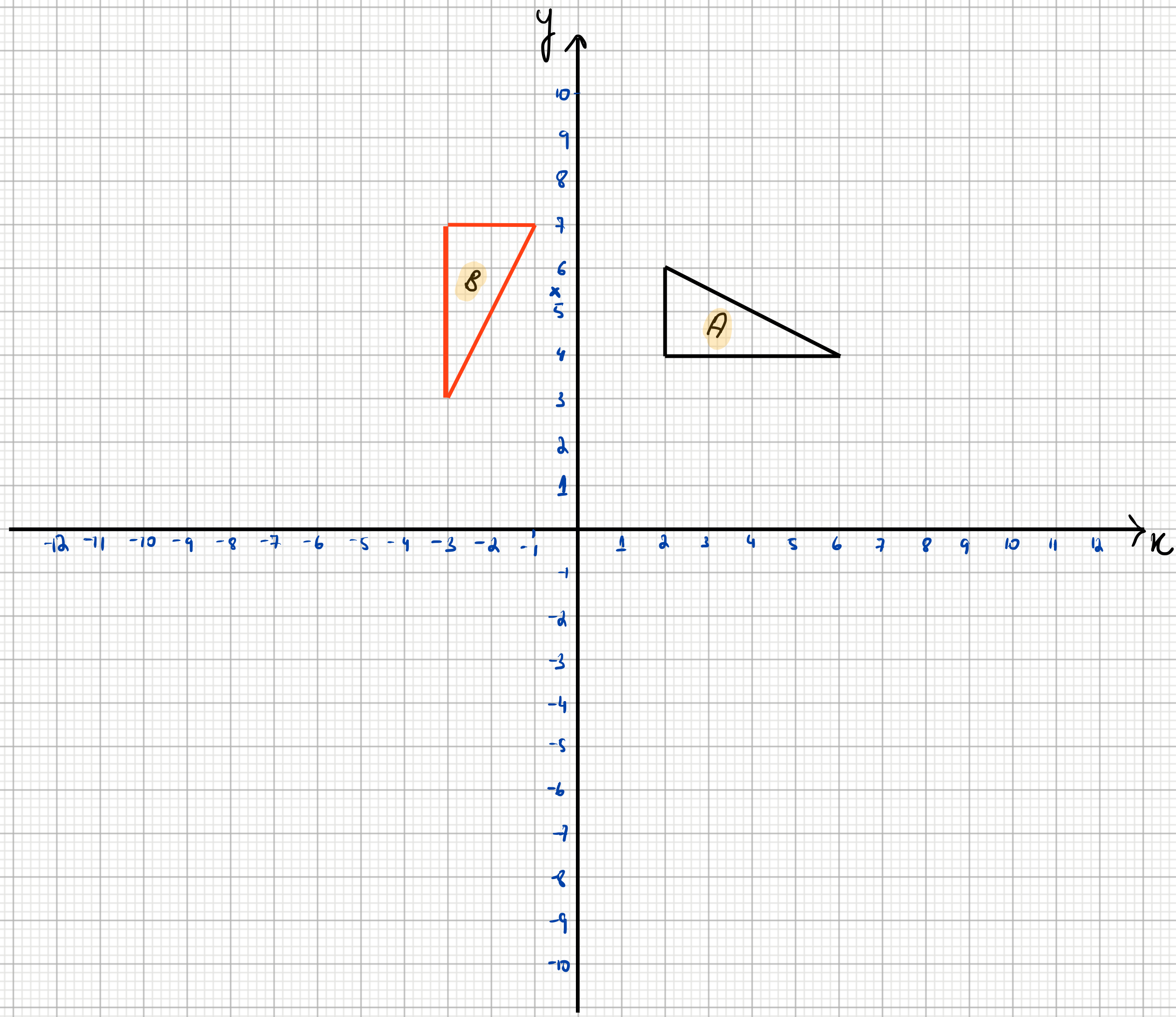
Angle 180° & ~~Direction~~ [1]



Example 1

$90°$

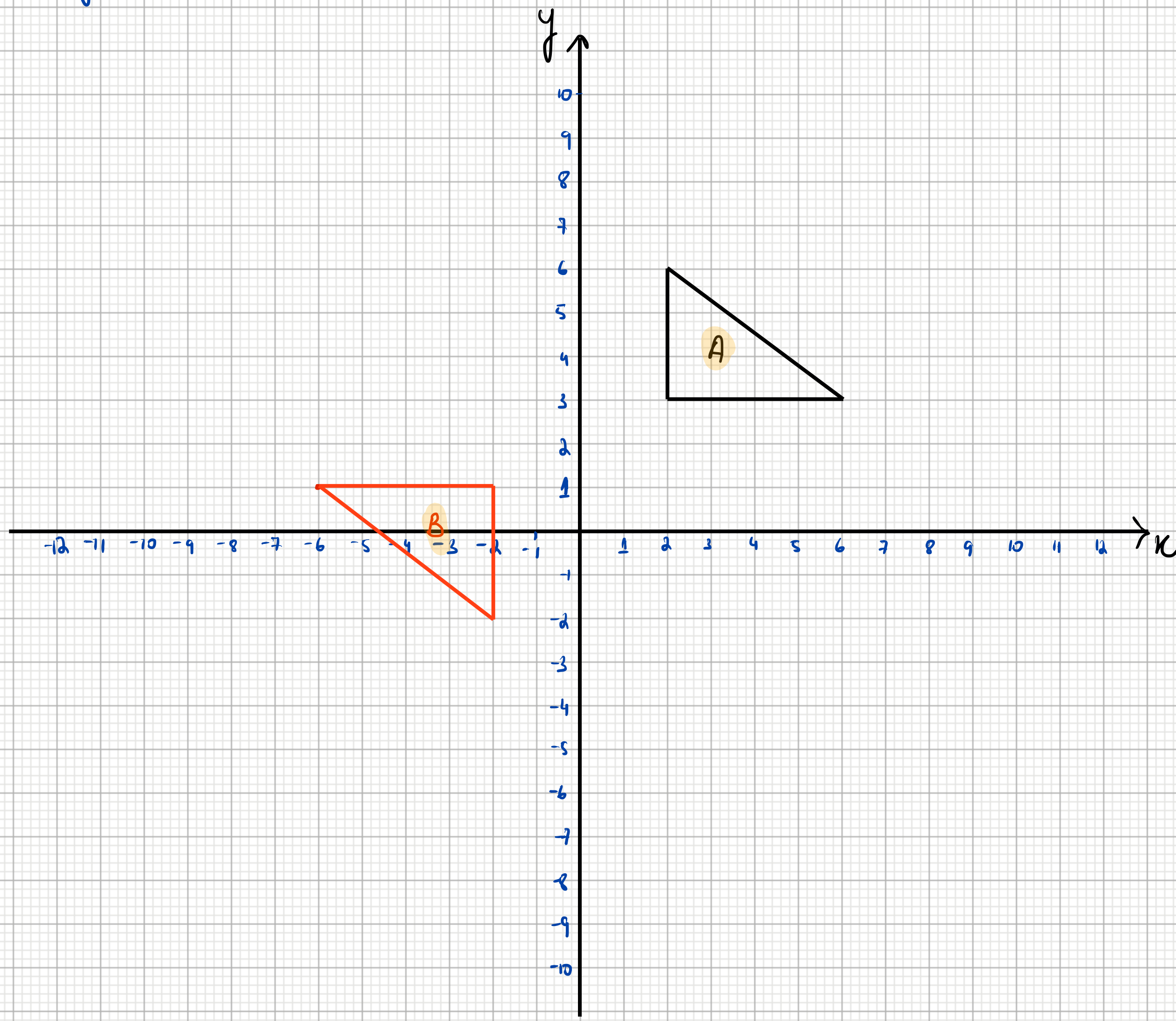
Describe fully the single transformation that maps ΔA onto ΔB



Example 1

180°

Q. Describe fully the single transformation that maps ΔA onto ΔB

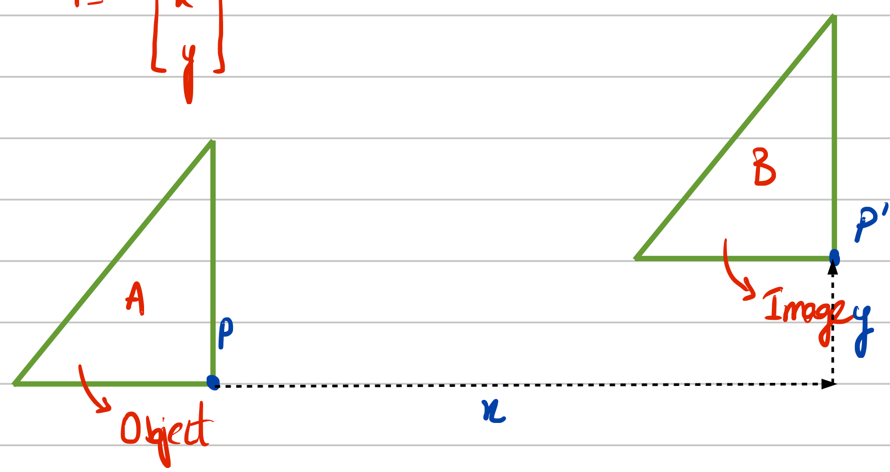


Translation

Translation: Moving or sliding an object without rotating or Reflecting it.

Defined by: Translation vector (No invariant Point)

$$T = \begin{bmatrix} x \\ y \end{bmatrix}$$



$I = O + T$

I: Image

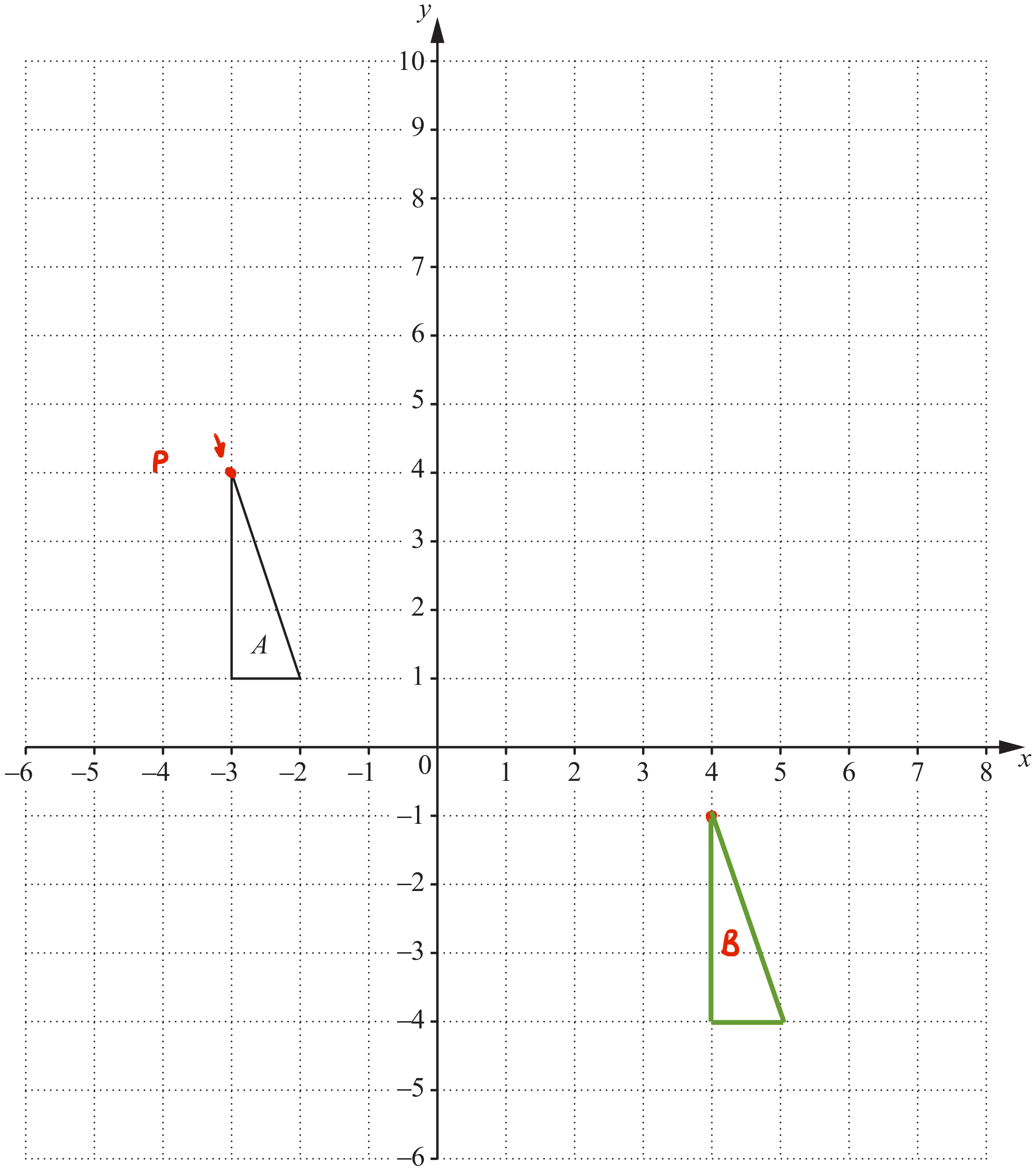
O: Object

T: Translation vector

$$T = \begin{bmatrix} x \\ y \end{bmatrix}$$

Example 1

9 **(a)** Triangle A is shown on the grid.



(i) Triangle A is mapped onto triangle B by a translation of $\begin{pmatrix} 7 \\ -5 \end{pmatrix}$. → 7 units ↓ 5 units

Draw and label triangle B on the grid. [2]

$P = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$

$I = O + T$

$P' = \begin{bmatrix} -3 \\ 4 \end{bmatrix} + \begin{bmatrix} 7 \\ -5 \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$

Example 2

(a) Write down the vector that represents the translation that maps triangle A onto triangle B.

Answer [1]

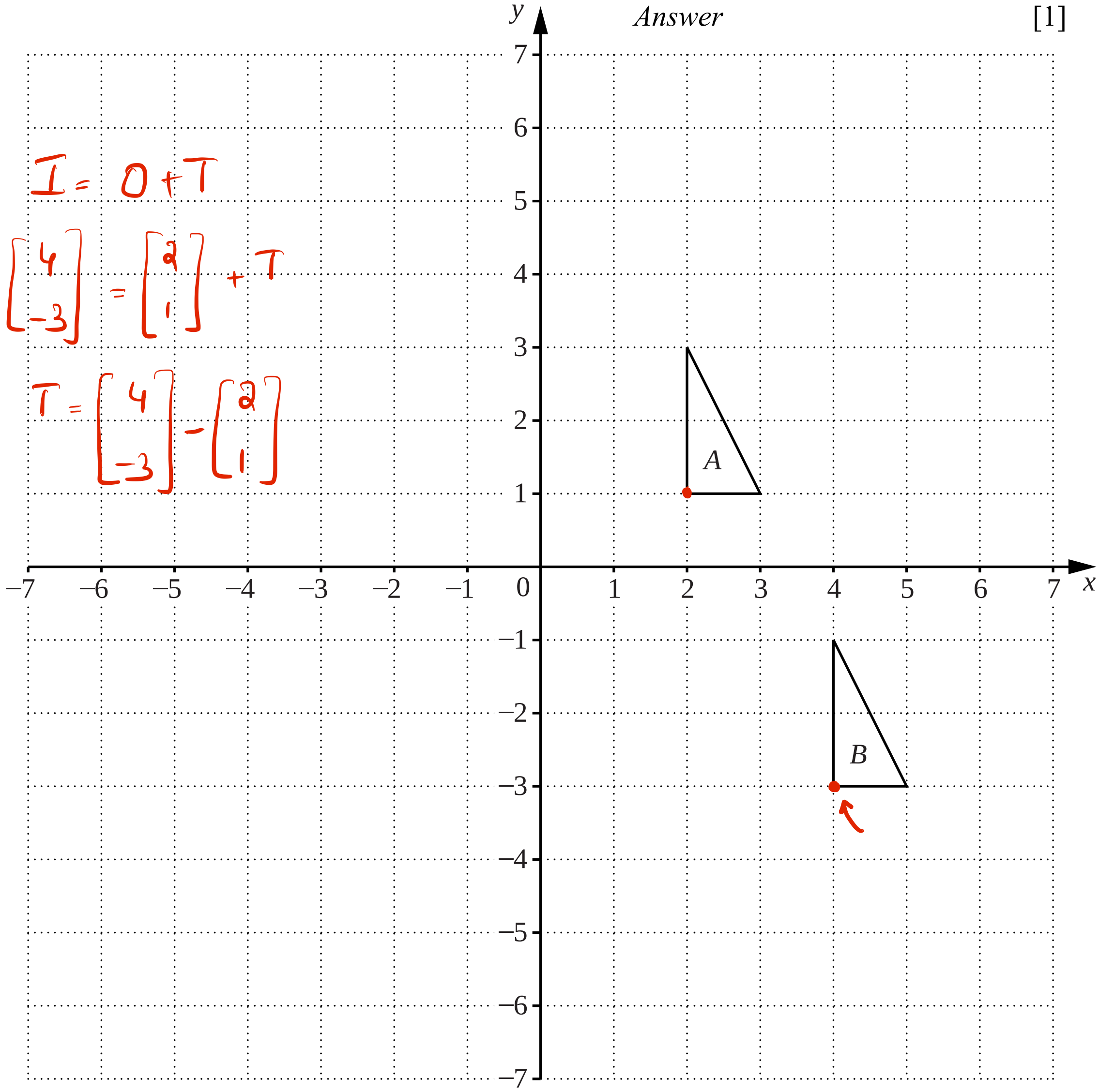
$I = O + T$

$\begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} + T$

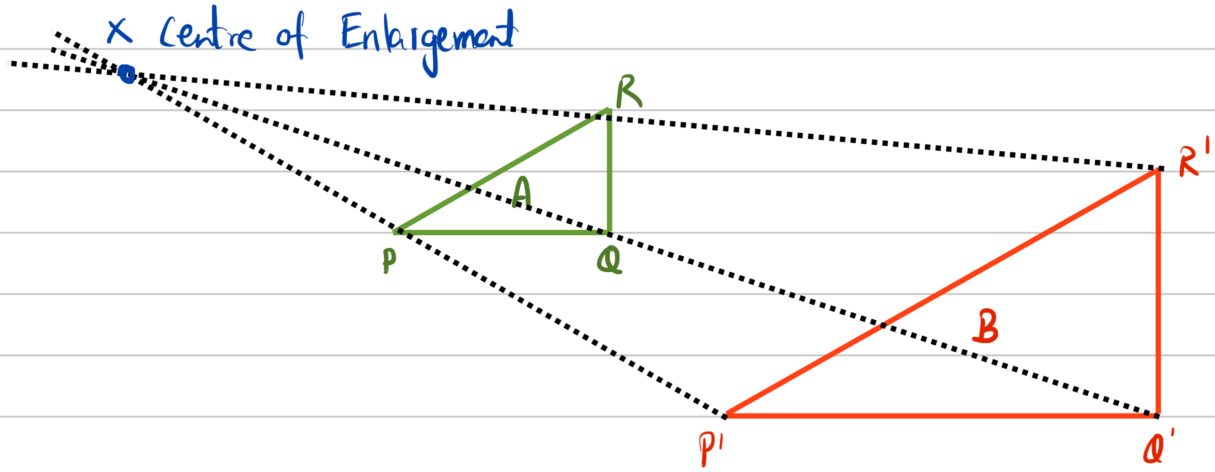
$T = \begin{bmatrix} 4 \\ -3 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -4 \end{bmatrix}$

$T = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$



Enlargement



ΔA is mapped onto ΔB by an enlargement

Defined by

① Centre of Enlargement

② Scale factor $= \dfrac{\text{Image's length}}{\text{Object's length}}$ OR $\dfrac{\text{Distance of image from the centre of Enlargement}}{\text{Distance of object from the centre}}$

$$\frac{P'Q'}{PQ} \quad \text{OR} \quad \frac{XP'}{XP}$$

Invariant Point: Centre of Enlargement

Positive Scale Factor

Enlarge ΔABC with centre origin & s.f

(i) 2

(ii) $\frac{1}{2}$

(i) $A(2,2)$ $B(6,2)$ $C(6,4)$

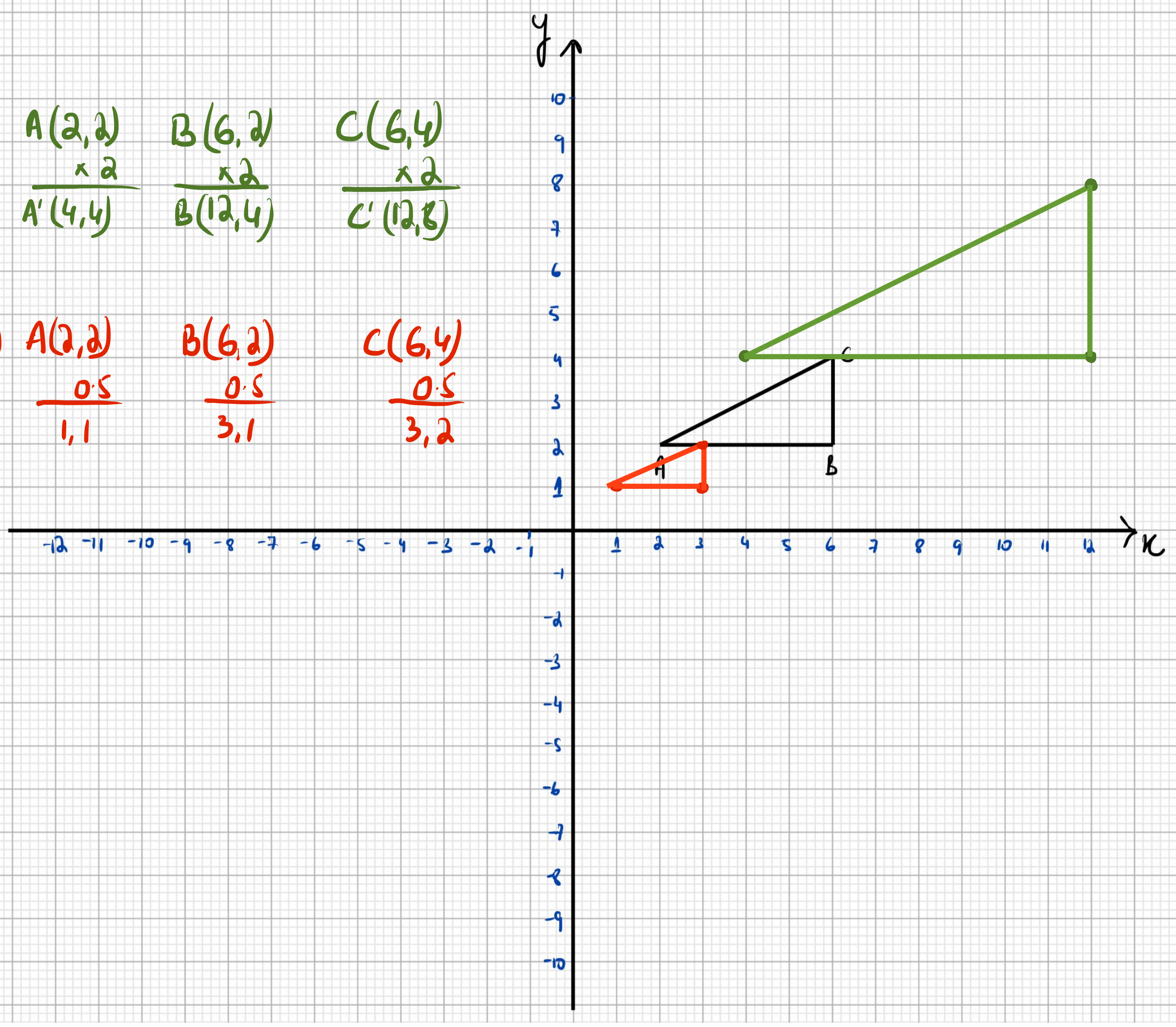
$\times 2$ $\times 2$ $\times 2$

$A'(4,4)$ $B(12,4)$ $C'(12,8)$

(ii) $A(2,2)$ $B(6,2)$ $C(6,4)$

0.5 0.5 0.5

$1,1$ $3,1$ $3,2$



Note: when the centre is origin, just multiply the coordinates of the object by the s.f.

Negative Scale factor

Enlarge $\triangle ABC$ with centre origin & s.f

(i) -2

(ii) $-\frac{1}{2}$

A(2,2) × −2 = −4, −4

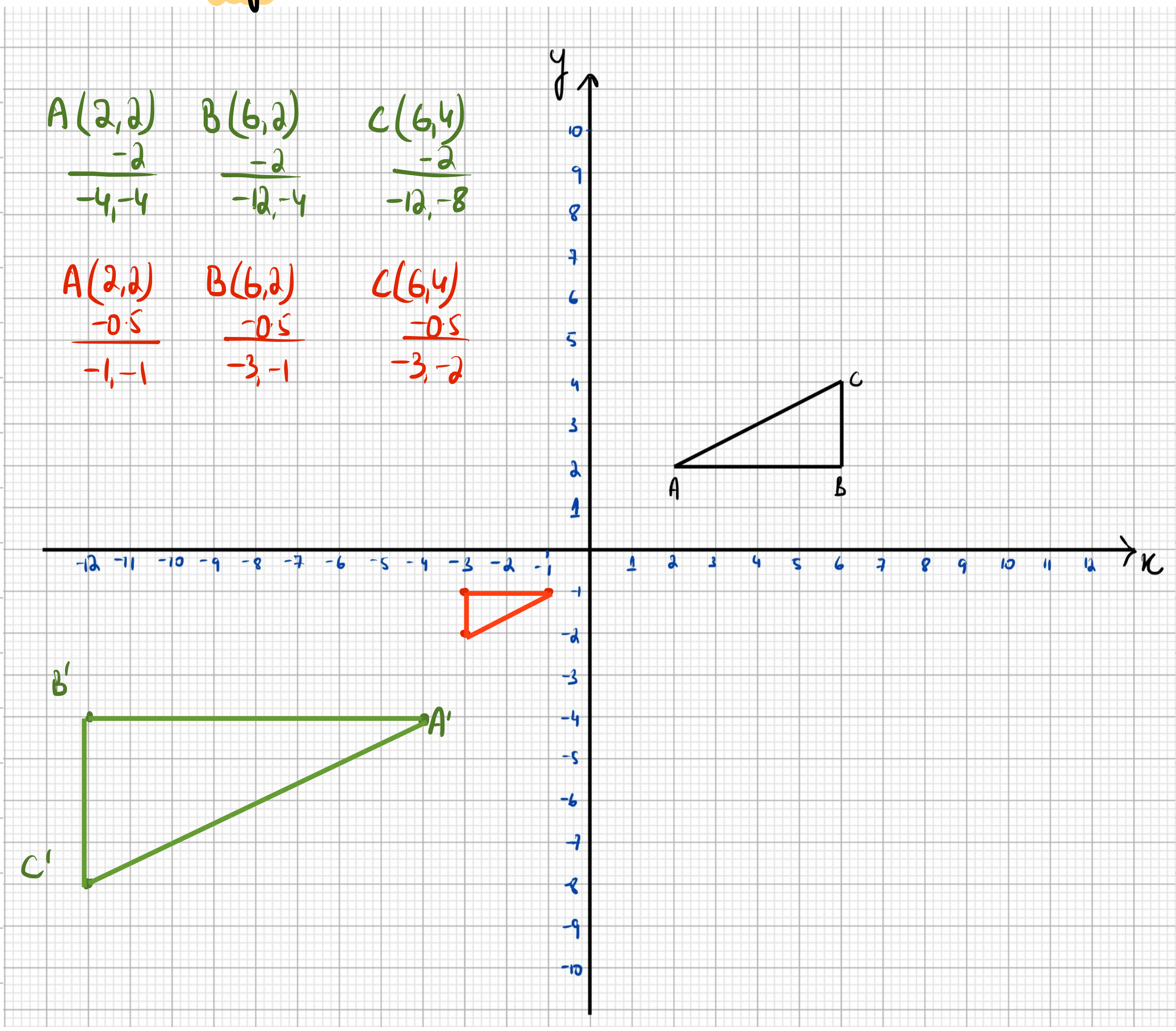
B(6,2) × −2 = −12, −4

C(6,4) × −2 = −12, −8

A(2,2) × −0.5 = −1, −1

B(6,2) × −0.5 = −3, −1

C(6,4) × −0.5 = −3, −2

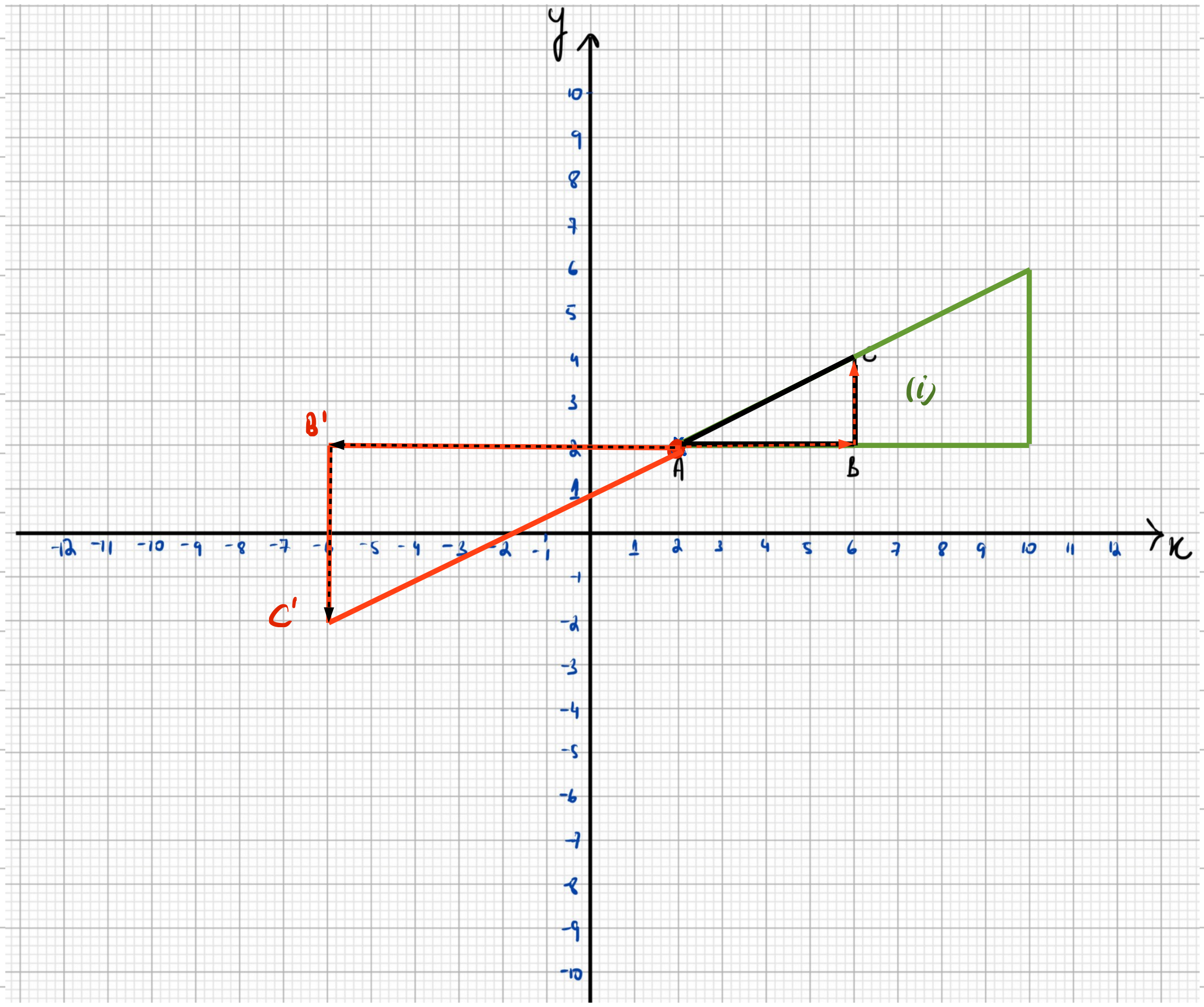


Centre other than Origin

Enlarge ΔABC with centre $A(2,2)$ & scale factor

(i) 2

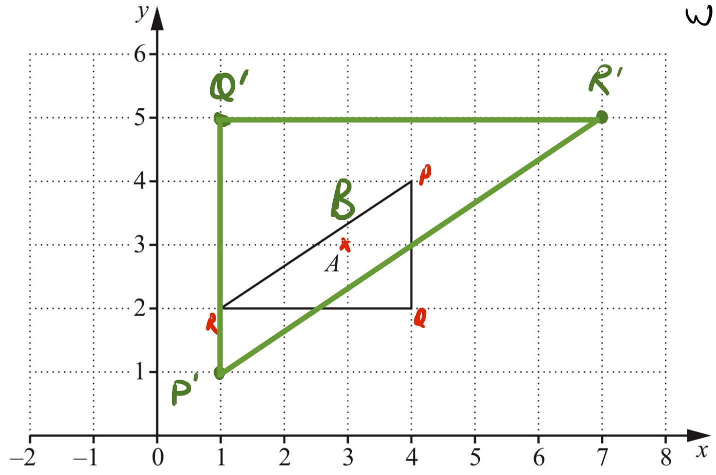
(ii) -2



Example 1

6

w16_qp12

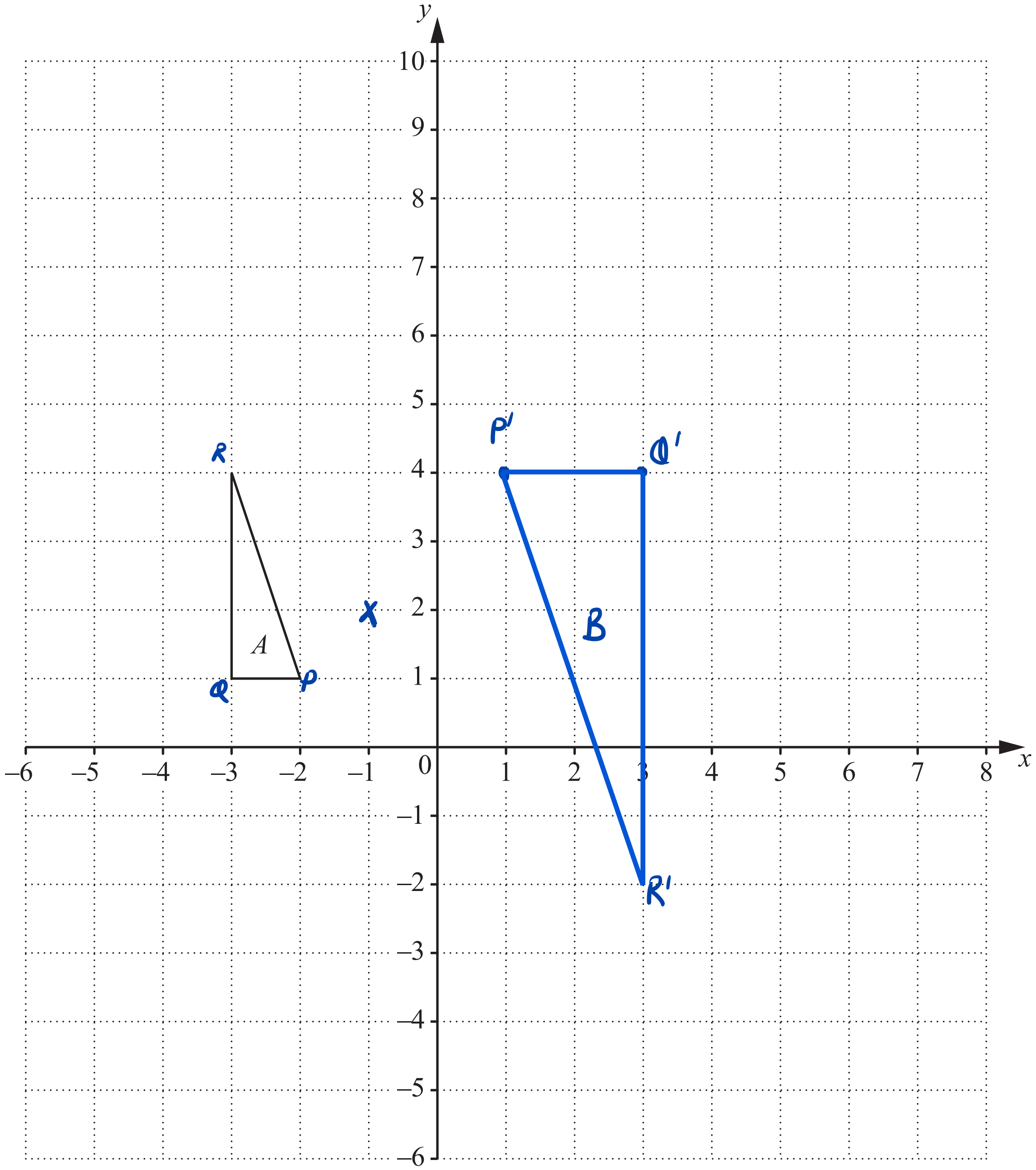


The diagram shows triangle A.
Triangle A is mapped onto triangle B by an enlargement.
The enlargement has centre (3, 3) and scale factor -2.

Draw and label triangle B. [2]

Example 2

9 **(a)** Triangle A is shown on the grid.

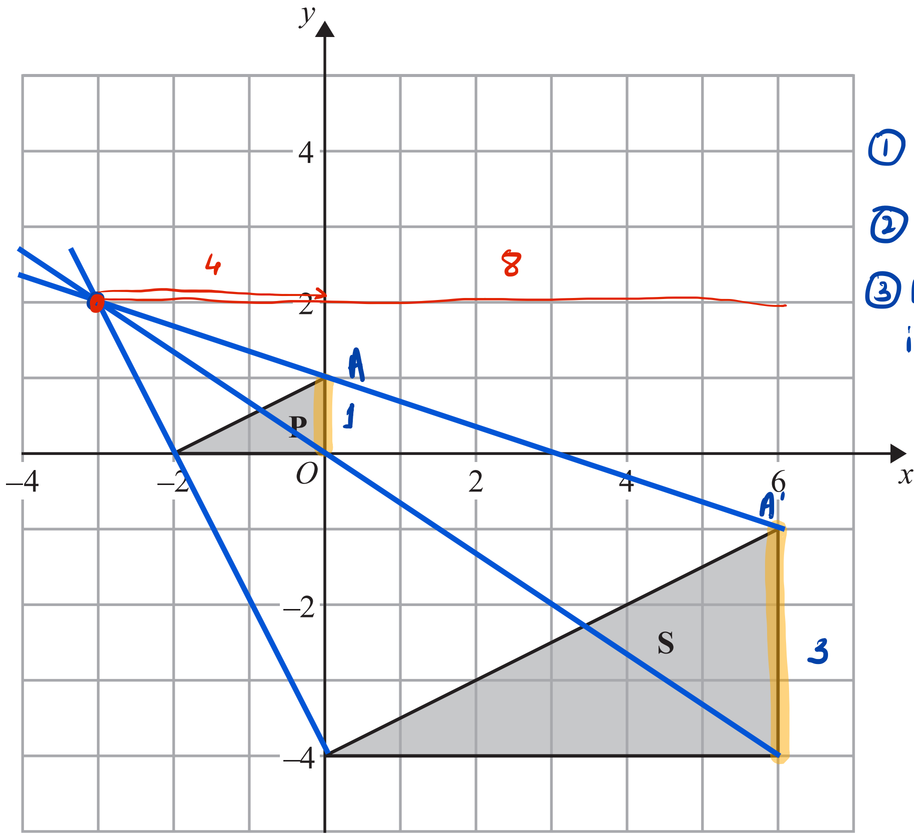


(ii) Triangle A is mapped onto triangle C by an enlargement scale factor -2, centre $(-1, 2)$.

Draw and label triangle C on the grid. [2]

How to Determine Enlargement?

Example 1



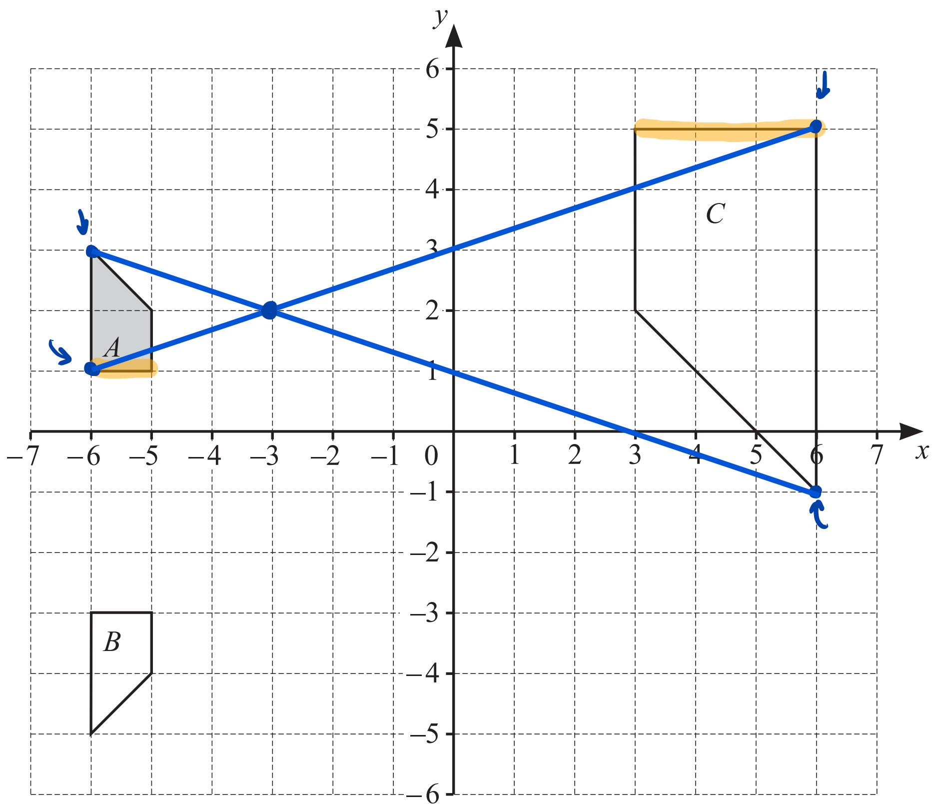
① Join A & A' & extend it
② Join B & B' & extend it
③ where ① & ② intersect is the centre of enlargement.

$\frac{8}{4} =$

(b) Describe fully the single transformation that maps triangle **P** onto triangle **S**.

Enlargement $S.f = \frac{3}{1} = 3$ Centre $(-3, 2)$

Example 2



Shapes A, B and C are drawn on the grid.

(b) Describe fully the **single** transformation that maps shape A onto shape C.

Enlargement s.f. = −3 Centre (−3, 2)

[3]

$\frac{3}{1} = 3\times$

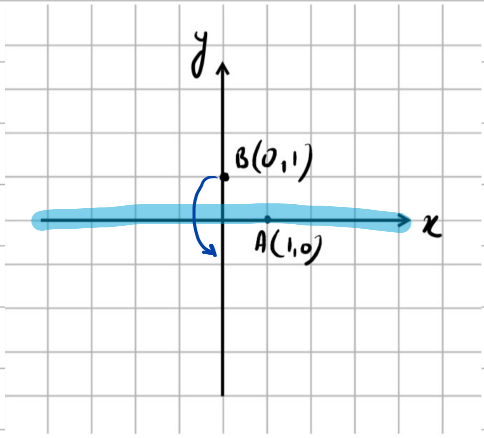
Enlargement

Summary

	Scale factor (k)	What happens to the image?
Positive	$k > 1$	Larger than the object (Same side)
	$0 < k < 1$	Smaller than the object (Same side)
Negative	$k < -1$	Larger than the object (Opposite side)
	$-1 < k < 0$	Smaller than the object (Opposite side)

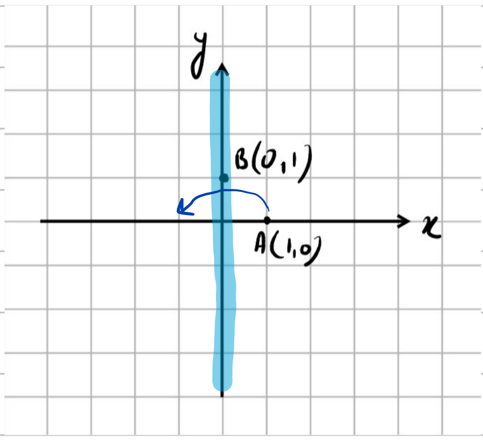
Reflection

(i) Along x-axis



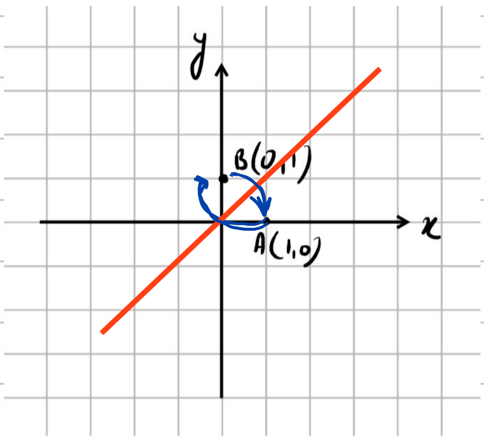
$$\begin{array}{cc} A & B \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{array}{cc} A' & B' \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

(ii) Along y-axis



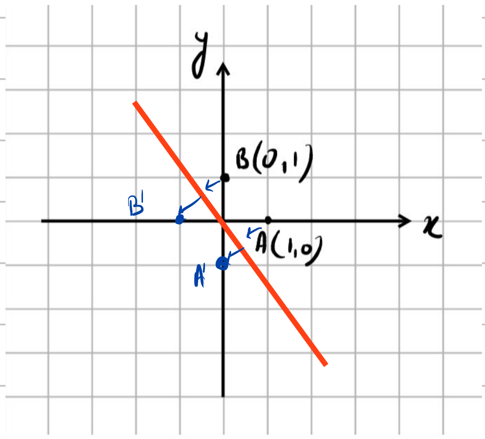
$$\begin{array}{cc} A & B \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{array}{cc} A' & B' \\ \end{array} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

(iii) Along y=x



$$\begin{array}{cc} A & B \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{array}{cc} A' & B' \\ \end{array} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

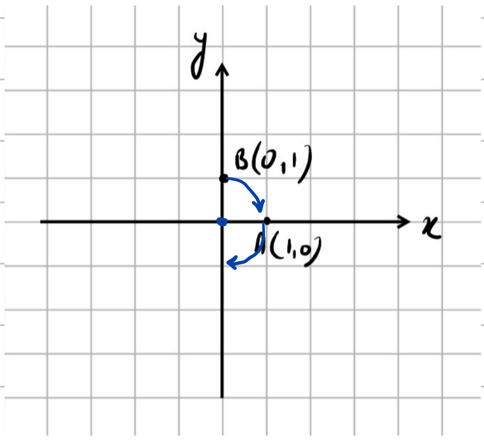
(iii) Along $y = -x$



$$\begin{array}{cc} A & B \\ \end{array} \qquad \begin{array}{cc} A' & B' \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

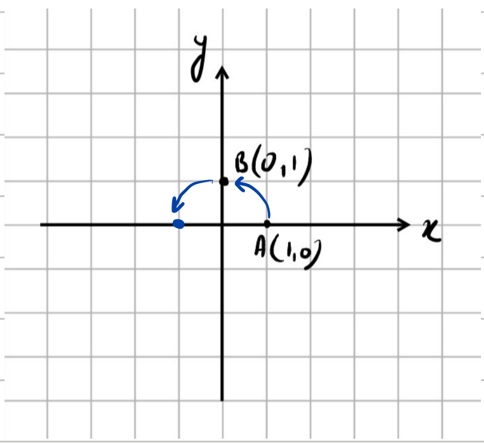
Rotation (Centre Origin)

(i) 90° C.W



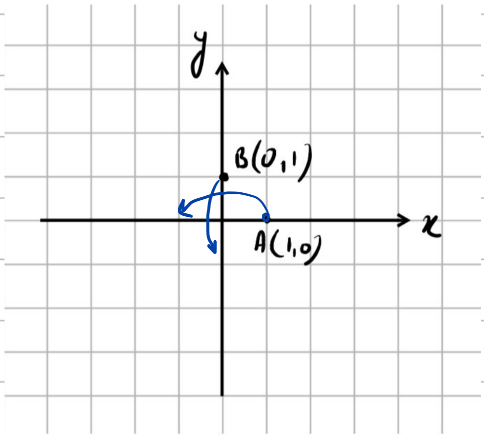
$$\begin{array}{cc} A & B \\ \end{array} \qquad \begin{array}{cc} A' & B' \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

(ii) 90° A.C.W



$$\begin{array}{cc} A & B \\ \end{array} \qquad \begin{array}{cc} A' & B' \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

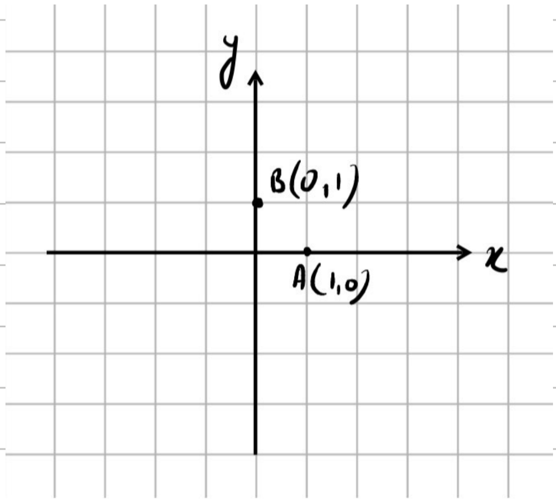
(iii) 180°



$$\begin{array}{cc} A & B \end{array} \quad \begin{array}{cc} A' & B' \end{array}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Enlargement (Centre Origin)



$$\begin{array}{cc} A & B \end{array} \quad \begin{array}{cc} A' & B' \end{array}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{S.f = k} \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \qquad k \neq 1$$

$S.f = 2 \quad \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ $\qquad S.f = -\frac{1}{2} \quad \begin{bmatrix} -\frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$

$S.f = 3 \quad \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

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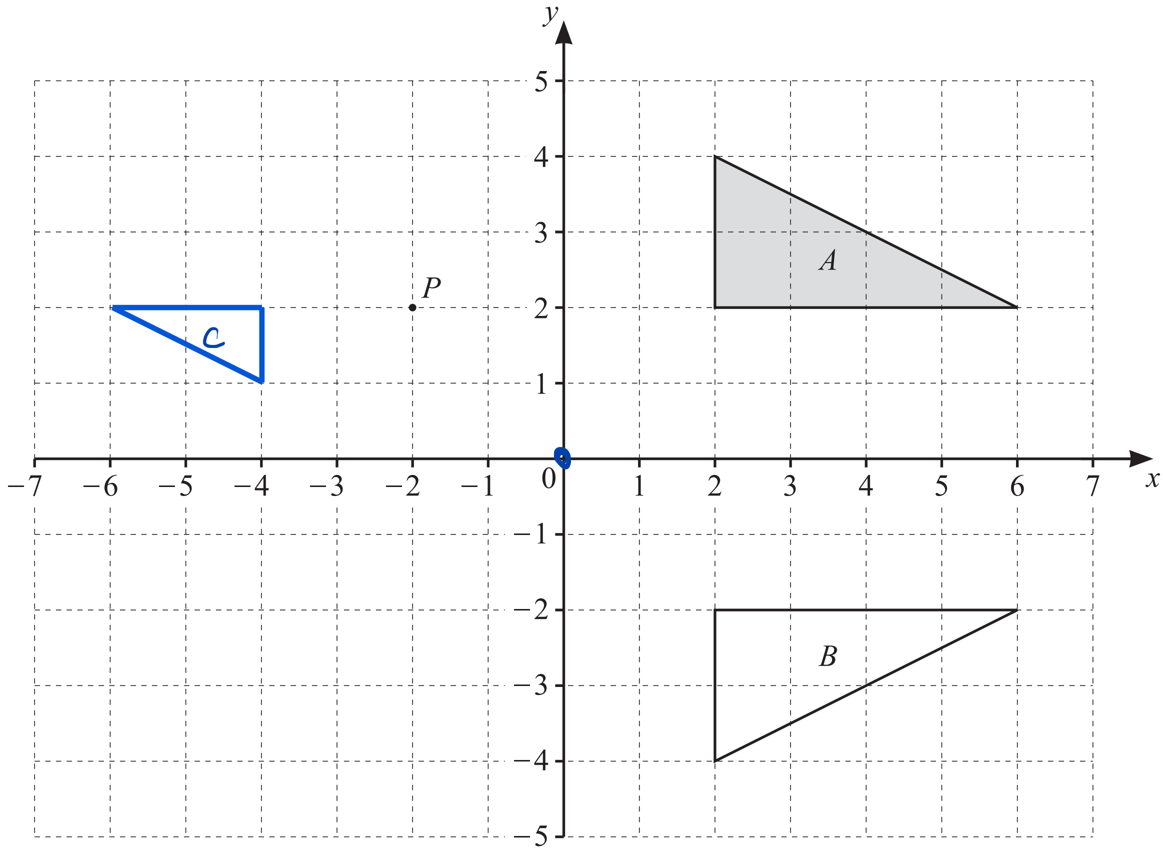
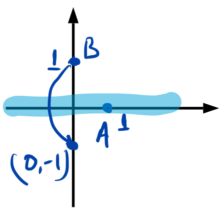


Triangle A, triangle B and the point P (–2, 2) are drawn on the grid.

(a) **(i)** Describe, fully, the **single** transformation that maps triangle A onto triangle B.

Reflection along x-axis or $y = 0$ .... [2]

(ii) Write down the matrix that represents this transformation.



$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ Ref along x-axis

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ [2]

(b) Triangle A is mapped onto triangle C by an enlargement, centre P, scale factor $-\frac{1}{2}$.

On the grid, draw and label triangle C. [2]

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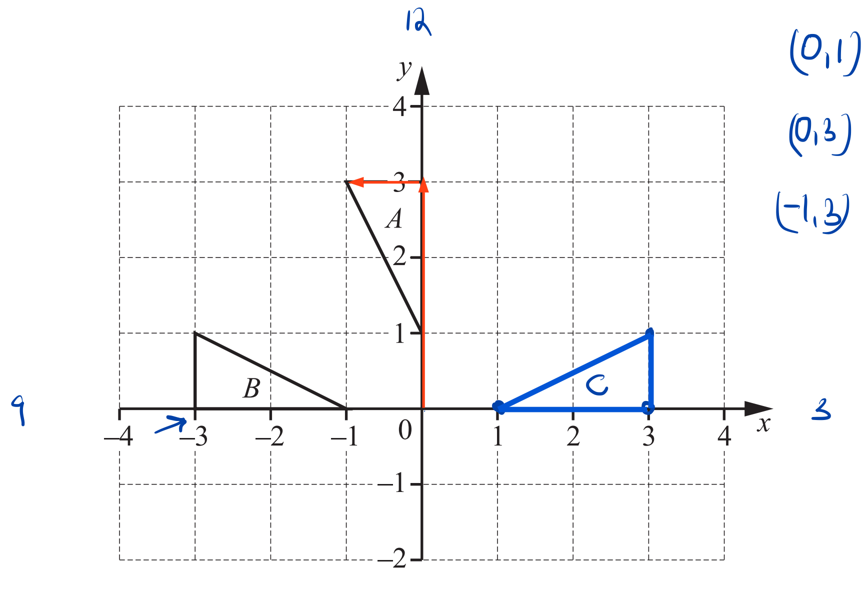


(0,1) → (−1,0)
(0,3) → (−3,0)
(−1,3) → (−3,1)

The diagram shows triangles *A* and *B*.

(a) Describe fully the **single** transformation that maps triangle *A* onto triangle *B*.

Reflection along the line y = −x ....

.... [2]

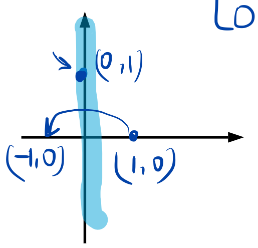
(b) Triangle *A* is mapped onto triangle *C* by a rotation, through 90° clockwise, centre (0, 0).

Draw, and label, triangle *C* on the diagram. [2]

(c) Triangle *B* is mapped onto triangle *C* by the transformation T.

Find the matrix that represents the transformation T.

$$\begin{array}{cc} A & B \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Answer $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ [1]

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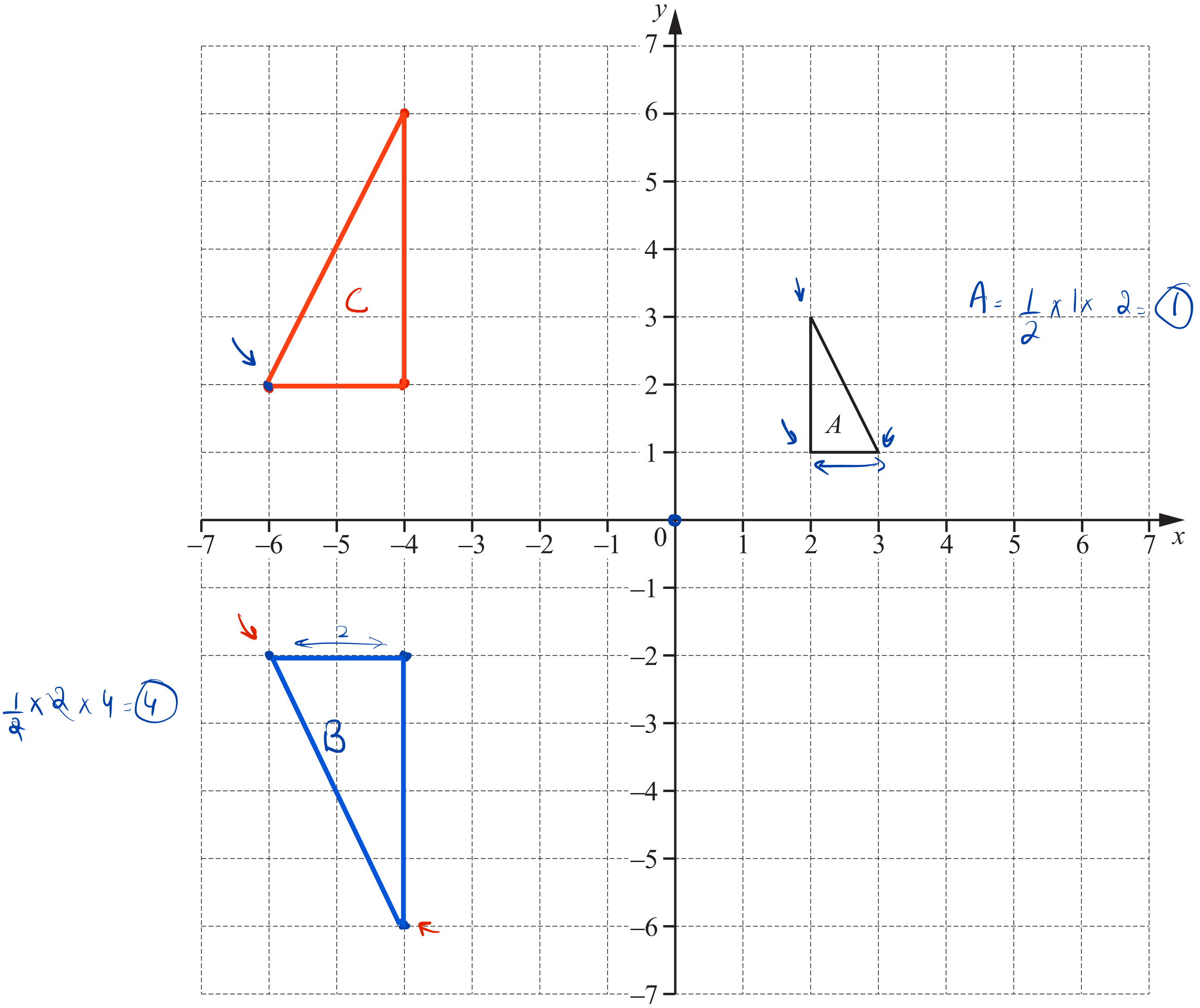


Triangle A is drawn on the grid.

(a) Transformation P is represented by the matrix $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$.

Enlargement

S.f = −2

P maps triangle A onto triangle B.

(i) Draw and label triangle B.

[2]

(ii) Describe fully the single transformation P.

Enlargement, s.f. = −2 centre origin

.... [2]

Similar Δ's

(iii) Write down the ratio area of triangle A : area of triangle B.

$\frac{A_1}{A_2} = \left(\frac{L_1}{L_2}\right)^2 \qquad \left(\frac{1}{2}\right)^2$

Answer ...1... : ...4... [1]

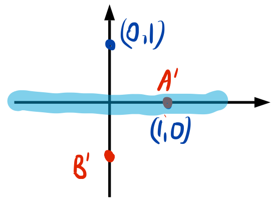
→ Reflection along x-axis

(b) Transformation Q is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Q maps triangle B onto triangle C.

Draw and label triangle C.

Q: Reflection along x-axis



$\begin{array}{cc} A & B \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow \begin{array}{cc} A' & B' \\ \end{array} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

[2]

$\boxed{\text{Backup Plan}}$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -6 & -4 & -4 \\ -2 & -2 & -6 \end{bmatrix}$

$-6+0$

$0+2$

$(-6,2)$

(c) Transformation Y is represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$.

Y maps triangle A onto triangle D.

Find the matrix that represents the transformation that maps triangle D onto triangle A.

$A \xrightarrow{Y} D$

$D \xrightarrow{Y^{-1}} A$

$Y = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$Y^{-1} = \frac{1}{3-0} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$\frac{1}{3} \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$5 \times 3 = 15$

$15 \times \frac{1}{3} = 5$

$\frac{1}{3} = 3^{-1}$

Answer $\frac{1}{3}\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}$ [2]

or

$\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$